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Probing the Strain-Rotation Balance in Non-Newtonian Turbulence with Inertial Particles

Michael Sinhuber, Joseph G. Ballouz, and Nicholas T. Ouellette*
Department of Civil and Environmental Engineering,
Stanford University, Stanford, California 94305, USA

It has long been thought that small amounts of polymer additives can alter the balance between strain and rotation in turbulent flow. Quantitative evidence for this idea, however, is scant, in part because measuring the velocity gradient in intense turbulence is very difficult. Here we take a different approach to investigating this question, using the well known preferential concentration effect of inertial particles in turbulence as a probe of the strain/rotation balance. By measuring the pair correlation function of weakly inertial particles in a turbulent water flow with varying concentrations of a high-molecular-weight polyacrylamide, we show that particle clustering is monotonically enhanced as the polymer concentration increases. Our results are consistent with the recently developed energy flux balance model for polymer turbulence, which demonstrates that the balance between strain and rotation is indeed modified by polymers and that this effect increases with the polymer concentration. Our results provide further support for the energy flux balance model as the proper description of polymer turbulence, and highlight the utility of inertial particle clustering as a probe for characterizing the small-scale dynamics of turbulence.

Since at least the 1940s, it has been known that even tiny amounts of additives such as long-chain polymers can significantly modify the properties of turbulent flows. The most well known effect of polymers on turbulence is the reduction of skin-friction drag in wall-bounded flows [1–3], but polymers can also modify other aspects of the flow, such as its heat-transfer properties [4]. The specific mechanisms by which polymer additives induce these flow changes are still somewhat controversial, but it is by now generally agreed that polymers modify the dynamical structure of the turbulence [3]. There is, for example, both experimental and numerical evidence that polymers tend to attenuate the small scales of the turbulent energy cascade even in isotropic turbulence far from walls [5–12]. But in addition to changing the energetics of the turbulence, polymers are also expected to alter the spatial structure of the flow. In particular, it is widely accepted that polymers change the dynamically set balance between strain and rotation in turbulence [7, 10, 13], suppressing intense rotation in favor of strain. However, there is little quantitative experimental evidence to support this hypothesis.

A large part of the difficulty in assessing the possible modification of strain and rotation by polymers stems from the challenges inherent in directly measuring the velocity gradient in turbulence [14, 15]. Thus, available experimental evidence for vorticity suppression by polymers is fairly qualitative [13]. We can make progress, however, via indirect measurements, as long as we have access some other property of the turbulence that is sensitive to the balance between strain and rotation in a quantitatively understood way. An appealing candidate for such a probe is the well known preferential concentration effect of inertial particles (that is, particles that are heavier than the fluid) in turbulence. Inertial particles are well known to avoid strongly rotational regions and thus accumulate in regions of high strain [16, 17]. Measurements of the statistical properties of the distribution of inertial particles in turbulence are therefore sensitive to the balance of strain and rotation in the flow, and in numerical studies it has been found that polymers change these properties in a nontrivial manner [18]. However, as we show below, we can relate these statistics to the polymer-induced modification of the turbulence without having to measure the velocity gradient tensor itself.

Here, we describe measurements of the two-point position statistics of weakly inertial particles in a turbulent water flow containing small amounts of a long-chain polyacrylamide. We find that the particle clustering, as quantified by the behavior of the pair correlation function at small scales, is monotonically enhanced as the concentration of the polymer is increased. Using theoretical results from Chun *et al.* [19], we relate this enhancement to the relative amount of strain and rotation in the turbulence, showing that the polymers do indeed suppress rotation relative to strain. We further show that our results are in excellent agreement with the recently proposed energy flux balance model for turbulence in dilute polymer solutions proposed by Xi *et al.* [11], which allows us to argue how the modification of the strain-rotation balance scales with the polymer concentration. Our results demonstrate compellingly that polymers alter the relative amounts of strain and rotation in turbulence in a concentration-dependent way, and provide further support for the energy flux balance model of turbulence in dilute polymer solutions. Additionally, our results highlight the efficacy of using the preferential concentration of inertial particles as a probe of small-scale turbulence dynamics.

The preferential concentration (often referred to as "clustering") of inertial particles in turbulence has been a very active area of research over the past few decades [20]. Although the precise details of the mechanisms responsible for

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* nto@stanford.edu

preferential concentration are still debated, it is generally agreed that inertial particles tend to be found in regions of high strain more commonly than in regions of high vorticity; and thus, because the particles dynamically avoid some regions of the flow, they appear to cluster together. This clustering is manifest as an enhancement of the particle pair correlation function $g(r)$ (sometimes also known as the radial distribution function) at small scales, such that $g(r)$ scales as a power law in r with an exponent that depends on the particle inertia [19, 21, 22]. $g(r)$, defined as

$$g(r) = \frac{\langle \sum_i \sum_{i \neq j} \delta(r - r_{ij}) \rangle}{\bar{n}} - 1 \tag{1}$$

where r_{ij} is the distance between particles i and j, is the ratio of the likelihood of finding a particle a distance r away from a target particle to the mean number density ($\bar{n}$) of particles in the fluid; and note that here, we subtract 1 from this ratio so that $g(r) = 0$ if the particles are homogeneously distributed. By perturbatively expanding the relative position and velocity of pairs of inertial particles in powers of the Stokes number St, which characterizes the inertia of the particles relative to the carrier flow, Chun *et al.* [19] argued that for small St, $g(r) \sim (r/\eta)^{-\xi}$, where η is the Kolmogorov length scale and

$$\xi \propto \mathrm{St}\ \tau_\eta^2 \langle Q \rangle_{\mathcal{L}}. \tag{2}$$

Formally, this relation holds for $r \ll \eta$, where the flow is nearly linear; however, in practice, it tends to hold for larger r as well [22]. Here, τ_η is the Kolmogorov time scale and $Q = \Omega_{ij}\Omega_{ij} - s_{ij}s_{ij}$ is the second invariant of the velocity gradient tensor, where s_{ij} is the strain rate tensor, Ω_{ij} is the rotation rate tensor, and summation is implied over repeated indices. The operator $\langle \cdot \rangle_{\mathcal{L}}$ is an average taken over ensembles of Lagrangian trajectories. ξ is therefore sensitive both to the relative amount of strain and rotation in the flow and to the way inertial particles sample these two fields. Thus, if polymers modify the balance between strain and rotation in the turbulence, we would expect ξ to change.

Since the original empirical results of Toms [1] and others, many models have been developed to try to explain the way that polymers modify turbulent flows. Of particular interest for non-wall-bounded flows are predictions for how the dynamical scales that characterize the turbulent energy cascade will change when polymers are introduced. Most of these models take advantage of standard analogies between polymer molecules and entropic springs that can be stretched by strain in the flow, storing elastic energy that is then released (presumably at smaller scales) when the molecules relax. What differs between various models is the assumption of which properties associated with these springs determine the coupling to the flow. Lumley, for example, conjectured that the most relevant property of the polymers was their relaxation time τ_p [2], while Tabor & de Gennes instead based their model on the amount of elastic energy stored in the polymer molecules [23, 24]. Neither of these models, however, appears to correctly explain experimental findings [9, 11, 12]. Lumley's model, for example, cannot explain the observed dependence on polymer concentration ϕ. The Tabor & de Gennes model does include a concentration effect, including a critical concentration below which the polymers are not expected to influence the turbulence, but has not been found to be quantitatively correct and is internally inconsistent [11, 25]. To remedy these issues, Xi *et al.* instead recently proposed a model based on balancing the *flux* of elastic energy into and out of the polymers with the flux of energy through the turbulence cascade rather than working with the energy itself [11]. This elastic energy flux balance model is much better supported by available experimental evidence, correctly predicting, for example, the dependence of turbulence statistics on polymer concentration [11, 12]. Of particular relevance here, the elastic energy flux balance models predicts that length scales should rescale with the polymer concentration by a factor of $\phi^{2/5}$ [11], and time scales with a factor of $\phi^{4/15}$ [12]. For example, the apparent Kolmogorov length scale in a polymer solution η^p is predicted to scale as $\eta^p \sim \eta^w \phi^{2/5}$, where η^w is the Kolmogorov scale in a Newtonian fluid under the same driving conditions, and the Kolmogorov time scale in a polymer solution to scale as $\tau_\eta^p \sim \tau_\eta^w \phi^{4/15}$.

We can use this model to make a prediction for how the scaling of $g(r)$ should change in the presence of polymers. Equation 2 tells us that the scaling exponent of $g(r)$ involves the Stokes number, the Kolmogorov time scale, and the second invariant of the velocity gradient tensor. As described above, we expect that in a polymer solution, $\tau_\eta^p \sim \tau_\eta^w \phi^{4/15}$. In turbulent flow, the Stokes number for a small spherical particle is given by

$$\mathrm{St} = \frac{1}{18} \frac{\rho}{\rho_f} \left(\frac{d}{\eta^w} \right)^2, \tag{3}$$

where ρ is the particle mass density, ρ_f is the fluid density, and d is the particle diameter. The only parameter here would we expect to change in the presence of polymers is the Kolmogorov scale, and so $\mathrm{St}^p \sim \mathrm{St}^w \phi^{-4/5}$. And finally, since the strain and rotation rates have units of inverse time, we would expect that $\langle Q \rangle_{\mathcal{L}}^p \sim \langle Q \rangle_{\mathcal{L}}^w \phi^{-8/15}$. Note that this prediction assumes that the amount of strain and rotation may change in the presence of polymers, but that the way a given kind of particle will sample the fields will not. This is a reasonable assumption in light of the

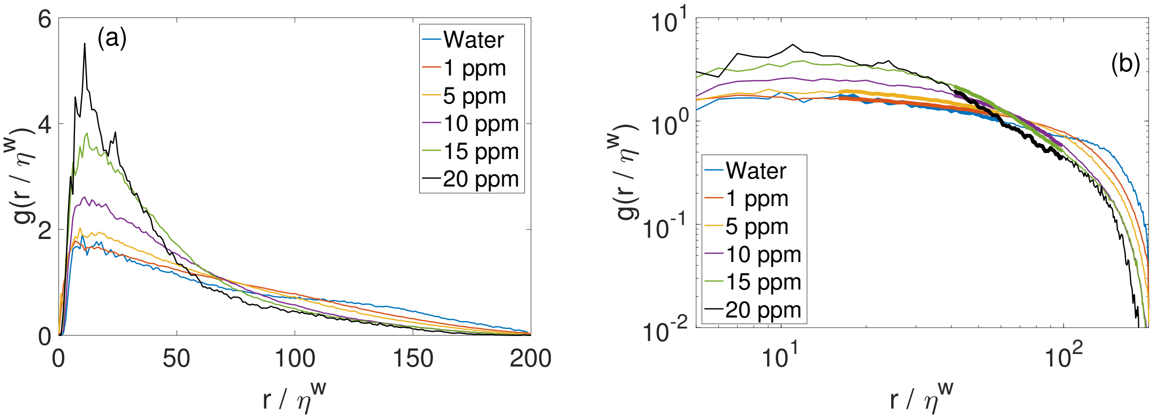

FIG. 1. (a) The pair correlation function $g(r)$ as a function of r/η^w for pure water and five different polymer concentrations. Note that, as mentioned in the text, we define $g(r)$ so that $g(r) = 0$ for homogeneously distributed particles. At small scales, the peak of $g(r)$ increases monotonically with concentration. (b) The same data shown on logarithmic scales. For small r, we observe roughly power-law behavior, which we fit to determine scaling exponents. We performed these fits between $r/\eta^w = 15$ to 60 for $\phi = 0$, 1, and 5 ppm and $r/\eta^w = 40$ to 100 for $\phi = 10$, 15, and 20 ppm, as highlighted by the bold curve segments

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significant amount of experimental evidence that typical turbulence scaling still holds, even in a Lagrangian sense, for dilute polymer solutions, and that the effect of the polymers is primarily felt as a rescaling of length and time scales [8, 9, 11, 12]. Thus, the concentration dependence of the rescaled Kolmogorov time and velocity gradient cancel. Since the theory of Chun *et al.* [19] was developed for a Newtonian fluid, $\xi \sim \mathrm{St}^w$ (see eq. 2), and the elastic energy flux balance model predicts that $\xi \sim \phi^{4/5}$. We note that this prediction only gives a form for the dependence of ξ on the concentration ϕ; the actual value of ξ will also depend on other flow and particle parameters, which may be Reynolds-number dependent.

To test this prediction, we conducted experiments in a von Kármán swirling flow between counter-rotating disks that we have described in detail elsewhere [12]. Briefly, we generated turbulence in a closed cylindrical tank with a diameter of 58 cm. The impellers that drive the flow have a diameter of 44 cm, and the distance between the impellers was 50 cm. To drive the polymers inertially rather than via boundary-layer shear (which could be altered via polymer drag reduction [26]), the disks were baffled with 5 cm straight vanes. To ensure that the polymer molecules were not torn apart by the turbulent shear, we controlled the rotation rate of the impellers and kept it fairly low; for this work, it was fixed at 0.23 Hz. The working fluid was deionized water, and its temperature was kept constant in time and uniform in space (to within 0.1 °C) by controlling the temperature of the aluminum top and bottom plates of the apparatus. Under these conditions in pure water, the Taylor-microscale Reynolds number was $R_\lambda = 420$, the integral length scale was $L = 14.2$ cm, the Kolmogorov length scale was $\eta^w = 124$ μm, and the Kolmogorov time scale was $\tau_\eta^w = 15.2$ ms [12]. We note that the effective viscous dissipation scales may be larger in the polymer solutions, but it is not yet fully understood how to capture them [9, 11, 12]; thus, we report scales for pure water only, as indicated by the w superscript.

To measure the behavior of the flow, we used three-dimensional Lagrangian particle tracking [27]. The fluid was seeded with 30-μm polystyrene microspheres containing a fluorescent dye. Under these conditions, the particle Stokes number was roughly 3×10^{-3}—small but not zero, and so ideal for applying the theory of Chun *et al.* [19], as it follows from a perturbative expansion in Stokes number. The particle fluorescence was excited with a Q-switched Nd:YAG laser pulsed at 10 kHz with an average power of 45 W. We tracked the motion of the particles in three dimensions with three Photron Fastcam SA5 cameras, each running at 2000 frames per second with a resolution of 1024×1024 pixels, arranged around the experiment using standard stereomatching and tracking techniques [27]. The full measurement volume in these experiments had a linear dimension of 2.3 cm, and was located in the center of the apparatus where the flow is closest to homogeneous. The typical number density of particles in the apparatus was roughly 10^{-3} mm^{-3}. We took care to appropriately normalize the contribution to $g(r)$ of particles near the edges of the measurement volume by accounting for the possibility of neighboring particles outside the field of view.

The polymer we used was a high-molecular-weight (18×10^6 a.m.u.) polyacrylamide (PolySciences 18522). We used polymer concentrations ϕ ranging from 1 to 20 parts per million (ppm) by weight. For this polymer, the radius of gyration R_g at equilibrium is roughly 0.5 μm, the fully stretched length is about 75 μm, and the Zimm relaxation time τ_p is 43 ms. The turbulent Weissenberg number $\mathrm{Wi} = \tau_p/\tau_\eta = 2.8$. Details of our polymer preparation methods and a characterization of the properties of the flow are described elsewhere [12].

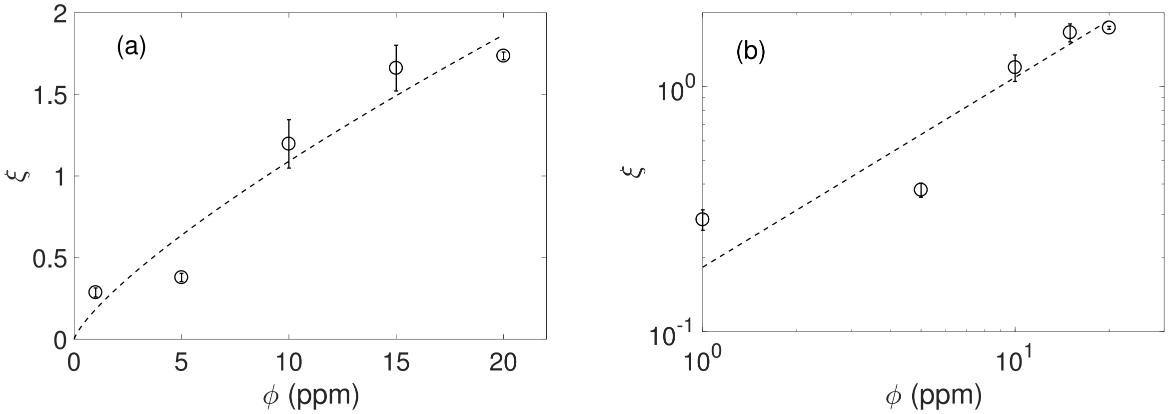

FIG. 2. Power-law scaling exponents ξ extracted from fits to the data shown in fig. 1 as a function of polymer concentration ϕ, plotted on both (a) linear axes and (b) logarithmic axes. Error bars are estimated by both considering the statistical uncertainties from the fits and from adjusting the ranges over which we fit. The data are consistent with a power-law dependence of ξ on ϕ; fitting the data gives a scaling exponent of 0.77 ± 0.04, as shown with the dashed line.

For each of these data sets, we computed the pair correlation function $g(r)$, as shown in fig. 1(a). Even though the Stokes number is very small, we still see some evidence of preferential concentration, given that the curve is not flat. Indeed, for the pure water experiments, our data is consistent with that of Saw *et al.* [22]. But even though the particles and Reynolds number are the same for all the experiments, we observe that the peak value of $g(r)$ increases monotonically with the polymer concentration ϕ, just as suggested by our theoretical arguments above. In fig. 1(b), we show the same data plotted on a logarithmic scale. Although the power-law scaling range is short, we can still estimate the scaling exponents $\xi(\phi)$ by fitting.

In fig. 2, we show the results of these fits for ξ. The relationship between ξ and ϕ is consistent with power-law scaling, as expected. When we fit a power law to the data, we obtain a scaling exponent of 0.77 ± 0.04, in excellent agreement with the prediction of 4/5 from the energy flux balance model and providing strong support for our theoretical arguments above. Thus, given that obtaining this scaling prediction required that the difference between the square of the rotation rate and the square of the strain rate was a function of ϕ, our results thus confirm that the balance between strain and rotation is indeed modified by polymers. Quantitatively, our results show that $\langle Q\rangle_{\mathcal{L}}^p = \langle \Omega_{ij}\Omega_{ij} - s_{ij}s_{ij}\rangle_{\mathcal{L}}^p \sim \phi^{-8/15}$, so that indeed rotation is suppressed relative to strain in a way that increases monotonically with polymer concentration.

In summary, we have measured the preferential concentration statistics of weakly inertial particles in a turbulent flow in a dilute polymer solution. We find that the preferential concentration increases monotonically with the polymer concentration, in a way that is quantitatively captured by the energy flux balance model for turbulence in polymer solutions. Because preferential concentration is a result of the balance between strain and rotation in turbulence, these results give us an indirect way to confirm that, as has long been thought, polymers modify the structure of the velocity gradient field in turbulence. Thus, in addition to illuminating the ways in which polymers modify turbulence, our results also point to the potential power of using inertial particles to probe the statistics of the difficult-to-measure velocity gradient, a method we expect to find further applications in turbulence.

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