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Abstract

Recent findings (Wu et al. 2017 PNAS vol. 114) reinforce earlier assertions (e.g., Falco 1991 Phil. Trans. Roy. Soc. A vol. 336) that the sublayer pocket motions play a distinctly important role in near-wall dynamics. In the present study, smoke visualization and axial velocity measurements are combined in order to establish the scaling behavior of pocket events in the viscous sublayer of the turbulent boundary layer. In doing so, an identical analysis methodology is employed over an extensive range of friction Reynolds numbers between $388 \leq \delta^+ \leq 2.2 \times 10^5$. Both the pocket width ($W$) and time interval between pocket events ($T$) increase logarithmically with Reynolds number when normalized by viscous units. Normalization of $W$ and $T$ by the Taylor microscales evaluated at a wall-normal location of about 100 viscous units, however, appears to successfully remove this Reynolds number dependence. The present results are discussed in the context of motion formation owing to the three-dimensionalization of the near-wall vorticity field, and concomitantly, the recurring perturbation of the viscous sublayer.

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I. INTRODUCTION

Understanding the physical mechanisms that sustain turbulence production and drag force generation has been of interest in turbulent boundary layer research at least since Theodorsen’s introduction of the hairpin vortex model\(^1\) over 65 years ago. Coherent vortical motions, including but not limited to quasi-streamwise vortices, hairpin vortices, and so-called typical eddies, are individually and collectively purported to play significant, but varying, roles in both turbulence production and momentum transport.\(^2\)\(^–\)\(^9\) Sublayer streaks have been believed to play an integral role in the near-wall self-sustaining bursting cycle that involves the generation and evolution of quasi-streamwise vortices.\(^10\)\(^–\)\(^12\) Alternatively, Falco and co-workers,\(^7,13\) portray streaks as the far-field signature of intermediate-scale wallward advecting vortical motions whose stronger near-field interaction culminates with the rapid formation of sublayer pockets. Consistent with this, recent analyses of direct numerical simulations by Wu et al.\(^14\), provide convincing evidence that viscous sublayer streaks are passive structures in the near-wall regeneration process, and largely symptomatic of the presence of “turbulent-turbulent spots” (referred to as TUTSs) in the inner layer. Deciphering the nature of the connection between inner-layer structures and their impact on the sublayer, and hence the skin friction, is of paramount importance for advancing turbulence theory and improving the accuracy of numerical models, especially as the Reynolds number becomes large.

Two main turbulence structures have been observed in the sublayer, streaks and pockets; see Robinson\(^6\) for an overview. Figure 1 illustrates these structures as would be seen by sublayer smoke visualization. A significant feature of the streak motions is that when normalized by viscous scales their average spanwise streak spacing, \(\ell_s^+\), is apparently invariant with Reynolds number, \(\ell_s^+ \approx 100\).\(^15,16\) Note that a superscript ‘+’ indicates normalization by the kinematic viscosity, \(\nu\), and the friction velocity, \(u_\tau\), (\(= \sqrt{\tau_{wall}/\rho}\)), where \(\tau_{wall}\) is the mean wall shear stress, and \(\rho\) is the fluid density. Conversely, while also known to be present in both high and low Reynolds number flows, the inner-normalized size of the pocket motions show variations with Reynolds number, \(\delta^+ = \delta u_\tau/\nu\), where \(\delta\) is the boundary layer thickness.\(^15\) If streaks are essentially passive entities in the sublayer as the results by Wu et al.\(^14\) suggest, then that would imply that pockets are the predominant dynamically active motions in the viscous sublayer. Such considerations motivate the present study of pocket
A considerable and growing body of evidence indicates that, even very close to the wall, a number of inner-normalized turbulence statistics exhibit Reynolds number dependencies. The physical origin of some of the observed phenomena has been shown to connect to inner/outer interactions, while other studies suggest that these $\delta^+$ dependencies are associated with the scaling structure of the mean dynamical equation, and accordingly the mechanisms of inertial turbulent transport. Here, a relevant observation is that many of these Reynolds number dependencies are approximately logarithmic.

As first observed in flow visualizations, when a tracer is continuously injected into the sublayer (via a tangential slot in the wall), pockets are identified in plan view as initially circular and then crescent shaped regions devoid of tracer. The laboratory measurements of Falco and co-workers, along with analysis of visualizations conducted on the salt playa of western Utah, indicate that the inner normalized pocket width ($W^+$) and time interval between pockets ($T^+$) increases approximately logarithmically with Reynolds number. Although both streaks and pockets are observed in the viscous sublayer, it is reasonable to conclude that, because of the difference in scaling properties between the two types of
coherent structures, the mechanisms responsible for low-speed streaks are not identical to those of pockets. Collectively, these considerations motivate the present focus on correlating pocket motion attributes with wall-layer turbulence structure.

Three different physical mechanisms capable of producing pocket formations have been described in the literature. Falco and co-workers primarily considered pockets as footprints of so-called “typical eddies” (identified from flow visualizations as spatially-compact regions of concentrated vorticity having a ring-like configuration) that propagate in the outer region, and in some cases advect toward the wall. Smith et al. observed pocket-type events during the formation and lift-up of hairpin vortices, in between the vortex legs and behind the vortex head. In his direct numerical simulation (DNS) interrogation, Robinson found that local regions of high pressure coincide with the spanwise divergence of streamlines in the sublayer, indicative of wallward moving fluid and pocket formations. This connection between pressure fluctuations and pocket events is corroborated by the high Reynolds number pressure measurements of Klewicki et al. In that study, the peak in the spectra of the fluctuating wall pressure gradient (both streamwise and spanwise gradients) was found to be consistent with the characteristic length scale of pockets at similar Reynolds number. Note, wall pressure gradients are important because they provide a mechanism for the surface flux of vorticity, and thus likely play a critical role in the near-wall self-sustaining process of the turbulence.

Potentially significant common attributes of the pocket forming physical mechanisms described above are the presence of positive absolute spanwise vorticity ($\omega_z$), i.e., having a sign opposite to that of the mean vorticity ($-\partial U/\partial y$), in conjunction with a high pressure perturbation giving rise to a localized region of high pressure gradients along the wall. Two-point correlations of spanwise vorticity for probe separations in the wall-normal direction at low Reynolds number reveal the regular occurrence of opposing sign $\omega_z$ motions in the near-wall region. Similar, albeit more limited, measurements provide evidence that this structural feature is preserved at $\delta^+ = O(10^5)$. These behaviors are compatible with the expected vorticity distribution of a typical eddy, as well as the generation of positive $\omega_z$ under a shear layer motion as it rolls-up to form a hairpin-like vortex. Consistent with the increasing probability of positive $\omega_z$ motions with wall normal distance, analysis of DNS data indicates that there is a rapid transfer of mean enstrophy to fluctuating enstrophy in the region $5 \lesssim y^+ \lesssim 50$, that coincides with the rapid three-dimensionalization of the
Accordingly, into the logarithmic region and beyond, the motions bearing positive $\omega_z$ increasingly gain parity with those bearing negative $\omega_z$.\cite{36,37}

In his physical model, Falco\cite{7} proposes that the positive $\omega_z$ motions characteristic of the outer region advect toward the surface at (generally) shallow angles, and through the subsequent interaction with the surface produce the pocket motions. In support of this picture, plan-view flow visualizations simultaneous with hot-wire measurements in the buffer layer reveal large negative contributions to the Reynolds shear stress (associated with the wallward movement of high momentum fluid) coincident with pocket formations.\cite{26} Outside of the sublayer ($y^+ > 10$), measurements at both high and low $\delta^+$ indicate that the duration of spanwise vorticity events with large positive fluctuation (magnitude greater than 1 rms) scale according to the local Taylor time scale.\cite{32,36} Note, this threshold level guarantees that nearly all of the positive detected events have positive $\omega_z$. The present paper provides evidence that the average pocket width ($W$) and time interval between pockets ($T$) also scale with the Taylor microscales evaluated in the near-wall region, but just beyond where the mean-to-fluctuating enstrophy transfer mentioned above occurs. An important feature of the the present study is that the friction Reynolds number varies over three orders of magnitude, achievable through a combination of laboratory and field experiments.

II. EXPERIMENTAL METHODS

The present flow visualization experiments were conducted in the boundary layer wind tunnel located in the Fluid Mechanics Laboratory at the University of Utah. Three different Reynolds numbers were investigated: $\delta^+ = 1131, 1977, 3050$. Theatrical fog was injected through a thin, nearly tangential slot in the wind tunnel floor and illuminated with a horizontal laser sheet using an argon-ion laser (5 W, Spectra-Physics 2000). The height of the laser sheet was positioned 2 mm above the floor of the wind tunnel, which corresponds to a wall-normal distance of 10–34 viscous units across the three Reynolds numbers investigated. Plan view flow visualization images were captured at 500 frames/s with a motion picture camera (Red Lake 16 mm) positioned above the transparent wind tunnel ceiling. A sample frame from the video is shown in Fig. 2. The friction velocity at each $\delta^+$ was determined using the Clauser method, based on mean velocity profile measurements from a hot-wire. Further details regarding the laboratory experiments are given by Fershtut.\cite{38}
The field experiments were performed in the atmospheric boundary layer that flows over the salt flats of Utah’s western desert. Flow visualizations of the sublayer were accomplished using floodlighting, a CCD camera (60 frames/s), and a buried reservoir that allowed theatrical fog to seep through a tangential slot made flush with the desert floor. Note, the camera lens was focused on the top surface of the slot. Plan view images of the sublayer were captured during sunset as the atmosphere transitions through neutral thermal stability (during which buoyancy effects become negligible). The local friction velocity was estimated using a hot-wire rake capable of measuring the velocity gradient near the surface. Further details of the field study are described by Klewicki et al., hereafter referred to as K95. The Reynolds number associated with the atmospheric boundary layer was estimated as $\delta^+ \approx 2.2 \times 10^5 \pm 11\%$ using the local friction velocity and kinematic viscosity along with an estimate of the surface layer depth based on the work of Metzger et al.\cite{39}

The average pocket width and time interval between pocket events were measured using identical analysis techniques for the laboratory and field data, namely those described in K95. Important features of this methodology are summarized here. To obtain pocket width data, a horizontal reference line was drawn on the video monitor at an arbitrary distance of 3 cm (wind tunnel) and 6 cm (atmosphere). For all pockets passing through the reference line, a
Precision rule was used to measure the maximum width between the centers of concentrated fog lines outlining the crescent-shaped pocket pattern. From the field data, 869 pockets were measured; while from the laboratory data, 200 pockets were measured at each $\delta^+$. The average time interval between pocket events was determined by dividing the total number of pockets detected by the total time elapsed during the interrogation period. The inner normalized time interval was then weighted by the ratio $2W^+/Z^+$, where $Z^+$ represents the inner normalized screen width. This weighting allows $T^+$ to be interpreted as the inner normalized time interval between events that would be detected by an imaginary probe located at a fixed $y^+$ in the measurement plane. Note, the exact wall-normal location of the measurement plane for the present flow visualization data remains somewhat ambiguous, but is believed to be in the region $10 \leq y^+ \leq 30$ based on agreement with the prior work of Falco$^{7,26}$. Although a slightly different method was used to determine $T^+$ in the study of K95, renewed analysis of their data using the present method$^{38}$ yield results essentially identical to those originally published in K95.

In the present study, Taylor microscales are used to normalize $W$ and $T$. Here, $\lambda$ and $\lambda_t$ denote the Taylor length and time scale, respectively. For reference, profiles of $\lambda^+$ and $\lambda_t^+$ as a function of $\delta^+$ and $y^+$ are shown in Fig. 3 over a range of Reynolds numbers spanning three decades. In the case of the Taylor length scale, $\lambda^+$ increases nearly monotonically with
$y^+$ at each Reynolds number, except for a short plateau region between $10 < y^+ < 100$. On the other hand, in the case of the Taylor time scale, the $\lambda_t^+$ versus $y^+$ profiles exhibit a local minimum near $y^+ \approx 50$. The data also reveal apparent Reynolds number trends. For any given $y^+ < \sim 5$, both $\lambda^+$ and $\lambda_t^+$ increase with $\delta^+$. One can observe different regimes where the Reynolds number behavior changes. For example, both $\lambda^+$ and $\lambda_t^+$ increase more dramatically with $\delta^+$ in the log region, $y^+ > \sim 200$; whereas, $\delta^+$ variations are less pronounced closer to the wall. In the viscous sublayer, $\lambda^+$ appears to be independent of Reynolds number. Later, evidence is given that the pocket width $W$ and time interval between pockets $T$ scale with the Taylor microscales using values for $\lambda$ and $\lambda_t$ in the region $\sim 50 < y^+ < \sim 100$. Note, this scaling region corresponds approximately with the region in the $y^+$ profile where $\lambda_t^+$ exhibits a local minimum and $\lambda^+$ exhibits a plateau.

Specific details regarding the laboratory and field experiments, from which the hot-wire data in Fig. 3 were taken, are provided elsewhere.\textsuperscript{40–42} Note, since the field data were acquired over different years, the friction Reynolds numbers for each data set are not the same. Specifically, the atmospheric flow visualizations are characterized by a Reynolds number of $\delta^+ = 2.2 \times 10^5$; whereas, the Taylor microscale measurements from the atmosphere are characterized by slightly larger Reynolds number (for the 1995 hot-wire data, $2.6 \times 10^5 \leq \delta^+ \leq 4.6 \times 10^5$; for the 1999 hot-wire data, $5.7 \times 10^5 \leq \delta^+ \leq 7.9 \times 10^5$; and for the 2002 hot-wire data, $5.9 \times 10^5 \leq \delta^+ \leq 6.5 \times 10^5$). The effect of this on scaling pocket width $W$ and the time between pocket events $T$ is discussed in the next section.

For both the laboratory and field studies, the Taylor length is estimated based on the relation $\lambda^2 = \langle u^2 \rangle / \langle (\partial u / \partial x)^2 \rangle$, assuming isotropic turbulence,\textsuperscript{43} where $u$ is the fluctuating velocity along the streamwise coordinate ($x$). Here, $\langle \cdot \rangle$ denotes a long-time temporal average. Furthermore, the instantaneous velocity signal is decomposed into its mean ($U$) and fluctuating components ($u$), such that $\langle u \rangle = 0$. The derivative is evaluated using Taylor’s frozen turbulence hypothesis along with a Savitzky-Golay filter. Note, the data of Klewicki\textsuperscript{42} indicate that outside of the buffer layer the assumption of isotropy provides a good estimate of $\lambda$. The Taylor time scale, on the other hand, is determined using an osculating parabola fit to the autocorrelation of $u$ at zero time lag. Therefore, calculation of $\lambda_t$ does not rely on the assumption of isotropy.
FIG. 4: Reynolds number dependence of the inner normalized pocket width ($W^+$) and time interval between pockets ($T^+$). ○ present, ◇ Falco,§ □ Klewicki et al.,15 - - logarithmic curve fits.

III. RESULTS

Figure 4 shows the present $W^+$ and $T^+$ data as a function of $\delta^+$. For comparison, the $W^+$ data from Falco7 over a Reynolds number range $400 < \delta^+ < 1500$ are also shown. The present low Reynolds number $W^+$ data agree remarkably well with those of Falco. Curve fits to the data reveal a logarithmic Reynolds number dependence of the inner normalized pocket width and time between pockets that are respectively well-approximated by:

\[
W^+ = 27.6 \log \delta^+ - 19.5, \quad (1)
\]
\[
T^+ = 6.6 \log \delta^+ - 1.1. \quad (2)
\]

The error bars in Fig. 4 represent 95% confidence intervals for each data point, based upon the standard deviation of the mean. The uncertainty in the field data is about twice as large as that of the laboratory data, due to the increased uncertainty in measuring the friction velocity associated with the atmospheric boundary layer. In addition, the width of the square markers used for the atmospheric data correspond to the uncertainty in $\delta^+$ for those data.

Since inner normalizations show mild (logarithmic) but apparent $\delta^+$ dependencies for $W$ and $T$, three other types of normalizations were investigated. These include (i) outer scaling using normalization based on by the boundary layer thickness $\delta$ and freestream velocity $U_\infty$, i.e., $W/\delta$ and $T U_\infty/\delta$, (ii) meso scaling using normalization based on intermediate
length and time scales defined by the geometric mean between the inner and outer scales, i.e., \( W^+ / \sqrt{\delta^+} \) and \( T^+ / \sqrt{\delta^+ / U_{\infty}^+} \); and (iii) Taylor scaling using normalization based on the Taylor microscales, i.e., \( W / \lambda \) and \( T / \lambda_t \). Note, meso scaling is, for example, inspired by the scalings associated with the mean momentum balance as first shown by Wei et al. 21

The performance of inner, outer, meso, and Taylor scales in terms of removing \( \delta^+ \) trends in \( T \) and \( W \) are presented in Fig. 5. The normalized data (denoted generically as \( \tilde{T} \), \( \tilde{W} \)) are plotted for convenience on log-log coordinates, which is not necessarily meant to imply power-law behavior. For example, the goodness-of-fit parameter \( (R^2) \) associated with logarithmic and power-law curve fits to \( W^+ \) are 0.9998 and 0.9936, respectively, indicating that a logarithmic function of \( \delta^+ \) fits the \( W^+ \) data slightly better than a power law. Clearly under the proper normalization, \( \tilde{T} \) and \( \tilde{W} \) versus \( \delta^+ \) will appear on the log-log plot as straight lines having zero slope.

In Fig. 5, the grey dashed lines representing the inner, outer, and meso scalings of \( \tilde{W} \) and \( \tilde{T} \) follow directly from the empirical expression for \( W^+ \) and \( T^+ \) in (1) and (2), respectively. For the outer and meso scalings of \( \tilde{T} \), values of \( U_{\infty}^+ \) from the hot-wire data were also used. The black markers represent Taylor scaling derived from the hot-wire data shown in Fig. 3. For each \( \delta^+ \) data set, \( W^+ \) and \( T^+ \) are determined based on the empirical relations in (1) and (2), respectively; then, these values are normalized by the corresponding \( \lambda^+ \) and \( \lambda_t^+ \) obtained from the hot-wire data. In Fig. 5, \( \lambda^+ \) and \( \lambda_t^+ \) are evaluated at a fixed location of \( y^+ = 100 \) for all \( \delta^+ \). However, one would obtain nearly identical results using values of \( \lambda^+ \) and \( \lambda_t^+ \) at any \( y^+ \) in the range \( 50 \leq y^+ \leq 200 \). This is highlighted below; and, the physics associated with this observation are discussed further.

The dashed lines in Fig. 5 represent power law relations of the form:

\[
\tilde{W} = \alpha (\delta^+)^A, \\
\tilde{T} = \beta (\delta^+)^B.
\]

In order for \( \tilde{W} \) and \( \tilde{T} \) to be independent of \( \delta^+ \), one seeks a normalization whereby \( A \approx 0 \) and \( B \approx 0 \), which would yield a line with zero slope (when plotted in log-log coordinates). As apparent, normalization by the Taylor microscales yields \( \tilde{T} \) and \( \tilde{W} \) values that are essentially independent of Reynolds number. From this, one can obtain the following Reynolds number
FIG. 5: Scaling the (left) pocket width and (right) time interval between pockets. Circles and triangles indicate laboratory data, and squares indicate field data. The dashed lines represent power-law curve fits.

Invariant scaling relations:

\[
\frac{W}{\lambda(y^+=100)} = C_1 \tag{5}
\]

\[
\frac{T}{\lambda_t(y^+=100)} = C_2 \tag{6}
\]

where \(C_1\) and \(C_2\) are constants with values (based on curve fits to the present data) of 0.8 and 3.8, respectively. Interestingly, Metzger et al.\textsuperscript{44} also found that the local Taylor microscale succeeds in removing both Reynolds number and \(y^+\) trends in the statistical mode of the bursting period (over a similar \(\delta^+\) range). The connection between bursts and pockets can be made loosely through the hairpin vortex packet paradigm.\textsuperscript{9} Another interesting correlation stems from the recent work of Klewicki et al.,\textsuperscript{45} which showed that the characteristic length associated with surface-pressure gradient perturbations also corresponds to the mean pocket width. Furthermore, Priyadarshana and Klewicki\textsuperscript{46} revealed that Reynolds number effects in the spanwise vorticity spectra at \(y^+=320\) can be scaled by normalizing the frequency with the local Taylor microscale (the data from their study also spanned over three decades in \(\delta^+\)). Note, in the context of the present discussion, the difference between \(y^+=320\) and \(y^+=100\) is relatively insignificant (see Fig. 6).

Since \(\lambda\) and \(\lambda_t\) are functions of \(y^+\), there is some question as to which \(y^+\) location should be used to evaluate the Taylor microscales for purposes of obtaining \(\tilde{T}\) and \(\tilde{W}\). Figure 6 shows the effect of the \(y^+\) location, at which the Taylor microscales are evaluated, on the
slope of the curve fits, defined as

\[ \frac{W}{\lambda(y^+)} \propto (\delta^+)^A(y^+), \]
\[ \frac{T}{\lambda_t(y^+)} \propto (\delta^+)^B(y^+). \]

The error bars indicate the uncertainty in the estimation of the slope based on the covariance matrix of the curve fit parameters obtained from the non-linear regression analysis, which accounts for uncertainties in the magnitudes of \( \delta^+ \), \( W \), and \( T \); see Metzger\(^{47} \) for an example application of this method. The case of \( A = B = 0 \) characterizes the proper normalization, i.e., \( \delta^+ \)-independence of \( \tilde{T} \) and \( \tilde{W} \). One can notice a range of \( y^+ \) values between about 50 and 200 wherein the curve fits yield nearly identical slope regardless of \( y^+ \). In this range, both slopes \( A \) and \( B \) are close to zero to within the uncertainty of the data and analysis. Note, part of the uncertainty stems from the fact that the pocket data and Taylor microscale measurements from the field studies were acquired at different friction Reynolds numbers, with the differences being as much as \( \Delta \delta^+ = 5 \times 10^5 \) in some cases. The root mean square (rms) of the residual associated with \( A \) and \( B \) over the range \( 50 \lesssim y^+ \lesssim 200 \) is around 3.5%. The rms of the residuals span between 1.6% and 5.6% over the entire \( y^+ \) range shown in Fig. 6. Therefore, the curve fits in (7) and (8) using \( \lambda \) and \( \lambda_t \) values in the range \( 50 \lesssim y^+ \lesssim 200 \), respectively, are deemed good fits to the data.
IV. DISCUSSION AND CONCLUSION

The present study along with K95 establish that, when inner-normalized, the length and time scales of coherent motions detected in the sublayer consist of both a Reynolds number invariant component (streaks) and a Reynolds number dependent component (pockets). The latter is likely a consequence of spatially compact motions above the sublayer that impart their signature on the sublayer. Apparently these motions play an increasingly important role on near-wall dynamics with increasing Reynolds number. Conceptually, one can think of the pocket size and time interval between pockets as having connection with some family of eddies that gain proximity to the wall. Possibilities include, but may not be limited to, typical eddies, the “heads” of hairpin-like vortices that lift out of the sublayer, and vortices associated with the more recently identified turbulent-turbulent spots. The present data indicate that if the scale of the (eddy) motion is measured in Taylor units, then the proximity of the eddy to the wall (as measured in viscous units), whereby a pocket formation may be induced, remains nominally invariant with Reynolds number. In fact, the present study suggests that this wall-normal location is $50 \lesssim y^+ \lesssim 200$.

Several interesting coincidences are noted. The location $y^+ = 100$ lies near the lower edge of the “traditional” log layer (observed in the inner normalized mean streamwise velocity profile), and also corresponds to the location at which a shoulder/plateau begins to emerge in the inner normalized rms streamwise velocity profile. Note, a compilation of streamwise velocity statistics over a very large Reynolds number range may be found elsewhere. The inner normalized rms spanwise vorticity profiles also exhibit a noticeable knee (change in slope) in the region $50 \lesssim y^+ \lesssim 120$ that appears to be nearly invariant with Reynolds number. In connection with this, we note that analysis of the mean enstrophy equation reveals that the region interior to $y^+ \approx 50$ is marked by a rapid decrease in the mean vorticity magnitude. This coincides with a transfer of mean to fluctuating enstrophy and thus physically attributed to the three dimensionalization of the vorticity field into more spatially compact motions. Consistent with this, observations of the pre-multiplied streamwise velocity spectra reveal that, over a very large Reynolds number range, the region of interest lies between the inner spectral peak (characteristic of the near-wall cycle), and the mid-layer peak associated with the motions that modulate the near-wall flow. In relation to coherent motions, we also note that Tomkins and Adrian found that merging vortex packets
FIG. 7: Inner normalized Kolmogorov length versus inner normalized distance from the wall. Markers are the same as that in Fig. 3, except all of the atmospheric data at $\delta^+ = 2.2 \times 10^5$ are plotted as black circles here. $-\cdots$, $\eta^+ = (\kappa y^+)^{1/4}$.

provide an important mechanism of spanwise growth in the boundary layer up to a distance of about $y^+ = 100$, for the $\delta^+$ range they considered. This is relevant to the present study because “merging structures are expected to leave a highly distinctive footprint or signature in the flow field”; and, sublayer pockets may reflect this signature. Finally, Stanislas et al.\(^5\) observed a difference in vortex characteristics and behavior above $y^+ \approx 100$, compared to the region below that. In particular, their measurements reveal that the characteristic size of eddies increases from the wall and, for $y^+ > 150$, tends to nearly a constant value, which remains independent of Reynolds number when scaled by the local Kolmogorov length.

Interestingly, however, Reynolds number dependencies in the present pocket width data do not scale with the local Kolmogorov length. Figure 7 attests to this fact by showing the Reynolds number invariance of the inner normalized Kolmogorov length, $\eta^+$, estimated using the local isotropy assumption, which has been shown to correspond closely to the full dissipation rate outside of the buffer layer.\(^5\) Therefore, in terms of pocket width scaling, $W/\eta$ behaves like $W^+$. If sublayer pocket events are indeed believed to be linked to the existence of eddies in the lower log layer, then the question arises as to why the Kolmogorov length fails to scale the pocket width, when previous work\(^5,5\) indicate that the radius of eddies in the log region does appear to scale with the Kolmogorov length.
Undoubtedly, many factors affect the response of the sublayer to the motion of eddies above it (i.e., in the form of pocket events). It may be rationally hypothesized, however, that the primary factors driving this interaction are the vortical intensity of the eddy, the size of the eddy, and the propagation velocity (magnitude/direction) of the eddy. The near-invariance of the $\omega_z^+$ and $\omega_y^+$ intensities with Reynolds number, suggests that the vortical intensity of an initiating eddy at $y^+ \approx 100$ does not vary appreciably with Reynolds number. Therefore, one might surmise that the Reynolds number dependence of the characteristic pocket scales has, less to do with the vortical intensity of the initiating eddy, and more to do with its size and/or propagation velocity. However, since eddy size normalized by the Kolmogorov length appears to be independent of Reynolds number at least in the log region, one is left deducing that the local propagation velocity provides the major influence in determining characteristic pocket scales.

Given the proposed hypothesis stated above along with the present evidence that the Taylor microscale at $y^+ \approx 100$ exhibits the same Reynolds number dependence as pocket scales, one may aptly infer that sublayer pockets represent the footprints of wallward-moving coherent vortical structures initially residing at $y^+ \approx 100$ independent of Reynolds number. The important modifier here is the wallward motion of the eddy. Further corroboration and qualification of this phenomenological picture is provided by Metzger et al. wherein the burst period (or frequency) scaled by the local Taylor microscale was found to be both independent of Reynolds number and wall-normal position. Note, burst events are generally accepted to consist of a “sweep” of high momentum fluid toward the wall accompanied by an “ejection” of low momentum fluid away from the wall. The equivalency in the scaling properties of (i) the time interval between sublayer pocket events and (ii) the burst period strongly suggests that sublayer pocket events may be attributed to and visually signify the occurrence of a sweep.

A simple scaling argument for pocket width $W^+$ is presented to examine this idea further. Consider the scenario illustrated in Figure 8 whereby an eddy of characteristic size $\ell^+$ and initially residing in the inner layer at a height of $y^+ \approx 100$ begins to propagate toward the wall at a velocity $\vec{V}_p^+$. When this structure impacts the wall, the footprint is expected to have a length $L^+$, greater than $\ell^+$ due to the actue angle of propagation $\theta$. From geometry, the length of the footprint is

$$L^+ = \frac{\ell^+}{\sin \theta}.$$ (9)
FIG. 8: Schematic of an eddy that initiates from the inner layer (at $y^+ \approx 100$) and propagates toward the wall with a velocity $\vec{V}_p^+$. For a given characteristic size $\ell^+$ of the eddy, the footprint at the wall (in the form of a pocket event) is expected to extend a length $L^+$ due to the acute angle of propagation $\theta$.

Assuming that the eddy propagates in a straight line, the propagation angle $\theta$ can be determined from the components of the propagation velocity, $\vec{V}_p^+ = u_p^+ \hat{i} + v_p^+ \hat{j}$, where $u_p$ and $v_p$ denote the horizontal and vertical velocity components respectively. We further assume that the width of the pocket is approximately equal to its length ($W^+ \approx L^+$). Plan view flow visualization videos reveal pockets to be quasi-circular structures, thus substantiating this assumption. This leads to a relation for pocket width of the following form

$$W^+ = \frac{\ell^+}{v_p^+} \sqrt{(u_p^+)^2 + (v_p^+)^2}.$$  \hspace{1cm} (10)

In the analysis below, we show how (10) can be manipulated to obtain a logarithmic Reynolds number dependence of the inner normalized pocket width consistent with observations from the experimental data in Figure 4. In order to proceed, we make a number of simplifying assumptions. First of all, it is assumed throughout that the wallward-moving eddy initially resides at $y^+ = 100$. Second, as mentioned earlier, Stanislas et al.\textsuperscript{51} indicate the mean diameter of vortices in the log layer normalized by the Kolmogorov scale is $\ell/\eta \approx 20$ independent of Reynolds number. Using the relation $\eta = (\kappa y^+)^{1/4}$ yields a characteristic eddy size of $\ell^+ \approx 50$ (based on values of $\kappa = 0.38$ and $y^+ = 100$). Third, it is reasonable to imagine that the propagation velocity components will be similar to the root mean square velocity of the turbulence at the initiating height of the eddy, taken here to be $y^+ = 100$. 

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This gives \( u_p^+ \approx (U^+)_{y^+=100} + (u^+)'_{y^+=100} \) and \( v_p^+ \approx (v^+)'_{y^+=100} \), where the uppercase and lowercase letters denote the mean and fluctuating (standard deviation) velocities, respectively. It is well known that \( U^+ \) is Reynolds number independent in the log layer\(^{18} \) with a value of approximately 17 at \( y^+ = 100 \). Similarly, \( v^+ \) has also been shown to be Reynolds number independent (for moderate to high Reynolds numbers) with a value of 1.16 in the log layer.\(^{48} \) Therefore, the only quantity in the relation describing \( W^+ \) that exhibits Reynolds number dependence is \( u^+ \). In fact, Marusic et al.\(^{54} \) suggest a logarithmic dependence of the form: \( (u^+)'^2 = B_1 - A_1 \log(y^+ / \delta^+) \). That study examined a number of experimental data sets yielding a range of values for \( A_1 \) and \( B_1 \). For purposes of the scaling analysis here, the average values of \( A_1 = 1.25 \) and \( B_1 = 2.2 \) are considered. Substituting this relation into (10) and performing some algebraic manipulation, one can show that to leading order

\[
W^+ \sim (\log(\delta^+))^{1/4},
\]

where the proportionality constant is equal to \( \ell^+ A_1^{1/4} \sqrt{2 (U^+)_{y^+=100} / (v^+)'_{y^+=100}} \). Plugging in the values stated above for each of the quantities yields a Reynolds number growth rate of 266 for the inner normalized pocket width, i.e., \( W^+ \approx 266 (\log(\delta^+))^{1/4} \) to leading order. This slightly underpredicts (by only a few percent) the behavior of the data, which exhibit a growth rate of 274 when curve fit to the scaling relation in (11). Note, the \( R^2 \) value of this curve fit, i.e., \( W^+ = a (\log(\delta^+))^{1/4} + b \), is 0.991 indicating the data fit well to this scaling model. The agreement between the scaling analysis and the data are quite remarkable considering the assumptions and simplicity of the propagation model utilized. Given this, it is not surprising that the Taylor microscale evaluated at \( y^+ \approx 100 \) succeeds in removing Reynolds number dependencies in the pocket width, since the Taylor microscale inherently incorporates information about the local streamwise turbulent velocity by definition.

In summary, the present results support the physical picture of the near-wall self-sustaining process based on the “parent-offspring autoregeneration mechanism” as described in the recent study of Wu et al.\(^{14} \) and in accordance with earlier work.\(^{9,27,55–57} \) In this picture, viscous sublayer streaks behave as passive entities, frequently perturbed by turbulent-turbulent spots (TUTS) that are generated locally within the viscous sublayer and buffer layer. TUTSs appear to originate from hairpin vortex packets occupying the near-wall region and, as such, are characterized by spatially intermittent concentration of small-scale vortices. Wu et al. revealed that TUTSs are structurally analogous to transitional-turbulent
spots that form as part of the secondary instability in laminar boundary layer transition. In their DNS visualizations, they noticed that indented sublayer streaks often simply form the edges of pockets that arise from the existence of TUTSs. The present results appear consistent with this connection between pockets and TUTSs. Specifically, we have shown that (i) pockets scale with the Taylor microscale measured at $y^+ \approx 100$, which is in the same wall-normal region where TUTSs largely reside, and (ii) the Reynolds number growth of the inner normalized pocket width ($W^+$) can be obtained from a simple model of a compact vortex located at $y^+ \approx 100$ that propagates obliquely toward the wall. It remains to be seen whether the disturbances leading to pocket formation in the sublayer are responsible for instigating the birth of infant TUTSs, or whether pockets are mere footprints of compact vortical motions from more mature TUTSs aloft. In either case, future work is needed to determine if there are specific events that initiate the wallward motion of pocket-producing eddies, and how this overall picture of the near-wall process is linked to the observation of a hierarchy of large-scale and very large-scale motions farther from the wall. In this regard, determining the relative contributions of TUTSs to wall-layer turbulence production (versus the self-sustaining process associated with streaks and quasi-streamwise vortices) would seem to be an important factor.

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