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Log-layer mismatch and modeling of the fluctuating wall stress in wall-modeled large-eddy simulations

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Log-layer mismatch refers to a chronic problem found in wall-modeled large-eddy simulation (WMLES) or detached-eddy simulation, where the modeled wall-shear stress deviates from the true one by approximately 15%. Many efforts have been put to resolve this mismatch. The often used fixes, which are generally ad-hoc, include modifying subgrid-scale stress models, adding a stochastic forcing, moving LES/wall-model matching location away from the wall, etc. An analysis motivated by the integral wall-model formalism suggests that ‘log-layer mismatch’ is resolved by the built-in physics-based temporal filtering. In this work we investigate in detail the effects of local filtering on ‘log-layer mismatch’. We show that both local temporal filtering and local wall-parallel filtering resolve ‘log-layer mismatch’ without moving the LES/wall-model matching location away from the wall. Additionally, we look into the momentum balance in the near-wall region to provide an alternative explanation of how LLM occurs, which does not necessarily rely on the numerical-error argument. While filtering resolves ‘log-layer mismatch’, the quality of the wall-shear stress fluctuations predicted by WMLES does not improve with the new remedy. The wall-shear stress fluctuations are highly underpredicted due to the implied use of LES filtering. However, good agreement can be found when the WMLES data are compared to the DNS data filtered at the corresponding WMLES resolutions.

I. INTRODUCTION

Turbulence is comprised of fluid motions of many scales. Resolving all such scales leads to direct numerical simulations (DNS). DNS provides the highest-fidelity solution to fluid problems, but its cost quickly goes beyond the capacity of the current supercomputers when deployed for practical high Reynolds number applications. Large-eddy simulations (LES) has been considered as a predictive and affordable alternative to DNS, as it resolves the flow-dependent, large-scale motions directly, but models the small, less-energetic eddies that are more amenable to universal and calibration-free parameterization [1–3]. In wall turbulence, however, the requirement that LES must resolve the energy-containing motions everywhere makes LES only little cheaper than DNS [4, 5], because the size of the energy-containing eddies in the near-wall region becomes progressively comparable to the smallest dissipative eddies toward the wall [6]. The problem is more exacerbated by the fact that the currently available subgrid-scale models underperform on coarse near-wall grids [7–9]. For application of LES to technologically-relevant problems, one therefore requires an additional way of compensating for the lack of modeled stresses on coarse grids. This leads to wall-modeled LES (WMLES), where the grid spacing is chosen to scale with the local boundary-layer thickness [10, 11], and an auxiliary way of augmenting the turbulent stress in the near-wall region is introduced. Following the taxonomy given in Ref. [12], WMLES is further categorized into two different types of approaches: wall-stress modeling and hybrid RANS/LES. In the former, a formal LES framework is maintained everywhere in the LES domain, while the momentum transport in the unresolved near-wall region is modeled through the modified wall-boundary condition. In the latter, the turbulence model switches from a Reynolds-Averaged Navier-Stokes (RANS) parameterization within the entire boundary layer to a LES-like one in the outer region [7]. The two approaches, at first glance, may look similar. However, the key difference is in how LES is coupled to the low-dimensional representation of the near-wall turbulence, and this difference leads to near-wall flow patterns and modeling challenges unique to each approach. Refs. [11, 12] provide detailed discussions on this subtle but important distinction between the two approaches. Despite such difference, a chronic problem has been found commonly in the two approaches, which is log-layer mismatch [13–15].

The discussion from this point on focuses solely on wall-stress modeling, and henceforth we use the terms WMLES and wall-stress modeling interchangeably. In a typical WMLES, the grid spacing (Δ) (especially in the wall-normal direction) is chosen to be a fraction of the local boundary-layer thickness (δ), typically at $\delta/\Delta \approx 20 \sim 30$ [16–20]. The lack of turbulent stress in this coarse near-wall grid is compensated for by the modeled wall-stress boundary condition, which augments the total stress in the inner-most region of boundary layers. Modeling the wall stress based on the most neighboring LES velocity has often led to underpredicted [21, 22]/overpredicted [23, 24] stresses, and this mismatch between the modeled and the real skin friction is the so-called log-layer mismatch (LLM). Many efforts have been put to resolve this mismatch. The early works have focused on modifying the SGS stress models [25–30]. Later investigations suggest that LLM is not solely due to an inaccurate SGS stress model. Kawai & Larsson [21] argued that the flow in the near-wall region of WMLES is inevitably underresolved, making the input to the wall model (traditionally from the wall-adjacent LES data) contaminated by the truncation and SGS modeling errors. They then proposed that in order to resolve LLM, one needs to abandon the practice of using the first off-wall grid point for wall modeling, and instead use the LES information slightly away from the wall.

This remedy by Kawai & Larsson has been useful for its robustness. However, its non-local nature appears to render this method impractical for flows involving complex wall geometry, by incurring significant user overhead in LES mesh preparation and wall-model implementation. First, ensuring the existence of a proper matching location (with a prescribed wall distance, or number of elements below it) at the meshing stage is almost always impossible when complex geometries are meshed with unstructured elements. This situation was encountered in the recent applications of WMLES to a 3-D aircraft configuration involving multiple wall junctures [31] and an industry-scale gas-turbine combustor (Sanjeeb T. Bose, Cascade Technologies, Inc., private communication, December 2016). Second, data exchange between the wall faces and their proper matching locations in parallel, unstructured-grid flow solvers requires non-local search and message passing at each time step, which degrades parallel performance of the flow solver. Lastly, this method unphysically detaches the wall stress from the closest LES velocity, failing to properly account for sharp turning angles or skewed velocity profiles commonly found in external aerodynamics applications [31]. Therefore, it is imperative that a local wall-modeling approach using the wall-adjacent LES data has to be developed, while resolving LLM appropriately. In addition to such practical concerns, the interpretations of how LLM is caused and subsequently resolved by the traditional remedy have been unclear, as it is based largely on a qualitative length-scale argument [21]. Although it is well demonstrated and understood that the near-wall solution in WMLES is underresolved, how this further develops into LLM has not been explored systematically.

This study aims at developing an alternative remedy for LLM that is better suited for complex-geometry applications, and also providing a more comprehensive explanation of how LLM is caused and can be resolved through a quantitative analysis. The starting point of this work is the recent work of Yang *et al.* [32], where it was observed that the physics-based time filtering of the wall-model input (taken from the wall-adjacent LES solution) effectively resolves LLM in their integral wall-model framework, without using the LES information away from the wall for wall modeling. The paper is organized as follows: In Sec. II, flow configuration, wall models, and computational method are described briefly. LLM and its resolution with a prevailing remedy are reproduced in Sec. III, and a new method for resolving LLM is proposed and tested in Sec. IV. Sec. V provides an alternative explanation of how LLM is caused and resolved by the remedies. Lastly, Sec. VI discusses briefly about the prediction of wall-shear stress fluctuations in WMLES with respect to the filtered DNS. Conclusions are given in Sec. VII.

II. COMPUTATIONAL DETAILS

We use the open-source code ‘LESGO’ to investigate the LLM problem. The code solves the filtered Navier-Stokes equations in a half channel with a staggered grid. The size of the computational domain is $2\pi\delta \times \delta \times 2\pi\delta$ in the streamwise (x), wall-normal (y), and spanwise (z) directions, respectively, where δ is the half channel height. The code uses a pseudo-spectral approach in the wall-parallel directions, with a second-order accurate central finite

difference scheme used in the wall-normal direction. Details of this code can be found in Ref. [33] and the references cited therein. A symmetric condition is used at the top boundary. Periodic boundary conditions are applied in the homogeneous directions (x and z). The channel is driven by a constant pressure gradient. The scale-dependent Lagrangian-dynamic Smagorinsky model [33] is used for subgrid-scale stress modeling. While not reported here, no significant difference in computed flow statistics was found when employing different dynamic SGS models (at least for the statistics of interest here).

In WMLES, the no-slip wall boundary condition is replaced with the shear-stress boundary condition, and the eddy viscosity is 0 at the wall. For the channel configuration investigated in the present study, a simple log-law like equilibrium wall model is employed.

$$\tau_x = \rho \left[\frac{\kappa u_{||}}{\log(h_{wm}/y_o)} \right]^2 \frac{u_{wm}}{u_{||}}, \quad \tau_z = \rho \left[\frac{\kappa u_{||}}{\log(h_{wm}/y_o)} \right]^2 \frac{w_{wm}}{u_{||}}, \quad (1)$$

where τ is the wall-shear stress, u_{wm} and w_{wm} are the LES velocities at the matching location (a distance $y = h_{wm}$ away from the wall) in x and z directions, respectively, $u_{||} = \sqrt{u_{wm}^2 + w_{wm}^2}$ is the modulus of the LES velocity at the matching location projected on the wall-parallel plane, and $\kappa = 0.4$ is the von Karman constant. $y_o \equiv \nu / (\bar{u}_\tau e^{\kappa B})$ is the effective roughness length and is kept constant at $1 \times 10^{-4} \delta$ and $6.77 \times 10^{-5} \delta$ depending on the cases, corresponding to the friction Reynolds numbers of $Re_\tau \approx 1400$ and $Re_\tau \approx 2000$, respectively. Here, $B = 5.0$ is the intercept in the logarithmic velocity profile, and \bar{u}_τ is the mean friction velocity in the channel flow which is known a priori in the present study. With the wall boundary condition provided by Eq. (1), the LES equations in the bulk region are integrated. At the next time step, the updated u_{wm} , w_{wm} are used in Eq. (1) again to supply the wall boundary condition for LES. Unless noted otherwise, lengths and velocities are normalized by the half channel height and the friction velocity.

III. LLM AND A PREVAILING REMEDY FOR ITS RESOLUTION

We first reproduce LLM problem in a turbulent channel flow, and show its resolution using a commonly-used method. It has long been known that wall models operating on the raw LES velocity at the first off-wall grid point give rise to the so-called ‘log-layer mismatch’ (LLM) problem, leading to up to 15% deviation of the modeled wall-shear stress from the correct value, independent of the spatial discretization used, the SGS model employed and the flow configuration of interest [21, 34]. This LLM is reproduced in the present half channel calculation in Fig. 1 (case ‘NFI’). The two cases in figure 1 use a grid of size 64^3 , with the grid spacing being uniform in each direction. As the present channel flow is driven by a fixed pressure gradient, the mean wall-shear stress is always correct, and LLM here manifests itself as an erroneous upward shift of the mean velocity profile (positive LLM). Note that LLM is not a problem specific to the simple wall model deployed in the present study. It is known to persist in WMLES using more comprehensive wall models as reported in Refs. [19, 21]. A prevailing remedy for LLM is to use the LES information slightly away from the wall (e.g., u_{wm} equals u_{LES} at the second, third, or fourth off-wall grid point). The hypothesis is that the LES solution at the first off-wall point is likely contaminated by the numerical and SGS modeling error. Based on this reasoning, Kawai and Larsson [21] showed that LLM is resolved when providing the LES data at the n^{th} off-wall point to the wall model ($n \geq 2$). We reproduce the same result in Fig. 1 (case ‘2nd pt’) as well. Using the LES data from the third or fourth off-wall points produced essentially the same result, consistent with the findings in Ref. [21]. It should be noted that the shape of the mean velocity profiles in the bulk region is a function of the wall boundary condition, and changes in the wall condition can lead to changes in the mean profile in the wake region.

Porte-Agel *et al.* [28] also reported that LLM can be resolved through a proper modification of the SGS model in the near-wall region. They argued that because the flow field is underresolved, the momentum flux through the top face of the first LES computational cell should be based on the imposed law of wall $du/dy = (u_\tau/\kappa y)(u_{wm}/u_{||})$, $dw/dy = (u_\tau/\kappa y)(w_{wm}/u_{||})$, instead of evaluating velocity derivatives directly with a finite difference/finite volume

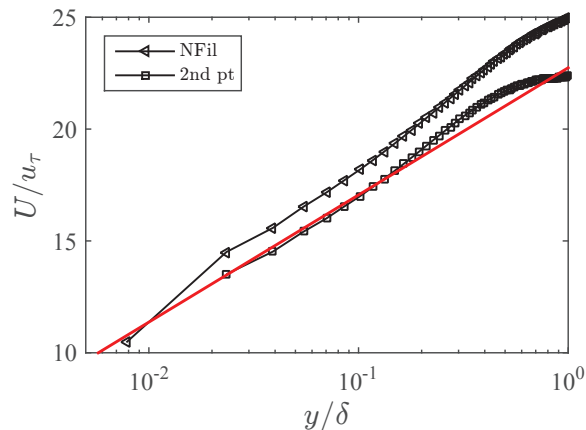


FIG. 1: Mean velocity profiles. The expected log law, $U/u_\tau = 1/\kappa \log(y/y_o)$, is indicated using a solid red line. For the cases ‘NFI’, the first LES grid point from the wall is used as the wall-model input. For the case ‘2nd pt’, the instantaneous LES velocity at the second LES grid from the wall is used as the wall-model input, with no filtering.

scheme. Because the velocity derivatives enter the SGS stress model and also the filtered Navier-Stokes equations, evaluating derivatives according to the imposed log law essentially reformulates the SGS model [35]. However, our numerical experiments show this reformulation has limited impact on LLM (not shown).

IV. A NEW REMEDY FOR LLM

In this section, we propose a new and simpler remedy for LLM, in which wall model operates with the filtered LES data at the first off-wall grid point. This is motivated by an observation in Ref. [32], where LLM is avoided by providing the time-filtered LES signal at the first off-wall grid point to a wall model, according to

$$u_{wm}^n = (1 - \epsilon)u_{wm}^{n-1} + \epsilon u_{LES}^n. \quad (2)$$

w_{wm} is obtained similarly by filtering the spanwise LES velocity. Here, u_{wm} is the velocity used in the wall model, the superscript n is the LES time step index, and u_{LES} is the instantaneous streamwise LES velocity at the first off-wall grid point ($y = h_{wm} = \Delta y/2$). The weight of time averaging is defined as $\epsilon = \Delta t/T_f$, where Δt is the LES computational time step, and T_f is a filtering time scale to be specified. We examine the effect of the filtering time scale on the WMLES prediction by considering the computational time step ($T_f = 2\Delta t$), a convective time scale at the matching location ($T_f = \Delta t_c \equiv \Delta x / \langle u(y = \Delta y/2) \rangle$), and the characteristic time scale for the wall-normal turbulent transport at the matching location ($T_f = T_i \equiv h_{wm}/\kappa u_\tau$). As seen in Fig. 2, increasing the filtering time scale reduces the strength of LLM, and the mismatch is essentially resolved (becomes sufficiently weak) with the filtering time scales $T_f > \Delta t_c$. Figure 3 quantifies the strength of LLM as a function of the filtering time scale. The strength of LLM is quantified here as $(\Delta U_{LES} - \Delta U_{log})/U_{2,log}$, where $\Delta U_{LES} = \langle u(y = 1.5\Delta y) \rangle - \langle u(y = 0.5\Delta y) \rangle$ is the velocity difference between the first and second LES grid points, $\Delta U_{log}/u_\tau = 1/\kappa \log(1.5\Delta y/y_o) - 1/\kappa \log(0.5\Delta y/y_o)$ is the velocity increment expected from the log law, and $U_{2,log}/u_\tau = 1/\kappa \log(1.5\Delta y/y_o)$ is the expected velocity at the second LES grid point. A few observations can be made. LLM persists when using a raw LES signal ($T_f = \Delta t$, no time filtering). The strength of LLM drops to $\approx 2\%$ with $T_f = \Delta t_c$, and to about 1% with $T_f = T_i$. Because the uncertainty in the von Karman constant is about 2.5% [36], taking $T_f = \Delta t_c$ already resolves LLM. It was also verified that LLM remains negligibly small with very large filtering time scales ($T_f \gg T_i$).

A temporal filtering with $T_f = \Delta t_c$ is expected to have a similar effect as a $2\Delta x$ wall-parallel filtering. It is of interest here to know whether the wall-parallel spatial filtering is an equally useful remedy for LLM. Three types of spatial filters are tested (the 9-point top-hat filter, the 9-point Gaussian filter, and the 5-point top-hat filter, all spatial

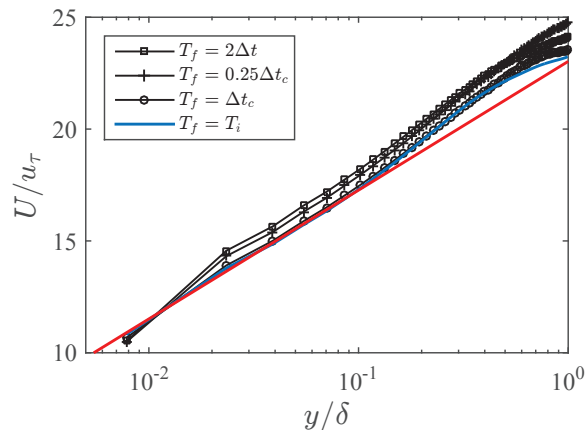


FIG. 2: Mean velocity profiles obtained with the temporally filtered wall-model input at the first off-wall LES point. All cases use a grid of size 64^3 and a roughness length scale $y_o = 0.0001\delta$. The log law is indicated using a solid red line.

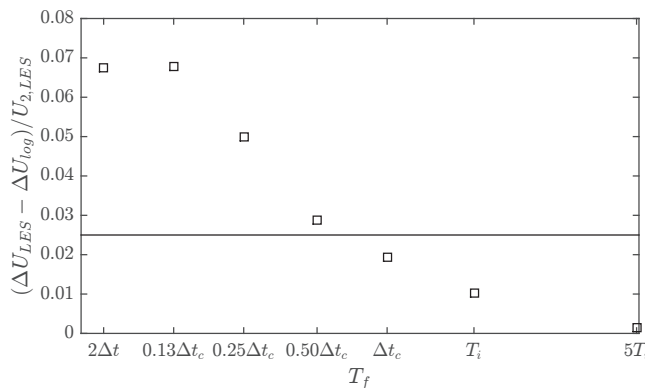


FIG. 3: Strength of LLM plotted against the filtering time scale. Δt is the LES time step size, $\Delta t_c = \Delta x / \langle u(y = 0.5\Delta y) \rangle$, and $T_i = h_{wm} / (\kappa u_\tau)$. The line at 0.025 indicates expected the uncertainty in evaluating the strength of LLM owing to the variation in the von Karman constant.

filtrations are performed within the wall-parallel plane located at $y = h_{wm}$) in half channel WMLES calculations with 32^3 and 64^3 grid points. As seen in Fig. 4, the filter type has no noticeable impact on the resolution of LLM. Grid convergence is found, suggesting that the filter width only needs to be a few times the grid spacings (Δx and Δz). Similarly to the finding in the temporally filtered cases, it is found that LLM remains weak with much larger filter widths. The test cases in this section have used a fixed grid aspect ratio $\Delta x = \Delta z = 2\pi\Delta y$. Although not shown, we conducted several numerical experiments where the grid aspect ratio is varied ($\Delta x = \Delta z = C\Delta y$ with C ranging from 1 to 10). LLM was again observed, but it was removed with the same filtering strategy.

As the low resolving power of a less accurate numerical scheme can act as an effective filter, it is of interest to examine whether LLM can be removed by deploying lower-order numerical methods for the discretization of the LES equations. To this end, we carried out the same set of numerical experiments using a finite difference channel flow code to investigate the effects of numerical methodology. In this code, the second-order finite difference scheme is used for spatial discretization, and the subgrid stress is modeled using the recently developed minimum dissipation model [37]. A grid of size $32 \times 64 \times 32$ is used for a channel of size $2\pi\delta \times 2\delta \times 2\pi\delta$ (note this is a full channel, and grid aspect ratio is kept the same as other cases). Figure 5 shows the mean velocity profiles as functions of the wall-normal distance. With the finite difference code (case FD), LLM occurs when the equilibrium model is applied directly to the first off-wall LES grid point, and we resolve LLM by applying a 5-grid-point spatial filtering (case FD, filt). The LESGO results are included for comparison. LLM of similar magnitude is found. Employing a less-accurate

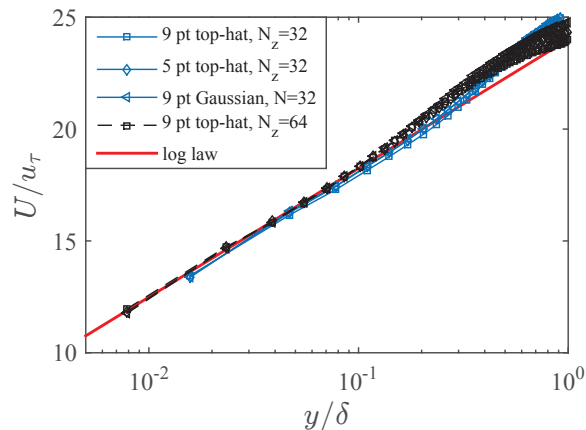


FIG. 4: Mean velocity profiles obtained with the spatially-filtered wall-model input at the first off-wall LES point. The effective roughness height is $y_o = 6.8 \times 10^{-5}\delta$, which corresponds to a $Re_\tau = 2000$ channel flow. The expected log-law, $U/u_\tau = 1/\kappa \log(y/y_o)$, is indicated using a solid red line. Solid blue lines are for a 32^3 grid and dashed black lines are for a 64^3 grid. Squares symbols are for 9-point top-hat filters, diamonds symbols are for 5-point top-hat filters, and left-pointing triangles symbols are for 9-point Gaussian filters.

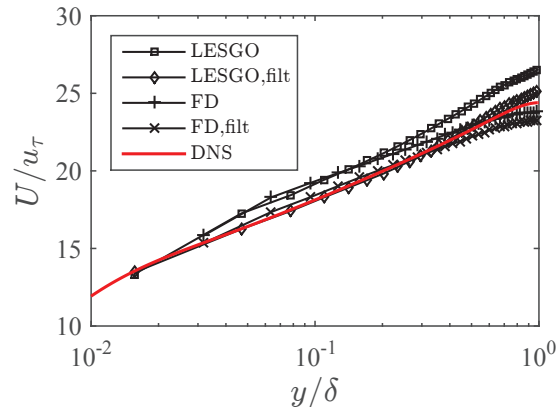


FIG. 5: Mean velocity profiles. $z_o = 6.8 \times 10^{-5}\delta$. LESGO and FD are obtained with the spectral and second-order finite difference codes, respectively, without filtering the wall-model input. “flt” means that the results are obtained with the 5-grid-point spatial filtering. The red line is the DNS results of Hoyas & Jimenez [38].

spatial discretization does not seem to affect LLM in the near-wall region nor the effectiveness of the proposed remedy. However, the flow in the bulk region is affected by the numerical methodology, where the law of wake is not captured at this particular resolution by the finite difference code.

Both the temporal filtering and wall-parallel spatial filtering allow a local wall-modeling procedure using the LES information right above the wall. We anticipate the temporal filtering would be of greater utility for practical applications. This is because the wall-parallel filtering can be inappropriate in highly accelerating/decelerating flows, and also due to the difficulty of constructing a discrete wall-parallel filter operating on unstructured mesh elements.

V. REVISTING THE CAUSE OF LLM

At this point, it is important to realize that the new remedy suggested in the previous section, as opposed to the prevailing remedy, neither introduces additional grid resolution near the wall nor uses the LES information away

TABLE I: A list of the relevant quantities near the wall. The nomenclature for the cases is the same as in figure 1. u is the instantaneous streamwise LES velocity at $y = h_{wm}$, and u' is the instantaneous streamwise velocity fluctuation. $\langle u'v' \rangle$ is the resolved Reynolds shear stress evaluated at the first LES grid point for all cases. All quantities are in wall units.

cases	h_{wm}	$\langle u \rangle_{y=h_{wm}}^2$	$\langle u'^2 \rangle_{y=h_{wm}}$	$\langle u'^2 \rangle / \langle u \rangle^2$	$-\langle u'v' \rangle_{y=\Delta y/2}$	LLM strength
NFil	$\Delta y/2$	1.1×10^2	4.7	4.2%	0.22	7%
Fil	$\Delta y/2$	1.1×10^2	5.7	5.2%	0.28	2%
2nd pt	$3\Delta y/2$	1.8×10^2	7.4	4.1%	0.27	2%

from the wall. This is in direct contradiction to the common perception that LLM is caused by the near-wall LES solution contaminated by the numerical and SGS modeling errors due to the too-large grid spacing [21]. With the new remedy, LLM is resolved even when the near-wall LES solution remains significantly underresolved. This implies that potentially a distinctly different mechanism beyond the naive numerical-error argument may be at work in producing LLM. Identifying the correct cause of LLM is of great importance in wall-stress modeling, because this would have significant implication on developing new wall-modeling strategies, as in the progress made in the hybrid RANS/LES community over the past decades for new model developments [7]. In this section, we propose an alternative explanation of how LLM is caused in WMLES based on detailed analysis of flow statistics near the wall.

We first consider the validity of the argument made by Bou-Zeid & Meneveau [33]. They argued that wall models can overpredict the wall stress even in the absence of the numerical and SGS modeling errors in LES. Based on the commonly-found scaling $\tau_{w,wm} \sim \rho u^2$ between the wall-model input and output, they deduce that the mean wall stress modeled by a wall model can be overpredicted due to the fluctuations inherent in the LES velocity, even if the LES velocity is correct in the mean sense: $\langle \tau_{w,wm} \rangle \sim \rho \langle u \rangle^2 + \rho \langle u'^2 \rangle > \rho \langle u \rangle^2 \sim \langle \tau_w \rangle$. Here, $\langle \cdot \rangle$ indicates ensemble averaging, τ_w is the true wall stress, $\tau_{w,wm}$ is the modeled wall stress, u' is the instantaneous velocity fluctuation, and u is the instantaneous velocity. This argument may explain a negative LLM (downward shift of the mean-velocity profile due to the overpredicted wall stress), but the positive LLM is clearly not explained by this mechanism. In fact, a close scrutinization of the near-wall flow statistics given in Table I shows that this scaling argument does not explain the negative LLM either. The mean velocity ($\langle u \rangle^2$) and the level of velocity fluctuations ($\langle u'^2 \rangle / \langle u \rangle^2$) show no appreciable change in the cases with and without LLM. Instead, comparing the case with LLM ('NFil') with the cases without LLM ('Fil' and '2nd pt'), a notable difference is found in the resolved Reynolds shear stress at the first off-wall grid point. We notice the Reynolds shear stress $-\langle u'v' \rangle$ is significantly smaller when LLM persists, suggesting that turbulence near the wall is less energetic. Considering a momentum balance at the first off-wall LES grid point, the total shear stress $-\langle u'v' \rangle + \langle (\nu + \nu_t) \partial u / \partial y \rangle$ is balanced by the imposed mean pressure gradient ($1/\rho \cdot dp/dx$). Therefore, a smaller $-\langle u'v' \rangle$ necessarily leads to an increased $\langle (\nu + \nu_t) \partial u / \partial y \rangle$. The increased $\langle (\nu + \nu_t) \partial u / \partial y \rangle$ may be due to a corresponding increase in either the LES eddy viscosity ν_t , the velocity gradient $d\langle u \rangle / dy$, or both. However, because $\langle u(y) \rangle \equiv 0 + \int_0^y \langle du/dy' \rangle dy'$, LLM is associated much more likely with the increased $d\langle u \rangle / dy$. This was confirmed from the WMLES solution, where the LES eddy viscosity at the first off-wall grid point is almost identical in the cases considered in Table I. We therefore conclude that the positive LLM is a result of underpredicted Reynolds shear stress. Because $-\langle u'v' \rangle$ is not sufficiently large, the velocity gradient $d\langle u \rangle / dy$ increases to compensate for the deficit, leading to a positive LLM.

A more fundamental question is then why the Reynolds shear stress is underpredicted when using the raw LES data at the first off-wall grid point as the wall-model input (as in the case 'NFil'). We consider how the modeled

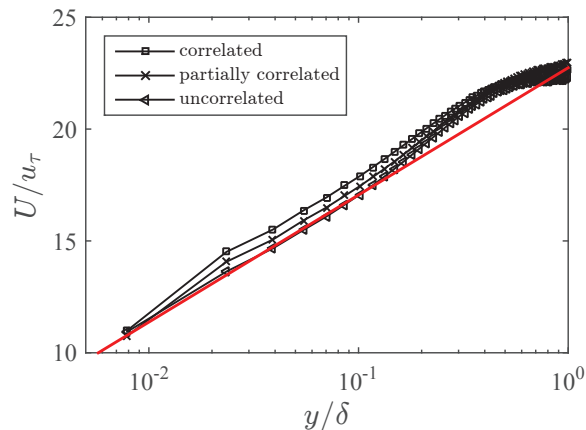


FIG. 6: Semi-log plot of the mean velocity profiles as functions of the wall-normal distance. The wall stresses are specified according to Eq. (4). $C = 0.2$ for the case ‘correlated’, $C = 0.1$ for the case ‘partially correlated’ and $C = 0$ for the case ‘uncorrelated’.

wall-shear stress and the input LES velocity is related at the instantaneous level.

$$\begin{aligned} \tau_{wm} &= \rho \left[\frac{\kappa}{\log(h_{wm}/y_o)} \right]^2 u^2 = \rho \left[\frac{\kappa}{\log(h_{wm}/y_o)} \right]^2 (\langle u \rangle^2 + 2 \langle u \rangle u' + u'^2) \\ &\approx \langle \tau_w \rangle + 2\rho \left[\frac{\kappa}{\log(h_{wm}/y_o)} \right] u_\tau u' = \langle \tau_w \rangle + C\rho \cdot u_\tau u'. \end{aligned} \quad (3)$$

Here, it is assumed that the term that is second order in fluctuations (u'^2) is negligibly small compared to others. Equation (3) suggests that the instantaneous fluctuations in the wall stress τ'_w is proportional to the instantaneous fluctuations in the input LES velocity (i.e., $\tau'_w \sim \rho u_\tau u'$). When u is taken from the first LES off-wall grid point, this implies that the wall model responds immediately to the change in u , instantaneously draining more momentum when $u' > 0$ and less momentum when $u' < 0$. This effectively “locks” the wall-adjacent LES velocity, damping the near-wall fluctuations. Consequently, the Reynolds shear stress $-\langle u'v' \rangle$ becomes less energetic. Overall, this problem is the direct consequence of the unphysical high correlation between the LES velocity at the first off-wall point (likely located at $y^+ \gg 100$ for high Reynolds number flows) and the wall-shear stress (defined at $y^+ = 0$). The correlation coefficient C in the present WMLES calculations ($N = 64^3$) can be estimated from Eq. (3) to be 0.2. Note that C , in the framework of the recent inner-outer interaction model [39], can be interpreted as the superposition effect of the velocity signal in the log-layer region on the wall-shear stress [40–42]. The coefficient C in the high Reynolds number experiment or DNS is found to be less than 0.1 at wall distances comparable to the matching locations in the present study [39].

This analysis suggests that LLM can be mitigated by disrupting the unphysical correlation between τ'_{wm} and u' in coarse WMLES. This hypothesis can be confirmed by a numerical experiment, where the imposed shear stress at the wall is constructed by a superposition of the known mean wall-shear stress with the fluctuations having a controllable correlation with the LES velocity at the first off-wall grid point. Note that the channel flow driven with the known pressure gradient allows the construction of such simple wall models, as the mean wall-shear stress is known *a priori*. For this purpose, we model the wall stress according to

$$\tau_{wm} = \langle \tau_w \rangle + C\rho u_\tau [u - \langle u \rangle]_{y=\Delta y/2}, \quad (4)$$

with $C = 0.2, 0.1$, and 0 . As seen in Fig. 6, a 7% LLM is found with $C = 0.2$. LLM reduces with $C = 0.1$, and LLM is essentially resolved with $C = 0$.

To summarize, the proposed mechanism of LLM is as follows. When a wall model operates with the LES velocity at the first off-wall grid point, the modeled stress fluctuations are instantaneously (and unphysically) correlated to

the LES velocity, causing damping of turbulence adjacent to the wall. Then, to satisfy the momentum balance, the reduced Reynolds shear stress $-\langle u'v' \rangle$ gives rise to an increased $\langle du/dy \rangle$, leading to LLM.

It becomes more clear now how filtering the LES signal provided to the wall model resolves LLM. Filtering disrupts the unphysical, direct coupling between τ'_w and u' by removing the velocity fluctuations u' in Eq. (3), and subsequently leading to a more energetic $\langle u'v' \rangle$, and eventually to a reduction of LLM. The same argument can be used to explain how the traditional remedy of using the LES information away from the wall, as advocated by Kawai & Larsson [21], helps resolving LLM. In this case, the wall-shear stress is correlated with the LES velocity far away from the wall, which effectively removes the damping or locking cycle between the wall-shear stress and the wall-adjacent LES velocity. This argument could also be used to explain how the control-based wall model developed in Ref. [34] resolves LLM, where the wall-shear stress is modified such that the near-wall RANS solution matches as closely as possible the near-wall LES solution. As reported in Ref. [34], the control strategy essentially decorrelate the modified stress from the first off-wall LES velocity, but correlates it with the second off-wall LES velocity.

VI. WALL STRESS FLUCTUATIONS

The discussion in the previous sections has focused on the first-order quantities. High-order quantities including the wall-stress fluctuations are of interest in a few applications [43, 44]. When compared to DNS, WMLES inevitably underpredicts the fluctuations in the wall-shear stress [45] owing to the implied use of LES filters. A fair comparison is between WMLES and the filtered DNS at the corresponding LES resolution.

Table II reports the root-mean-square (*rms*) values of the wall-stress fluctuations from WMLES and the filtered DNS. The grid sizes of the WMLES are 32^3 , 64^3 and 128^3 , and the DNS is filtered at the corresponding LES resolutions. The input LES velocities to the equilibrium wall model is filtered with a 9-point top-hat filter. The DNS is a channel flow at $Re_\tau \approx 2000$. The DNS resolution is $\Delta x^+ = 12.3$, $\Delta y_{min}^+ = 0.22$, $\Delta y_{max}^+ = 6.1$, and $\Delta z^+ = 6.1$. Details of this DNS dataset can be found in Ref. [38]. Based on the data in table II, we reconfirm that WMLES underpredicts the fluctuations in the wall stress; meanwhile we find that the *rms* wall-stress fluctuations from WMLES agrees well with those from the DNS filtered at the corresponding LES resolution.

VII. CONCLUSIONS

Log-layer mismatch (LLM) in WMLES refers to the erroneous shift of the mean velocity profile below or above the logarithmic layer, when wall-stress models are operated with the LES velocity at the wall-adjacent control volumes. In boundary layers, LLM leads to the skin friction error up to 15%, not admissible in application of WMLES for practical engineering applications. A prevailing remedy for resolving LLM in WMLES is based on a hypothesis that the near-wall LES solution is contaminated by the inherent truncation and SGS modeling errors owing to the use of too large grid spacings near the wall, especially in the wall-normal direction. This remedy suggests providing the wall model the LES data at least 2~3 cells away from the wall. This method has proven to be robust, resolving LLM with little effort when using structured grids. However, this remedy was recently found to be not only impractical in terms of the LES mesh preparation for complex geometries (with corners and junctures) and implementation in the unstructured, parallel flow solvers, but also inaccurate in accounting for the complex velocity profiles found in external aerodynamics (e.g., skewed 3-D mean velocity profile).

In this paper, we propose a new, simpler remedy for LLM that is better suited for application to complex flows. This method resolves LLM by simply providing the wall model the filtered LES data in the wall-adjacent control volumes. It is shown that both spatial and temporal filtering work.

Generally, LLM is a function of the wall model, the spatial and temporal discretization used in the code, the LES grid, the SGS model, and the flow configuration. The perception of LLM being attributable simply to the numerical error [21] does not explain how the new method resolves LLM, as the new remedy still uses the LES data right above

TABLE II: Root-mean-square of the streamwise wall stress fluctuations. We test three wall models: the equilibrium model used at the first off-wall LES grid point, with a 9-point top-hat filtering of the wall-model input; the equilibrium wall model used at the second off-wall LES grid point (no filtering); the integral wall model in its original form [32] (with a temporal filtering of the first off-wall LES data). For each wall model, WMLES at 3 grid sizes are conducted. The *rms* of the DNS stress fluctuations is 0.42. The last three rows are for the filtered DNS data. All numbers are in wall units.

LES grid	corresponding LES grid size	wall model	rms of the streamwise wall stress
32 ³	N/A	equilibrium model 1st grid	0.16
64 ³	N/A	equilibrium model 1st grid	0.21
128 ³	N/A	equilibrium model 1st grid	0.25
32 ³	N/A	equilibrium model 2nd grid	0.13
64 ³	N/A	equilibrium model 2nd grid	0.19
128 ³	N/A	equilibrium model 2nd grid	0.24
32 ³	N/A	integral model	0.15
64 ³	N/A	integral model	0.20
128 ³	N/A	integral model	0.26
N/A	32 ³	N/A	0.14
N/A	64 ³	N/A	0.19
N/A	128 ³	N/A	0.25

the wall. We propose a new mechanism explaining the cause of LLM based on quantitative analysis of the near-wall flow statistics in WMLES of turbulent channel flow, summarized as follows. LLM is thought to be a consequence of the underpredicted Reynolds shear stress $|\langle u'v' \rangle|$ near the wall. This is compensated for by the increase in the velocity gradient ($\frac{\partial \langle u \rangle}{\partial y}$) near the wall, eventually leading to the erroneous shift of the mean velocity profile (that is, LLM). The reason why the Reynolds shear stress is underpredicted when wall models are given the wall-adjacent LES data is first explained based on how the wall stress from the wall model is coupled to the wall-adjacent LES velocity at the instantaneous level. It is proposed that the unphysically strong coupling between the wall-adjacent LES velocity (defined likely at $y^+ \gg O(100)$ in WMLES) and the wall stress (defined at $y^+ = 0$) makes the wall-stress fluctuations an effective damper of the near-wall turbulence. According to this analysis, resolving LLM requires de-correlating the wall stress and the wall-adjacent LES velocity, and numerical experiments further carried out (Fig. 6) provide support to this argument. The proposed mechanism can also explain how both the traditional remedy and the new method proposed here help removing LLM. The LES flow field in the near wall region is admittedly underresolved, but we conclude that this is not necessarily the culprit of LLM.

We reconfirm that WMLES does not predict the wall-shear stress fluctuations well. However, it is found that WMLES captures the rms of the filtered DNS wall stress. Better description of the fluctuating wall stress requires adequate modeling of the unresolved SGS wall-stress fluctuations. Although a direct path to such SGS modeling is not known, the insights and understanding gained from the superposition/modulation model [46–52] might be a promising starting point for the finer-scale stress modeling.

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