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Correlation of pressure fluctuations in turbulent wall layers

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The mean square pressure fluctuations ⟨pp⟩⁺ in the inner and outer regions are correlated for various Reynolds numbers using published DNS data. It is found that the overlap matching region has logarithmic behavior. The outer region overlap law is ⟨pp⟩⁺(y/h) = (1/ǫ) ln(y/h) + Do. In the inner region, the correlation has two terms and is of the form; P(y⁺) ≡ ⟨pp⟩⁺(y⁺, Reτ) − ⟨pp⟩⁺(0, Reτ). The inner matching law is P(y⁺) = (1/ǫ) ln(y⁺) + Di. In the inner region, P(y⁺) is the same for channel flow and boundary layers, however, in the outer region, ⟨pp⟩⁺(y/h) for channel flow and boundary layers are quite different. A compatibility equation, obtained by equating the matching laws, is a relation for the wall pressure in terms of the matching constants; ⟨pp⟩⁺(0) = (1/ǫ) ln(Reτ) + Do − Di. Alternately, an expression becoming logarithmically infinite may be expressed as two finite terms with a mixed scaling;

⟨pp⟩#(0) ≡ ⟨pp⟩/ρ²u²τUο = ⟨pp⟩⁺(0)uτ/Uο = α + βuτ/Uο(Reτ)

Channel flow has an extensive overlap region while boundary layers have a very limited overlap region. The wall-pressure spectra E_{pp}(k_x h), as a function of the streamwise wavenumber k_x h, does not have a (k_x h)⁻¹ overlap region. Nevertheless, trends show this behavior might occur at higher Reynolds numbers.

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I. INTRODUCTION

In this paper, the fluctuating pressure is correlated in the spirit of matched asymptotic expansions. The inner region expression is a function of $y^{+} = yu_{\tau}/\nu$ (y distance from wall, $u_{\tau}$ friction velocity, and $\nu$ kinematic viscosity) with gauge functions that depend on the friction Reynolds number $Re_{\tau} = u_{\tau}h/\nu$ ($h$ channel half-width). Similarly, the outer region expression is a function of $y/h$ with gauge functions that depend on the Reynolds number. Matching these expressions in an overlap region produces a ‘common part’ matching law.

Fluctuating pressures are an interesting property in turbulent wall layers. However, experimental measurements, either within the flow or on the wall, are hampered by instrument errors, facility flow unsteadiness, and contamination from acoustic noise. Much research on pressure fluctuations in boundary layers has been motivated by practical issues on rockets, airplanes, submarines, and ships. Sound reception and radiation, laser transmission, and wall vibration are important issues.

Many papers review the literature on pressure fluctuations. Particularly comprehensive and useful are the papers by Farabee and Casarella\textsuperscript{1}, Tsuji \textit{et al.}\textsuperscript{2}, and Klewickett, Priyadarshana, and Metzger\textsuperscript{3}. Additionally, some recent experiments are by Gravante \textit{et al.}\textsuperscript{4}, and modeling by Aupoix\textsuperscript{5}.

Most of the data used in this paper comes from the direct numerical simulations (DNS) of channel flow by Lee and Moser\textsuperscript{6}. This work offers data from flows with Reynolds number up to $Re_{\tau} \approx 5200$, and including $Re_{\tau} \approx 180, 550, 1000, \text{ and } 2000$. Data from the lower Reynolds number cases are consistent with previous DNS studied by Kim, Moin, and Moser\textsuperscript{7}, Moser, Kim, and Mansour\textsuperscript{8}, del Álamo \textit{et al.}\textsuperscript{9}, Hoyas and Jiménez\textsuperscript{10}. Also, there is general agreement with a previous paper specifically on pressure fluctuations by Hu, Morfey, and Sandham\textsuperscript{11} which has DNS up to $Re_{\tau} = 1400$. Data for boundary layers used in this paper has been presented by Schlatter and Örlü\textsuperscript{12}.

The numerical scheme used by Lee and Moser\textsuperscript{6} computes the velocities and vorticities without involving the pressure. Subsequently, the pressure field is found by solving a Poisson equation given the velocity fields.

$$\frac{\partial^{2}p}{\partial x_{k} \partial x_{k}} = -\rho \frac{\partial^{2}u_{i}u_{j}}{\partial x_{i} \partial x_{j}}$$  \hspace{1cm} (1)

A Neumann boundary condition is obtained from the Navier-Stokes equation in the wall-
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normal direction by applying no-slip and no-penetration conditions.

\[
\frac{\partial p}{\partial y} \text{(wall)} = \frac{1}{Re} \frac{\partial^2 v}{\partial y^2} \tag{2}
\]

The spectral Galerkin method in streamwise and spanwise direction and seventh order B-spline is used in the wall-normal direction to solve (1).

Since the fluctuation pressure \( p \) is related to the known fluctuation velocities \( u_i \) by linear equations, one can arbitrarily divide the pressure into three parts;

\[
p = p_r + p_s + p_{st} \tag{3}
\]

Here; \( p_r \) is a “rapid” pressure, also known as the turbulence-mean shear component, \( p_s \) is a “slow” pressure, also known as the turbulence-turbulence component, and \( p_{st} \) is a “Stokes” pressure, a harmonic component. Equations governing these pressures are (Mansour, Kim, and Moin\textsuperscript{13}):

\[
\frac{\partial^2 p_r}{\partial x^2} + \frac{\partial^2 p_r}{\partial y^2} + \frac{\partial^2 p_r}{\partial z^2} = -2\rho \frac{\partial v}{\partial x} \frac{\partial U}{\partial y}, \quad \frac{\partial p_r}{\partial y} \text{(wall)} = 0 \tag{4}
\]

\[
\frac{\partial^2 p_s}{\partial x^2} + \frac{\partial^2 p_s}{\partial y^2} + \frac{\partial^2 p_s}{\partial z^2} = -\rho \frac{\partial u_i}{\partial x} \frac{\partial u_j}{\partial x_j}, \quad \frac{\partial p_s}{\partial y} \text{(wall)} = 0 \tag{5}
\]

\[
\frac{\partial^2 p_{st}}{\partial x^2} + \frac{\partial^2 p_{st}}{\partial y^2} + \frac{\partial^2 p_{st}}{\partial z^2} = 0, \quad \frac{\partial p_{st}}{\partial y} \text{(wall)} = \frac{1}{Re} \frac{\partial^2 v}{\partial y^2} \tag{6}
\]

If desired one can combine the harmonic component with either of the other pressures.

One of the first analysis of DNS pressure data was by Kim\textsuperscript{14}. Although the Reynolds number was very low, \( Re_\tau = 180 \), the trends he described are consistent with current data. Importantly, he found that both the rapid and slow pressures are significant. At that time, the accepted argument was that only the rapid pressure was significant. Hoyas and Jiménez\textsuperscript{10} verified that the Stokes pressure is never significant. From the boundary condition (6) it is seen that the Stokes pressure is zero at infinite Reynolds number.

The rapid pressure depends on the mean velocity and vertical fluctuating velocity. These velocities scale well with the friction velocity \( u_\tau \). However, the situation is not so simple with the slow pressure as it depends on all the fluctuating velocities. There has been considerable uncertainty about scaling the streamwise velocity \( u \). DeGraaff and Eaton\textsuperscript{15} made boundary layer measurements and found that \( \langle uu \rangle \) scaled with the mixed velocity \( u_\tau U_o \). Atmospheric measurements at very high Reynolds numbers by Klewicki, Priyadarshana, and Metzger\textsuperscript{3}
support this. Marusic and Kunkel\textsuperscript{16} produced a curve fit to the data using mixed scaling, and patching the inner and outer expressions; rather than employing a matching law as is customary in matched asymptotic expansions. A two-term asymptotic expansion, the first term scaling with $u_\tau U_o$ and the second with $u_\tau^2$, was proposed by Panton\textsuperscript{17}. Some current ideas on the scaling of $\langle uu \rangle$ are by Monkewitz and Nagib\textsuperscript{18}. With these considerations, one does not expect a simple scaling of the slow pressure in the very near wall region.

Bradshaw\textsuperscript{19} proposed that the wall-pressure spectrum $E_{(pp)}^+$, in terms of streamwise wavenumber, has two regions with different scaling; for low and high wavenumbers respectively. The low wavenumbers are impressed from the outer region and have a length scale $h$, whereas the length scale for the high wavenumbers is the viscous scale $\nu/u_\tau$. The matching law between these regions has a slope of minus one. We can derive this as follows. Assume that the proper non-dimensional spectrum at low wavenumber $E_{(pp)}^+(k_x h)$ is scaled by $u_\tau$ and $h$. At high wavenumbers the inner region dominates the spectrum and the proper length scale is the viscous scale $\nu/u_\tau$. There is a scale change for the dependent quantity $E_{(pp)}^+(k_x^+)$ as well as for the wavenumber $k_x^+=k_x h/Re_\tau$. If the Reynolds number is high enough for these regions to separate, then the matching law occurs. Equating the functions and their slopes produces the matching laws $C/(k_x h)$ and $C/k_x^+$ respectively. Furthermore, the mean-square wall pressure is the integral of $E(k_x h)$. At some high wavenumber $k_{x,0}^+$ the spectrum drops rapidly and no longer contributes to the integral. Hence, the upper limit of integration is effectively $(k_x h)_0=k_{x,0}^+Re_\tau$ and the resulting mean square pressure has the form

$$\langle pp \rangle^+(0) = A \ln Re_\tau + B$$

The origin of the logarithmic term, in this instance, is the fact that the overlap region of the spectrum grows linearly with the logarithm of the Reynolds number. This was also proposed by Bradshaw\textsuperscript{19}. Regardless of the theoretical origin, it is common to correlate the wall pressure with this logarithmic form. Klewicki, Priyadarshana, and Metzger\textsuperscript{3} present a comprehensive review of the experimental evidence for the $k_x^{-1}$ behavior (as well as for the vorticity-pressure correction).

Also, several other turbulence properties, for example the fluctuating wall shear stress, have been expressed with a logarithmic dependence on $Re_\tau$. Many researchers prefer to start an asymptotic expansion with an order-one term. In other words, the dependent quantity is properly scaled to be finite in the parameter limit. This can be done as follows. Consider
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a relation for some dependent variable \( f \) of the form

\[
    f = A \ln \text{Re}_\tau + B. \quad (8)
\]

Recall that matching the mean-velocity profiles produces the wall shear-stress relation

\[
    \frac{U_o}{u_\tau} = \frac{1}{\kappa} \ln \text{Re}_\tau + C_{io} \quad (9)
\]

Eliminating the logarithmic term between these equations, and then rescaling the dependent quantity by the gauge function \( \frac{u_\tau}{U_o}(\text{Re}_\tau) \) gives a two-term expansion. For the pressure this is

\[
    \langle pp \rangle^#(0) \equiv \frac{\langle pp \rangle}{\rho^2 u_\tau^3 U_o} = \langle pp \rangle^+(0) \frac{u_\tau}{U_o} = \alpha + \beta \frac{u_\tau}{U_o}(\text{Re}_\tau)
\]

here

\[
    \alpha = \kappa A \text{ and } \beta = (B - \kappa AC_{io}) \quad (10)
\]

The rescaling makes \( \langle pp \rangle^# \) finite as \( \text{Re}_\tau \to \infty \) and introduces the mixed velocity \( \sqrt{u_\tau U_o} \) into the dependent scale. Thus, one has a choice of either of these two equivalent representations. One a hypothetical value at \( \text{Re}_\tau=1 \) plus an addition for finite Reynolds number. It grows without bound as the Reynolds number increases. The other representation has a finite value for infinite Reynolds number, minus a correction for finite Reynolds number.

The first section of this paper is an analysis of data from channel flow DNS calculations. The primary source is Lee and Moser\(^6\). The second section deals with constant pressure boundary layers and employs the DNS calculations of Schlatter and Örlü\(^12\). Summary and conclusions comprise the final section.

II. CHANNEL FLOW

Channel flow DNS data now extend to \( \text{Re}_\tau = 5186 \). Fig. 1a shows the mean square pressure in the inner variable \( y^+ \) with curves for each \( \text{Re}_\tau \). The overall level rises as the Reynolds number increases. For any Re curve, the level rises as one moves away from the wall, peaking at about \( y^+ = 30 \) before dropping off in a logarithmic decline. Pressure profiles in the outer variable \( y/h \) are in Fig. 1b. Here, all the curves for different Reynolds numbers collapse together quite well away from the wall. The logarithmic matching law (common part denoted by subscripts \( cp \)), \( \langle pp \rangle^+_cp \) that fits in the overlap region is also shown in Fig. 1b.
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The constants were determined by fitting the \( Re_\tau = 5186 \) data at \( y^+ = 200 \) and 1000.

\[
\langle pp \rangle^+(y/h) = \frac{1}{\epsilon} \ln(y/h) + D_o; \quad \frac{1}{\epsilon} = -2.5625, \ D_o = 0.2703
\]

The outer function \( \langle pp \rangle^+(y/h) \) deviates from this common part, beginning about \( y/h = 0.8 \), as it flattens to zero slope to match with the other side of the channel.

Next, consider the inner function. In the overlap region, it must produce Eq (11) in the inner variable. Thus, to within a constant, the inner overlap law for \( \langle pp \rangle^+ \) is

\[
\langle pp \rangle^+(y^+ \to \infty) = \frac{1}{\epsilon} \ln(y^+) - \frac{1}{\epsilon} \ln(Re_\tau) + D_o
\]

In light of Eq(12) it is useful to define a function that absorbs the \( Re_\tau \) dependence in \( \langle pp \rangle^+ \).

\[
\phi(y^+) \equiv \langle pp \rangle^+(y^+, Re_\tau) + \frac{1}{\epsilon} \ln(Re_\tau) - D_o
\]

Note that Eqs. (12) and (13) imply that

\[
\phi(y^+ \to \infty) = \frac{1}{\epsilon} \ln(y^+)
\]

A perfect correlation will produce a single curve for \( \phi(y^+) \). Fig. 2a displays \( \phi(y^+) \) for all \( Re_\tau \). There is a good correlation for all \( y^+ \) except for some deviations near the wall; \( y^+ < 30 \).

Shift the origin by defining the inner pressure function \( P(y^+) \)

\[
P(y^+) \equiv \phi(y^+) - \phi(0)
\]
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As an average value at the wall we take \( \phi(0) = -12 \). Observe that \( \mathcal{P}(y^+) \) has a defect form and by definition \( \mathcal{P}(0) = 0 \). In terms of the pressure fluctuations \( \mathcal{P}(y^+) \) is

\[
\mathcal{P}(y^+) = (pp)^+(y^+, Re_\tau) - (pp)^+(0, Re_\tau)
\]

(16)

Fig. 2b displays values for \( \mathcal{P}(y^+) \). Observe that the common part begins about \( y^+ = 200 \).

It will be shown that as a condition of matching the inner and outer functions, the wall pressure should have a logarithmic dependence on \( Re_\tau \) with a slope \( 1/\epsilon \). The data for the wall pressure and the slope of the overlap region do not meet this requirement exactly. By setting \( \phi(0) = -12 \) the overlap requirement is met, but the wall pressures do not correlate with exactly the correct slope.

Because of the way in which it is constructed, \( \mathcal{P}(y^+) \) approaches the same logarithmic curve for all Reynolds numbers. If the curves had been forced to agree at the wall, then they would approach slightly different logarithmic lines at infinity. Since the curves collapse exactly in the outer variable, it is appropriate to continue the good agreement into the inner region. From a physical standpoint this is consistent with the fact that complicated events, summarized by Hoyas and Jiménez\(^{10}\), occur in this inner region. A more accurate inner function might be formed by adding a higher-order term to \( \mathcal{P}(y^+) \) which modifies the near-wall behavior. If such a term dies out exponentially in \( y^+ \), it will not enter matching with the outer function. It will only modify the near-wall region.
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![Graph showing correlation of pressure fluctuations](image)

FIG. 3. Indicator function for logarithmic common part

The inner matching law is

\[ P_{cp}(y^+) = \frac{1}{\epsilon} \ln(y^+) + D_i; \quad D_i = 12 \]  \hfill (17)

Matching between the inner and outer regions satisfies equations for the values,

\[ P_{cp}(y^+) + \langle pp \rangle^+(0, Re_\tau) = \langle pp \rangle^+_{cp} \left( y/h \rightarrow \frac{y^+}{Re_\tau} \right) \]  \hfill (18)

and for the derivatives,

\[ y^+ \frac{dP_{cp}}{dy^+} = \left( y/h \right) \frac{d\langle pp \rangle^+_{cp}}{d(y/h)} = \frac{1}{\epsilon} \]  \hfill (19)

where \( \epsilon \) is a constant. The traditional indicator function for \( 1/\epsilon \) is shown in Fig. 3. An interesting side issue is that numerically \( 1/\epsilon \) and \( -1/\kappa \) (\( \kappa \) is the von Kármán constant determined from the computed velocity profiles) are nearly the same. The dashed line on Fig.4 corresponds to \( \kappa = 0.384 \). We do not know of any theoretical reason for these coefficients to be equal. The physics governing the mean velocity and the pressure fluctuations is quite different.

The methodology described above was applied to the rapid and slow pressure components. The results for \( \phi(y^+) \) for the rapid and slow components are shown in Fig. 4. Away from the wall, the curves approach a logarithmic dependence with different slopes, and near the wall there are some trends with \( Re_\tau \), especially prominent for the slow component. The slow pressure (turbulence-turbulence term) has more variation in \( y \) than the rapid pressure (turbulence-mean-shear term) and is mainly responsible for the maximum around \( y^+ = 40 \).
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FIG. 4. Inner correlation for rapid and slow pressure components; (a) rapid pressure component, \( \phi_r = \langle pp \rangle_r^+ - 0.63592 \ln(Re_\tau) - 0.068265 \); (b) slow pressure component, \( \phi_s = \langle pp \rangle_s^+ - 1.7260 \ln(Re_\tau) - 0.24166 \)

One can also observe that the trends with \( Re_\tau \) near the wall are opposite for the two components.

The matching laws for the inner and outer regions are (11) and (17) respectively. Eliminating \( \langle pp \rangle^+ \) between these equations yields

\[
\langle pp \rangle^+(0) = -\frac{1}{\epsilon} \ln(Re_\tau) + D_o - D_i
\]

Thus, the overlap logarithmic laws produce an expression for the wall pressure as a function of the Reynolds number. This is another derivation of a logarithmic law for the wall pressure distinct from that of Bradshaw. Fig. 5a shows various recommended relations for the wall pressure variance along with the equations and DNS data. The channel DNS data of Lee and Moser are given as points, while a line is given for (20) (where constants from the overlap law were used). Constants are listed in table I. Although channel flow and boundary layer data are given on the same chart, one should not expect them to have the same levels. Boundary layer data will be discussed subsequently. The alternate representation of the DNS data in mixed scaling, that is in the form of (10), is displayed on Fig. 5b.

The spectrum \( E_{\langle pp \rangle^+}(k_x h) \) for the wall pressure at low wavenumbers is given in Fig. 6a. The computational box length, \( 8 \pi h \), corresponds to a wavenumber of \( k_x h = 0.25 \). The higher
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**FIG. 5.** Wall pressure for various flows (a) in inner scaling (b) in mixed scaling

**TABLE I.** Parameters in log-law for wall-pressure; \( \langle pp \rangle^+(0) = A \ln(Re_\tau) + B \)

<table>
<thead>
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<th>Type</th>
<th>Detail</th>
<th>( A )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel</td>
<td>DNS (Lee and Moser(^6), Datafit)</td>
<td>2.24</td>
<td>-9.18</td>
</tr>
<tr>
<td>Channel</td>
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<td>2.56</td>
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</tr>
<tr>
<td>Channel</td>
<td>DNS (Hu, Morfey, and Sandham(^11))</td>
<td>2.60</td>
<td>-11.25</td>
</tr>
<tr>
<td>ZPGBL</td>
<td>Exp. (Farabee and Casarella(^1))</td>
<td>1.86</td>
<td>-4.30</td>
</tr>
<tr>
<td>ZPGBL</td>
<td>Exp. (Klewicki, Priyadarshana, and Metzger(^3))</td>
<td>2.30</td>
<td>-6.86</td>
</tr>
<tr>
<td>ZPGBL</td>
<td>DNS (Schlatter and Örlü(^12))</td>
<td>2.42</td>
<td>-8.96</td>
</tr>
</tbody>
</table>

\( Re_\tau \) curves correlate roughly with a peak at \( k_x h = 2 \) to 3. This corresponds approximately to a wavelength of \( \lambda_x h = 3 \) to 2. The high wavenumber spectra \( E_{(pp)}^+(k_x^+) \) at different Reynolds numbers collapse when plotted against \( k^+ \), as shown in Fig. 6b. They all begin to drop rapidly at about \( k_x^+ = 0.02 \) to 0.03; \( \lambda^+ = 200 \) to 300. As the Reynolds number increases, the low wavenumber spectra increase, and a power-law dependence develops at intermediate wavenumbers (linear in the log-log plot of Fig. 6b). However, the slope does not become the theoretical minus one value for the Reynolds numbers shown here. The spectrum chart, Fig. 7a, displays the pre-multiplied values \( k_x^+ E_{(pp)}^+(k_x^+) \). As is well known,
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FIG. 6. Wall-pressure spectrum

when the abscissa $k_x^+$ is logarithmic and the ordinate $k_x^+ E^+(pp)_+$ is linear, the physical area below a curve section indicates the contribution of that section to the mean square $\langle pp \rangle^+$. The peak of the contribution occurs around $k_x^+ = 0.02$ to 0.03; $\lambda^+ = 200$ to 300. If the ordinate on this graph were the mixed scale variable $E^#(pp)_+ = E^+(pp)_+ u_\tau / U_0$ as shown in Fig. 7b, the level of the peaks would collapse better than for $E^+(pp)_+$. Nevertheless, the most important aspect of Fig. 7 is the curve for $Re_\tau = 5186$. This curve shows contributions from the low wavenumbers beginning to separate from the high wavenumber correlation region. A complete separation with a horizontal line on this graph would validate Bradshaw’s argument about separation of high and low wavenumber events.

Pre-multiplied spectra at $Re_\tau = 5186$ for various positions from the wall are given in Fig. 8. The wall curve is the same as shown on Fig. 7a. The curve for $y^+ = 30$ is where the largest value of $\langle pp \rangle^+$ occurs, and it peaks at the same wavenumber as does the wall value. This near-wall region is where the largest values of velocity fluctuations occur. As one moves away from the wall, the high wavenumber contribution vanishes and the peaks move progressively to lower wavenumbers as the distance from the wall increase. The spectral peak at the centerline is at $k_x^+ = 0.0009$ corresponding to $\lambda_x/h = 1.34$. It is interesting to note that all curves closer to the wall than $y/h = 0.2$ lie together when the wavenumber is less than $k_x^+ = 0.001$; that is wavelength greater than $\lambda_x/h = 1.2$. This is consistent with
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FIG. 7. Premultiplied wall-pressure spectrum

FIG. 8. Premultiplied pressure spectrum for various $y$ at $Re_T = 5186$

the concept that the influence of small scale fluctuations is confined near the wall, while large scale fluctuations in the outer region impose their pressure on the wall.
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FIG. 9. Pressure fluctuation profiles of ZPGBL from Schlatter and Örlü\textsuperscript{12}; (a) in inner distance variable, $y^+$ (b) in outer variable, $y/\delta$ (c) inner correlation function, $\mathcal{P} = \phi(y^+) - \phi(0)$

FIG. 10. Pressure fluctuation profiles of ZPGBL from Schlatter and Örlü\textsuperscript{12}; inner correlation function, $\mathcal{P} = \phi(y^+) - \phi(0)$

III. ZERO PRESSURE GRADIENT BOUNDARY LAYER

The data used in this section is from Schlatter and Örlü\textsuperscript{12}. Pressure profiles from progressive stations in a developing boundary layer are presented in Fig. 9a in inner variables.
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The corresponding wall values are given as a function of Reynolds number in Fig. 5a. A fit of (20) to this data gives

$$\langle pp \rangle^+(0) = 2.4161 \ln(Re_\tau) - 8.964$$  \hspace{1cm} (21)

Fig. 5a also has the recommended equations of Farabee and Casarella\(^1\), who reviewed experimental data and of Klewicki, Priyadarshana, and Metzger\(^3\), who added very high Reynolds number data. The Reynolds number for boundary layers is based on the overall thickness \(\delta\) while channel flow data uses \(h\). These are not exactly comparable so one should not expect channel and boundary layer curves for \(\langle pp \rangle^+(0)\) to correlate.

The first observation is that the high Reynolds number DNS profiles do not have an obvious logarithmic region. Since it was difficult to determine an outer overlap law from the profiles, another method was devised. The slope \(1/\epsilon = -2.4161\) was taken from the wall pressure relation above and the constant \(D_o = 2.3866\) was found by evaluating the \(Re_\tau = 1300\) profile at \(Y = 0.15, y^+ = 190\). This is approximately the location of the beginning of the logarithmic region in channel flow. The outer overlap law is then

$$\langle pp \rangle^+(Y \to 0) = \frac{1}{\epsilon} \ln(y/\delta) + D_o; \quad \frac{1}{\epsilon} = -2.4161, \ D_o = 2.3866$$  \hspace{1cm} (22)

This curve is given on figs. 9b.

Fig. 9b displays the data in the outer variable \(y/\delta\). There is a good correlation when the Reynolds number is greater than 900. The outer function rises above the log overlap
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law somewhat before \( y/\delta = 0.2 \), decreases and crosses the log law at about \( y/\delta = 0.5 \) as it drops toward zero about \( y/\delta = 1.4 \). This behavior is also seen on Fig. 11 where only the \( Re_\tau = 1300 \) curve is shown.

Processing the data in the same manner as for the channel gives the inner pressure function \( P(y^+) \) presented as Fig. 10. The correlation in the boundary layer, especially near the wall, is better than for the channel. The final chart, Fig. 11, compares the inner pressure function for the channel \( (Re_\tau = 5186) \) and the boundary layer \( (Re_\tau = 1271) \). They are essentially the same in the inner wall layer, but completely different in the outer layer.

IV. SUMMARY AND CONCLUSION

In channel flow, outer region profiles of the fluctuating pressure variance, \( \langle pp \rangle^+(y/h) \), correlate very well for different Reynolds numbers. In the inner wall region, the correlation is a defect form \( P = \langle pp \rangle^+(y^+, Re_\tau) - \langle pp \rangle^+(0, Re_\tau) \). Matching these expressions produces logarithmic overlap laws:

\[
\langle pp \rangle^+(y/h) = \frac{1}{\epsilon} \ln(y/h) + D_o \quad (23)
\]

\[
P = \frac{1}{\epsilon} \ln(y^+) + D_i \quad (24)
\]

For channel flow these laws begin at about \( y^+ = 200 \) and extend to \( y/h = 0.8 \). The inner profile \( P(y^+) \) rises from zero to a maximum at about \( y^+ = 40 \), then drops into the logarithmic overlap behavior. The correlation is not perfect at and near the wall. This is mainly due to some unaccounted for trends in the slow pressure component, which are also responsible for the rising values.

Combining these laws yields a law for the wall pressure.

\[
\langle pp \rangle^+(0) = \frac{1}{\epsilon} \ln(Re_\tau) + D_o - D_i \quad (25)
\]

This is an alternate derivation of an equation originally proposed by Bradshaw based on wavenumber spectra. The new aspect of the equation above is that both constants for the wall pressure relation are not arbitrary, but come from the logarithmic overlap laws.

It is of interest to contrast the mean velocity and pressure profiles. The inner region velocity expansion has one term and it is matched to a two-term defect form in the outer expansion. The reference term in the defect is \( U_o/u_\tau \), and the matching produces a relation,
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\((U_o/u_\tau) = (1/\kappa) \ln(Re_\tau) + C_{io}\). For the pressure profile, it is the opposite; the inner expansion has two terms and the outer expansion one term. The inner reference term is the wall pressure \((pp)^+(0)\) and the matching produces the Reynolds number relation above.

The one-dimensional streamwise wavenumber spectra \(E_{pp}(k_x h)\) correlates across Reynolds numbers at a maximum of about \(\lambda_x = 2h - 3h\). These low wavenumbers are most important for generating surface vibration and acoustic sound.\(^{20}\) High wavenumbers are emphasized by the viscous scaling \(E^+(k_x^+)\). The data for different Reynolds numbers correlate and drop off sharply about \(k_x^+ = 0.03; \lambda^+ = 200\). In the mid-range of wavenumbers the slope increases as the Reynolds number increases, but does not become the theoretical minus-one value.

Contribution to the mean square \((pp)^+\) are displayed geometrically by the pre-multiplied spectra \(k_x^+ E_{pp}^+(k_x^+)\) which peaks around \(k_x^+ = 0.02 - 0.03; \lambda^+ = 300 - 200\). In this regard note that Klewicki, Priyadarshana, and Metzger\(^3\) found that a peak in pressure gradient fluctuations occurred at a convection length of \(\lambda^+ = 285\).

The pre-multiplied spectrum for \(Re_\tau = 5186\) begins to show a separation between low and high wavenumber events. This is consistent with the results of Lee and Moser\(^6\) who found a similar separation beginning to appear in the fluctuating velocity spectra. In a plot of the pre-multiplied spectrum, a plateau would indicate a \(k^{-1}\) region, indicative of scale separation.

Correlating the data for zero-pressure-gradient boundary layers shows marked differences from the channel flow. The outer region behavior is, as one would anticipate, quite different from that of the channel. The profiles of \((pp)^+(y)\) do not reveal a logarithmic region. Nevertheless, processing the data in the same manner as was done for the channel gives a curve in the inner region that match the channel curve up to \(y^+ = 150\). One should recall that the ZPGBL data extends to only \(Re_\tau = 1271\). Trends indicate that a logarithmic region is likely at higher Reynolds numbers.

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