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Large-scale flow and Reynolds numbers in the presence of boiling in locally-heated turbulent convection

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We report on an experimental study of the large-scale flow (LSF) and of Reynolds numbers in turbulent convection in a cylindrical sample with height equal to its diameter and heated locally around the center of its bottom plate (locally heated convection or LHC). The sample size and shape were the same as those of Narezo Guzman *et al.* [J. Fluid Mech. **787**, 331 (2015); **795**, 60 (2016)] (NG). Measurements were made at a nearly constant Rayleigh number as a function of the mean temperature, both in the presence of controlled boiling (2-phase flow) and for the superheated fluid (1-phase flow). Superheat values $T_b - T_{on}$ up to about 11 K (T_b is the bottom-plate temperature and T_{on} is the lowest T_b at which boiling is observed) were used. The LSF was less organized than it is in (uniformly heated) Rayleigh-Bénard convection (RBC) where it takes the form of a single convection roll. LSF-induced sinusoidal azimuthal temperature variations (such as are found for RBC) could be detected only in the lower portion of the sample, indicating a less organized flow in the upper portions. Reynolds numbers were determined using the elliptic model (EM) of He and Zhang [Phys. Rev. E **73**, 055303(R)]. We found that for our system the EM is applicable over a wide range of space and time displacements, as long as these displacements are within the inertial range of the temporal and spatial spectrum. At three locations in the sample the results showed that the vertical mean-flow velocity-component is reduced while the fluctuation velocity is enhanced by the bubbles of the 2-phase flow. Enhancements of velocity fluctuations up to about 60% were found at the largest superheat values. Local temperature measurements within the sample revealed temperature oscillations which also were used to determine a Reynolds number. These results were generally consistent with the mean-flow EM results, and showed a 2-phase-flow enhancement of up to about 30%.

I. INTRODUCTION

Boiling usually involves the process of heterogeneous vapor-bubble nucleation, where the bubble formation in a heated liquid is initiated by microscopic foreign objects such as suspended dust particles or by chemical or geometric inhomogeneities at the heated surface. Here we are concerned with boiling in a liquid contained between two parallel horizontal plates, with the lower one heated to a temperature T_b above a characteristic temperature T_{on} where vapor-bubble formation first starts when nucleation centers are present. The upper surface was cooled to T_t well below T_{on}. In the absence of boiling this system is well known as Rayleigh-Bénard convection (RBC) where the fluid undergoes vigorous turbulent single-phase flow (for a general introduction to RBC, see *e.g.* [1] or [2]; for more detailed reviews see *e.g.* [3–5]).

In RBC a large-scale circulation (LSC) (see *e.g.* [6–10]) co-exists with intense smaller-scale fluctuations of the temperature and the velocity. In the case of samples with nearly equal horizontal and vertical size the LSC takes the form of a single convection roll, with up-flow and down-flow on opposite sides of the sample near the side wall. The dynamics of the LSC is driven by the smaller-scale fluctuations, and thus the LSC amplitude and azimuthal orientation θ_0 vary irregularly in time [11–15]. The time-averaged temperature in the interior (bulk) of the fluid varies only weakly with vertical and horizontal position [16–22], and two boundary layers (BLs), one below the top and the other above the bottom plate, sustain most of the applied temperature difference [23–26]. Hot thermal plumes emanate from the bottom and cold ones emerge from the top BL. They are carried by, and by virtue of their buoyancy in turn drive, the LSC [3, 4, 27].

In a recent study it was found that the excess heat transport due to the formation of liquid droplets ("rain") under a solid top plate confining a RBC sample of vapor was very reproducible [28]. For this and other reasons the authors concluded that the droplet formation occurred *via* homogeneous nucleation in the top BL and did not involve nucleation centers on the top surface *per se*. In contrast, the same study showed that boiling over the heated bottom plate of a RBC sample of liquid was very *irreproducible* and initiated by heterogeneous nucleation. The irreproducibility is attributable to the uncontrolled microscopic nature of the roughness of the macroscopically smooth top surface of the bottom plate. The irreproducibility of heterogeneous nucleation interferes with a systematic, quantitative study of the boiling process.

The irreproducibility problem mentioned above was overcome recently by Narezo-Guzman *et al.* [29, 30] (to be referred to as NG). The top surface of the bottom plate of their sample was that of a thin silicon wafer. Etched into the wafer was a triangular lattice of cylindrical holes that were 100 μm deep and had a diameter of 30 μm. The holes, when filled with vapor while the sample above them was almost entirely liquid, acted as reproducible nucleation centers of vapor bubbles. When the holes were filled with liquid, no nucleation took place and the super-heated single-phase RBC flow could be studied for comparison with the same externally applied parameters such as the excess bottom-plate temperature $T_b - T_{on}$ (known as the "superheat") and the temperature difference $\Delta T = T_b - T_t$ across the sample.

Initial experiments by NG [29] with a silicon wafer supported and heated over its entire area by a copper plate revealed that nucleation took place also at the corner where the sidewall meets the bottom plate (see Fig. 1 below). For this reason the authors then used a composite bottom plate. It consisted of a 10 cm diameter silicon wafer, but only the central circular area of 2.54 cm diameter was supported by a copper anvil where heat was applied to the sample. This system differs significantly from RBC where the entire bottom plate is heated, and we shall refer to it as locally-heated convection (LHC). Since only the center of the bottom plate is heated, the corresponding conductive heat flux of the LHC cell is different from that of the classical RBC system [29, 30]. The lattice of nucleation centers extended only over the heated central area. The remainder of the wafer was supported on its underside by a plastic disc that reached from the anvil to the sidewall. The low conductivity of the plastic allowed a horizontal temperature gradient to develop in the bottom plate. Thus, the junction of the bottom plate and the sidewall was well below T_{on} and did not lead to nucleation even when the 2.54 cm diameter central area was well above T_{on}. The high conductivity of the supporting copper anvil allowed nucleation at a controlled nearly uniform temperature. The excess heat transport [29] due to boiling was measured as a function of the superheat and of the nucleation-center lattice-spacing, and many aspects of the bubble nucleation and dynamics [30] were studied using high-speed videography.

One of the conclusions of NG [30] was that the large-scale flow (LSF) in LHC differs in a major way from that in RBC, and that it is significantly enhanced by the buoyancy of the rising bubbles. Much of the heat-transport enhancement due to boiling was found to be due to this enhanced flow with only a relatively small fraction due to the latent heat carried by the bubbles. In the present paper we report on a study in the presence and absence of boiling for the same sample geometry as that used by NG. However, our sample contained a much larger number of local temperature probes that enabled us to obtain information about the LSF structure and about the local mean-flow velocity U (see Eq. 5 below) and fluctuation velocity V (Eq. 7 below) and their corresponding Reynolds numbers (Eqs. 6 and 8 below).

The system parameters and experimental details are provided in Sect. II and Sect. III respectively.

Local temperature measurements were made at ten locations within the bulk of the sample (see Table I). The corresponding temperature auto- and cross-correlation functions were used to obtain local values of U and V using the predictions of the elliptic approximation (EA) of He and Zhang [31] (for a recent review see Ref. [32]) and the assumption that temperature behaves as a passive scalar [33, 34] and exhibits self-similarity scaling in the inertial range [35]. We discuss the application of the EA results to our data, and summarize the EA predictions [31, 36–38] relevant to our work in Sect. IV.

We carried out heat-transport measurements (Sect. V A) the results of which largely agreed with those of NG and thus showed that the insertion of local temperature probes into the sample did not significantly alter this global property. Results for the LSF reported in Sect. V B indicate that the flow structure for LHC is more disorganized than it is for RBC, and that this structure is significantly influenced by the vapor-bubble injection into the bulk. The results for the Reynolds numbers Re_U and Re_V corresponding to U and V are given in Sect. V C. Several other Reynolds numbers can be defined based on oscillation frequencies observed within the sample. The results for them are given in Sect. V D 1 and V D 2.

The paper concludes with a discussion and summary of the most important results.

II. CONTROL AND RESPONSE PARAMETERS

We studied convection in cylindrical cells of diameter D and height L. The horizontal bottom plate was heated to a temperature T_b and the parallel top plate was cooled to a temperature T_t. When $\Delta T = T_b - T_t$ is not too large, fluid properties can be assumed to be constant and the Oberbeck-Boussinesq approximation applies [39, 40]. In that case the Rayleigh number

$$\mathrm{Ra} = \frac{\alpha g \Delta T L^3}{\nu \kappa} \tag{1}$$

and the Prandtl number

$$\mathrm{Pr} = \frac{\nu}{\kappa} \tag{2}$$

specify the state of the system. Here g is the gravitational acceleration, and α, κ and ν are the isobaric thermal expansion coefficient, the thermal diffusivity, and the kinematic viscosity at the center temperature. In addition, the aspect ratio

$$\Gamma \equiv D/L \tag{3}$$

specifies the geometry of the cylindrical sample.

A global response of the system to the applied temperature difference is the heat flux from the bottom to the top plate as expressed by the Nusselt number

$$\mathrm{Nu} = \frac{QL}{A \lambda \Delta T} \tag{4}$$

which relates the overall heat-current density Q/A to the purely conductive heat flux $\lambda \Delta T / L$ in the absence of convection (λ is the thermal conductivity of the fluid).

As discussed in the Introduction, in our system only a central circular area A_h smaller than A was heated while the cooling area at the top plate A_c was equal to A. In Eq. 4 we used A_h, as was done also by NG [29].

Another system response is a turbulent flow with a vertical velocity component $u(t)$ which has a mean value

$$U = \langle u(t) \rangle \tag{5}$$

($\langle ... \rangle$ denotes the time average). This mean velocity will vary with the location within the sample. In RBC where the LSC corresponds to a single convection roll it vanishes at the sample center and its absolute value reaches a maximum near (but not too near) the sidewall (see, for instance, Ref. [8]). It is described by the dimensionless Reynolds number

$$\mathrm{Re}_U = \frac{UL}{\nu}. \tag{6}$$

The local velocity, in addition to having a non-zero mean value almost everywhere, is also highly fluctuating. Thus another quantity of interest is the root-mean-square fluctuation velocity

$$V = \langle [u(t) - U]^2 \rangle^{1/2} \tag{7}$$

and the associated Reynolds number

$$\mathrm{Re}_V = \frac{VL}{\nu}. \tag{8}$$

In RBC V is much more uniform throughout the sample than is U.

Reynolds numbers also can be defined in terms of the periods or frequencies of several experimentally accessible signals. One of these is the period $\mathcal{T}^{ac}$ of oscillations in the temperature time-series measured with thermistors imbedded in the sidewall. In addition, the period $\mathcal{T}^{sl}$ of a lateral displacement of the LSC circulation plane (the sloshing mode) can be deduced from a Fourier analysis of eight azimuthally distributed sidewall temperatures. Finally, an oscillation frequency f_0 can be determined from temperature time series determined locally within the fluid. These Reynolds numbers are give by

$$\mathrm{Re}_{ac} = \frac{L^2}{\nu \mathcal{T}^{ac}}, \mathrm{Re}_{sl} = \frac{L^2}{\nu \mathcal{T}^{sl}}, \mathrm{Re}_{f_0} = \frac{L^2 f_0}{\nu}. \tag{9}$$

It is found that Re_{sl}, Re_{ac} and Re_{f_0} are equal to each other within experimental error.

We shall describe locations within the sample by the vertical distance z or z/L from the bottom plate and by the radial location

$$\xi \equiv 1 - r/R \tag{10}$$

where r is the horizontal distance from the vertical center line and $R = D/2$ is the sample radius.

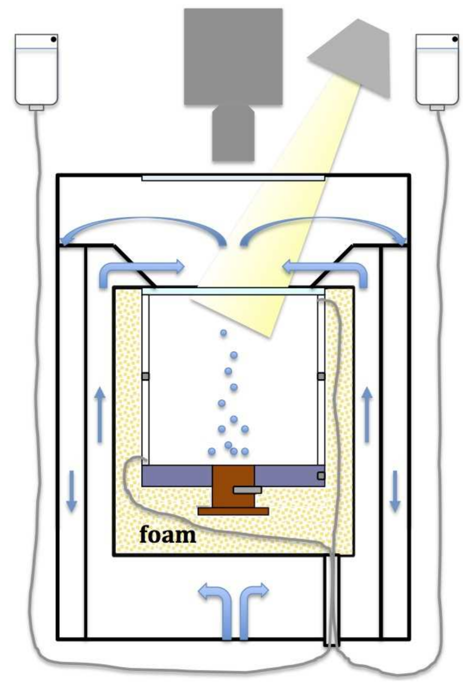

FIG. 1. Schematic drawing of the apparatus.

III. EXPERIMENTAL PROCEDURE

1. *The apparatus*

The apparatus is shown schematically in Fig. 1 and was described before by NG [29]. The cylindrical cell in which the convection took place was located in a dry can and was surrounded by air. All empty spaces within the can were filled with foam to prevent convection of the air and to thus reduce radial heat flux out of the cell. The temperature of the sapphire top plate was held constant by a circulating-water bath of temperature T_t^*. The outflow of the circulating water surrounded the can, thus providing a constant-temperature environment. A copper cylinder with a diameter $D_h = 2.54$ cm was glued to the center of the silicon bottom plate and was heated to a temperature T_b^*. Because T_t^* and T_b^* are the temperatures of the plates at the outside of the cell, a small correction was made to obtain the temperatures of the bottom plate T_b and top plate T_t in contact with the fluid (see NG [29]).

The central heated area of the bottom plate coincided with an area of the silicon wafer covered by cylindrical cavities with a diameter of 30 μm and a depth of 100 μm, arranged on a triangular lattice. The center-to-center distance between two neighboring cavities was $l = 600$ μm, resulting in a total of $N = 1570$ cavities. Bubble formation at the cavities is illustrated in Fig. 2 by a snapshot taken from above.

We used two cells (A and B) described in detail below in Sect. III 2 and III 3. They were of the same size and shape but had different arrangements of local tempera-

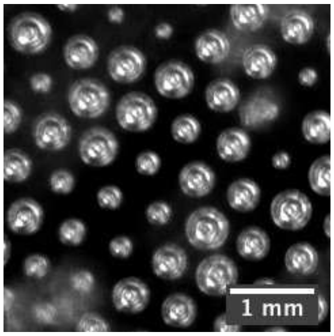

FIG. 2. Image of bubble formation at bottom-plate cavities as viewed from the top (from NG [29]). Bubbles form only at the cavities on a triangular lattice.

FIG. 3. Photograph of a type 111-104HAK-H01 (0.36 mm diameter) thermistor mounted through its 0.9 mm diameter ceramic rod. Adapted from Fig. 2 of Ref. [21].

ture probes. They had a height $L = 88.3$ mm and an aspect ratio $\Gamma = 1.00$.

The applied temperature difference was nominally constant, but actually ranged from about 15.8 K to about 16.8 K. The bottom temperature was varied from about 29.9°C to about 41.7°C so as to cover a range of superheat values. The nominally constant Rayleigh number varied from about 1.7×10^{10} to about 2.0×10^{10}.

The mean-flow Reynolds number Re_U in RBC varies with position and thus comparison between various measurements is complicated. However, in the bulk of RBC the fluctuation Reynolds number Re_V depends only weakly on position and comparison with other work is more direct. Thus we mention here that Re_V was measured for RBC over a wide range of Pr and Ra by Lam *et al.* [43]. Their data could be represented by $\mathrm{Re}_V = 0.84\mathrm{Ra}^{0.40}\mathrm{Pr}^{-0.86}$. For our typical value $\mathrm{Ra} = 1.85 \times 10^{10}$ and our Pr close to 8 this yields $\mathrm{Re}_V \simeq 1800$.

2. *Cell A*

As described elsewhere (see *e.g.* Ref. [13]), twenty four thermistors were located in blind holes in the sidewall of cell A, with each of them yielding a local temperature measurement. Groups of eight were each equally spaced azimuthally at the three heights $z/L = 0.25$, $z/L = 0.50$ and $z/L = 0.75$. These vertical locations will be identified as $k = b, m$, and t respectively. Relative to an arbitrarily chosen origin the thermistors were located at azimuthal positions $\theta_{t,i} = (2i + 1)\pi/8, i = 0, ..., 7$ in the counter-clockwise direction when viewed from above. The blind holes had a depth of 5.3 mm, which made the distance between the thermistors and the fluid as small as possible. This cell had no thermistors located in its interior.

The analysis for cell A was based on twenty four simultaneous temperature time-series obtained from the twenty four thermistors for each set of external constraints (superheat value, 1- or 2-phase flow). The time series consisted of typically 2×10^4 data points for each thermistor at time intervals of $\delta t \simeq 1.9$ s, thus covering a time span of more than 10 h.

3. *Cell B*

As for cell A, cell B had eight thermistors at the same azimuthal positions and in blind holes in the sidewall at the mid-height $z/L = 0.50$ (but not at $z/L = 0.25$ and 0.75). In addition there were ten Honeywell type 111-104HAK-H01 thermistors inside the fluid at the locations given in Table I. As shown in Fig. 3, they were mounted on ceramic rods with a diameter of 0.80 mm as described in Refs. [21, 22, 41]. We did not observe measurable effects of bubble-probe interactions. The rods were inserted through holes, also of nominally 0.80 mm diameter, in the 6.4 mm thick plexiglas side wall. While the vertical hole locations in the wall were known with high accuracy, there were two possible contributions to the uncertainties of the vertical thermistor locations. One came from any tilt of the rods relative to a line orthogonal to the cylinder axis. We believe this to be very small, probably no more than 0.2 mm on the sample axis ($\xi = 1.00$) and much less near the side wall ($\xi = 0.125$). Another possible contribution came from the suspension of the thermistors by their 0.10 mm diameter platinum leads (see Fig. 3). Because of the fragile nature of these leads the vertical position of the thermistor center is estimated to be uncertain by about 0.3 mm. Thus the total uncertainty is close to 0.5 mm. Similarly we estimate that the uncertainty of the distance δz between adjacent thermistors is about 0.7 mm. This number will be used below for the error bars in Fig. 7.

As indicated in Table II, there were three sets, each consisting of three thermistors. For each set there was a vertical distance of about 5.1 mm between the lower and middle thermistor and about 6.3 mm between the middle and upper thermistor. Thus three cross-correlation functions could be obtained from each set, as indicated in Table II. The middle thermistor of the two sets on the vertical center line (*i.e.* $\xi = 1.00$) was located at $z/L = 0.25$ and $z/L = 0.50$. The third set was located near the sidewall at $\xi = 0.125$, with its middle thermistor at $z/L = 0.50$. Finally, a single thermistor was located on the center line ($\xi = 1.00$) at $z/L = 0.75$.

For cell B the time interval of consecutive temperature measurements was $\delta t = 0.06$ s. In that case a single data segment could gather 50000 data points per ther-

TABLE I. The identifiers ID, and the radial and vertical locations, of the internal thermistors in cell B. The sample height was $L = 88.3$ mm and the radius was $R = 44.13$ mm. The angle θ_t (in rad) is the azimuthal location, measured in a counter-clockwise direction when viewed from above, of the insertion of the thermistor.

ID	$R - r$ (mm)	ξ	z (mm)	z/L	θ_t
V0-1	44.15	1.00	17.0	0.192	0
V0-2	44.15	1.00	22.1	0.250	0
V0-3	44.15	1.00	28.4	0.322	0
V0-4	44.15	1.00	39.1	0.443	0
V0-5	44.15	1.00	44.1	0.500	0
V0-6	44.15	1.00	50.5	0.572	0
V1-4	5.52	0.125	39.1	0.443	π
V1-5	5.52	0.125	44.1	0.500	π
V1-6	5.52	0.125	50.5	0.570	π
V0-7	44.15	1.00	66.2	0.750	π

TABLE II. The identifiers ID_1 and ID_2 of thermistor pairs used to evaluate the time intervals τ_p and τ_d. The spacings between the members of the pairs are δz, and the mean vertical locations of the pairs are $\bar{z}/L$.

Set	ID_1	ID_2	ξ	δz (mm)	$\bar{z}/L$	θ_t
1	V0-1	V0-2	1.00	5.1	0.221	0
1	V0-2	V0-3	1.00	6.3	0.286	0
1	V0-1	V0-3	1.00	11.4	0.272	0
2	V0-4	V0-5	1.00	5.0	0.472	0
2	V0-5	V0-6	1.00	6.4	0.536	0
2	V0-4	V0-6	1.00	11.4	0.508	0
3	V1-4	V1-5	0.125	5.1	0.472	π
3	V1-5	V1-6	0.125	6.4	0.536	π
3	V1-4	V1-6	0.125	11.4	0.508	π

mistor, corresponding to about 50 minutes. At each superheat twenty such segments were measured, and correlation functions and spectra were calculated from each segment and averaged over all twenty.

4. Working fluid and bubble nucleation

The working fluid was Novec$^{\text{TM}}$-7000 manufactured by 3M$^{\text{TM}}$. Its physical properties are well documented [44]. It was chosen because it has the experimentally convenient relatively low boiling temperature $T_\phi \simeq 34°$C at atmospheric pressure. The fluid in the cell was connected to a reservoir placed at an elevation of 1.16 m above the bottom plate. The surface of the fluid in the reservoir was exposed to atmospheric pressure, and at the bottom plate provided a hydrostatic pressure of 16.0 ± 0.3 kPa in addition to atmospheric pressure. The properties needed to evaluate Ra, Pr, Nu, and Re were evaluated at the measured center temperature T_c (for details see [29]). The Prandtl number was close to eight for all measurements.

The filling procedure of the cell and the measurement protocol were described in detail by NG [29]. During all of our measurements the bubbles emanating from the bottom plate re-dissolved before reaching the top plate; thus no vapor layer ever formed above the liquid. Our goal was to better understand the flow profile in 1- as well as in 2-phase flow and to measure the corresponding Reynolds numbers for different values of superheat, while keeping the thermal forcing ΔT constant.

The data presented in the paper are all well-converged. However, unavoidably there are also systematic errors. For those connected with the sample-filling process, etc., the scatter of the data provides an estimate on the error of the measurements.

IV. PREDICTIONS AND TESTS OF THE ELLIPTIC APPROXIMATION

A. Range of applicability of the elliptic approximation

The EA [31] is a second-order Taylor-series expansion of the correlation function $C(\tau, \delta z)$ in space and time about $C(0, 0) = 1$. Thus it is a controlled approximation valid for sufficiently small time and space increments τ and δz. It shows that, for small τ and δz, contours of equal correlation in the $\tau - \delta z$ plane are ellipses. In the derivation of local velocities from temperature correlation functions it is assumed that temperature behaves as a passive scalar; this is the case in the bulk of RBC as shown in Refs. [33, 34]. The EA is valid in turbulent systems with strong fluctuations, such as RBC, where the Taylor frozen-flow approximation (see, *e.g.* [45]) is not applicable.

As can be seen from Figs. 4 and 6, the maximum values of the cross-correlation functions are well below unity. Thus one might expect that the EA would fail for data such as those in these figures. However, He and Zhang [31, 36] argued that the applicability of the EA extends to exceptionally large values of τ and δz because of the Kolmogorov similarity hypothesis which implies that, in the inertial range of δz and τ, the contours of equal correlation should remain elliptic with the same axis ratio and orientation as those found by the EA. The relevant predictions of the EA are determined entirely by the axis ratio and orientation; thus these predictions should be applicable over the entire inertial range even though the value of the correlation function on a given contour is well below unity and for that reason may differ significantly from that predicted by the EA. Their analysis [36] of channel-flow data from numerical simulation supported their argument. There they found that even data with a maximum value of $C(\tau, \delta z)$ as small as about 0.4

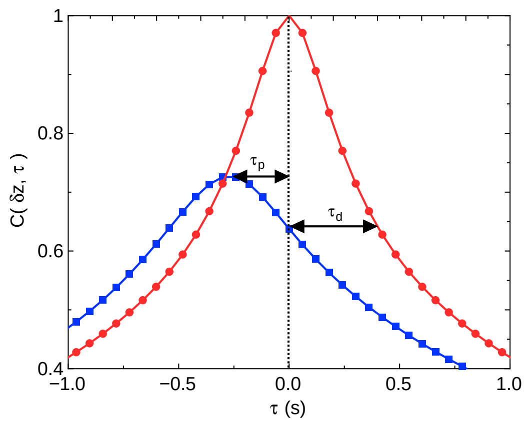

FIG. 4. The auto-correlation function (circles, red) and cross-correlation function (squares, blue) for $T_b - T_{on} = 10.6$ K and 2-phase flow of thermistors V1-5 and V1-6 ($\delta z = 6.4$ mm, see Tables I and II). The auto-correlation function is the average of those at the two locations. The time intervals $\tau_p = -0.266$ s and $\tau_d = 0.407$ s are illustrated by the arrows.

conformed to the predictions of the EA. We shall refer to the extension of the EA predictions to this extended range of space and time as the elliptic *model* (EM).

B. Predictions of the elliptic approximation

Detailed derivations and summaries of relevant EA predictions were given in Ref. [41] and the supplementary material of Ref. [42]. Here we summarize the results needed to evaluate the present measurements.

It follows from the EA that $C(\delta z, \tau)$ can be written as a function of a single variable z_E:

$$C(\delta z, \tau) = C(z_E, 0) \ . \tag{11}$$

Here z_E is the re-scaled length

$$z_E = \sqrt{(\delta z - U\tau)^2 + (V\tau)^2} \tag{12}$$

where U is a component of the mean- flow velocity (Eq. 5) and V is the root-mean-square deviation from U (Eq. 7). Further predictions of the EA are given in terms of two time intervals τ_p and τ_d that are obtained from the auto-correlation functions at two points in space (which, for a homogeneous system are equal to each other) and the corresponding cross-correlation function. Figure 4 shows an example. As illustrated, τ_p is the location along the time-delay axis of the maximum of the cross-correlation function, while τ_d is the time at which the auto-correlation functions have decayed to the same value as that of the cross-correlation function at $\tau = 0$.

For δz small enough for the EA predictions to be applicable τ_p and τ_d are proportional to δz:

$$\tau_d = \alpha_0 \delta z, \ \tau_p = \alpha_p \delta z \ . \tag{13}$$

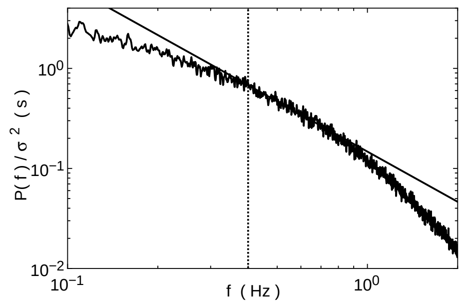

FIG. 5. The normalized temperature power spectrum $P(f)/\sigma^2$ as a function of the frequency f in Hz for $T_b - T_{on} = 10.6$ K and 2-phase flow at the sample center. The solid line is a power-law fit with the exponent fixed at -5/3. The vertical dotted line indicated an estimate of the lower limit of an inertial range at about 0.4 Hz.

In terms of α_p and α_0 the EA predicts that

$$V_{eff} \equiv \sqrt{U^2 + V^2} = 1/\alpha_0 \ , \tag{14}$$

$$U = \alpha_p/\alpha_0^2 \ , \tag{15}$$

and

$$V = \frac{\sqrt{1 - (\alpha_p/\alpha_0)^2}}{\alpha_0} \ . \tag{16}$$

We used Eqs. 15 and 16 to determine U and V. The results for U and V and Eq. 12 were used to compute z_E.

Based on the EA one can derive a space-time equivalence which makes it possible to obtain the spatial temperature spectrum $E(k)$ from the temporal temperature spectrum $P(f)$. The equivalence is obtained by transforming the time coordinate to the spatial coordinate according to

$$z = V_{eff}\tau \tag{17}$$

where V_{eff} is given by Eq. 14.

C. Expected range of applicability of the EM for the experimental conditions of this study

We show in Fig. 5 the normalized power spectrum of the temperature measured at the sample center (other locations yield consistent results but are less definitive because of the appearance of a peaks in the spectrum from the sloshing and/or torsional modes in the lower decade of the inertial range). One sees that there is no wide range of the frequency over which a unique power law gives a good fit. This is due primarily to the onset of viscous dissipation already at frequencies somewhat below one Hz. However, as indicated by the solid line, there is a range over which the Obukhov-Corrsin spectrum [46, 47] with an exponent of -5/3 fits the data. Deviations from this power law at low frequencies occur below about 0.4 Hz, and this frequency might be used as an estimate of the lower limit f_I of the inertial range. A similar analysis, but using an exponent of -7/5 as suggested by Bolgiano and Obukhov [48, 49] yields a similar, albeit slightly lower, value of about 0.3 Hz. Similar results are obtained for all of our runs. Thus we estimate that in our experiment the inertial range covers time intervals smaller than $1/f_I = \tau_I \simeq 2.5$ s.

For the example of Fig. 5 the EA analysis yielded $V_{eff} = 9.5$ mm/s (for all of our runs we found $V_{eff} \gtrsim 6$ mm/s), thus indicating (Eq. 17) that the inertial range in this case would extend up to about $\delta z_I = \tau_I V_{eff} \simeq 24$ mm. If the arguments of He and Yang regarding the range of applicability of the EA are valid, then we would expect that our values of $\delta z \lesssim 12$ mm and our results for $|\tau_p| \lesssim 0.9$ s and $\tau_d \lesssim 1.8$ s are small enough.

D. Tests of the applicability of the elliptic approximation to our measurements

Several previous experiments used the EA predictions to derive velocities from correlation-function measurements [37, 38, 41, 50, 51]. Some of these have demonstrated the applicability of the EA predictions to time and space increments τ and δz well beyond those expected for a second-order expansion of $C(\tau, \delta z)$. Here we examine the applicability of the EA predictions to our measurements.

1. *Correlation functions and homogeneity*

One of the assumptions of the EA is that the sample is homogeneous over the distances between the probes used in the measurements. Thus, one obvious requirement is that the two auto-correlation functions used to determine τ_d (see Fig. 4) should coincide. In real experiments the homogeneity assumption will generally not be satisfied exactly, and the issue then becomes whether deviations from it are sufficiently small for the EA to yield useful results for U and V.

In Fig. 6 we show examples for two locations in our sample (see Table II) of auto-correlation functions $C(\tau, 0)$ and cross-correlation functions $C(\tau, \delta z)$ for 2-phase flow with a superheat of 7.82 K. Results at the third location, for other superheat values, and for 1-phase flow are similar. From the auto-correlation functions one sees that there are significant deviations from homogeneity. It is difficult to estimate the errors introduced by this. In the determination of τ_d (see Fig. 4) we used the average of the two auto-correlation functions that corresponded to the cross-correlation function under consideration. The average of the two values of τ_d obtained by using the two auto-correlation functions separately had essentially the same value. This procedure cannot be expected to completely cancel errors due to spatial inhomogeneity because this inhomogeneity introduces odd terms into the Taylor-series expansion of $C(\tau, \delta z)$, and the effect of these terms is difficult to estimate. We do not think that these errors are very large because the overall analysis gave consistent results over the entire range of $T_b - T_{on}$, of δz (see Fig. 7), and for 2-phase and 1-phase flow (see Sect. V C below).

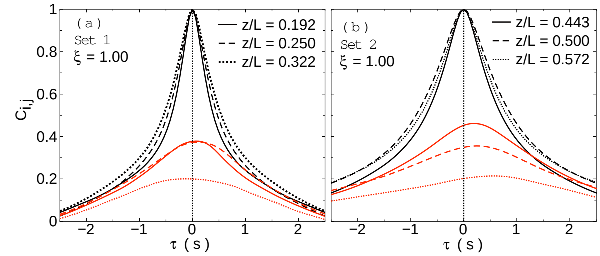

FIG. 6. Examples of correlation functions. In each set the upper three curves are the auto-correlation functions at the locations z/L and ξ given in the figure (see also Table I). The lower three curves (red) are the three corresponding cross-correlation functions. In (a) they are for (see Table II) $\bar{z}/L = 0.221$ and $\delta z = 5.1$ mm (solid line), $\bar{z}/L = 0.286$ and $\delta z = 6.3$ mm (dashed line), and $\bar{z}/L = 0.272$ and $\delta z = 11.4$ mm (dotted line). In (b) they are for $\bar{z}/L = 0.472$ and $\delta z = 5.0$ mm (solid line), $\bar{z}/L = 0.536$ and $\delta z = 6.4$ mm (dashed line), and $\bar{z}/L = 0.508$ and $\delta z = 11.4$ mm (dotted line). All results are for 2-phase flow and $T_b - T_{on} = 7.82$ K.

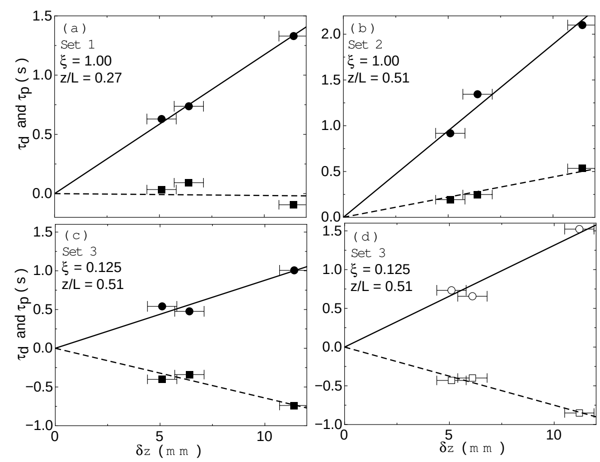

FIG. 7. Plots of τ_p (squares) and τ_d (circles) as a function of δz. (a), (b), and (c) are for 2-phase flow and $T_b - T_{on} = 7.82$ K (run 1511203). (d) is for 1-phase flow and $T_b - T_{on} = 8.45$ K (run 1512075). The data yield (a): $\alpha_p = -0.002$, $\alpha_0 = 0.117$ s/mm; (b): $\alpha_p = 0.044$, $\alpha_0 = 0.189$ s/mm; (c): $\alpha_p = -0.064$, $\alpha_0 = 0.088$ s/mm; (d): $\alpha_p = -0.075$, $\alpha_0 = 0.131$ s/mm.

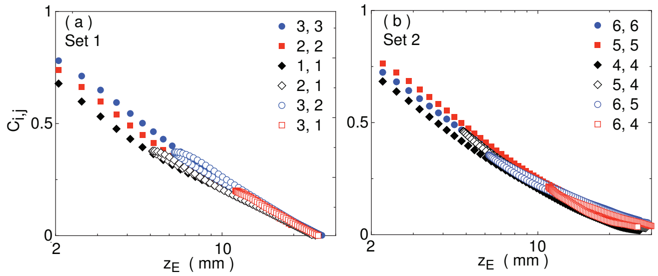

FIG. 8. Examples of plots of correlation functions $C_{i,j}(\tau, \delta z)$ as a function of z_E (Eq. 12). The values of i and j are given in the figure. All data are for 2-phase flow and $T_b - T_{on} = 7.82$ K.

2. *The dependence of τ_p and τ_d upon δz*

An important test of the applicability of the EM is the dependence of τ_p and τ_d upon the displacement δz; these time intervals should be proportional to δz (Eq. 13). Representative results are shown in Fig. 7. The horizontal error bars correspond to the uncertainty of δz, which are evaluated from the estimated absolute errors of δz that is approximately 0.7 mm (see Sect. III 3). Contributions from measurement errors to the uncertainties of the time intervals are negligibly small (less than 0.01 s); but systematic errors due to the spatial inhomogeneities discussed above may be significant but are difficult to estimate. The data are generally consistent with the expected proportionality to δz, thus supporting the applicability of the EA results to relatively large time and space intervals which are, however, well within the inertial range indicated by the temperature spectra (see Fig. 5).

It is interesting to note that the deviations from the fitted straight lines in Figs. 7 (c) and (d) are very similar. These measurements correspond to the same spatial locations, but (c) is for 2-phase and (d) is for 1-phase flow. This suggests that the errors are due primarily to errors of δz and only to a lesser extent to errors of τ_p and τ_d.

3. *The dependence of the correlation functions upon z_E*

Another test of the EA is based on a central result of the EA which shows that $C(\delta z, \tau)$ can be written as a function of a single variable z_E. Having determined U and V from Eqs. 13 to 16 and data such as those in Fig. 7, we can obtain z_E as a function of τ and δz from Eq. 12. Figure 8 shows results corresponding to the correlation functions in Fig. 6. The deviations from homogeneity are seen in the plots of the auto-correlation functions (solid symbols). However, the results for the three cross-correlation functions fall within the range of the auto-correlation functions and suggest that deviations from a unique curve for all functions are likely to be caused by the small inhomogeneity of the sample over the measurement distance rather than a failure of the EA.

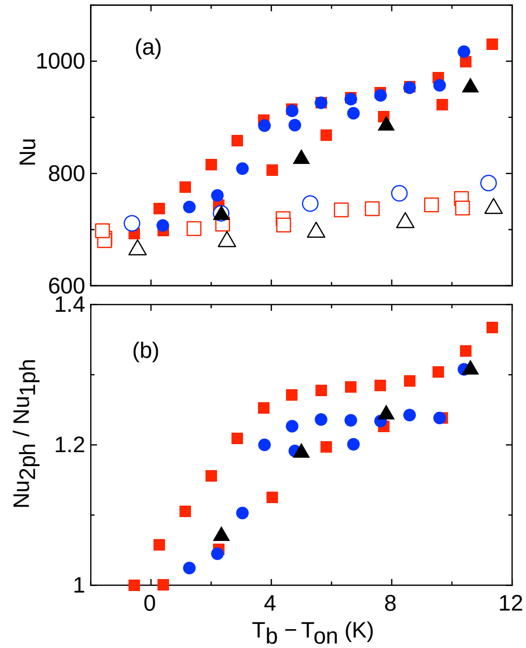

FIG. 9. (a) The Nusselt number for 1-phase flow (Nu_{1ph}, open symbols) and 2-phase flow (Nu_{2ph}, solid symbols) and (b) the ratio $\mathrm{Nu}_{2ph}/\mathrm{Nu}_{1ph}$ as a function of the superheat $T_b - T_{on}$. Circles (blue): Cell A. Triangle: Cell B. Squares (red): NG [29].

V. RESULTS

A. Heat transport

The only differences between the two samples A and B on the one hand and those of NG [29] on the other were the numbers and locations of thermistors. To check whether this difference impacted the heat transport, Nusselt numbers were measured for both cells in the 1-phase and 2-phase flows. The results are compared with those of NG in Fig. 9. For both cells they fall within the spread between the different data sets of the earlier work.

B. The large-scale flow

1. *Qualitative nature of the large-scale flow*

Shadowgraph flow-visualizations by NG [30] indicated that, in the presence of bubbles (2-phase flow), there was a large-scale flow (LSF) at least in the lower portion of the sample. It carried hot plumes, emitted mostly by the thermal boundary layer just above the central heated part of the bottom plate, to one side where the plumes then rose near the side wall toward the top plate. This is illustrated in Fig. 10 (a) and more clearly by Movie

3 of Ref. [30]. Cold plumes generated similarly under the entire top plate tended to move toward the other side of the cell where they descended; but their signature became weaker as the bottom plate was approached and it is not clear whether the plume motion was indicative of a closed-loop large-scale circulation (LSC) in the form of a single convection roll.

In the absence of bubbles (1-phase flow) hot plumes were rising more or less vertically from the bottom plate and their motion did not reveal any evidence of a LSF. Cold plumes did suggest some lateral LSF just under the top plate, but their descent through the bulk did not reveal any evidence for a closed LSC roll. This is shown in Fig. 10 (b) and more clearly in Movie 4 of Ref. [30].

2. *Quantitative measurements of the large-scale flow*

We searched for a LSF using the thermistors immersed in the sidewall. Each set of eight thermistors, azimuthally distributed uniformly around the circumference, yielded temperatures $T_{k,i}$, $i = 0, 1, 2, ..., 7$ at a given measurement time. The function

$$T_f = T_{0,k} + \delta_k \cos\left[i\pi/4 - \theta_{0,k}\right] \qquad (18)$$

was fit to them, separately at each height $k = b, m, t$ and at each measurement time. The least-squares adjusted parameters $T_{0,k}$, δ_k, and $\theta_{0,k}$ describe an azimuthally uniform background temperature, a "temperature amplitude" of the LSF, and the azimuthal orientation of a large-scale up-flow respectively (see *e.g.* Refs. [11, 13] for more details).

Results for $\langle\delta_k\rangle/\Delta T$ are shown in Fig. 11 as a function of the superheat. One sees that any coherent structure leading to an azimuthal cosine variation, if present at all, is strongest near the bottom of the sample ($k = b$, circles, red). However, at the Rayleigh numbers $Ra \simeq 2 \times 10^{10}$ of the experiment measurements for (uniformly heated) RBC at the same Pr and $\Gamma = 1.00$ [52] give $\langle\delta_k\rangle/\Delta T \simeq 0.017$, which is significantly larger than the data in Fig. 11 for LHC. The indication of an azimuthally varying structure for $k = b$ ($z/L = 0.25$, $\langle\delta_b\rangle/\Delta T$) is consistent with the visualization shown in Fig. 10(a) for 2-phase flow, but Fig. 10(b) for 1-phase flow in general gives no indication of an LSF. Indeed, we see from the results for $\langle\delta_b\rangle/\Delta T$ and 1-phase flow (open circles, red) that $\langle\delta_b\rangle/\Delta T$ values near those for 2-phase flow (solid circles, red) are reached only in some of the runs, and not for instance for superheats of -0.6 K and 5.3 K. Inspection of the time series for 1-phase flow and superheats of -0.6 K and 5.3 K suggests that the data are statistically reliable (*i.e.* not seriously affected by inadequately long averaging times). We believe that they are suggestive of multistability; but a more detailed investigation of this issue would be desirable. The overall indication of a poorly defined LSF, especially for single-phase flow, is consistent with the flow visualization illustrated in Fig. 10 and the corresponding movies in the supplementary material of Ref. [30].

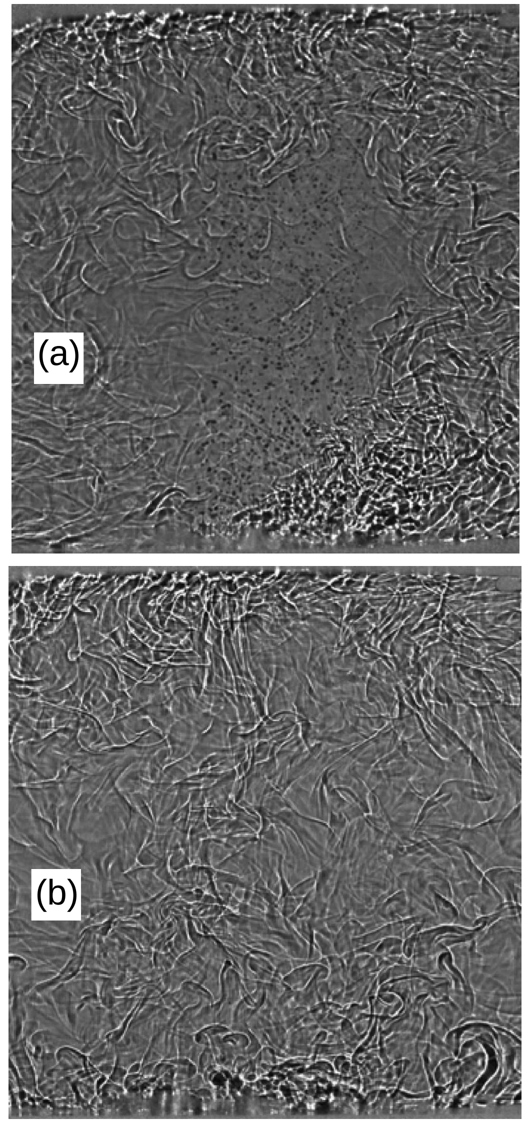

FIG. 10. Shadowgraph images taken from the side for (a) 2-phase and (b) 1-phase flow (from NG [30]). They show a horizontal average over the entire width of the sample. The tiny dark dots in (a) (hardly visible) rising through the center are the bubbles. The corresponding movies in the supplementary material of Ref. [30] reveal a more detailed picture of the flow dynamics.

A useful additional indicator of the LSF strength is based on a Fourier decomposition of the $T_{k,i}$ at a given k [53–55]. This yields the Fourier coefficients $A_{k,j}$ (the cosine coefficients) and $B_{k,j}$ (the sine coefficients) where $j = 1, ..., 4$ identifies the $N = 4$ Fourier modes accessible with eight temperatures. The "energy"

$$E_{k,j}(t) = A^2_{k,j}(t) + B^2_{k,j}(t) \qquad (19)$$

for each mode, as well as the sum $E_{k,tot}(t)$ of the four

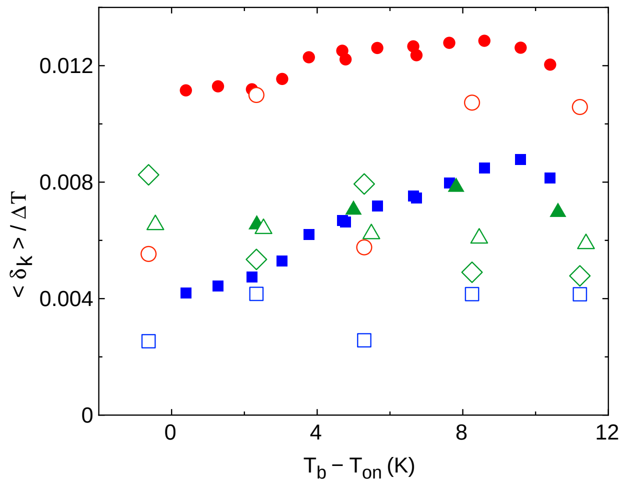

FIG. 11. The scaled temperature amplitude $\langle\delta_k\rangle/\Delta T$ of the LSF. Circles (red), diamonds (green), and squares (blue) correspond to the lower ($k = b$), middle ($k = m$), and upper ($k = t$) set of thermistors in cell A respectively. The triangles (green) are for $k = m$ of cell B. Open symbols: 1-phase flow. Solid symbols: 2-phase flow.

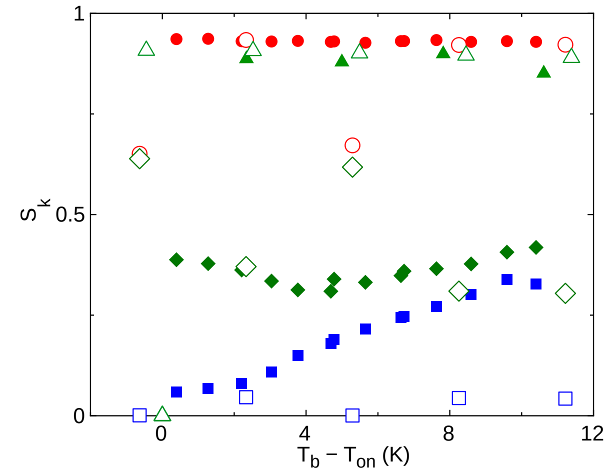

FIG. 12. The parameter S_k (Eq. 20) describing the strength of the LSF. Circles (red), diamonds (green), and squares (blue) correspond to the lower ($k = b$), middle ($k = m$), and upper ($k = t$) set of thermistors in cell A respectively. The triangles (green) are for $k = m$ of cell B. Open symbols: 1-phase flow. Solid symbols: 2-phase flow.

energies, were calculated. The parameter

$$S_k = MAX\left[\frac{(N\langle E_{k,1}\rangle/\langle E_{k,tot}\rangle - 1)}{(N-1)}, 0\right] \qquad (20)$$

was proposed by Stevens et al. [54] as a useful indicator of the LSF strength and coherence. A value of S_k close to unity corresponds to a LSF where the cosine wave is well defined and dominates over fluctuations. Smaller values of S_k suggest a strongly fluctuating LSF structure, and it was suggested in Ref. [54] that $S_k \lesssim 0.5$ is indicative of no dominant coherent structure with a characteristic length scale close to the sample height.

Results for S_k are shown in Fig. 12 as a function of the superheat $T_b - T_{on}$. As in Fig. 11, the open (solid) symbols are for 1-phase (2-phase) flow. One sees that for cell A only the lower part of the sample ($k = b$, circles, red) yielded values of S_m close to one as expected for a coherent structure extending over the diameter of the sample, and then reliably only for 2-phase flow (solid circles, red). Consistent with the $\langle\delta_b\rangle/\Delta T$ results (Fig. 11), the 1-phase data yielded S_m close to one only for some of the runs. At higher levels in cell A ($k = m$ and t) the measurements provide no support for a coherent structure. One concludes that there is no coherent convection roll as is found in RBC (which leads to $S_k \simeq 1$ at all levels, see *e.g.* [52])), and only up-flow at a particular azimuthal orientation and near the sample bottom which dissipates at larger heights.

The results for S_m of cell B (triangles, green in Fig. 12) differ significantly from those of cell A (measurements for $k = b$ and t were not made in B, see Sect. III 3). Even at the mid-plane ($k = m$, $z/L = 0.50$) the azimuthal structure is well defined for both 1-phase and 2-phase flow, with S_m close to one. This surprising result suggests that the formation of a coherent structure in cell B is *enhanced* by the obstructions due to the thermistors that were inserted into the sample (cell A had no internal thermistors), while one might have expected the opposite. We note, however, that the amplitudes $\langle\delta_m\rangle/\Delta T$ for B (Fig. 11, triangles, green) are smaller than those for A and of similar size to $\langle\delta_m\rangle/\Delta T$ for A where the results for S_m (Fig. 12) indicate the absence of a well defined structure. Compared to RBC for the same Ra and Γ [52] we see that $\langle\delta_m\rangle/\Delta T$ for B is smaller by a factor of nearly three.

C. Reynolds numbers based on the elliptic model

1. *General remarks and results*

We used the elliptic model (EM) of He and Zhang [31, 36] to determine the mean-flow velocity U (Eq. 5) and root-mean-square fluctuation velocity V (Eq. 7) from simultaneous temperature time-series taken at several internal locations of cell B at time intervals of 0.06 s (see Sect. III 3). The relationships derived from the EM that are relevant to our analysis were summarized in Sect. IV A. In Sect. IV C we discussed the ranges of time and space displacements τ and δz over which we expect the EM to be applicable based on the assumption of Kolmogorov self-similarity. In Sect. IV D we presented several tests of the applicability of the EM under the conditions of our experiment. These tests do not reveal any major problems with using the EM to analyze our data.

The measurement locations were at the same radial position and separated vertically by distances δz (see Table

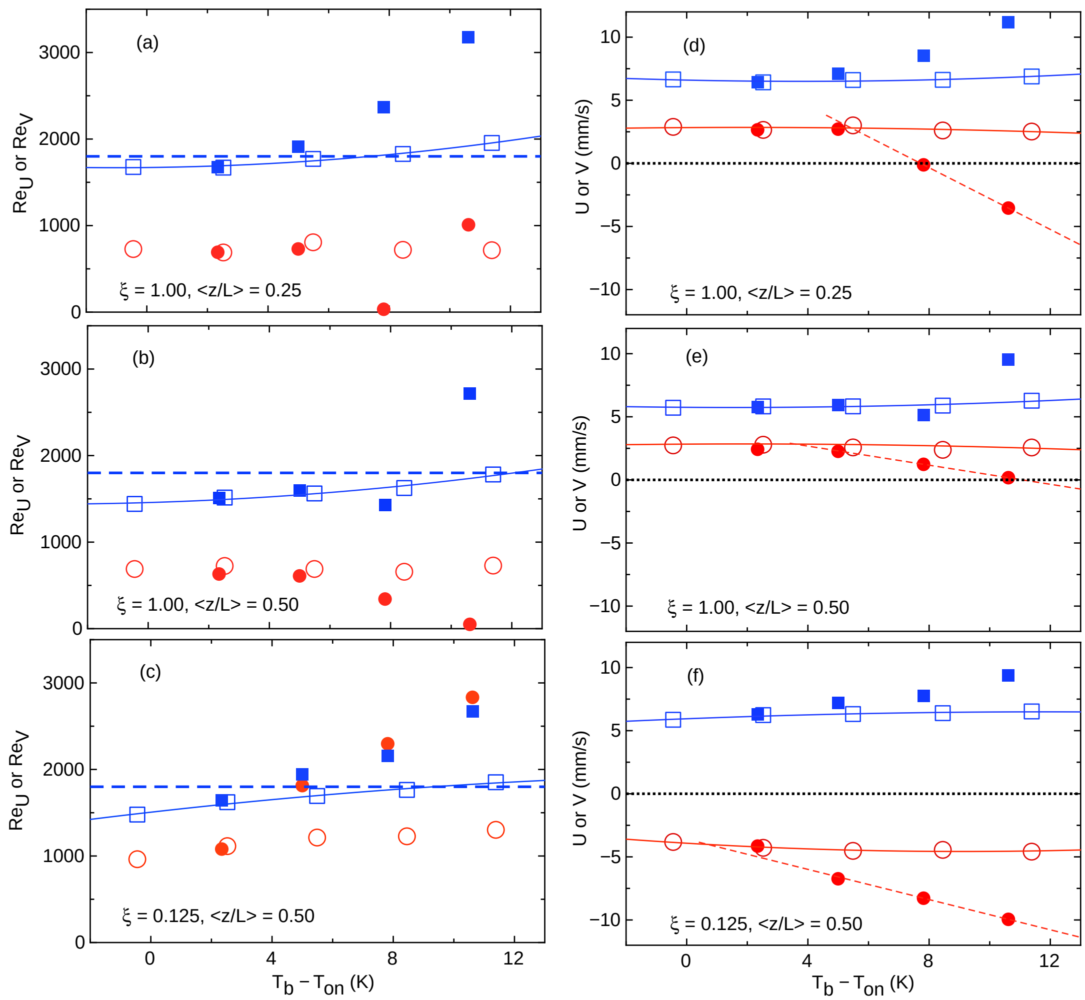

FIG. 13. (a), (b), and (c): Reynolds numbers Re_U (circles, red) and Re_V (squares, blue) as a function of the superheat $T_b - T_{on}$. (d), (e), and (f): Mean-flow velocity U (circles, red) and root-meant-square fluctuation velocity V (squares, blue) as a function of the superheat $T_b - T_{on}$. Open symbols: 1-phase flow. Solid symbols: 2-phase flow. Horizontal dashed lines (blue): Re_V for RBC. Solid lines: Fits of a quadratic equation to the data. Short dashed lines (red): Fits of a straight line to the two points with the largest $T_b - T_{on}$.

I). The velocity determinations required obtaining two time intervals τ_p and τ_d from measurements at pairs of nearby locations (see Fig. 4); these pairs and their δz values are listed in Table II. Each velocity determination was based on three sets of τ_p, τ_d, and δz, with each set derived from measurements at three vertical positions as indicated in Table II and described in Sect. IV A. The sets are associated with averaged locations $\bar{z}/L$ and ξ equal to 0.25 and 1.00 for set 1, 0.50 and 1.00 for set 2, and 0.50 and 0.125 for set 3 (the precise and detailed locations follow from Table II). The measurements thus are of the time-averaged vertical velocity and velocity-fluctuation components $U(\bar{z}/L, \xi)$ and $V(\bar{z}/L, \xi)$ (Eq. 7). The velocities were converted to Reynolds numbers Re_U (Eq. 6) and Re_V (Eq. 8) using the kinematic viscosity ν at the measured center temperatures T_c.

Results for $\mathrm{Re}_U, U, \mathrm{Re}_V$, and V are shown in Fig. 13. The circles (red) are for Re_U and U, while the squares give the results for Re_V and V. The top pair (a) and (d), middle pair (b) and (e), and bottom pair (c) and (f) are for sets 1, 2, and 3 respectively. The left column gives the Reynolds numbers, while the right one shows the corresponding velocities in mm/s. It is helpful to show the velocities as well as the Reynolds numbers because U can be negative [see Fig. 13(f)] or even pass through zero as $T_b - T_{on}$ changes [see Fig. 13(d)] while $\mathrm{Re}_U \geq 0$. In order to facilitate the comparison between 1-phase flow (open symbols) and 2-phase flow (solid symbols), we fitted a quadratic polynomial to the 1-phase data and show the fits as solid lines in the figure.

2. *Discussion of 1-phase flow*

Let us first consider the case of 1-phase flow (open symbols). As expected, neither Re_U (open circles) nor

Re_V (open squares) shows a significant dependence on the superheat because Ra does not change significantly (see Sect. III 1). Comparison of Re_U with other measurements for RBC is not easy because Re_U depends strongly on the position in the sample. For RBC it is largest in the bulk near the side wall while it vanishes at the sample center. For our case of local heating with cooling over the entire top plate, we find that $\mathrm{Re}_U \simeq 800$ [Fig. 13(b)] and that there is up-flow [$U \simeq 2.5$mm/s > 0, Fig. 13(e)] at the sample center. This is consistent with the movie of NG [30] and Fig. 10(b) which show more or less vertically rising plumes distributed broadly in the horizontal direction rather than a closed single-cell LSC. Consistent with this flow, the data closer to the bottom plate and on the sample center line for Re_U and U do not change much, as indicated by the results for set 1 shown in Fig. 13(a) and (d).

Closer to the side wall the 1-phase results for set 3 [$\xi = 0.125$, Fig. 13(c) and (f)] indicate a significant down-flow ($U \simeq -4$ mm/s < 0). This is not unexpected because there must be down-flow at least at some azimuthal locations when there is up-flow through the central part of the sample.

As is the case for RBC, the results for the fluctuation velocity V and Re_V in 1-phase flow do not depend very much on the location within the sample. Averaged over all measured superheats, one has $V = 6.63$ mm/s and $\mathrm{Re}_V = 1780$ for set 1 ($\xi = 1.00, \bar{z}/L = 0.25$), $V = 5.90$ mm/s and $\mathrm{Re}_V = 1585$ for set 2 (sample center, $\xi = 1.00, \bar{z}/L = 0.50$), and $V = 6.26$ mm/s and $Re_V = 1682$ for set 3 (near the side wall, $\xi = 0.125, \bar{z}/L = 0.25$). These results are quite close to the result $\mathrm{Re}_V \simeq 1800$ estimated for RBC at the same Ra and Pr (see Sect. III 1) and shown in Figs. 13 (a), (b), and (c) by the horizontal dashed lines (blue).

3. Discussion of 2-phase flow

For 2-phase flow the data in Fig. 13 reveal a number of remarkable features. We note that the rising bubbles (solid symbols) at sufficiently large superheat modify not only the mean-flow velocity U and Re_U, but also the fluctuation velocity V and Re_V. Further, the influence of the bubbles does not begin as soon as they are formed when $T_b - T_{on}$ exceeds zero; rather there is a finite onset value δT_{2ph} of $T_b - T_{on}$ below which the rising bubbles have no influence at the measurement positions of the present work. This is most clearly seen from the results for U in (d), (e), and (f). The results for the onset of enhancement of V are not as clear; but within our resolution the onset is the same as it is for U. There seems to be an exception or complication for set 2 [Fig. 13 (b) and (e)] where the result for V at a superheat of 7.9 K is lower in the 2-phase system than it is for the corresponding 1-phase flow even though based on all other data we would have expected an enhancement. We have no explanation for this outlier. The onset shift is largest (with δT_{2ph} a little over 5 K) along the center line in the lower part of the sample. At the sample center we find $\delta T_{2ph} \simeq 4$ K, while near the sidewall δT_{2ph} seems to be close to zero.

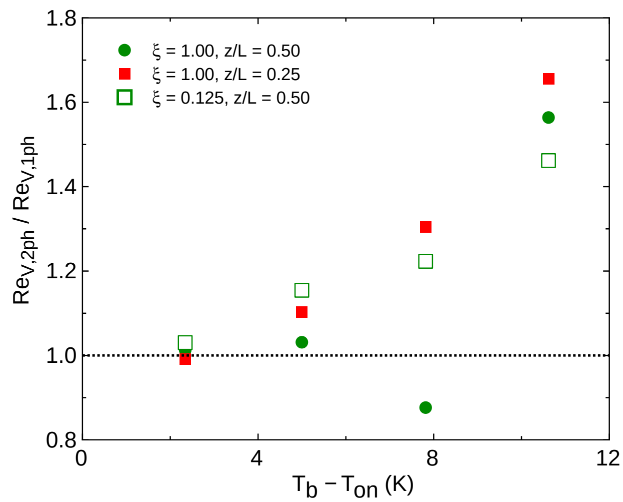

FIG. 14. The ratio $\mathrm{Re}_{V,2ph}/\mathrm{Re}_{V,1ph}$ as a function of the superheat $T_b - T_{on}$.

For $T_b - T_{on} > \delta T_{2ph}$, the amplitude of U is *reduced* by the bubbles of 2-phase flow for the set 1 and set 2, and along the centerline (where it was positive for 1-phase flow) even changes sign; whereas the amplitude of U in set 3 increases with superheat. It is not possible to say whether this effect is due directly to bubble influence, or indirectly to changes in the large-scale flow structure induced by the bubbles.

The fluctuation velocity V (as well as Re_V), on the other hand, is *enhanced* by the bubbles of the 2-phase flow [except for the anomaly mentioned above and seen in Fig. 13 (b) and (e)]. This effect is consistent with direct numerical simulations with bubble injection [56–59] which also found that velocity fluctuations were enhanced in 2-phase flow, with the enhancement increasing with increasing rate of bubble injection. Note that in these simulations the vapor-bubble nucleation-process is not modeled. Instead, a certain and controlled number of nuclei is imposed. These then grow or shrink, depending on the local conditions. The experimentally observed enhancement of velocity fluctuations stands in contrast to the *reduction* of temperature fluctuations found by Narezo Guzman *et al.* [29]. The diminished temperature fluctuations may be attributed to a smoothing of the temperature field due to the enhanced mixing by the larger velocity fluctuations. This reduction of the temperature fluctuations by the bubbles was found also in numerical simulations [59], where the temperature field became much less intermittent in the presence of the vapor bubbles due to the smoothing of sharp temperature fronts.

In Fig. 14 we provide a more quantitative comparison of Re_V in 1-phase and 2-phase flow by showing the ratio $\mathrm{Re}_{V,2ph}/\mathrm{Re}_{V,1ph}$ as a function of the superheat. At the larger superheats one sees that the relative fluctuation enhancement is similar at all three locations in the sample, and reaches values as large as 60% or so.

D. Reynolds numbers from frequency measurements

It has long been known from single-point determinations in RBC samples of the temperature or the velocity [8, 9, 60–67] that the dynamics of turbulent RBC includes oscillations with a characteristic frequency which seems to be related to the turnover time of the LSC (see, however, [68, 69]). One can use this frequency to determine Reynolds numbers Re_{ac}, Re_{sl}, or Re_{f_0} (see Eq. 9). It turns out that all three are equal to each other within experimental resolution; thus the three measurements may be regarded as alternative experimental methods to study the same phenomenon.

It is now known that the origin of these oscillations in RBC can be found, at least over a wide parameter range, in both a torsional and a sloshing mode of the LSC. The torsional mode is a time-periodic twist of the circulation plane of the LSC that consists, at a given moment, of a rotation in opposite azimuthal directions in the top and the bottom half of the sample [70, 71]. The sloshing mode is a periodic vertically and radially uniform lateral displacement of the circulation plane [72–74]. These two modes have the same frequency.

In our case of LHC we found that a well developed LSC extending as a single convection roll over the entire volume of a $\Gamma = 1$ sample does not seem to exist (see Sect. V B). Thus it would not have been surprising if local temperature oscillations also were absent. This, however, is not the case, and in the remainder of this section we report on such frequency measurements. Even though the results have common features with those for RBC, their interpretation in terms of a torsional and sloshing mode obviously has to be done with caution.

1. Reynolds numbers from correlation functions

In this section we present measurements of a frequency using the sidewall thermistors. We used the auto-correlation functions

$$\tilde{C}^{k,k}_{i,i}(\tau) = \langle [T_{k,i}(t) - \langle T_{k,i}\rangle] \times [T_{k,i}(t+\tau) - \langle T_{k,i}\rangle]\rangle \tag{21}$$

with the normalization

$$C^{k,k}_{i,i}(\tau) = \tilde{C}^{k,k}_{i,i}(\tau)/\tilde{C}^{k,k}_{i,i}(0) \tag{22}$$

calculated from the temperature time series $T_{k,i}(t)$ of the thermistors in the sidewall. They reveal an oscillatory signal very similar to that found in RBC [75]. The eight correlation functions corresponding to $i = 0, ..., 7$ at a given k can be averaged to yield $\bar{C}^{k,k}(\tau)$. An example from cell A is shown in Fig. 15a. The data clearly reveal oscillations also for our case of LHC.

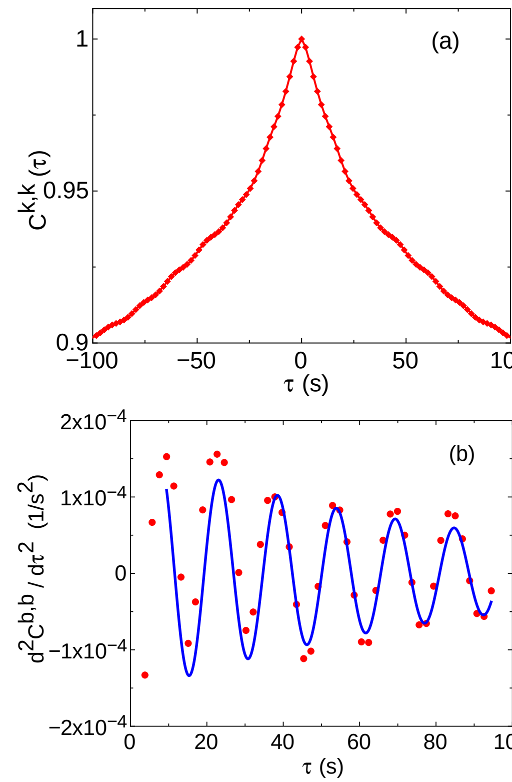

FIG. 15. (a): Azimuthally averaged auto-correlation function for the largest superheat $T_b - T_{on} = 10.4$ K and 2-phase flow in cell A for $k = b$. (b): Points: The second derivative of the averaged auto-correlation function for $k = b$ in (a) with respect to τ. Line: Fit of Eq. 23 to the data.

The oscillatory contribution to $\bar{C}^{k,k}(\tau)$ is weak, and it is difficult to extract a reliable frequency directly from the data. As noted in Ref. [75], the oscillations become more obvious in the second derivative of $\bar{C}^{k,k}(\tau)$. This is shown in Fig. 15b. The function

$$d^2C/d\tau^2 = c_1 \exp(-\tau/\mathcal{T}^{bg}) \cos(2\pi\tau/\mathcal{T}^{ac}) \tag{23}$$

provides a reasonable fit and gives both the oscillation period $\mathcal{T}^{ac}$ and the exponential decay time $\mathcal{T}^{bg}$ of the oscillations. Using $\mathcal{T}^{ac}$, Re_{ac} is given by Eq. 9.

Results for Re_{ac} at the lower plane $k = b$ of cell A are shown in Fig. 16 as circles (blue) and as a function of the superheat. Results for $k = m$ and $k = t$ differ by less than 1 % from the $k = b$ values and are not shown. The data indicate that Re_{ac} of 1-phase flow (open symbols) is enhanced significantly by the bubbles of the 2-phase flow (solid symbols). At the larger superheat values this enhancement reaches values up to about 25 %. We note, however, that this is much smaller than the 60 % enhancement found for the bubbles of Re_V (see Sect. V C 3 and Fig. 14).

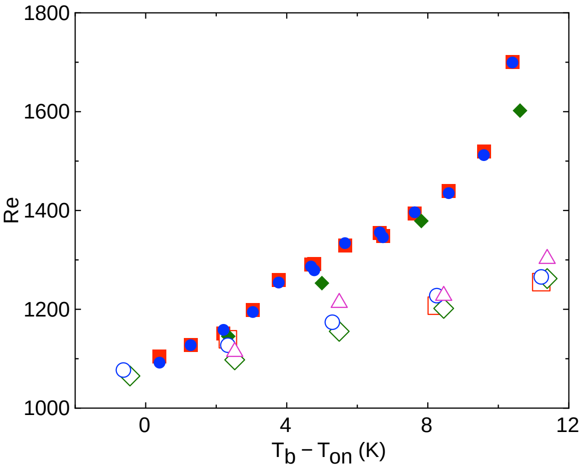

FIG. 16. Reynolds numbers as a function of the superheat $T_b - T_{on}$. Circles (blue): Re_{ac} at the lower plane $k = b$ of cell A. Squares (red): Re_{sl} based on the sloshing-mode oscillations at the lower plane $k = b$ of cell A. Diamonds (green): Re_{f_0} from temperature oscillations in the interior of cell B. Triangles (purple): Re_U for $\xi = 0.125$, $\bar{z}/L = 0.50$ from Fig. 13 (c). Open symbols: 1-phase flow. Solid symbols: 2-phase flow.

2. Reynolds numbers from the sloshing mode

As discussed at the beginning of Sect. V D, the LSC in turbulent RBC undergoes both torsional oscillations and a periodic lateral displacement referred to as the sloshing mode. The antisymmetric sine terms in a Fourier decomposition of the eight sidewall temperatures at a given height can characterize the off-center displacement inherent in this mode [74]. The corresponding sinusoidal Fourier amplitudes are given by

$$D_n = \sum_{i=1}^{8} \{[T - T_0 - \delta \cos(\theta_i - \theta_0)] \sin[n(\theta_i - \theta_0)]\} \ . \quad (24)$$

The contribution $D_2 \neq 0$ corresponds to a temperature profile that is tilted in the azimuthal direction so that the temperature extrema, located at θ_h and θ_c, come closer together, corresponding to a slosh displacement. In the absence of a sloshing mode θ_h and θ_c are separated by π. Thus it is convenient to define the slosh displacement angle as $\theta' = (\theta_h - \theta_c - \pi)/2$. It satisfies the equation [74]

$$\delta \sin \theta' = 2 D_2 \cos(2\theta') \ . \quad (25)$$

We calculated θ' only at the lower plane $k = b$ of cell A because we found no coherent azimuthal variation at the other levels (see Fig. 12). The power spectrum of θ' is shown in Fig. 17. In addition to a broad contribution due to the driving of the LSC dynamics by the turbulent background [14, 76], it reveals a remarkably sharp peak at the sloshing frequency f_{sl}. A similar spectrum was obtained also from the time series for D_2 and yielded the

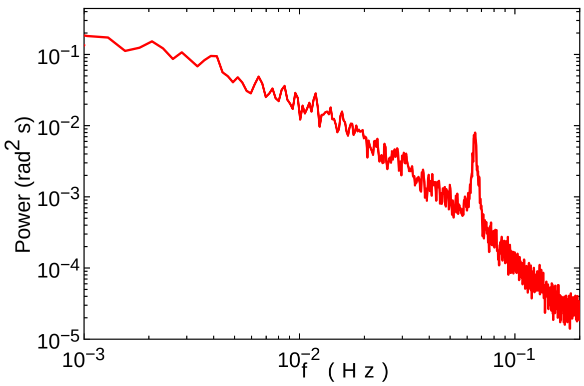

FIG. 17. The power-spectral density of the displacement angle θ' at the lower plane $k = b$ in cell A for 2-phase flow at a superheat $T_b - T_{on} = 10.4$ K.

same f_{sl}. The period $\mathcal{T}_{sl} = 1/f_{sl}$ was used with Eq. 9 to calculate Re_{sl}. The results are shown as squares (red) in Fig. 16. As expected from measurements for RBC, they agree very well with the results for Re_{ac} (see Sect. V D 1) that are shown as circles (blue) in the same figure.

3. Reynolds numbers from local temperature oscillations

The power spectra derived from the internal thermometers of cell B are shown in Fig. 18 for the largest superheat $T_b - T_{on} = 10.62$ K in 2-phase flow. Spectra obtained for 1-phase flow looked very similar. The spectra for the thermistors near the wall [$\xi = 0.125$, Fig. 18 (b)] all show a strong peak at the same frequency $f_0 = 0.062$ Hz (left vertical dotted line), and a second weak peak at the second harmonic $2f_0 = 0.124$ Hz (right vertical dotted line). The peaks in the spectra on the cell axis [$\xi = 1.00$, Fig. 18 (a)] are smaller, but still quite discernible for the thermistors close to the bottom and top plate. Near the sample center they are only barely visible.

The data show that there is a unique frequency $f_0 = 0.062$ Hz at all locations where the spectral peak can be identified. It is used to calculate the corresponding Reynolds number Re_{f_0} from Eq. 9. These results are shown as diamonds (green) in Fig. 16. They agree well with those for Re_{ac} and Re_{sl}, in the 1-phase as well as the 2-phase state.

VI. DISCUSSION AND SUMMARY

We reported on measurements that characterize boiling in a cylindrical sample of fluid with Pr $\simeq 8$ and aspect ration $\Gamma = 1.00$ heated from below. The sample geometry was the same as that of NG, except that we

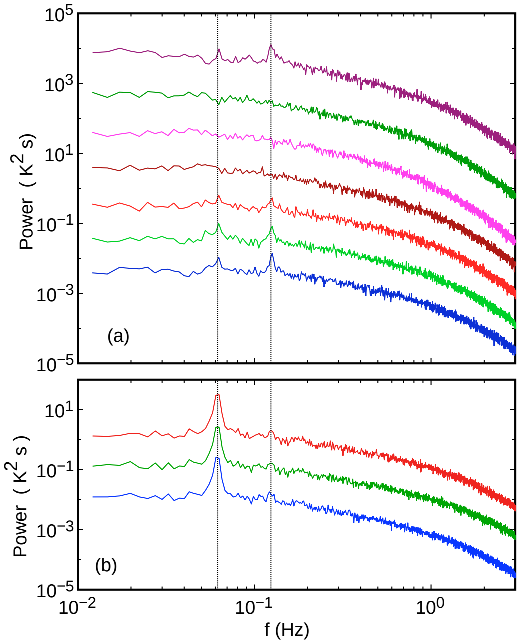

FIG. 18. Power spectra for cell A at the locations within the fluid given in Table I. These results are for 2-phase flow and $T_b - T_{on} = 10.62$ K. (a): Spectra along the sample centerline ($\xi = 1.00$). (b): Spectra near the side wall ($\xi = 0.125$). In both cases the lowest line (blue) is for the lowest thermistor at that radial location, and the higher ones are successively shifted upward by a factor of ten. The vertical dotted lines correspond to $f_0 = 0.062$ Hz and $2f_0 = 0.124$ Hz.

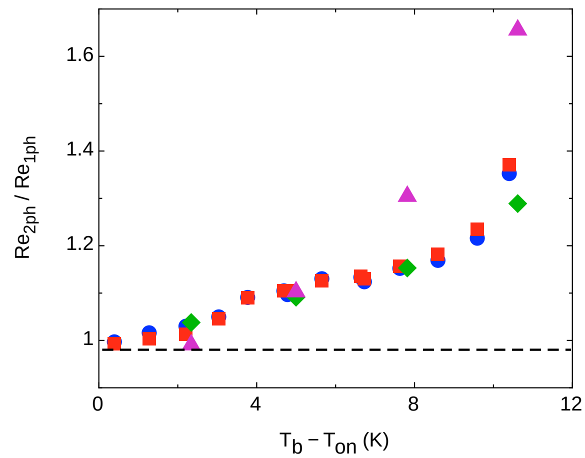

FIG. 19. The relative increase of Reynolds numbers in 2-phase flow compared to 1-phase flow as a function of the superheat. Circles (blue): Re_{ac}. Diamonds (green): Re_{f_0}. Squares (red): Re_{sl}. Triangles (purple): Re_V for $\bar{z}/L = 0.25, \xi = 1.00$ from Fig. 14.

installed nine internal thermometers for local temperature measurements in one sample (cell B, Sect. III 3) and 24 thermometers mounted in the sidewall of another (cell A, Sect. III 2); NG only had two internal devices. The additional thermometers made it possible to measure temperature-oscillation frequencies, as well as Reynolds numbers using temperature correlation-functions and the elliptic model (EM) of He and Zhang [31]. Only a central circular area of 2.54 cm diameter of the bottom plate (of 8.83 cm diameter) was heated and was covered by a lattice of controlled bubble-nucleation centers. This geometry differs from RBC where the entire bottom plate is heated; we refer to it as locally heated convection (LHC).

Heat-transport measurements were made in both 1-phase and 2-phase flow. They yielded Nusselt numbers which agreed with those of NG, thus indicating that global properties were not changed much by the insertion of additional thermometers.

Using the sidewall thermometers, we studied the large-scale flow in the turbulent system. The LSF did not consist of a single well-developed convection roll as in RBC. In the bottom portion of the sample there was a flow structure that yielded a dominating sinusoidal azimuthal temperature variation as in RBC. This sinusoidal mode was stronger for 2-phase flow than for 1-phase flow, but in both cases weaker than for RBC. At the mid plane and in the top portion this mode did not dominate and higher Fourier modes contained more energy. This finding indicates a mean-flow structure that is much more complex and disordered than the single-roll mean flow in RBC.

We measured temperature cross-correlation functions (CCFs) using temperature probes separated vertically by up to 12 mm. They were used to determine both mean-flow velocities U and fluctuation velocities V [31, 33–35] using the elliptic approximation (EA) of He and Zhang [31, 36]. The applicability of the EA to our system is supported by the agreement of our resuts with several general results of the EA (Eqs. 11, 12, 13). They yielded internally consistent results and fluctuation Reynolds numbers that agree within ten percent or so with those measured by other means for RBC (see Sect. V C 2).

We found that the EA predictions were valid even in cases where the maximum value of the CCF $C(\delta z, \tau)$ was as small as 0.2 or so. One would not expect *a priori* that the elliptic approximation, a second-order series expansion of $C(\delta z, \tau)$ about $C(0, 0) = 1$, would be valid in such a case. Our result supports the relevance also for thermally driven convection of the assertion by He and Zhang [31, 36], based on the Kolmogorov similarity hypothesis, that the results of the EA should be valid for all time and space displacements within the inertial range. Indeed we showed that all our values of δz and τ were within the inertial range of our measured fluctuation spectra.

We saw that three methods of determining Reynolds numbers from frequency measurements (Re_{ac}, Re_{sl}, and Re_{f_0}) all give the same results within our resolution, both in 1-phase and in 2-phase flow. This strongly suggests that the measured frequencies have the same origin, as they are believed to do in RBC. In RBC we would argue that they all come from the synchronous torsional and sloshing modes of the LSC; but in our case of LHC we found that a well developed large-scale flow does not exist. Thus the agreement between the three Reynolds numbers is remarkable and its origin needs further elucidation.

The comparison of the frequency-based Reynolds numbers with those derived from the velocities obtained from the elliptic model is more complex. Since in RBC the frequencies are believed to correspond to an inverse turnover time of the LSC, the comparison should be made with Re_U rather than Re_V. But since U varies strongly with position within the sample, its maximum value near (but not too near) the side wall should be used. Based on this consideration, we made the comparison using Re_U of 1-phase flow at $\bar{z}/L = 0.50$, $\xi = 0.125$ (Fig. 16). There was good agreement between Re_U and the frequency-based Re. This agreement is somewhat surprising because U depends on the LSF structure and orientation. As discussed above, the LSF structure of LHC is not the same as that of RBC.

For all three frequency-based Re the same enhancement due to the bubbles of 2-phase flow is found (Fig. 19); it reaches values near 20 or 30 % at the largest superheats. Any enhancement of Re_U due to the bubbles is not meaningful since in addition to any direct flow-speed enhancement the bubbles of 2-phase flow also alter the LSF structure and orientation and thereby indirectly change U. The enhancement of Re_V (Fig. 14 and Fig. 19) at the larger superheats is larger than that of the frequency-based Re, and reaches values in excess of 60 %. This effect is consistent with direct numerical simulations with bubble injection [56–59].

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