Effect of viscosity ratio on the shear-driven failure of liquid-infused surfaces

Ying Liu, Jason S. Wexler, Clarissa Schönecker, and Howard A. Stone

Phys. Rev. Fluids 1, 074003 — Published 17 November 2016

DOI: 10.1103/PhysRevFluids.1.074003
The effect of viscosity ratio on the shear-driven failure of liquid-infused surfaces

Ying Liu,¹ Jason S. Wexler,¹ Clarissa Schönecker,² and Howard A. Stone¹,∗

¹Department of Mechanical and Aerospace Engineering,
Princeton University, Princeton, New Jersey 08544, USA
²Max Planck Institute for Polymer Research, Mainz, Germany
(Dated: September 6, 2016)

Abstract

Liquid-infused surfaces (LIS) display great advantages such as omniphobicity, anti-icing, anti-fouling and self-cleaning. They can also reduce the drag of an object moving through an immiscible fluid since there is slip along the fluid-fluid interface. However, the trapped liquids are susceptible to the shear of the external flow, which will drain the lubricants from the surfaces and hinder their practical use. In this study, we investigate the shear-driven failure of liquid-infused surfaces under a broad range of ratios of the viscosity of the external fluid to that of the lubricant. The effect of viscosity ratio on the steady-state lubricant retention is characterized experimentally and analyzed analytically. The model offers a possible way to estimate the shear-driven failure of surfaces filled with different lubricants and even air-infused superhydrophobic surfaces in the limit where the external fluid is much more viscous than the infused liquid.

* hastone@princeton.edu
I. INTRODUCTION

Liquid-infused surfaces (LIS) are surfaces with microstructures filled with lubricating liquids [1–4], for which some inspiration comes from the inner surface of the Nepenthes pitcher plant [3, 5]. After chemically functionalizing the surfaces, the lubricants can wick into the microstructure and remain there due to capillary forces. When these surfaces contact with another liquid, which is immiscible and energetically unfavorable with the substrate compared with the lubricants, the surfaces display ultra-low adhesion of that external liquid [1–3]. The lack of pinning also brings other benefits, such as self-cleaning [4], anti-icing [6, 7] and anti-fouling [8–10]. In addition, LIS outperform superhydrophobic surfaces (SHS), which are filled with air, in their pressure-stability and self-healing capabilities [3]. In addition, similar to SHS [11], LIS also displays drag reduction capability in both laminar [12] and turbulent [13] flows because the existence of the lubricants (air for SHS) allow slip at the fluid-fluid interface.

Here, we focus on the drag reduction application of LIS, where the drainage of the lubricant caused by the shear of the external flow becomes a serious problem. Once drainage occurs, the surfaces can lose their favorable properties, the drag can even increase because without lubricants, the microstructures of the surfaces exposed to the external flow may trigger or intensify the turbulence [14, 15]. In order to obtain a prediction of drag reduction, it is necessary to study the shear-driven drainage of LIS. One of the most important parameters in this problem is the viscosity ratio, $N = \mu_{\text{ext}}/\mu_{\text{int}}$, between the viscosity of the external fluid, $\mu_{\text{ext}}$, and the viscosity of the infused fluid, $\mu_{\text{int}}$. Recent work [16–18] has studied the shear-driven drainage of oil trapped in longitudinal grooves in the limit $N \ll 1$. The lubricant depletion in LIS with both regular geometries and random roughness under different viscosity ratios has also been reported experimentally [19], where large $N$ has been found to delay the depletion of the lubricants. However, how $N$ influences the shear-driven drainage of LIS has not been studied quantitatively.

In this study, we first explore the viscosity ratio dependent shear-driven drainage of LIS experimentally in §II and report a steady-state of the lubricant length, $L_\infty$, which depends on the external flow rate or shear stress and the viscosity ratio $N$. Then in §III, we develop an analytical model to understand the experimental data. We first use scaling arguments and a force balance to provide a scaling of $L_\infty$ (see III A) and we then solve a model of
FIG. 1. (a) Side view of the experimental setup. A glycerol and water solution (blue) with viscosity $\mu_{\text{ext}}$ flows into the microfluidic channel through the inlet port and exists through the outlet slot, draining the oil (green) with viscosity $\mu_{\text{int}}$ out of the microgrooves at the top of the channel. The oil is connected to a reservoir of oil at the cell terminus in order to avoid overflow cascades. The drainage of oil in the grooves is observed under a LED light with a vertically-mounted camera. (b) Top view of the microfluidic device. The rectangle with the dotted edges is the area covered by fifty streamwise microgrooves. The green lines with length $L(t)$ represent the part of the grooves filled with oil. (c) A schematic of two of the fifty streamwise microgrooves (width $w$, spacing $d$ and height $h$) filled with oil. The direction of longitudinal external flow is shown by the arrow.

the flow field in detail and obtain an explicit expression of $L_\infty$ (see III B). The results of the scaling arguments and those of the detailed theory are consistent. In §IV, we show that there is good agreement between theoretical and experimental results and discuss the applications of the current model for the drag reduction purpose.
II. EXPERIMENTS

A. Protocol

We study experimentally the effects of viscosity ratio on the failure of liquid-infused surfaces by using the microfluidic device design shown in Fig. 1. The configuration is the same as used in our previous work on shear-driven drainage [16–18]. Figs. 1(a) and (b) show, respectively, the side and the top views of the microfluidic device, which has a main chamber with height $H = 180 \mu m$, width $W = 7 \text{ mm}$ and length $45 \text{ mm}$. There are fifty streamwise microgrooves with height $h = 10.0 \mu m$, width $w = (9.0 \pm 0.2) \mu m$, length $L_0 = 33 \text{ mm}$ and spacing $d = 10.0 \mu m$ at the the center of the top side of the main chamber (see Fig. 1(c)). Norland Optical Adhesive (NOA 81), a UV-curable epoxy, is used to construct both the main chamber and the microstructures, by using the microfluidic sticker technique [20]; the epoxy’s large rigidity meets the requirement that the device be capable of sustaining large pressures.

This geometry provides a means to theoretically study the drainage behavior of liquid-infused surfaces. Here, the infused liquid in the experiments is silicone oil (Gelest PDM-7040) with viscosity, $\mu_{int} = 42.7 \text{ mPa}\cdot\text{s}$, and density, $\rho_{int} = 1.06 \times 10^3 \text{ kg/m}^3$, to which is added a small amount of fluorescent dye (Tracer Products TP-4300) for visualization. In order to achieve various viscosity ratios,

$$N = \frac{\mu_{ext}}{\mu_{int}},$$

aqueous glycerol solutions with different concentrations are used as the external fluids. Table I lists the four external fluids we use and their measured properties. The significant change of the viscosity $5.4 \text{ mPa}\cdot\text{s} \leq \mu_{ext} \leq 1.0 \times 10^3 \text{ mPa}\cdot\text{s}$ provides a wide range of viscosity ratios $0.13 \leq N \leq 23$.

To initialize the experiment, we first close the outlet slot and from the inlet port fill the main chamber and the microgrooves with silicone oil. Then using a syringe pump (Harvard Apparatus, PHD-2000), the aqueous glycerol solution is pumped into the device through the inlet port with flow rate $Q = 2.5 \mu \text{L/min}$, slowly washing the oil in the main chamber out the filling port, which only leaves the oil trapped in the microgrooves. After the glycerol-oil interface reaches the midpoint between the outlet slot and the filling port, the filling port is closed and the outlet is opened so that the external flow is redirected to the outlet and a
FIG. 2. Steady-state length $L_\infty$ under various external flow rates $Q$ with four different viscosity ratios, $N = 0.13$ (red diamonds), $N = 1.2$ (blue circles), $N = 4.2$ (green triangles), $N = 23$ (orange squares). The results show $L_\infty$ is inversely proportional to $Q$.

A reservoir of oil appears at the cell terminus (see Figs. 1(a,b)). This step avoids the overflow cascades of oil at the groove end [18].

We then increase the flow rate of the aqueous glycerol solutions and thus the shear stress applied on the glycerol-oil interface (see Fig. 1(c)). Using a Nikon AF Micro-Nikkor 200 mm lens and a Tiffen Yellow filter, we observe the drainage of oil in the grooves under a LED light with peak wavelength 395 nm. The length of the oil $L(t)$ is measured from the upstream end, which is defined as the location with strongest intensity contrast under the macroscopic view, to the outlet slot (see Figs. 1(a,b)). Even though $L(t)$ varies between different grooves, the variation is less than 10 $\mu$m, which is less than 10% of the smallest $L(t)$ in our experiments.

B. Results

We made systematic measurements of the length of the oil in the grooves as a function of the flow rate and time. We observe that $L(t)$ first decreases and then reaches a steady-state length $L_\infty$. For a single experiment, we fix the flow rate and track both $L(t)$ and $L_\infty$. After the steady state is reached, the flow rate is increased stepwise and $L_\infty$ is recorded as a function of the external flow rate $Q$ and the viscosity ratio $N$.

We first report the raw data of $L_\infty$ versus $Q$ under four viscosity ratios $N$ as shown in
TABLE I. The measured properties and calculated parameters of the four different external fluids used in the experiments.

<table>
<thead>
<tr>
<th>wt% of aqueous glycerol solution</th>
<th>µ_{ext} (mPa·s)</th>
<th>N = µ_{ext}/µ_{int}</th>
<th>p_{ext} (×10^3 kg/m^3)</th>
<th>γ (×10^{-3} N/m)</th>
<th>θ_r (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50%</td>
<td>5.4</td>
<td>0.13</td>
<td>1.13</td>
<td>28.2</td>
<td>32 ± 2</td>
</tr>
<tr>
<td>80%</td>
<td>50</td>
<td>1.2</td>
<td>1.21</td>
<td>29.5</td>
<td>28 ± 2</td>
</tr>
<tr>
<td>90%</td>
<td>1.8 × 10^2</td>
<td>4.2</td>
<td>1.24</td>
<td>30.0</td>
<td>29 ± 2</td>
</tr>
<tr>
<td>100%</td>
<td>1.0 × 10^3</td>
<td>23</td>
<td>1.26</td>
<td>24.0</td>
<td>30 ± 2</td>
</tr>
</tbody>
</table>

Fig. 2. Two conclusions can be drawn: First, $L_\infty$ is approximately inversely proportional to $Q$ for a specific viscosity ratio $N$. Second, $L_\infty$ depends on $N$: if $\mu_{int}$ is constant, for a given flow rate $Q$, $L_\infty$ is larger for lower viscosity ratios $N$. The latter observation is reasonable because less viscous external fluid applies a lower shear stress, which leads to a smaller drainage.

III. MODELING THE DRAINAGE

In order to understand our experimental data, we first use scaling arguments to provide physical insight and then we develop a theoretical model to solve the problem in detail. The shear stress applied by the external flow and the external pressure gradient drive the drainage of the oil, while the interface changes shape in the streamwise direction so that there is surface tension-generated pressure gradient, i.e., Laplace pressure gradient, that drives a backflow. Thus, the steady-state length of the oil in a groove can be considered to arise from a balance among the effects of the shear stress, the external pressure gradient and the surface tension. Next, we model each of these forces.

A. Scaling

The pressure-driven flow problem is three-dimensional because of the distribution of fluid-solid and fluid-fluid boundaries along one wall, as shown in Fig. 3(a). In order to address this configuration, we look at a representative velocity profile at the middle of a groove, i.e. $z = 0$, sketched in Fig. 3(b). We note that the external velocity profiles sketched in
FIG. 3. (a) A schematic of the cross section of the channel. (b) The pressure-driven velocity profile of the external flow and the infused liquid at the center of a groove, i.e. $z = 0$. (c) A close-up of the velocity profile near the fluid-fluid interface at $z = 0$, where the internal flow is consisted of a downstream flow driven by the external flow and a backflow driven by the Laplace pressure gradient. We note that the external velocity profiles sketched in (b) and (c) are based on the experimental results of Schäffel et al. [21] for pressure-driven flow over a superhydrophobic surface.

(b) are from the experimental results of Schäffel et al. [21] for pressure-driven flow over a superhydrophobic surface. We are interested in the limit $w/H \ll 1$. In this limit, the external flow can be divided into two parts (see Fig. 3(a,b)): I) an inner region influenced by the slip at the fluid-fluid interface, with height denoted $h_1 = O(w)$ [23]; II) an outer region without the influence of the slip, with height $H - h_1 \simeq H$. The reason that $h_1$ is of the order of $w$ is that the inner region, i.e., the perturbation of the external flow, is caused by the slip at the interface with width $w$. Hence, we also choose $w$ as the characteristic length scale of $h_1$.

Region I and the internal flow inside the groove constitute a slip-influenced region as shown in the red rectangular box in Fig. 3(b). We assume that region II applies a shear stress $\tau_{\infty}$ at the boundary between the two regions and a pressure gradient $\partial p_e/\partial x$ to the slip-influenced region. We estimate $\tau_{\infty} \approx H \frac{\partial p_e}{\partial x}$ (see Appendix A).
1. Shear stress at the fluid-fluid interface

Let $z$ be the direction transverse to the flow direction. In order to determine the shear-driven flow, it is necessary to obtain the shear stress at the fluid-fluid interface, $\tau_s(z)$, which varies across the groove. For simplicity, we take $\tau_s(0)$ as the characteristic interfacial shear stress, $\tau_s$.

We assume the velocity in Fig. 3(b,c) at the top boundary of region I is $u_\infty$, the slip length of the external flow is $\beta$ and that of the internal flow is $b$ [22]. From velocity and shear stress continuity at the fluid-fluid interface, we find $\beta = Nb$ [22]. The scaling of the shear stresses are given by $\tau_\infty \sim \mu_{ext} u_\infty / h_1$ and $\tau_s \sim \mu_{ext} u_\infty / (h_1 + \beta)$. If we rewrite $h_1 = w/a$, where $a$ is a constant, and define the normalized maximum slip length $D = b/w$, we can get the relation between the two shear stresses by combining the three equations above:

$$\tau_s \sim \frac{1}{aDN + 1} \tau_\infty .$$  \hspace{1cm} (2)

2. Force balance

In the steady state of the length of the oil, we expect a force balance between the surface tension and the sum of shear stress and the external pressure gradient, i.e. $\gamma w L_\infty \sim \phi_1 \frac{\tau_s}{h} + \phi_2 \frac{\partial p_e}{\partial x}$, where $\phi_1$ and $\phi_2$ are dimensionless coefficients considering the ratios of different force terms. Combining with $\partial p_e / \partial x \simeq 2 \tau_\infty / H$, we obtain the characteristic steady-state length,

$$L_\infty \approx \frac{aDN + 1}{2\phi_2 h H aDN + \phi_1 + 2\phi_2 h H \tau_\infty w} \frac{\gamma h}{\tau_\infty w}.$$ \hspace{1cm} (3)

$$= f(N) \frac{\gamma h}{\tau_\infty w} .$$ \hspace{1cm} (4)

The scaling arguments demonstrate that $N$ influences $L_\infty$ by introducing a prefactor, $f(N)$, as shown in Eq. (4). In the next subsection, we will show that the constants $a$, $\phi_1$ and $\phi_2$ depend on geometry and the wettability of the solid substrate by the two liquids. These constants can be obtained by solving for the flow field.

B. Theory

The flow of the oil inside the grooves is studied in this subsection (see Figs. 4(a, b)) and a general expression for the steady-state length is given. The total flow in the groove
FIG. 4. (a) The schematic of one groove, showing the morphology of the interface and the recirculation of the oil in the groove. At the upstream end of the groove wetted by oil, the radius of curvature of the interface reaches its smallest value $r_{\text{min}}$ and the deformation reaches its largest value $\delta_{\text{max}}$. (b) The total velocity profile of the oil at the center of the groove and its decomposition: flow caused by the slip velocity $u_{sl}$ at the interface, by the pressure gradient in the external flow $\partial p_e/\partial x$, and by the Laplace pressure gradient $\partial p_L/\partial x$.

FIG. 5. (a) Confocal microscope image of the groove cross-section. Pure glycerol (black at top part of the images) flows over silicone oil (red area) at rate $Q = 100 \mu\text{L/min}$. The straight green line is the flat substrate between two grooves and the margin between the black and red represents the interface concave into the groove with a deformation, $\delta(x)$ and radius of curvature $r$. The receding contact angle of the oil, $\theta_r$ is the angle between the interface and the vertical groove wall. (b) The steady-state deformation of the fluid-fluid interface at the center of the groove, $\delta(x)$, measured along the longitudinal direction of the groove. The results show that $\delta(x) \propto x$ for $N = 0.13$ (red diamonds), 1.2 (blue circles), 4.2 (green triangles) and 23 (orange squares) under flow rate $Q = 2000, 500, 200$ and 100 $\mu\text{L/min}$ respectively.
consists of two distinct contributions, one from a slip velocity and a second from the pressure distribution, \( p_{int} \), in the trapped liquid. As discussed more below, an approximate normal stress balance yields the pressure contributions in the liquid-filled groove,

\[
p_{int} = p_e + p_L ,
\]  

(5)

where \( p_L \) is the Laplace pressure given by,

\[
p_L = -\frac{\gamma}{r} .
\]  

(6)

Here, \( r \) is the radius of curvature of the interface and \( \gamma \) is the interface tension.

Thus, we find it convenient to discuss the total flow consisting of three components: 1) the flow driven by the slip velocity at the interface, with a flux denoted as \( q_{sl} \); 2) the flow driven by the pressure gradient caused by the external flow, with a flux denoted as \( q_{pe} \); 3) the back flow driven by the Laplace pressure gradient caused by surface tension and the deformation of the glycerol-oil interface, with a flux denoted as \( q_{pL} \).

1. **External flow**

The first two fluid fluxes of the trapped oil result from the external flow, which we solve for first and use the results to determine the internal flow. As mentioned above, the external flow can be treated as a channel flow. Given \( W \gg H \), we estimate the effective Reynolds number, \( Re = \left( \frac{H}{W} \right) \left( UH\rho_{ext}/\mu_{ext} \right) \), where the characteristic velocity is \( U = Q/WH \) and the density \( \rho_{ext} \approx 1.0 \times 10^3 \text{kg/m}^3 \). The largest Re corresponds to \( \mu_{ext} = 5.4 \text{mPa-s} \) and \( Q = 7500 \mu\text{L/min} \), which yields \( Re \leq 0.085 \). The influence of the slip on the external flow can be neglected (see Appendix A). Hence, the external flow can be determined as a Hele-Shaw flow, yielding the external pressure gradient in terms of the flow rate \( Q \),

\[
\frac{\partial p_e}{\partial x} = -\frac{12\mu_{ext}Q}{WH^3} .
\]  

(7)

The corresponding wall shear stress of the model flow is

\[
\tau_\infty = \frac{6\mu_{ext}Q}{WH^2} .
\]  

(8)
2. Slip-velocity-driven flux and external pressure-driven flux

Now we use the above results to obtain \( q_{sl} \) and \( q_{pe} \). We consider a surface periodically patterned with grooves filled with lubricant under shear of a streamwise external flow with shear stress \( \tau_\infty \). As shown in Fig. 1, the slipping interface fraction of the surface is \( \alpha = w/(w + d) \) and the aspect ratio of the groove is \( A = h/w \).

The oil flow inside the grooves satisfies a low Reynolds number flow condition (see Appendix C). By solving for the flow in a rectangular groove (see Appendices C 1 and C 2), the slip-driven flux and the external pressure-driven flux are expressed by

\[
q_{sl} = \frac{2D}{1+2DN} \frac{c_{sl}w^3\tau_\infty}{\mu_{int}} \quad \text{and} \quad q_{pe} = -\frac{c_pwh^3}{\mu_{int}} \frac{\partial p_e}{\partial x},
\]

where \( c_{sl} \) and \( c_p \) are two dimensionless geometric parameters depending only on \( A \), and \( D = b/w \) is the normalized maximum local slip length depending on \( A \) and \( \alpha \). Note that Eq. (9a) is equivalent to \( \tau_s \simeq \tau_\infty/(1 + 2DN) \), which is consistent with the result in the scaling arguments if \( a = 2 \) in Eq. 2 and is also consistent with the detailed analysis given by Schönecker and Hardt [22].

3. Laplace pressure-driven flux

To better clarify the Laplace pressure-driven flow, a schematic of the morphology of the glycerol-oil interface is shown in Fig. 4(a). Also, a representative confocal microscope image of the cross section of a groove is shown in Fig. 5(a), where the red area represents the fluorescent oil and the black area above represents the aqueous glycerol solution. The interface shape is approximately an arc of a circle with radius \( r = (w/2) \sec \theta \), where \( \theta \) is the angle of the oil on the epoxy in the environment of the aqueous glycerol solution. The deformation of the curved interface, denoted as \( \delta(x) \), is measured with a confocal microscope along the longitudinal direction, \( x \) (see Fig. 5(b)). A concave fluid-fluid interface into the groove corresponds to \( \delta > 0 \). The results show that for all of the aqueous glycerol solutions, \( \delta(x) \) varies approximately linearly with \( x \), i.e. \( \delta \propto x \), which means the interface is flat at the downstream end of the groove and increasingly concave into the groove in the upstream direction, i.e. \( r(x) \) increases with \( x \). At the upstream end of the oil, \( \delta(x) \) reaches a maximum \( \delta_{max} \), and \( r \) and \( \theta \) reach their minima, respectively, \( r_{min} \) and \( \theta_r \), the receding contact angle,
which represents a material property of the solid substrate. For a given geometry, \( r_{\text{min}}, \delta_{\text{max}} \) and \( \theta_r \) are related by the equation

\[
r_{\text{min}} = \frac{w \sec \theta_r}{2} = \frac{\delta_{\text{max}}^2 + \left(\frac{w}{2}\right)^2}{2\delta_{\text{max}}}.
\] (10)

Thus, the Laplace pressure \( p_L \), given by Eq. (6), increases with \( x \). The positive Laplace pressure gradient, \( \partial p_L / \partial x \), drives a back flow and helps retain the oil within a groove.

The Laplace pressure-driven flux can be described similar to Eq. (9b) except using \( p_L(x) \) as the pressure distribution,

\[
q_{pL} = -\frac{c_p w h^3 \partial p_L}{\mu_{\text{int}}} \frac{\partial x}{\partial x}.
\] (11)

Thus, we integrate Eq. (11) along \( x \) by assuming \( q_{pL} \) is constant, use Eqs. (6, 10, 11) and assume a linear dependence of \( \delta(x) \) on \( x \), as observed in the experiments (see Appendix C 3), to obtain an expression for the Laplace pressure-driven flux,

\[
q_{pL} = -\frac{\chi c_p w h^2 \gamma}{\mu_{\text{int}} L}.
\] (12)

Here, the dimensionless prefactor \( \chi \) depends on the geometry of the groove’s cross section and the wettability of the solid substrate by the two liquids. In particular, \( \chi \) is given by

\[
\chi = \frac{2\delta_{\text{max}}}{h^2 r_{\text{min}} \left( (h - \delta_{\text{max}})^{-2} - h^{-2} \right)^{-1}}.
\] (13)

4. Steady-state length of the trapped liquid

When a steady state is established, the total flux of the oil is zero, i.e.,

\[
q_{sl} + q_{pc} + q_{pL} = 0.
\] (14)

The steady-state length of the oil, \( L_\infty \), is obtained by solving Eq. (14) combined with Eq. (9) and (12), yielding

\[
L_\infty = \frac{\chi \gamma}{h} \left[ \frac{2D}{1 + 2DN} \frac{c_{sl}}{c_p} \left( \frac{w}{h} \right)^2 \frac{\tau_\infty}{h} - \frac{\partial p_c}{\partial x} \right]^{-1}.
\] (15)

The two terms in the square brackets of Eq. (15) represent the shear-driven drainage and the pressure-driven drainage respectively. Note as both \( \tau_\infty \) and \( \partial p_c / \partial x \) are proportional to \( Q \), we obtain \( L_\infty \propto 1/Q \), which agrees with the experimental results shown in Fig. 2(a).
IV. EXPERIMENTAL MEASUREMENTS, THEORY AND DISCUSSIONS

In this section, we compare the theoretical and experimental results and show that there is good agreement. We start by substituting Eqs. (7-8) into (15), and after reorganization obtain an expression for the steady-state length in the form of Eq. (4),

$$L_\infty \sim \frac{aDN + 1}{2\phi_2 h aDN + \phi_1 + 2\phi_2 k \tau_\infty w} \frac{\gamma h}{\tau_\infty w},$$

(16)
where

\[ a = 2, \quad \phi_1 = \frac{c_w}{c_p} \frac{2Dw}{\chi h} \] and \[ \phi_2 = \frac{h}{\chi w}. \]  
(17a, b, c)

In order to isolate the effects of \( \tau_\infty \) and \( N \) on \( L_\infty \), we plot both \( L_\infty/f(N) \) versus \( \tau_\infty \) (see Fig. 6(a)), where \( f(N) \) is defined in Eq. (4), and \( L_\infty (\tau_\infty w)/ (\gamma h) \) versus \( N \) (see Fig. 6(b)). In Fig. 6(a), the data for different \( N \) all collapse onto the same line (dashed black), which represents the theoretical prediction of Eqs. (16 - 17). Since \( \tau_\infty \propto Q \), the results are consistent with \( L_\infty \propto Q^{-1} \). The results shows that higher shear stress or an external pressure gradient causes more drainage if other variables are fixed. We observe that there is good agreement between the theoretical predictions and the experimental results, which verifies the current model.

Next, we emphasize the effect of the viscosity ratio, \( N \), and remove the effects of shear stress and external pressure gradient by introducing the normalized steady-state length \( L_\infty (\tau_\infty w)/ (\gamma h) \). The results are plotted versus \( N \) in Fig 6(b). The theoretical predictions and the experimental data are denoted respectively by the solid blue curve and the red squares. Clearly, there is excellent agreement.

In addition, we also consider the limit when \( N \ll 1 \), i.e. the external fluid is much less viscous than the internal fluid such as air flowing over water. For this case, the fluid-fluid interface can be assumed as a no-slip condition for the external flow and flow inside the groove is solved in Wexler et al. [16], which yields

\[ L_\infty \frac{\tau_\infty w}{\gamma h} = \frac{c_p^*}{c_{sh} r_{\min}} \frac{h}{r_{\min}}. \]  
(18)

Here, \( c_p^* \) and \( c_{sh} \) are two dimensionless parameters depending only on the groove aspect ratio, \( A \). In our case, for \( A = 1 \), \( c_p^* \approx 0.0572 \) and \( c_{sh} \approx 0.0762 \). The prediction from Eq. (18) is represented by the dashed green line in Fig. 6(b). When \( N \to 0 \), the solid blue line and the dashed green line are of the same magnitude, differing by a constant prefactor about 1.5. The difference arises because the current model (solid blue line) employs the slip-velocity condition at the upper boundary of the internal flow in the groove, while the asymptotic model of Eq. (18) (dashed green line) uses the shear stress condition there.

The results in Fig. 6 show that for constant \( \mu_{ext} \) and \( \tau_\infty \), \( L_\infty \) increases with \( N \) if other variables are fixed. The detailed explanation comes from a force balance: the shear stress at the interface, \( \tau_s \), decreases with increasing \( N \), as shown in Eq. (2), thus a lower Laplace
pressure gradient, $\partial p_L/\partial x$, is needed to balance the shear stress and the external pressure gradient. Since $\partial p_L/\partial x \sim \gamma/wL_\infty$, a higher maximum retention length $L_\infty$ is obtained.

V. CONCLUSION

We have demonstrated that the maximum lubricant retention length $L_\infty$ of a LIS with longitudinal grooves depends on the viscosity ratio $N$. In particular, we obtained an analytical viscosity ratio-dependent expression for $L_\infty$ by considering both the shear stress and the external pressure gradient. The manner in which $N$ influences $L_\infty$ is via the drainage driving force, which refers to the shear stress at the fluid-fluid interface, $\tau_s \simeq 1/(1+2DN)\tau_\infty$, where $\tau_\infty$ is the macroscopic wall shear stress. In addition, similar lubricant retention behavior has been experimentally observed for surfaces with some other geometries, such as those with random pillars, for small $N$ [16]. Therefore, we expect that for any $N$, there is always some lubricant retained in other surfaces geometries.

The results give guidance on the applications of LIS in two aspects: first, a higher shear stress or external pressure gradient results in lower lubricant retention; second, for a given external fluid and a fixed shear stress $\tau_\infty$, the less viscous lubricants achieve higher retention because $\tau_s$ decreases when $N$ increases. However, there is a limitation that large $N$ can cause an instability of the fluid-fluid interface, which also triggers the failure of LIS.

One of the limitations of the current model is that it is developed for steady viscous flow. Whether and how it can be applied to turbulent flow, which is highly unsteady, remains to be answered. Additional topics include relating $\tau_\infty$ in the current viscous flow model to the mean wall shear stress, $\tau_w$, in turbulent flows. In addition, the fluctuations in turbulent flows might trigger another failure mechanism of LIS via the instability of the interface. In such cases, it will be necessary to understand the effects of the Weber number, which is the ratio of the inertial to surface tension effects, on the instability-driven failure and the conditions under which either failure mechanism dominates. Finally, future studies should probe the influence of flow on superhydrophobic surfaces, under the effects of the pressure, shear and other perturbations. It will be worthwhile to study these failure mechanisms of superhydrophobic surfaces in both laminar and turbulent regimes.
VI. ACKNOWLEDGEMENTS

This work was supported under Office of Naval Research (ONR) Multidisciplinary University Research Initiative (MURI) Grants N00014-12-1-0875 and N00014-12-1-0962 (Program Manager Dr. Ki-Han Kim), and funding from German Academic Exchange Service (DAAD). We thank Dr. Z. Zheng for helpful advice on the model, Prof. A. J. Smits, Prof. M. Hultmark and all the other team members of our MURI project for their valuable feedback.

Appendix A: The effects of the slip on the external flow

The macroscopic effects of heterogeneous slip and no-slip boundary condition on the external flow can be considered by introducing an effective slip length, $\beta_{\text{eff}}$, which can be obtained from Schönecker et al. [23] for the groove aspect ratio, periodicity of the grooves and fluid interface fraction as a function of the viscosity ratio of the two fluids. Hence, the far field external flow in the grooved area in our experiments can be considered as that over a homogeneous slip surface with slip length $\beta_{\text{eff}}$.

With the width of the main channel being much larger than its height, i.e. $W \gg H$, the flow rate through this channel can be approximated as the sum of the flow over the grooved part (width $W_2 = 1$ mm) and over the ungrooved parts (width $W_1 = 6$ mm):

$$Q = \frac{1}{2\mu_{\text{ext}}} \frac{\partial p_e}{\partial x} \left( \frac{H^3}{3} - \frac{H^4}{2(H + \beta_{\text{eff}})} - \frac{H^3\beta_{\text{eff}}}{H + \beta_{\text{eff}}} \right) W_2 - \frac{H^3W_1}{6}. \quad (A1)$$

On the ungrooved part, the flow is a standard Poiseuille flow with a no slip condition at the upper and lower walls. On the grooved part, there is an effective slip length $\beta_{\text{eff}}$.

As mentioned in §III A above, the external flow can be divided into a slip-influenced inner region with height $h_1$ and an outer region without the influence of slip. In the spirit of an asymptotic expansion, we have for the velocity in the outer region

$$u_{\text{out}} = \frac{1}{2\mu_{\text{ext}}} \frac{\partial p_e}{\partial x} \left( y^2 - \frac{H^2}{H + \beta_{\text{eff}}} (y + \beta_{\text{eff}}) \right), \quad (A2)$$
corresponding to a Poiseuille flow with an effective-slip condition at the lower wall and a no-slip condition at the upper wall.

In the inner region, $\tau_\infty$ is approached as $y \to h_1$ (or $y \to \infty$ if using an inner coordinate system). The corresponding matching condition for this shear stress coming from the outer
region can be written as
\[ \tau_\infty = \mu_{\text{ext}} \frac{\partial u_{\text{out}}}{\partial y} \bigg|_{y=0} = -\frac{H^2}{2(H + \beta_{\text{eff}})} \frac{\partial p_e}{\partial x}. \] (A3)

The expressions for \( \beta_{\text{eff}} \) for closed longitudinal grooves [23] are used, since this corresponds to the final state with the oil recirculating in the grooves. In our experiments, we have \( \beta_{\text{eff}} \) ranging between 2 \( \mu \)m for \( N = 23 \) and 0.1 \( \mu \)m for \( N = 0.13 \). Since \( \beta_{\text{eff}} \ll H \), we can simplify (A1) and (A4) to
\[ Q = \frac{H^3 W}{12 \mu_{\text{ext}}} \frac{\partial p_e}{\partial x}, \quad \text{and} \quad \tau_\infty = -\frac{H}{2} \frac{\partial p_e}{\partial x}. \] (A5a, b)

From Eq. (A1), we can also evaluate the friction reduction, which is defined as the relative reduction of the pressure drop for a channel with LIS compared with that of a channel without LIS under a fixed flow rate \( Q \). The friction reduction for different viscosity ratios \( N \) is shown in Figure 7. We can see that the friction reduction increases with the viscosity ratio. The maximum friction reduction is 0.47% corresponding to largest \( N \approx 23 \). In addition, since only small ratio of the bottom wall of the channel is covered with LIS and the other part is no-slip surface, the response is very nearly an effective no-slip boundary condition and the friction reduction is quite small in the configuration considered here.

**Appendix B: Slip velocity**

The velocity field of a fluid flowing over a plate with a longitudinal finite-slip groove and that with a regular array of longitudinal finite-slip grooves are studied in previous work [22, 23]. The velocity at a finite-slip interface can be expressed as that at a perfect-slip interface [24] times a prefactor depending on the viscosity ratio \( N \) and the normalized maximum local slip length, \( D = b/w \), as
\[ u_{\text{sl}}(z) = \frac{2DN}{1 + 2DN} \frac{w\tau_\infty}{2\mu_{\text{ext}}} \sqrt{1 - \left( \frac{2z}{w} \right)^2}. \] (B1)

Here, we use the elliptic shape of the slip velocity in the single groove case but take \( D \) from a study of the periodic geometry [23]. This approach allows for an analytic solution of the slip-velocity driven flow. At the same time, it is also a reasonable approximation of both the shape and the amplitude of the velocity profile in the periodic case. In order to
show the latter claim, we compare the velocity in the current simplified model calculated by Eq. (B1) with that of the periodic case (dashed red) [23] (see Fig. 8). The results show that Eq. (B1) underestimates the velocities at the interface between 1% and 9% for the lowest to the highest viscosity ratios in our experiments. The comparison of the slip velocities at the center of the groove for $N = 3.7$ is shown in Fig. 8, where the solid line and the dashed line represent respectively the result of Eq. (B1) and that for the periodic case [23]. The good agreement between the different results verifies the reasonableness of our assumption.

We note that the normalized maximum local slip length $D$ is only a geometric parameter depending on $\alpha$ and $A$ [23]. In our case (longitudinal closed grooves with backflow, $A = 1$, $\alpha = 0.5$), $D \simeq 0.217$.

### Appendix C: Flow in a groove

Now, we analyze the flow of the infused liquid inside a groove. Given the Bond number, $\text{Bo} = gw^2(\rho_{\text{ext}} - \rho_{\text{int}}) / \gamma \sim O(10^{-6})$, and the Reynolds number of the internal flow, $\text{Re} = UL\rho_{\text{int}}/\mu_{\text{int}} \leq 0.023$, we can neglect the gravitational effect and use Stokes’ equation to
FIG. 8. The comparison of the slip velocity of the current model, \( u_{sl}(z) \) (Eq. (B1)) (solid), and that of periodic grooves when, \( u_{slp}(z) \) [23] (dashed) \( \alpha = 0.5 \), on the half of the interface when \( N = 3.7 \).

solve the internal flow.

The flow problem is described in Fig. 9 (a). The total flow profile satisfies the Poisson equation,

\[
\mu_{int} \nabla^2 u = \frac{\partial p}{\partial x},
\]

with a slip velocity given by Eq. (B1) at \( y = h \) and no-slip boundary conditions at the other three walls. The total flow is divided into three parts, which are shown in Fig. 9(b) and (c).

1. **Slip-velocity-driven flow**

The problem setting of the slip-driven flow is shown in Figure 9(b). The flow satisfies the Laplace equation \( \mu_{int} \nabla^2 u = 0 \) subject to no-slip boundary conditions at the side and bottom walls and the slip condition (B1) at the top surface. We use separation of variables to write the streamwise velocity, \( u(y, z) \), as a Fourier series,

\[
u(y, z) = \frac{2DN}{1 + 2DN} \sum_{n=0}^{\infty} a_n \cos \left( \frac{2\lambda_n z}{w} \right) \sinh \left( \frac{2\lambda_n y}{w} \right),
\]

where

\[
\lambda_n = \pi \left( \frac{1}{2} + n \right),
\]

and

\[
a_n = \frac{\pi \tau_{\infty}}{2\mu_{ext}} \frac{J_1(\lambda_n)}{\lambda_n \sinh \left( \frac{2\lambda_n h}{w} \right)}.
\]
FIG. 9. (a) The scheme of the total longitudinal flow in a groove shown in a cross-section of the groove. (b) Slip-driven flow with no pressure gradient. (c) Pressure-driven flow with no-slip condition on all boundaries.

By integrating the velocity \((C2)\) over the cross-section of the groove, we obtain an expression of the slip-driven flux,

\[
q_{sl} = \frac{2D}{1 + 2DN} \frac{c_{sl} \tau_{\infty} w^3}{\mu_{int}},
\]

with

\[
c_{sl} = \frac{\pi}{4} \sum_{n=0}^{\infty} \lambda_n^{-3} (-1)^n J_1(\lambda_n) \times \left[ \coth \left( \frac{2\lambda_n h}{w} \right) - \csch \left( \frac{2\lambda_n h}{w} \right) \right].
\]

In our case, \(w/h = 1\), thus \(c_{sl} \approx 0.108\).
2. External pressure-driven flow

The pressure-driven velocity is obtained by solving the Poisson equation (C1) with no-slip condition on all boundaries. Integrating the velocity [25] over the cross-section, we arrive at the expression of pressure-driven flux

\[ q_p = -\frac{c_p w h^3}{\mu_{int}} \frac{\partial p}{\partial x}, \]  
\[ (C7) \]

with

\[ c_p = \frac{1}{12} - \frac{h}{2w} \sum_{n=0}^{\infty} \lambda_n^{-5} \tanh \frac{\lambda_n w}{h}. \]  
\[ (C8) \]

For \( w/h = 1 \), \( c_p \simeq 0.0351 \). The external pressure-driven flux can be obtained by setting \( \partial p/\partial x = \partial p_e/\partial x \) in Eq. (C7).

3. Laplace pressure-driven flow

Note that Eq. (C7) is valid for any pressure-driven flow, including the Laplace-pressure-driven flow. However, because the \( \partial p_L/\partial x \) is not constant along \( x \), we have to solve \( q_{pL} \) by integrating Eq. (C7) from \( x = -L \) to \( x = 0 \) with the assumptions, \( \delta(x) = -\delta_{max} x/L \) and \( h(x) = h - \delta(x) \), which leads to

\[ \Delta p = \frac{1}{c_p \chi w h^2 r_{\min}} \frac{\mu_{int} q_{pL} L}{}, \]  
\[ (C9) \]

where \( \chi \) is given by Eq. (13).

The interface is flat downstream, so the Laplace pressure difference between the upstream and the downstream is given by \( \Delta p = \gamma/r_{\min} \). Thus, we get the back flow driven by Laplace pressure

\[ q_{pL} = -\frac{c_p \chi w h^2 \gamma}{\mu_{int} L}. \]  
\[ (C10) \]


