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Crackling to periodic dynamics in granular media

Aghil Abed Zadeh* and Robert P. Behringer

Department of Physics & Center for Non-linear and Complex Systems, Duke University, Durham, NC, USA

Jonathan Barés†

*Department of Physics & Center for Non-linear and Complex Systems, Duke University, Durham, NC, USA and
Laboratoire de Mécanique et Génie Civil, Université de Montpellier, CNRS, Montpellier, France*

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We study the local and global dynamics of sheared granular materials in a stick-slip experiment, using a slider and a spring. The system crackles, with intermittent slip avalanches, or exhibits irregular or periodic dynamics, depending on the shear rate and loading stiffness. The global force on the slider during shearing captures the transitions from the crackling to the periodic regime. We deduce a novel dynamic phase diagram as a function of the shear rate and the loading stiffness and associated scaling laws. Using photoelastic particles, we also capture the grain-scale stress evolution, and investigate the microscopic behavior in the different regimes.

I. INTRODUCTION

Sheared amorphous materials yield and flow, when sufficiently loaded [1–3]. The flow can be spatially heterogeneous and erratic in time. This intermittent behavior has been observed in phenomena as diverse as seismicity [4–6], fracture [7–9], damage [10, 11], friction [12, 13], plasticity [14, 15], magnetization [16, 17], wetting [18, 19], neural activity [20, 21] and granular avalanches [22–26].

In granular media, the intermittent dynamics, also called ‘crackling’ [27], is associated with the transition between jammed and unjammed states [26, 28] around yielding: when slowly sheared, a granular system may stick and slip, with slip sizes spanning a wide range of scales [29–32]. Meanwhile, under certain conditions, some sheared systems do not obey this crackling behavior. Instead, slips occur ‘periodically’ with a narrow size distribution [33–36]. For other systems, the oscillations may be damped or the grains flow continuously. In addition to safety and industrial challenges to control these dynamics, understanding these different behaviors is crucial in granular physics and many other associated fields, exhibiting similar dynamics.

The crackling response of granular media is of particular interest [2, 26, 37] and the effect of the system’s parameters on the dynamics, their scaling laws and avalanche shapes have been investigated [38–41]. Other studies concentrated on understanding the transition between the periodic stick-slip and steady sliding regimes [36, 42]. However, the transitions from crackling to periodic dynamics have not been studied. Consequently, fundamental issues remain: Do crackling and periodic dynamics arise from fundamentally different systemic behaviors, identifiable at a microscopic scale, or do they occur as bifurcations controlled by system-scale parameters? Are they the only dynamical regimes? What is

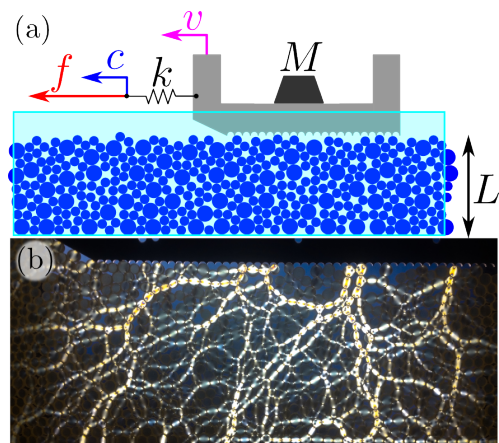


FIG. 1. (color online) (a) Sketch of the experiment from the side: A slider is pulled over a 2D vertical granular bed of bi-disperse photoelastic discs. The slider is pulled at a constant speed c by means of a spring of stiffness k . The pulling force, f , is measured by a force sensor, connected to the spring. The system is lit from behind by polarized light and observed from the front by a fast (120 fps) camera equipped with a crossed polarizer. (b) Photoelastic response of the granular bed during loading.

the interplay between the grain-scale mechanics and the macroscopic system parameters?

The study reported here addresses these questions. Experiments on sheared granular materials reproduce crackling and periodic regimes, with an ‘irregular’ regime in between. Adjusting the driving rate and the system stiffness causes the system to transition between different behaviors, and yields a dynamic phase diagram. The system observables, such as loading force are analyzed in both regimes. We find that the periodic behavior of macroscopic force is accompanied by erratic microscopic stress dynamics and is observed only with finite loading stiffness.

* aghil.abed.zadeh@duke.edu

† jb@jonathan-bares.eu

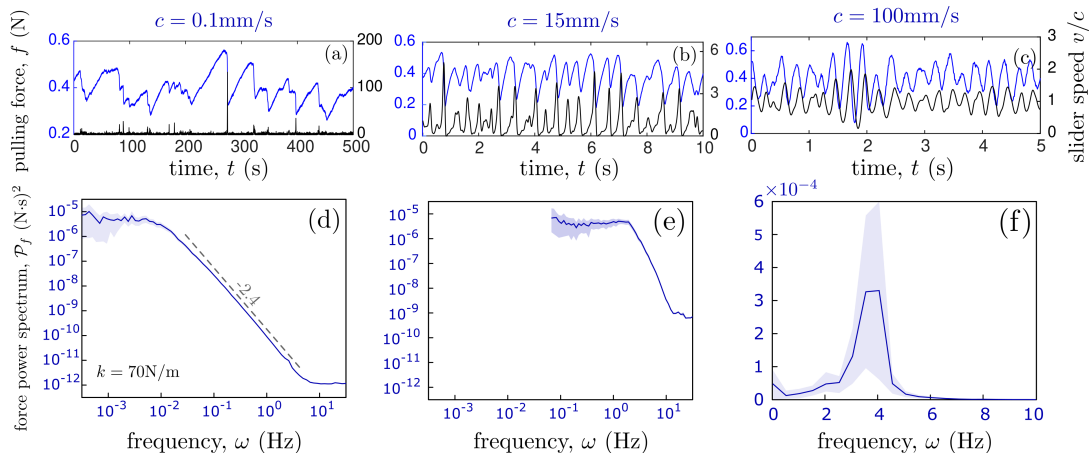


FIG. 2. (color online) Typical signals. Evolution of the slider speed $v(t)$ (black, lower) and pulling force $f(t)$ (colored, upper) for different loading speeds ($c = 0.1, 15$ and 100 mm/s) displaying different dynamics: crackling at low speed (a), irregular at medium speed (b) and periodic at high speed (c). (d),(e),(f): corresponding power spectral density of the force signal \mathcal{P}_f . A dashed line shows a slope corresponding with the exponent -2.4 ± 0.2 fitted on the power-law obtained for $c = 0.1$ mm/s. The shaded areas show the 95% confidence interval of the curves. (d) and (e) are in log-log scale, while (f) is in linear scale. Experiments are carried out with a spring of stiffness $k = 70$ N/m.

II. EXPERIMENTS

The experimental device provides data both at the global and local scales, similarly to [36, 43]. As shown in fig.1(a) [44], a stage pulls a 2D frictional slider of fixed length 25 cm and variable mass M . The stage, which moves at constant speed c , pulls the slider by means of a linear spring of stiffness k , which represents the loading stiffness. The slider rests on a vertical bed of fixed depth $L = 9.5$ cm, and length 1.5 m consisting of bi-disperse cylindrical photoelastic particles with diameters 0.4 cm and 0.5 cm (small/big ratio of 2.7) to avoid crystallization. Unless specified, the experiments are made with a slider of mass $M = 85$ g. The slider+particles system is sandwiched between two dry-lubricated glass plates. The slider bottom is toothed to enhance the friction with the grains. The force, f , applied to the spring, is measured by a sensor at a frequency of 1 kHz. The system is designed to be at constant pressure, and the slider can move in either the horizontal or vertical directions. However, the granular bed is prepared flat enough that it stays mostly horizontal, while pulled. Due to free granular surface, a small pile of granular matter develops in front of the slider. The front of the slider is designed with an angle and without teeth to reduce the force applied by the pile. We also prepare the initial front pile same as the steady state such that the dynamics is stationary. The system is lit from behind by a polarized light source. In front, a camera with a crossed polarizer images the grains and slider at a frequency of 120 Hz (see fig.1(b)). The photoelastic response of the media provides a local measure of the stress from the image intensity I [29]. We analyze the mean response of granular medium as the fast imaging is not accurate enough to provide grain-level stress measurement.

III. RESULTS

Fig.2 shows the evolution of the pulling force, $f(t)$ (colored), and of the slider speed $v(t)$ (black), for three typical c . At low speed (a) $c = 0.1$ mm/s, the system crackles [27, 43]. The slider is immobile most of the time, as it loads up elastically. It then undergoes erratic sudden jumps. These unloading events are accompanied by slider slips and irreversible grain flows. At high speed (c) $c = 100$ mm/s, the slider never stops, and exhibits smooth periodic oscillations. In between (b) $c = 15$ mm/s, most of the time, the slider exhibits slow noisy displacements with irregular jumps, smoother and less intense than in the crackling case. Fig.2(d) shows the power spectral density (PSD) of the force signal \mathcal{P}_f for these three different cases. At low c , above a flat lower cut-off, \mathcal{P}_f follows a power-law spanning more than two decades in ω with exponent -2.4 ± 0.2 , similar to Brownian noise [45]. At a medium c , \mathcal{P}_f is constant for a large range of ω , like white noise, and decays rapidly above a cut-off frequency (≈ 10 Hz). For higher c , \mathcal{P}_f develops a peak with a characteristic frequency which depends on c , and differs from the constant inertial frequency of the system, $\omega_{\text{sys}} = \frac{1}{2\pi} \sqrt{\frac{k}{M}} = 4.6$ Hz.

The system can transit from the *crackling* to *irregular* to *periodic*. Fig.3(a) shows the evolution of \mathcal{P}_f , as c varies, for fixed k [44]. Considering the crackling regime (small c) and ignoring the flat part of the PSD at high frequencies (related to the medium damping), \mathcal{P}_f is fitted to:

$$\mathcal{P}_f(\omega) \propto (1 + \omega/\omega_{\min})^{-\beta} e^{-\omega/\omega_{\max}},$$

where ω_{\min} and ω_{\max} are the lower and upper power-law cut-offs respectively, and $\beta = 2.4 \pm 0.2$. The

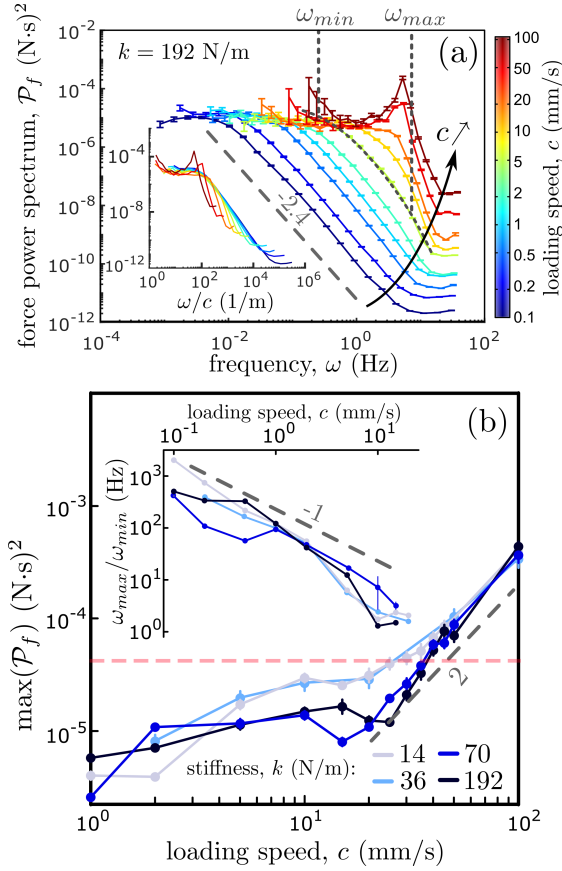


FIG. 3. (color online) (a): Force power spectra $\mathcal{P}_f(\omega)$ for different loading speeds, $c \in [0.1, 100]$ mm/s with $k = 192$ N/m. For $c = 2$ mm/s, \mathcal{P}_f is fitted by a gamma function displayed by a dashed line with lower (ω_{min}) and upper (ω_{max}) cut-off positions. A straight dashed line shows a slope corresponding with the exponent -2.4 fitted to the lowest c curve. Inset: \mathcal{P}_f with frequency scaled by c . (b) Maximum of \mathcal{P}_f as a function of c , for power spectra with a bump. The red horizontal line represents the threshold used to define the periodic regime in Fig. 4. A dashed line with slope 2 is given to guide the eye. Inset: Spreading of the power-law regime (if any) $\omega_{max}/\omega_{min}$ as a function of c . A dashed line with slope -1 is given to guide the eye. Different curves correspond to different k .

number of decades for which \mathcal{P}_f obeys a power-law, $\log_{10}(\omega_{max}/\omega_{min})$, decreases as c increases. This is quantified in the inset of fig.3(b), showing that this number is roughly inversely proportional to c : $\omega_{max}/\omega_{min} \propto 1/c$. Moreover, in the inset of fig.3(a), \mathcal{P}_f curves, except for their upper cut-offs, collapse when ω is scaled by c . This implies that $\omega_{min} = \kappa c$, where $\kappa \approx 10^2$ (1/m) is a characteristic wave number of the system, and independent of c . For higher c , a bump appears close to ω_{min} . This indicates the onset of oscillations at a frequency ω_c . The inset of fig.3(a) shows that, unlike ω_{min} , ω_c does not scale with c , since the peaks do not collapse. Fig.3(b) shows the value of \mathcal{P}_f at the peak. This scaling quantifies the oscillation strength.

Fig.4(a) presents the phase diagram that quantifies

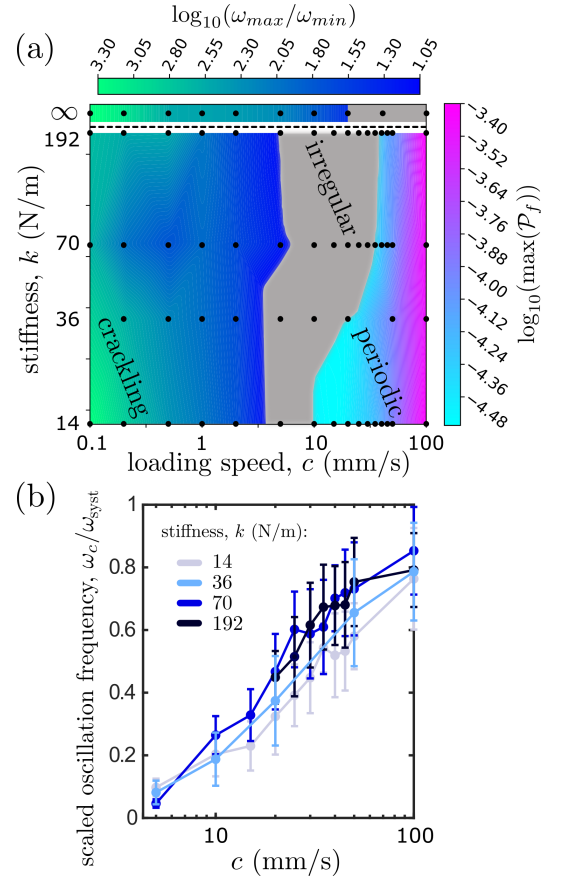


FIG. 4. (color online) (a) Dynamic phase diagram as a function of k and c . The color plot of the crackling regime uses the color bar on the top. It shows the PSD power-law range $\log_{10}(\omega_{max}/\omega_{min})$ (when larger than 1). The color plot of the periodic regime uses the color bar on the right. It is a map of the maximum of \mathcal{P}_f , when larger than $10^{-4.6}$ (N.s)². The irregular region, shown in gray is the complementary. Its complex boundary geometries encompass the measurement uncertainty. Black dots are experimental measurement points, the rest of the map is linearly interpolated (in the log space). The top line shows $k = \infty$, as the slider is pulled with a chain. (b) Evolution of PSD peak frequency, ω_c/ω_{sys} , vs. c on log-lin scales. The scaling collapses the curves for different stiffnesses within the errorbars. The errorbars represent the width of the peaks in power spectra.

crackling and periodicity as a function of c and k . The behavior is classified as crackling when there is a full decade or more of power law decay ($\log_{10}(\omega_{max}/\omega_{min}) > 1$), a commonly accepted definition [46, 47]. Conversely, the value of \mathcal{P}_f at the peak is higher for higher c . In the domain where $\mathcal{P}_f > 10^{-4.6}$ (N.s)² is above the low frequency plateau, the periodic regime lies. Between these regions lies the irregular regime, where there is neither a clearly defined power-law span nor a well defined spectral peak, but a white noise PSD. Moreover, our experimental precision does not provide a clear critical c between crackling and periodic regimes. Fig.4b shows how the oscillation frequency evolves with c for different k 's.

We observe a vanishing frequency and consequently a diverging period near the transition. This relation along with the fast and slow processes in stick-slip dynamics is reminiscent of a homoclinic bifurcation with c [48] as the bifurcation parameter. Moreover, the dependence of frequency on c , shows that the periodicity is not trivially induced by loading stiffness or can be described by a simple 1D frictional model [44], and the granularity of medium plays a crucial role in the dynamics. This relation might be due to interplay of granular dilation with speed and the effective friction [33]. Furthermore, the curves for different stiffnesses approximately collapse, within the experimental uncertainty, when ω_c is scaled by $\omega_{\text{syst}} = \frac{1}{2\pi} \sqrt{\frac{k}{M}}$ [44].

We also investigate the force signal probability distribution function (PDF), $P(f)$. Fig.5(a) shows $P(f)$ for different c , and fixed k . Regardless of the shear rate, these PDF's are well fitted by a Gaussian function of standard deviation $\sigma(c, k)$. Their mean value is independent of c in the range of speeds explored here. Similarly, as shown in the upper inset of fig.5(a), the force fluctuations, quantified by σ , remain constant ($\sigma \approx 0.065$ (N)) in the crackling and irregular regimes. However, σ starts to increase when the periodic behavior is first observed (from $c \approx 20$ mm/s), accompanied by the decoupling of the spring force and the granular friction force on the slider. This broadening is most likely due to the smooth turning points in the force curve for the periodic system, which imply that more time is spent at the extreme values of the force than in the crackling regime.

We also explore the effect of other macroscopic parameters. Unlike previous findings [39], changing the granular layer depth, L , does not change the system's behavior [44] to our experimental precision. The slider motion creates a narrow shear band, and particle flow is limited to the few top layers of grains. The narrowness of this shear band, where plastic granular flow occurs, is likely responsible for the fact that L does not play a significant role in our experiment. However, the granular global pressure, from the slider weight, $W = Mg$, changes $P(f)$ significantly. The lower inset of fig.5(a) shows these PDF's when the force is scaled by the slider mass, f/W . $P(f/W)$ for different M collapse on a single curve, indicating that M controls both the mean and fluctuations of f [44]. This collapse also implies a linear relation between the global pressure, \mathcal{P} , and the mean global shear stress, τ , and its fluctuations σ_τ . This provides a constant average friction coefficient, $\mu = \bar{\tau}/\mathcal{P} \approx 0.47$, for the slowly sheared medium.

We also study the relation between the global dynamics, as characterized by the $f(t)$, and the aggregated local dynamics determined by the granular stress. The aggregated local stress is computed from the polarized images mean intensity, I [44], in a region which covers all granular layers and is ~ 1.5 slider's length wide to include all force evolution.. Fig.5(b) shows the image intensity PSD, \mathcal{P}_I , for several c at fixed $k = 70$ N/m. Like \mathcal{P}_f in the crackling regime, \mathcal{P}_I follows a power-law with exponent

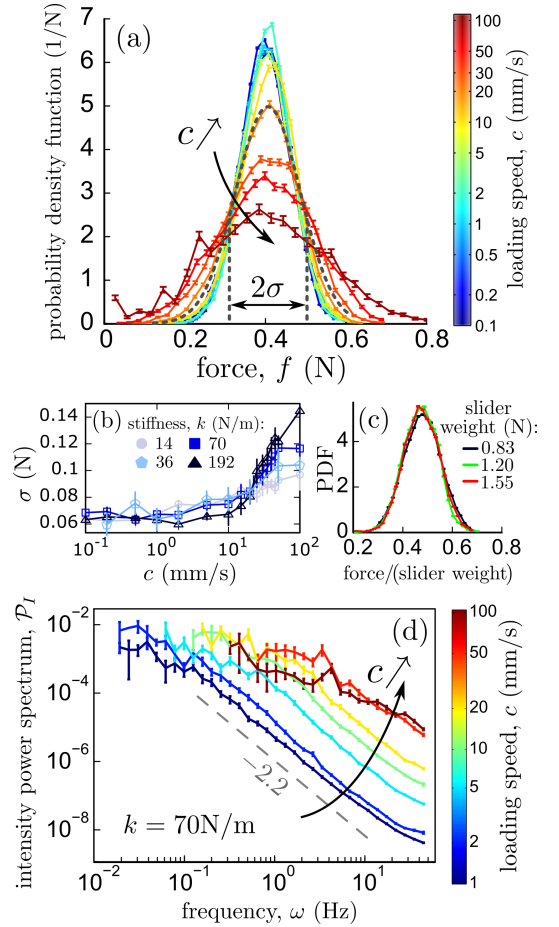


FIG. 5. (color online) (a) Probability density function of force $P(f)$, for different loading speeds, c , at fixed stiffness $k = 192$ N/m. For $c = 20$ mm/s, $P(f)$ is fitted by a Gaussian displayed by a dashed line. The standard deviation is σ . (b) σ vs. c for different stiffnesses, $k \in \{14, 30, 70, 192\}$ N/m. (c) PDF of the force signal scaled by the slider weight for different weights, $Mg \in \{0.83, 1.2, 1.55\}$ N with $c = 0.5$ mm/s and $k = 192$ N/m. (d) Image intensity ($I(t)$) power spectra, $\mathcal{P}_I(\omega)$, for different $c \in [1, 100]$ mm/s with $k = 70$ N/m. A straight dashed line shows a slope corresponding with the exponent -2.2 ± 0.2 fitted to the lowest speed.

-2.2 ± 0.2 , and the frequency interval of this power-law regime decreases as c increases. However, as c increases, we do not observe a clear peak in \mathcal{P}_I , that would indicate a periodic regime. For high c , we observe much faster stress fluctuations in the granular medium, compared to the slider dynamics [44]. The high stiffness and low inertia of particles may induce this difference. In this case, we observe a decoupling between the granular medium mean stress and the mean flow of particles, which is correlated with the slider motion. These erratic stress fluctuations show a non-periodic friction force on the slider, moving periodically. This granular boundary friction force, when coupled with finite loading stiffness [44] and slider's inertia, results in periodic oscillations at the macroscopic scale.

IV. CONCLUDING DISCUSSION

The experiments reported here demonstrate crackling, irregular and periodic dynamics while shearing granular matter via a slider and a spring. The system behavior depends on the stage speed, c , and the loading stiffness, k . The transitions between regimes occur over regions of $c - k$ -space, without sharp crossovers. The other main observations of these experiments are: (i) The shearing force PSD provides a useful tool to indicate the dynamics of the system for a given set of system control parameters, namely loading speed, system stiffness, global pressure, and system size. (ii) The transition to periodic regime shows a diverging period, which is similar to a homoclinic bifurcation, where c is the primary control parameter. (iii) The shearing force PDF follows a Gaussian distribution redwith an increasing standard deviation in the periodic regime, (iv) No periodic behavior is observed for the local stress dynamics, which implies the important role of the stiffness and inertia of the loading boundary in the periodic regime.

These observations are in agreement with various numerical simulations. Lacombe et al. [42] introduced a simple friction model, similar to our experiment, with two degrees of freedom, considering dilation. They reproduced stick-slip, inertial oscillations and sliding regimes by increasing the shear rate. However, no crackling dynamics was observed, since their model lacks stochasticity. Our phase diagram, presented in fig.4, may be a detailed version of the stick-slip domain of their diagram. In an experimental study, Kaproth et al. [35] observed only periodic stick-slip and sliding regimes, and

the driving rate only affects the slip event period. We believe they do not observe crackling dynamics because of mono-dispersity of their granular layers. In other studies, Aharonov et al. [38] simulated a system, very similar to ours, using the discrete element method. They also observed crackling, oscillatory and sliding regimes by changing the loading speed and the slider mass. However, the effect of the stiffness was not tested and the exact domain of the crackling regime were not investigated. Liu et al. [39] also studied the effect of the driving rate on avalanches. As in our experiments, they observed that increasing the driving rate decreases the power-law range of the avalanche size distribution, *i.e.* the number of decades over which the avalanche size PDF obeys a power-law. Avalanches have also been observed in depinning models and a phase diagram of dynamical regimes for such a model has been demonstrated [49] as a function of c , k and L .

The findings of this letter can inform a number of open problems in other fields, such as avalanche dynamics for non-zero driving rate in crackling systems and depinning transition, and fluctuating dynamics of amorphous media, including seismicity. Future work is required to investigate global and local (space-time) avalanche statistics.

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