Disordering, clustering, and laning transitions in particle systems with dispersion in the Magnus term

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Disordering, Clustering, and Laning Transitions in Particle Systems with Dispersion in the Magnus Term

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We numerically examine a two-dimensional system of repulsively interacting particles with dynamics that are governed by both a damping term and a Magnus term. The magnitude of the Magnus term has one value for half of the particles and a different value for the other half of the particles. In the absence of a driving force, the particles form a triangular lattice, while when a driving force is applied, we find that there is a critical drive above which a Magnus-induced disordering transition can occur even if the difference in the Magnus term between the two particle species is as small as one percent. The transition arises due to the different Hall angles of the two species, which causes their motion to decouple at the critical drive. At higher drives, the disordered state can undergo both species and density phase separation into a density modulated stripe that is oriented perpendicular to the driving direction. We observe several additional phases that occur as a function of drive and Magnus force disparity, including a variety of density modulated diagonal laned phases. In general we find a much richer variety of states compared to systems of oppositely driven overdamped Yukawa particles. We discuss the implications of our work for skyrmion systems, where we predict that even for small skyrmion dispersions, a drive-induced disordering transition can occur along with clustering phases and pattern forming states.

I. INTRODUCTION

There is a wide variety of systems that can be effectively modeled as an assembly of interacting particles that undergoes structural transitions under some form of external driving. In the presence of a random or periodic substrate, the particles can exhibit a depinning transition [1, 2] such as that found for superconducting vortices [3, 4], colloidal particles [5–7], or sliding friction [8]. Disordering or ordering transitions can occur in the absence of quenched disorder as a function of dc or ac shearing [9–13]. In many cases, the particles have a uniform size and particle-particle interaction force, but when the particle sizes or interactions become polydisperse, order-disorder transitions can appear even in the absence of driving or shearing [14–16]. Disordering transitions and other dynamical phases can also arise in systems with monodisperse particle-particle interactions if some of the particles have different dynamics than others. A well-studied example is oppositely driven repulsive particles, which can form static ordered states or undergo fluctuating disordered flow that is followed at higher drives by a transition to a laned state consisting of multiple partially phase separated oppositely moving stripes [17–26]. Similar ordering and laning transitions appear when the particles move at different velocities in the same direction [27]. Experimentally, laning transitions have been realized for colloidal particles [28, 29] and dusty plasmas [30, 31].

Here we study whether a disordering transition or lane formation can occur for an assembly of bidisperse particles that are all driven in the same direction when each particle species has a different non-dissipative Magnus term. We consider repulsively interacting particles that form a triangular lattice in the absence of a driving force or substrate. A dissipative force of magnitude $\alpha_d$ aligns the velocity of particle $i$ with the net direction of the external forces acting on that particle, while a Magnus term of magnitude $\alpha_m$ aligns the particle velocity perpendicular to the external forces. When we introduce an applied driving force of magnitude $F_D$, we find that if the Magnus term $\alpha_m = 0$ and the dissipative term $\alpha_d = \alpha_d$ for all $i$, the system forms a triangular lattice that moves parallel to the driving direction. If the Magnus term is nonzero but equal for all particles, $\alpha_m = \alpha_m$, a triangular lattice still forms, but it moves at a Hall angle $\theta_{Sk} = \theta_{Sk}^{int}$ with respect to the driving direction, where the intrinsic Hall angle $\theta_{Sk}^{int} = \arctan(\alpha_m/\alpha_d)$. If the Magnus term is bidisperse, with a value of $\alpha_m^a$ for half of the particles and $\alpha_m^b > \alpha_m^a$ for the other half, we find that when $\alpha_m^a = 0$ and $\alpha_m^b = 0$, a triangular lattice appears that moves elastically at an angle $\theta_{Sk}^{int}/2$ for small drives. Above a critical drive $F_c$, dislocation pairs proliferate in the lattice and a dynamical disordering transition occurs when the two species move at different velocities in the directions parallel and perpendicular to the drive due to the drive dependence of $\theta_{Sk}^b$. When $\alpha_m^a \neq 0$ and $\alpha_m^a < \alpha_m^b$, this behavior persists since the drive dependence of $\theta_{Sk}^b$ differs from that of $\theta_{Sk}^a$; however, $F_c$ increases as the difference $\alpha_m^a - \alpha_m^b$ decreases. At high drives, the disordered state transitions to a cluster or stripe state in which the particles phase separate into a single stripe oriented perpendicular to the drive with one species on each side of the stripe. The stripe becomes denser with increasing $F_D$ since $\theta_{Sk}^a - \theta_{Sk}^b$ increases and species $a$ piles up behind species $b$. Due to the increasing compression of the stripe, eventually an instability occurs in which the system can lower its particle-particle interaction energy by forming a more uniform state that we call a diagonal laned phase. In some cases, the diagonal laned state exhibits strong density modulations as well as additional transitions to a larger number of thinner lanes. All of these transitions
are associated with changes and jumps in the average velocity both parallel and perpendicular to the drive, as well as changes in the amount of six-fold ordering in the system.

Our results have implications for driven magnetic skyrmions, which are particlelike magnetic textures that interact repulsively with each other and form a triangular lattice [32–34]. Skyrmions can be set into motion by the application of a current [34–40], and due to the Magnus term they move at an angle with respect to the driving force known as the skyrmion Hall angle [34, 41–45]. Within a given sample, there can be dispersion in the size of skyrmions, and different species of skyrmions with different dynamics may be able to coexist with each other [38, 45–50], so it is important to understand how polydispersity affects the collective motion of skyrmions.

Our results suggest that if there is the slightest dispersion in the Magnus term, a drive-induced disordering transition from a crystal to a liquidlike state can occur even in the absence of quenched disorder. In systems with strong quenched disorder, monodisperse skyrmions depin into a plastically flowing disordered state, but at higher drives they dynamically reorder into a moving crystal state [41, 51, 52] similar to that found for superconducting vortices [2–4, 53]. Our results indicate that when there is a dispersion in the Magnus term, such behavior is reversed and the skyrmions disorder at higher drives. In addition, clustering or species segregation can occur. Recent continuum and particle-based simulations of monodisperse skyrmions showed that clustering transitions can occur in samples containing strong pinning or quenched disorder [52, 54]. Our results demonstrate that clustering can also occur in the absence of pinning when there is any disorder in the skyrmions that produces differences in the Magnus term. These results may also be relevant for soft matter systems in which Magnus forces are important, such as magnetic particles in solutions [55–57] or spinning colloidal particles [58, 59], where different size particles could experience different effective Magnus forces.

The paper is organized as follows. In Sec. II we describe our simulation details. Section III introduces the Magnus-induced disordering transition. In Sec. IV we focus on cluster and stripe formation, and show that a nonequilibrium conformal crystal structure can spontaneously emerge in the system. Section V describes the effect of varying the ratio of the damping term for the two species. In Sec. VI we vary the ratio of the Magnus term of the two species for fixed and equal damping terms. We show the results of introducing Magnus terms of opposite sign in Sec. VII. In Sec. VIII we briefly describe the effect of changing other variables and provide a general discussion. A summary of the work appears in Sec. IX.

II. SIMULATION

We consider a two-dimensional system of size $L \times L$ with periodic boundary conditions in the $x$ and $y$-directions containing $N_a$ particles of species $a$ and $N_b$ particles of species $b$ for a total of $N = N_a + N_b$ particles. The particle density is $n = N/L^2$, where $L = 36$. Unless otherwise noted, we take $N_a = N_b = N/2$. The equations of motion for particle $i$ of species $\gamma = a$ or $b$ is

$$\gamma \alpha_i \mathbf{v}_i + \gamma \alpha_{m} \hat{z} \times \mathbf{v}_i = \mathbf{F}_i^{ss} + \mathbf{F}^D$$

(1)

where the particle velocity is $\mathbf{v}_i = d\mathbf{r}_i/dt$. All particle-particle interactions have the same pairwise form of a modified Bessel function, $\mathbf{F}_i^{ss} = \sum_{j} K_1|\mathbf{r}_{ij}|\hat{r}_{ij}$ that falls off exponentially for large $r$. Here $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ is the distance between particles $i$ and $j$, and $\hat{r}_{ij} = (\mathbf{r}_i - \mathbf{r}_j)/r_{ij}$. This interaction potential has been used previously for particle-based models of skyrmions, and in the absence of pinning it causes the particles to form a hexagonal lattice [37, 41, 51, 54, 60]. For computational efficiency we cut off the skyrmion-skyrmion interaction beyond a length of $r_{ij} = 7.0$ when it becomes negligible. The driving force $\mathbf{F}^D = F_D \hat{x}$ is the same for all particles. We increase the drive in increments of $\delta F^D = 0.002$ and wait $10^4$ simulation time steps between increments to ensure we are in a steady state. The damping term $\alpha_i^\gamma$ aligns the particle velocity in the direction of the net applied forces, while the Magnus term $\alpha_{m}^\gamma$ generates velocity components that are perpendicular to the net applied forces. As in previous work, unless otherwise noted we normalize the two coefficients such that $(\alpha_i^\gamma)^2 + (\alpha_{m}^\gamma)^2 = 1.0$ [41, 42, 60]; however, we also consider systems with fixed $\alpha_{m}^\gamma$ and varied $\alpha_i^\gamma$ that are not subject to this constraint. The intrinsic Hall angle for species $\gamma$ is $\theta_{\text{int}}^\gamma = \text{arctan}(\alpha_{m}^\gamma/\alpha_i^\gamma)$. It is known from previous work on skyrmion systems with disorder that $\theta_{\text{int}}^\gamma$ depends on the velocity of the particles and can be written as $\theta_{\text{int}}^\gamma = \tan^{-1}(\langle V_y^\gamma \rangle/\langle V_x^\gamma \rangle)$ where $\langle V_x^\gamma \rangle = N_a^{-1}\sum_i^N \mathbf{v}_i \cdot \hat{x}$ and $\langle V_y^\gamma \rangle = N_a^{-1}\sum_i^N \mathbf{v}_i \cdot \hat{y}$, with the average taken over time in the steady state [41, 42, 51, 61]. In a system with $\alpha_i^a = \alpha_i^b = 0$, $\theta_{\text{int}}^{a} = \theta_{\text{int}}^{b} = 0$. We initialize the particles in a triangular lattice and assign $N_a$ randomly selected particles to be species $a$, with the remaining particles set to species $b$. We measure $\langle V_x^a \rangle$, $\langle V_y^a \rangle$, $\langle V_x^b \rangle$, and $\langle V_y^b \rangle$, along with $\theta_{\text{int}}^{a} = \theta_{\text{int}}^{b} = 1.5$. For reference, in the representative skyrmion-supporting material MnSi, a typical skyrmion lattice constant is $50 \text{ nm}$, typical driving currents range from $10^8$ to $10^9 \text{ A/m}^2$, and the skyrmion Hall angle ranges from $50^\circ$ to $83^\circ$ [37].

III. MAGNUS INDUCED DISORDERING TRANSITION

We first consider a system containing $N = 572$ particles at a density of $n = 0.4413$ which forms a tri-
are now moving at different velocities. (c) The fraction $P_a$ for the system forms a moving triangular crystal (MC) and the velocities are locked in both directions, while for $F_D \geq 0.1175$, the transverse velocity curves split with $(V^a_2)$ increasing more rapidly with $F_D$ than $(V^b_2)$, indicating that the two species are now moving at different velocities. (c) The fraction $P_b$ of sixfold-coordinated particles vs $F_D$. For $F_D < 0.1175$, $P_b \approx 1$ as expected for a triangular lattice, while $P_b$ drops for $F_D \geq 0.1175$, indicating a disordering of the system. The letters a and b indicate the values of $F_D$ at which the images in Fig. 2 were obtained. The vertical dashed line marks the transition from the moving crystal (MC) to the moving liquid (ML) state.

![Fig. 1](image1.png)

**FIG. 1:** $(V^a_1)$ (red) and $(V^b_1)$ (blue) vs $F_D$ in a sample with $\alpha_m^a = 0$, $\alpha_d^a = 1$, and $\alpha_m^b = 0.3$. (b) The corresponding $(V^a_2)$ (red) and $(V^b_2)$ (blue) vs $F_D$. For $F_D < 0.1175$, the system forms a moving triangular crystal (MC) and the velocities are locked in both directions, while for $F_D \geq 0.1175$, the transverse velocity curves split with $(V^a_2)$ increasing more rapidly with $F_D$ than $(V^b_2)$, indicating that the two species are now moving at different velocities. (c) The fraction $P_b$ of sixfold-coordinated particles vs $F_D$. For $F_D < 0.1175$, $P_b \approx 1$ as expected for a triangular lattice, while $P_b$ drops for $F_D \geq 0.1175$, indicating a disordering of the system. The letters a and b indicate the values of $F_D$ at which the images in Fig. 2 were obtained. The vertical dashed line marks the transition from the moving crystal (MC) to the moving liquid (ML) state.

![Fig. 2](image2.png)

**FIG. 2:** Images of particle positions for species a (red) and b (blue) for the system in Fig. 1 with $\alpha_m^a = 0$, $\alpha_d^a = 1.0$, and $\alpha_m^b = 0.3$ at the drives marked a and b in Fig. 1(c). (a) At $F_D = 0.05$, the system forms a triangular solid that is moving in the positive $x$ and negative $y$ directions. (b) At $F_D = 0.25$, the system is in a moving liquid phase.

![Fig. 3](image3.png)

**FIG. 3:** The Hall angles $\theta_{sk}^a$ (pink) and $\theta_{sk}^b$ (green) vs $F_D$ for the system in Fig. 1 with $\alpha_m^a = 0$, $\alpha_d^a = 0$, and $\alpha_m^b = 0.3$. In the moving crystal (MC) phase, the species are locked together and all the particles have $\theta_{sk}^a = \theta_{sk}^b = -8.73^\circ$, while in the disordered moving liquid (ML) state, the magnitude of $\theta_{sk}^a$ decreases toward $0^\circ$ while $\theta_{sk}^b$ gradually approaches $\theta_{sk}^{int} = -17.45^\circ$, marked by a dashed line.

angular solid when $F_D = 0$. We fix species a in the overdamped limit with $\alpha_m^a = 0$ and $\alpha_d^a = 1.0$, while species b has $\alpha_m^b = 0.3$ and $(\alpha_m^b)^2 + (\alpha_d^b)^2 = 1.0$. In Fig. 1(a) we plot the average velocities $(V^a_1)$ and $(V^b_1)$ versus $F_D$, and in Fig. 1(b) we show the corresponding $(V^a_2)$ and $(V^b_2)$ versus $F_D$ curves. In Fig. 1(c) we plot $P_b$, the overall fraction of sixfold coordinated particles, versus $F_D$. Here $P_b = N^{-1} \sum_i \delta(z_i - 6)$, where $z_i$ is the coordination number of particle $i$ obtained from a Voronoi tessellation. When $F_D < 0.1175$, we have $(V^a_2)/(V^b_2) = (V^a_1)/(V^b_1) = 1.0$ and $P_b = 1.0$, indicating that the system forms a triangular lattice which moves elastically in the positive $x$ and negative $y$ directions. Here, the skyrmions do not exchange neighbors as they move, and any local fluctuations in position are invariant with respect to the global drift velocity.

We illustrate the particle positions at $F_D = 0.05$ in Fig. 2(a), where a moving crystal (MC) phase appears. At $F_D = 0.1175$, a disordering transition occurs that is associated with a drop in $P_b$ caused by the proliferation of 5-7 defect pairs. At this same drive, the $(V^a_2)$ and $(V^b_2)$ curves in Fig. 1(b) split, and the $(V^a_2)$ curve increases more rapidly with $F_D$ than the $(V^b_2)$ curve, indicating that the disordering transition is triggered by a partial decoupling of the two species, which now move at different transverse velocities. In Fig. 2(b) we show the particle positions at $F_D = 0.25$ where the system has disordered but the particle species remain mixed. In this moving liquid (ML) phase, the particles undergo continual dynamical rearrangements. Despite the fact that $\alpha_m^a = 0$, $(V^a_1)$ continues to increase with increasing $F_D$ in the ML phase, indicating that although the two species are no longer fully coupled, species b is able to...
drift at which the MC-ML transition occurs, versus \( F \) drives, and we find that the critical force can be fit to \( \alpha b \) with fixed \( \alpha b \) and \( \alpha m \).

FIG. 4: (a) \( Fc \), the drive at which the MC-ML transition occurs, vs \( \alpha b \) for the system in Fig. 1 with \( \alpha m = 0 \) and \( \alpha m = 1.0 \). The solid line is a power law fit to \( Fc \propto C(\alpha m)^{3} \) with \( C = 0.0323 \) and \( \beta = -1.05 \). (b) \( Fc \) vs \( \alpha m \) in a system with fixed \( \alpha m = 0.7 \) for \( \alpha m = 1.0 \), showing a divergence at \( \alpha m/\alpha m = 1.0 \).

In Fig. 4(a) we plot the critical force \( Fc \), equal to the drive at which the MC-ML transition occurs, versus \( \alpha m \) for the system in Fig. 1 with \( \alpha m = 0 \) and \( \alpha m = 1.0 \). As \( \alpha m \) increases, the disordering transition shifts to lower drives, and we find that the critical force can be fit to \( Fc \propto C(\alpha m)^{3} \) with \( C = 0.0323 \) and \( \beta = -1.05 \). These results show that even when the Magnus term is very small, application of an external drive can induce a disordering transition. We note that under our imposed normalization constraint, \( \alpha m < 1.0 \). In skyrmion systems, all the particles have a finite Magnus term, so in Fig. 4(b) we plot \( Fc \) versus \( \alpha m \) for a system in which we vary \( \alpha m \) while fixing \( \alpha m = 0.7 \) with \( \alpha m = 1.0 \). Here \( Fc \) diverges at \( \alpha m/\alpha m = 1.0 \) according to \( Fc \propto |\alpha m/\alpha m|^{3} \).

FIG. 5: (a) \( \langle V_{||}^{a} \rangle \) (red) and \( \langle V_{||}^{b} \rangle \) (blue) vs \( F_D \) for the system from Fig. 1 with \( \alpha m = 0 \), \( \alpha m = 1.0 \), and \( \alpha m = 0.3 \). (b) The corresponding \( \langle V_{s}^{a} \rangle \) (red) and \( \langle V_{s}^{b} \rangle \) (blue) vs \( F_D \). (c) The corresponding \( Pb \) vs \( F_D \). MC is the moving crystal state. There is a transition at \( F_D = 2.5 \) from the moving liquid (ML) phase to a perpendicular stripe (PS) or cluster phase.

IV. CLUSTER AND STRIPE FORMATION

In Fig. 5(a,b) we plot \( \langle V_{||}^{a} \rangle \), \( \langle V_{||}^{b} \rangle \), \( \langle V_{s}^{a} \rangle \), and \( \langle V_{s}^{b} \rangle \) versus \( F_D \) for the system in Fig. 1 with \( \alpha m = 0 \), \( \alpha m = 1.0 \), and \( \alpha m = 0.3 \), while in Fig. 5(c) we show the corresponding \( Pb \) versus \( F_D \) curve. Here we consider values of \( F_D \) that are much higher than those presented in Fig. 1 in order to access the transition from the ML state to a phase separated (PS) cluster state. The ML phase ends at \( F_D = 2.5 \), where we find a jump of \( \langle V_{s}^{a} \rangle \) to \( \langle V_{s}^{a} \rangle = 0 \), indicating that the motion of species \( a \) has locked to the \( x \) direction, parallel to the applied driving force. This jump coincides with a jump in \( \langle V_{s}^{b} \rangle \) to more negative values, and both curves are much smoother above the jump, indicating that fluctuations are reduced in the PS state compared to the ML flow. The ML-PS transition is also associated with a jump up in \( P_b \) from \( P_b = 0.55 \) in the ML phase to \( P_b = 0.9 \) in the PS phase as the system becomes more ordered into a moving perpendicular stripe.

In Fig. 6(a) we show the particle configurations from the sample in Fig. 5(a) at \( F_D = 4.0 \) in the PS phase, where the particles undergo species and density phase separation into a partially clustered state consisting of a stripe aligned in the \( y \) direction, perpendicular to \( F_D \). The particle density is highest at the center of the stripe. Species \( a \) moves only along the \( x \)-direction, causing it to pile up behind species \( b \), which is moving in both the positive \( x \) and negative \( y \) directions. As a result, the entire pattern translates in the positive \( x \) direction, and...
the two halves of the pattern shear against each other along the y direction. In Fig. 5(a), \( \langle V^a_\parallel \rangle \) is slightly higher than \( \langle V^b_\parallel \rangle \) in the ML phase, but in the PS phase \( \langle V^a_\parallel \rangle \) and \( \langle V^b_\parallel \rangle \) become locked together.

There is considerable local sixfold ordering of the particles in the PS state illustrated in, Fig. 6(a), but due to the density gradient the lattice is distorted into conformal arch-like patterns. Conformal crystals arise in two-dimensional systems of repulsive particles in the presence of some form of density gradient, such as magnetic particles in a gravitational field [62], vortices in a Bean state [63–66], and colloidal particles under a gradient that is imposed by the system geometry [67]. The conformal crystals are the result of a competition between the local sixfold ordering favored by the repulsive particle-particle interactions and the need to spatially vary the interparticle spacing in order to accommodate the density gradient. Most conformal crystals have been observed under equilibrium conditions, while the conformal crystal structure illustrated in Fig. 6(a) is a strictly nonequilibrium state. As \( F_D \) increases, the conformal stripe state becomes more compressed along the x direction. Increasing \( \alpha^b_m \) also compresses the PS stripe, as illustrated in Fig. 6(b) for a system with \( \alpha^a_m = 0 \), \( \alpha^b_D = 1.0 \), and \( \alpha^b_m = 0.7 \) at \( F_D = 5.0 \). The width of the stripe is controlled by both \( \alpha_m \) and \( F_D \). For fixed \( F_D \), the stripe decreases in width with increasing \( \alpha_m \), while for fixed \( \alpha_m \), the stripe decreases in width with increasing \( F_D \). We also note that Fig. 6(b) shows an increase in the fluctuations along the interface of the two species for the thiner stripes. This is a result of the increase in the strength of the repulsive skyrmion-skyrmion interactions as the stripes become thin, which eventually leads to the instability of the stripe and the transition to a configuration with a more uniform density.

In Fig. 7(a) we plot \( \langle V^a_\parallel \rangle \) and \( \langle V^b_\parallel \rangle \) versus \( F_D \) for the system in Fig. 6(b) with \( \alpha^b_m = 0.7 \) in the vicinity of the ML-PS transition, while in Fig. 7(b) we show the corresponding \( \langle V^a_\perp \rangle \) and \( \langle V^b_\perp \rangle \) versus \( F_D \) curves. There is a clear parallel velocity locking, with \( \langle V^b_\parallel \rangle / \langle V^a_\parallel \rangle = 1.0 \) in the PS state, while \( \langle V^a_\perp \rangle \) locks to zero velocity at the transition at the same time as a jump in \( \langle V^b_\perp \rangle \) to a more negative value appears. The velocity fluctuations are reduced in the more ordered PS phase compared to the disordered ML state.

When \( F_D \) or \( \alpha^b_m \) is increased, the stripes become more compressed and the particle-particle interactions become strong enough to generate an instability that causes the system to enter a different phase consisting of multiple lanes with a more uniform particle density. In Fig. 8 we plot \( \langle V^a_\parallel \rangle \), \( \langle V^b_\parallel \rangle \), \( \langle V^a_\perp \rangle \), and \( \langle V^b_\perp \rangle \) versus \( F_D \) for a system with \( \alpha^b_m = 0.75 \). There is a large jump in the velocities near \( F_D = 7.75 \), where the system undergoes a transition from the PS state to what we term a diagonal lanced (DL) state. As illustrated in Fig. 9(a) for \( F_D = 8.0 \), this state is composed of multiple stripes of particles oriented at an angle to the driving direction. Within each stripe, the density gradient is reduced compared to what is observed in the PS state. At the transition into the DL state, \( \langle V^a_\perp \rangle \) jumps up since species b no longer blocks the motion of species a along the x direction. At the same time, there is a downward jump in \( \langle V^b_\perp \rangle \) since species b is no longer being pushed as hard in the x direction by species a. Interestingly, a small downward jump in \( \langle V^b_\perp \rangle \) occurs at the PS-DL transition, indicative of negative differential conductivity. This jump is produced by a Magnus force induced velocity imparted by species a.
At the same time, species $a$ contributes to the $y$ direction velocity component of species $b$. In the PS phase, species $a$ pushes against species $b$ along the $x$ direction, and due to the Magnus term, additional velocity components arise for species $b$ in the negative $y$-direction. In the DL state, the tilt of the stripes diminishes the magnitude of the $x$-direction push from species $a$ on species $b$, which reduces the Magnus force contribution to the $y$-direction velocity of species $b$. At the same time, species $a$ now has a negative $y$ direction component of force on species $b$ that generates a negative $x$ direction velocity component of species $b$ through the Magnus term. This produces the drop in $\langle V^b_{||} \rangle$ at the PS-DL transition. As $\alpha_m^b$ increases, the PS-DL transition shifts to lower values of $F_D$, and when $\alpha_m^b \geq 0.85$, the system passes directly from the ML to the DL phase without forming a PS state. In Fig. 9(b) we illustrate the DL state at $\alpha_m^b = 0.85$ and $F_D = 8.0$, where the number of stripes has increased. Within the DL phase, the stripes become more compressed as $F_D$ increases until a transition occurs to a new diagonal lanced state, DL2, containing a larger number of stripes with fewer rows of particles in each stripe.

In Fig. 10(a) we plot $\langle V^a_{||} \rangle$, $\langle V^b_{||} \rangle$, $\langle V^a_\perp \rangle$, and $\langle V^b_\perp \rangle$ versus $F_D$ for a system with $\alpha_m^b = 0.9$, which transitions directly from the ML to the DL phase. A second transition appears near $F_D = 13.5$, as indicated by the jumps in the velocity curves, which corresponds to a rearrangement into the more uniform lanced state DL2. In Fig. 11(a) we illustrate the DL state for the system in Fig. 10(a) at $F_D = 10.3$, where the particles form a series of diagonal stripes, each of which is composed of three rows of each species on either side. At $F_D = 14.75$ in the DL2 phase, as shown in Fig. 11(b), there are still two to three rows of particles on each side of each stripe, but the stripes are much more spread out so that the particle density is considerably more uniform. In Fig. 10(b), we plot $\langle V^a_{||} \rangle$, $\langle V^b_{||} \rangle$, $\langle V^a_\perp \rangle$, and $\langle V^b_\perp \rangle$ versus $F_D$ for a system with $\alpha_m^b = 0.998$ where $\alpha_m^b/\alpha_d^b = 15.79$, in which the same phases appear but the DL-DL2 transition is
shifted to a lower drive of $F_D = 6.5$. Figure 11(c) shows an image of the DL phase for the system in Fig. 10(b) at $F_D = 5.0$, where dense tilted stripes appear, each of which contains four to five rows of particles on each side. In Fig. 11(d), the DL$_2$ phase for the same system at $F_D = 7.5$ has a larger number of lower density stripes containing two rows of particles on each side, giving a more uniform particle density. The exact particle configurations in the DL states are not unique, but generally we find a small number of diagonal stripes with three or more rows of particles on each side of each stripe in the DL phase, while the DL$_2$ phase is more uniform and each of the more numerous stripes has two to three rows of particles on each side. As $F_D$ is further increased, the stripes in the DL$_2$ phase become more compressed, and an additional transition can occur to a DL$_3$ state in which each stripe has only a single row of particles on each side.

From the features in the transport curves and changes in the particle configurations, we construct the dynamical phase diagram shown in Fig. 12 as a function of $F_D$ versus $\alpha_m^b/\alpha_d^b$ for samples with fixed $\alpha_m^b = \alpha_b^d = 1.0$ and $\alpha_m^a = 0$. The extent of the MC phase increases as $\alpha_m^b/\alpha_d^b$ goes to zero, and similarly the ML-PS transition shifts to higher $F_D$ as $\alpha_m^b/\alpha_d^b$ decreases. The PS phase only occurs when $\alpha_m^b/\alpha_d^b < 0.85$, while for higher values of $\alpha_m^b/\alpha_d^b$ the system transitions directly from the ML phase to the DL state. The DL$_2$ state appears when $\alpha_m^b/\alpha_d^b \geq 0.9$. 

### FIG. 11: Images of particle positions for species $a$ (red) and $b$ (blue). (a) The DL state for the system in Fig. 10(a) with $\alpha_a^a = 1.0$, $\alpha_m^a = 0$, and $\alpha_m^b = 0.9$ at $F_D = 10.35$. (b) The DL$_2$ state from Fig. 10(a) at $F_D = 14.75$. (c) The DL state from the system in Fig. 10(b) with $\alpha_a^a = 1.0$, $\alpha_m^a = 0$, and $\alpha_m^b = 0.998$ at $F_D = 5.0$. (c) The DL$_2$ state from the system in Fig. 10(b) at $F_D = 7.5$. 

### FIG. 12: Dynamic phase diagram as a function of $F_D$ vs $\alpha_m^b/\alpha_d^b$ for the system in Fig. 10 with $\alpha_a^a = 1.0$ and $\alpha_m^a = 0$. MC: moving crystal; ML: moving liquid; PS: perpendicular stripe; DL: diagonal laned phase; DL$_2$: second diagonal laned phase. 

### FIG. 13: (a) $\langle V_{\parallel}^a \rangle$ (red) and $\langle V_{\parallel}^b \rangle$ (blue) vs $F_D$ for a system with $\alpha_m^a = \alpha_m^b = 0$, $\alpha_b^a = 1.0$, and $\alpha_b^d = 2.0$. (b) The corresponding $\langle V_{\perp}^a \rangle$ (red) and $\langle V_{\perp}^b \rangle$ (blue) vs $F_D$, which both fluctuate around zero but show a clear change in the magnitude of the fluctuations near $F_D = 2.1$. (c) The corresponding $P_b$ vs $F_D$ showing the moving crystal (MC), moving liquid (ML), and laning states.
have a smaller damping coefficient, move faster in the di-

ing liquid phase in which the species

As the drive increases, there is a transition to a mov-

an elastic triangular solid in the moving crystal state.

\[ \alpha = \alpha_0 \text{ and } \beta = \beta_0 \] for all the particles versus \( F_D \), where we find that \( P_b = 1.0 \) in the ordered MC phase. At the transition to the ML phase, \( P_b \) drops, but for drives above \( F_D = 2.1 \), there is a recovery of order to \( P_b \approx 0.9 \) as the system enters a uniform lanced state of the type illustrated in Fig. 14(a) for \( F_D = 2.5 \). This lanced state is the same as that found for oppositely driven Yukawa particles [19], and it is relatively ordered due to the large patches of triangular lattice that appear. Sliding of adjacent lanes past each other is made possible by the presence of aligned 5-7 dislocation pairs that can glide parallel to the driving direction, which give the state a smectic character. The zero Magnus term lanced state is distinct from the DL and DL2 states that appear for a finite Magnus term in that the lanes are aligned with the driving direction and the particle density is uniform for all values of \( F_D \) and \( \alpha_d \), unlike the DL and DL2 states, which can show strong density modulations. In Fig. 14(b) we illustrate that the lanced state for \( \alpha_d = 10 \) has features similar to the lanced states that form at lower \( \alpha_d \). In Fig. 15 we construct a dynamic phase diagram as a function of \( F_D \) versus \( \alpha_d/\alpha_0 \), and highlight the MC, ML, and lanced states. As \( \alpha_d/\alpha_0 \rightarrow 1 \), there is a divergence in the value of \( F_D \) at which the MC-ML transition occurs, along with a similar divergence of the transition from the ML to the lanced state.

\[ \langle V_x^a \rangle |\perp \rangle = \langle V_y^a \rangle |\perp \rangle = 1 \] for all the particles versus \( F_D \) and \( \alpha_d \), and in Fig. 16(b) we show the corresponding \( \langle V_x^b \rangle \) and \( \langle V_y^b \rangle \) versus \( F_D \). When \( F_D < 0.6 \), we observe a disordered or partially disordered state, as illustrated in Fig. 17(a) at \( F_D = 0.5 \), where a diagonal stripe is beginning to form.

For \( F_D > 0.6 \) the system enters a DL state as shown in Fig. 17(b) for \( F_D = 1.25 \), where the lanes are aligned at an angle of roughly \( \theta = 60^\circ \) to the driving direction. Within the DL state, \( \langle V_x^a \rangle \) and \( \langle V_y^a \rangle \) are both positive, indicating that species \( a \) is sliding along the positive \( x \) and positive \( y \) directions due to the orientation of the lane. In contrast, \( \langle V_x^b \rangle \) and \( \langle V_y^b \rangle \) are both negative, indicating that species \( b \) is moving opposite to the direction of the applied drive, a phenomenon that is known as absolute negative mobility [68–71]. The strong Magnus force of the species \( b \) particles rotates the \( F_D \) component of the velocity mostly into the negative \( y \) direction, leaving a residual positive \( x \) direction velocity. The overdamped species \( a \) particles move parallel to the driving direction.

\[ \alpha = \alpha_0 \text{ and } \beta = \beta_0 \]

\[ \langle V_x^a \rangle |\perp \rangle = \langle V_y^a \rangle |\perp \rangle = 1 \] for all the particles versus \( F_D \) and \( \alpha_d \), and in Fig. 16(b) we show the corresponding \( \langle V_x^b \rangle \) and \( \langle V_y^b \rangle \) versus \( F_D \). When \( F_D < 0.6 \), we observe a disordered or partially disordered state, as illustrated in Fig. 17(a) at \( F_D = 0.5 \), where a diagonal stripe is beginning to form.

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that species \( \langle V^b \rangle \) becomes more negative as \( F_D \) increases, meaning that species \( b \) is exhibiting absolute negative mobility.

![Image](https://via.placeholder.com/150)

**FIG. 17:** Images of particle positions for species \( a \) (red) and \( b \) (blue) for the system in Fig. 16 with \( \alpha^a = \alpha^b = 1.0, \alpha^a_m = 0, \text{ and } \alpha^b_m = 6.0 \). (a) The moving liquid state at \( F_D = 0.5 \), just before the transition into the diagonal lanced (DL) state. (b) The DL state at \( F_D = 1.25 \). (c) The DL state at \( F_D = 3.0 \) where the stripes are more compressed. (d) The DL₂ state at \( F_D = 3.75 \).

![Image](https://via.placeholder.com/150)

**FIG. 18:** \( \langle V^a \rangle \) (pink) and \( \langle V^b \rangle \) (blue) vs \( F_D \) for the system in Fig. 17 with \( \alpha^a = \alpha^b = 1.0, \alpha^a_m = 0, \text{ and } \alpha^b_m = 6.0 \) showing jumps at the PS-DL, DL-DL₂, and DL₂-DL₃ transitions. (b) Images of particle positions for species \( a \) (red) and \( b \) (blue) in the DL₃ state at \( F_D = 10.5 \). and pile up behind the species \( b \) particles, exerting a force of magnitude \( F_{ss} \) on them in both the positive \( x \) and negative \( y \) directions with components \( F_{ss} \cos \theta_l \) and \( F_{ss} \sin \theta_l \), respectively. Under the Magnus force rotation, the \( F_{ss} \cos \theta_l \) portion of the particle-particle interaction produces a species \( b \) velocity contribution that is mostly in the negative \( y \) direction, while the \( F_{ss} \sin \theta_l \) portion of the interaction gives a velocity contribution that is mostly in the negative \( x \) direction. Since the positive \( x \) direction velocities from \( F_D \) and \( F_{ss} \cos \theta_l \) are small, the net value of \( \langle V^b \rangle \) is negative, resulting in the absolute negative mobility that we observe. As \( F_D \) increases, the drift velocity of the entire diagonal lane in the positive \( x \) direction increases due to the increase in \( \langle V^a \rangle \), but at the same time the lane becomes more compressed, decreasing the distance between the species \( a \) and species \( b \) particles at the center of the lane, and increasing the particle-particle interaction force that is responsible for generating the negative value of \( \langle V^b \rangle \). As a result, \( \langle V^b \rangle \) becomes more negative with increasing \( F_D \).

We note that in previous studies of overdamped systems in which negative mobility is observed, the particles are coupled to some type of asymmetric substrate [68–71]. Magnus force-induced negative mobility was predicted to occur for monodisperse skyrmions, but only when the skyrmions are coupled to a substrate [72]. The bidisperse system we consider here is unique in that a negative mobility can appear in the absence of a substrate.

The compression of the diagonal lane becomes more intense with increasing \( F_D \), as illustrated in Fig. 17(c) at \( F_D = 3.0 \) for the \( \alpha^b_m = 6.0 \) system, until a transition occurs at sufficiently high drive to the more uniform DL₂ state shown in Fig. 17(d) at \( F_D = 3.75 \). The DL-DL₂ transition is accompanied by a jump up in \( \langle V^a \rangle \) and a drop in \( \langle V^b \rangle \) since the lanes are now aligned closer to the driving direction at an angle of \( \theta_l = 20^\circ \). This change in lane orientation also causes \( \langle V^b \rangle \) to abruptly jump from a negative value to a positive value at the DL-DL₂ transition.
uid in which it transitions with increasing drive into a moving liquid (ML) phase. At low drives we find an additional jump in both $\langle V_\parallel^a \rangle$ and $\langle V_\parallel^b \rangle$ due to rearrangements that transform the DL3 structure into what we call the DL5 state, illustrated in Fig. 18(b) for $F_D = 10.5$. The DL3 lanes are very thin and are nearly aligned with the $x$ direction.

In Fig. 19 we construct a dynamic phase diagram as a function of $F_D$ versus $m$ for the system in Figs. 17 and 18. We find that the width of the moving crystal phase diverges as $m \to 0$, while the PS phase only occurs when $m < 0.5$. These results show that the phases we observe are generic in systems where the damping coefficient is fixed but the Magnus term varies.

VII. MAGNUS TERMS OF OPPOSITE SIGN

Up to now we have focused on systems in which the Magnus term is zero for one or both species, or where both species have a finite Magnus force with the same sign but different magnitude. Here we examine the case where both species have a nonzero Magnus term of equal magnitude that is opposite in sign. In Fig. 20(a) we plot $\langle V_\parallel^a \rangle$, $\langle V_\parallel^b \rangle$, $\langle V_\perp^a \rangle$, and $\langle V_\perp^b \rangle$ versus $F_D$ for a system with $\alpha^a_m = 1.0$, $\alpha^b_m = -1.0$, and $\alpha^a_d = \alpha^b_d = 1.0$ at $F_D = 1.5$ showing the formation of a uniform perpendicular laned (PL) state oriented transverse to the driving direction. (b) A system with $\alpha^a_m = 1.0$, $\alpha^b_m = -1.0$, $\alpha^a_d = 1.0$, and $\alpha^b_d = 3.0$, in which the PL state shows compression or clustering.

In Fig. 20(b) we show the corresponding $P_b$ versus $F_D$. At low drives we find a moving crystal (MC) state, followed by moving liquid (ML) and perpendicular laned (PL) phases.

FIG. 20: (a) $\langle V_\parallel^a \rangle$ (dark red), $\langle V_\parallel^b \rangle$ (dark blue), $\langle V_\perp^a \rangle$ (pink), and $\langle V_\perp^b \rangle$ (light blue) vs $F_D$ in a system with $\alpha^a_m = 1.0$, $\alpha^b_m = -1.0$, and $\alpha^a_d = \alpha^b_d = 1.0$. (b) The corresponding $P_b$ vs $F_D$. At low drives we find a moving crystal (MC) state, followed by moving liquid (ML) and perpendicular laned (PL) phases.

FIG. 21: Images of particle positions for species $a$ (red) and $b$ (blue). (a) The system in Fig. 20 with $\alpha^a_m = 1.0$, $\alpha^b_m = -1.0$, and $\alpha^a_d = \alpha^b_d = 1.0$ at $F_D = 1.5$ showing the formation of a uniform perpendicular laned (PL) state oriented transverse to the driving direction. (b) A system with $\alpha^a_m = 1.0$, $\alpha^b_m = -1.0$, $\alpha^a_d = 1.0$, and $\alpha^b_d = 3.0$, in which the PL state shows compression or clustering.
long as $\alpha_d^a = \alpha_d^b$, even when $\alpha_m^a$ or $F_D$ are very large. By setting the damping terms of the two species to different values, it is possible to induce clustering in the PL state with $\alpha_m^a = -\alpha_m^b$, as illustrated in Fig. 21(b) for a system with $\alpha_m^a = 1.0$, $\alpha_m^b = -1.0$, $\alpha_d^a = 1.0$, and $\alpha_d^b = 3.0$, where the perpendicular lanes are now compressed. This compression arises because species $a$, which has a smaller damping term than species $b$, moves faster and collides with the band of slower moving particles, while the opposite sign of the Magnus terms for the two species is responsible for creating the perpendicular banding.

In Fig. 22 we construct a dynamic phase diagram as a function of $F_D$ versus $\alpha_m^a$ for the system in Fig. 20 with $\alpha_m^b = -\alpha_m^a$, highlighting the fact that the widths of the MC and ML phases diverge upon approaching the overdamped limit of $\alpha_m^a = 0$. The ML phase also increases in extent with increasing $\alpha_m^a$ when $\alpha_m^a > 5.0$.

VIII. OTHER VARIABLES AND DISCUSSION

We have considered several other variables such as varied damping and varied Magnus ratios, and in general find the same phases described above with some minor variations. For example, in Fig. 23(a) we plot $(V_{||}^a)$, $(V_{||}^b)$, $(V_{\perp}^a)$, and $(V_{\perp}^b)$ versus $F_D$ for a system with $\alpha_d^b = 1.0$, $\alpha_d^a = 2.0$, $\alpha_m^a = 0.1$, and $\alpha_m^b = -1.0$, where we find a series of pronounced velocity jumps at the transitions into different DL and DL2 phases. The same generic phases persist when the system density is varied, as shown in the dynamic phase diagram as a function of $F_D$ and $n$ in Fig. 23(b) for a system with $\alpha_d^a = \alpha_d^b = 1.0$, $\alpha_m^a = 0$, and $\alpha_m^b = 0.3$. As $n$ increases, the transitions between the phases shift to higher values of $F_D$. We observe a similar trend for higher values of $\alpha_m^b$. We have also examined systems in which $N_a \neq N_b$ and find similar dynamic phases, as illustrated in Fig. 24(a,b) for a sample with $N_b/N_a = 0.9$, $\alpha_d^a = \alpha_d^b = 1.0$, $\alpha_m^a = 0$, and $\alpha_m^b = 3.0$, where we show that in the ML phase, some species segregation occurs, while in the DL phase, thinner stripes appear. These results indicate that the general features we observe are robust for a wide range of parameters. We have tested systems with different densities and find that the same general phases occur; however, in denser systems, the increased skyrmion-skyrmion repulsion shifts the transition to the thinner stripe state up to larger values of the Magnus term and driving force. We have checked the effect of system size and find similar results in larger samples. In the single stripe state, a periodic ar-
ray of stripes can form in larger samples that has a stripe spacing larger than the $L = 36$ samples which are our focus here. In general the width and periodicity of the stripes depend on the values of the Magnus term, density, and drive. Increasing the Magnus term and drive tends to decrease the stripe width, while increasing the density tends to increase the stripe width.

In this work we have only considered bidisperse particles; however, it would also be interesting to study three or more particle species or even a continuum range of species with a Gaussian distribution of types. In such assemblies, it is possible that the system would generally form disordered or moving liquid phases; however, other new types of pattern formation could appear. We have only utilized particle-based simulations since these allow us to simulate a large number of particles over a wide range of parameters for long times, but it would also be interesting to perform continuum based simulations of multiple species or sizes of skyrmions to see whether similar effects arise when the internal degrees of freedom of the skyrmions are included. In our system, the Magnus term leads to the appearance of a skyrmion Hall angle, but there are recent studies which show that some skyrmion systems can exhibit motion that is more toroidal in addition to other types of complex dynamics [49, 50]. It would be interesting to study such systems in the presence of multiple skyrmion species. Further effects could arise if the applied driving were ac rather than dc, since the multiple species could organize into different types of patterns under cyclic driving.

IX. SUMMARY

We have examined a bidisperse system of particles with uniform pairwise interactions under dynamics that include both a damping term and a Magnus term. When both species have equal damping but only one species has a finite Magnus term, we find that the triangular lattice which forms under zero drive moves elastically at low drives with a Hall angle equal to the average Hall angle of the two species. At a critical drive, a Magnus induced disordering transition occurs in which each species moves with a different velocity in the direction perpendicular to the drive. The critical drive at which the disordering transition appears diverges as the Magnus term goes to zero. At higher drives, there is a transition to a perpendicular stripe or cluster state with both density and species phase separation. The stripes become more compressed as the drive or difference in Magnus terms increases, and another transition occurs to a density modulated diagonal lined state at even higher drives. The transitions are associated with pronounced jumps and locking of the transverse and longitudinal velocities of each species as well as changes in the global particle structures. We also find that multiple transitions can occur within the diagonal lined phase, each of which is accompanied by a reduction in both the number of particles in each row and the angle between the stripe and the driving direction, giving rise to a rich variety of different types of patterns. In some cases, one of the species can exhibit absolute negative mobility in which the particles move in the direction opposite to that of the applied drive due to the Magnus-induced rotation of the interaction forces between the two particle species. When both species have different damping terms but zero Magnus terms, we find dynamic phases that are very similar to those observed for oppositely driven Yukawa particles, which form uniform lined states. For equal damping terms and Magnus terms that are equal in magnitude but opposite in sign, uniform density states appear containing lanes that are perpendicular to the applied drive. We show that the diagonal lined and density modulated states are robust for a wide range of parameters and densities, and should be generic features of systems with dispersity in the Magnus force. We discuss the relation between our results and studies of skyrmion systems with dispersion in the Magnus force, where we predict that a disordering transition should occur as a function of increasing drive, and that a variety of clustered and pattern forming states could be observed. Similar effects may arise in soft matter systems containing Magnus terms, such as spinning magnetic particles in solution.

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[7] T. Bohlein, J. Mikael, and C. Bechinger, Observation of


