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Clogging and Depinning of Ballistic Active Matter Systems in Disordered Media

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We numerically examine ballistic active disks driven through a random obstacle array. Formation of a pinned or clogged state occurs at much lower obstacle densities for the active disks than for passive disks. As a function of obstacle density we identify several distinct phases including a depinned fluctuating cluster state, a pinned single cluster or jammed state, a pinned multicluster state, a pinned gel state, and a pinned disordered state. At lower active disk densities, a drifting uniform liquid forms in the absence of obstacles, but when even a small number of obstacles are introduced, the disks organize into a pinned phase-separated cluster state in which clusters nucleate around the obstacles, similar to a wetting phenomenon. We examine how the depinning threshold changes as a function of disk or obstacle density, and find a crossover from a collectively pinned cluster state to a disordered plastic depinning transition as a function of increasing obstacle density. We compare this to the behavior of nonballistic active particles and show that as we vary the activity from completely passive to completely ballistic, a clogged phase-separated state appears in both the active and passive limits, while for intermediate activity, a readily flowing liquid state appears and there is an optimal activity level that maximizes the flux through the sample.

I. INTRODUCTION

There is a wide range of soft and hard matter systems that can be modeled as collectively interacting particles which, when driven over quenched disorder, exhibit pinning-depinning behavior as well as transitions between different types of sliding regimes [1, 2]. Such dynamics occur for vortex motion in type-II superconductors [3, 4], sliding charge density waves [5], depinning of classical Wigner crystals [6, 7], current driven motion of skyrmions in chiral magnets [8, 9], colloids interacting with random [10–15] or ordered substrates [16], sliding in frictional systems [17], magnetic domain wall motion [18, 19], erosion [20], granular matter [21, 22], driven pattern forming systems [23], geophysical models of plate tectonics [24], and the motion of dislocations in crystalline materials [25]. The substrate may be random, ordered, or partially ordered, and it can be modeled as localized pinning sites with a finite trapping strength or as impenetrable obstacles. Under an applied drive, the particles exhibit a variety of pinned and moving order-disorder transitions that can be characterized by the moving structure, pattern formation features, changes in the velocity force curves, and fluctuation phenomena [1, 2]. In the systems listed above, the particles themselves are passive or experience only thermal fluctuations, so the driving is strictly externally applied; however, recently a growing number of studies have focused on what are called active matter systems containing self-driven particles with an activity that is often modeled as arising from driven diffusive or run-and-tumble dynamics [26, 27]. In the absence of a substrate, active disks exhibit a transition from a uniform gas or liquid state to a phase-separated or cluster state consisting of a high density solid coexisting with a low density active gas [28–34]. This transition occurs for fixed activity as a function of increasing density or for fixed density with increasing ac-

tivity level. Active matter systems have also been studied in the context of particle shape effects [27, 35], active rotators [36], passive and active mixtures [37–39], boundary effects [40–44], and ratchet effects [45–47].

Several studies have examined different types of active matter systems coupled with ordered [48–50] or disordered substrates [33, 51–66]. Numerical simulations show that when a drift force is applied to run-and-tumble disks moving through a random obstacle array, the flux through the system is non-monotonic as a function of activity level, indicating that there is an optimal run length or run correlation time that maximizes the flux of disks through the obstacles [33]. At small run lengths, the disks behave thermally and easily become trapped behind the obstacles, giving a low disk flux. As the activity level or run length increases, the disks can more readily move around the obstacles, increasing the flux; however, when the run length is too large, a self-pinning effect occurs in which the disks self-cluster around the obstacles, reducing the flux [33, 65]. As a result, the flux is maximized when the disks are active and the self-clustering is weak, while it is reduced when the activity becomes high enough for significant clustering or self-trapping to occur. When the run lengths are very long, the average flux under an applied drive is strongly reduced, but it never reaches zero since there are still long-time dynamical rearrangements that produce avalanches or intermittent flow of disks in the direction of the drift force [65]. Analytic and theoretical studies of active systems without a drift force also indicate that there is an optimal activity level that maximizes the diffusion in the system [64]. If the obstacles are replaced by pinning sites, the onset of clustering can have the opposite effect of increasing rather than decreasing the flux, since the clusters act like large rigid objects that are poorly trapped by individual pinning sites [60]. When the activity is low, a uniform liquid state appears and individual disks can be trapped

readily by individual pinning sites [60]. These works also showed that within the phase separated state, increasing the number of obstacles or pinning sites produces a disorder-induced transition from a phase-separated state to a uniform disordered state [58, 60].

In addition to the run-and-tumble models, several simulation studies of swarming or flocking active systems with quenched disorder show that there is an optimal noise level for the appearance of flocking [51, 54], and that increasing the disorder strength can induce a transition from a flocking to a non-flocking state [53]. Experiments on colloidal rollers that exhibit flocking behavior have also revealed a transition from a drifting flocking state to a non-flocking state as a function of increasing obstacle density, where the flow becomes increasingly filamentary as the disordered state is approached [57].

In these systems, the activity is stochastic in nature. For run-and-tumble particles, after each run interval the particle randomly reorients and runs in a new direction, while in driven diffusive models, there is a noise term controlling the rate of rotational diffusion [27]. The deterministic limit of active matter is purely ballistic flow where the swimming direction of each particle is permanently fixed. This can be achieved in run-and-tumble disk systems by setting the running time to infinity, or in driven diffusive systems by setting the rotational diffusion coefficient to zero [52]. In the deterministic regime, the particles can form a cluster state even at very low densities, and Bruss *et al.* [67] argue that phase separation occurs when the mean time between collisions is smaller than the mean duration of an individual collision. Previous studies [52] of an active ballistic system showed that the disks can form a frozen cluster state where almost all the fluctuations are lost. This frozen state arises due to the lack of stochastic behavior in the system, and the resulting cluster can be regarded as an absorbed state. In contrast, at long but finite run times, the cluster can gradually evolve over time due to the possibility of rare stochastic reorganizations [65].

Previous studies of the active ballistic limit did not include an applied drift force, so no pinning-depinning phenomena occurred. In this work we consider active ballistic disks driven through an obstacle array, and we measure the average drift mobility $\langle V \rangle$ of the disks in the direction of the driving force F_D . We find that the system can evolve toward a completely pinned or clogged state with $\langle V \rangle \approx 0$. We describe our simulation in Sec. II. In Sec. III we compare the active pinning or clogging to that observed in the zero activity or passive limit. The critical density of obstacles needed to induce the formation of a clogged state is much higher in the passive system than in the active ballistic system, and we show that in general the active ballistic disks are much more susceptible to forming clogs than the passive disks. In the active system, clogging is associated with the formation of a cluster state. As a function of increasing obstacle density, we identify four phases: a sliding state with either a fluctuating cluster or a liquid structure; a pinned single cluster or

jammed state consisting of a large cluster held in place by a small number of obstacles; a pinned multicluster state containing several distinct pinned clusters; and a pinned disordered state in which the disk density remains spatially uniform. As the active disk density increases, we also find what we term a pinned gel state where the disks form a percolating labyrinth structure. For low active disk densities where a flowing uniform liquid appears in the absence of obstacles, we find that introduction of a small number of obstacles causes a transition to a pinned cluster state, which we compare to the active wetting of clusters around an obstacle. In Sec. IV we examine the velocity-force relations and the behavior of the critical depinning force F_c . The depinning threshold for a cluster is finite, and there is a pronounced increase in F_c at the transition from the pinned single cluster state to the pinned disordered state, reminiscent of the “peak effect” observed in superconducting vortex systems at a transition from a collectively pinned ordered or quasiordered vortex crystal to a vortex glass state [2, 3]. In Sec. V we show how passive clogging can be connected to the ballistic clogging limit by considering finite but increasing run times for run-and-tumble active disks in obstacle arrays. In both the passive and ballistic clogged states, the disks form a cluster, while between these two limits a flowing liquid structure appears. There is an optimal run time at which the disk flux is maximized, and the flux decreases with increasing run length until the disks form a completely clogged state at infinite run times. We discuss how these results can be related to granular jamming transitions [68–70] and jamming in systems with quenched disorder [71, 72], where the activity or the obstacle density represent an additional set of parameters that can be used to induce a jammed state. In Sec. VI we summarize our results.

II. SIMULATION

We consider a two-dimensional system with periodic boundary conditions in the x and y -directions containing N_a active or mobile disks that interact through a stiff repulsive harmonic spring, $\mathbf{F}_s = k(d - 2R)\Theta(d - 2R)\hat{\mathbf{d}}$, where d is the distance between two disks, $\hat{\mathbf{d}}$ is the displacement vector, $k = 100$ is the spring constant, and the disk radius is $R = 0.5$. For this value of k the disk-disk overlaps remain very small, allowing us to define the active disk density in terms of the area covered by the disks, $\phi_a = N_a\pi R^2/L^2$, where the system size is $L = 100$. In the limit of no activity and no obstacles, the disks form a hexagonal lattice at $\phi_a = 0.9$. In addition to the mobile active disks, we introduce N_{obs} obstacles that are identical to the active disks but are permanently fixed in place. The obstacles are initially placed in a hexagonal lattice and are randomly diluted until we reach the desired obstacle density of $\phi_{\text{obs}} = N_{\text{obs}}\pi R^2/L^2$. Placing the obstacles in an initial hexagonal lattice ensures that there is a fixed minimum distance between any two ob-

stacles, thereby avoiding the rare obstacle density fluctuations such as large gaps or tight obstacle clustering that can arise from a completely random obstacle placement and dominate the dynamics. The total disk coverage of both active and fixed disks is $\phi_{\text{tot}} = \phi_{\text{obs}} + \phi_a$. The dynamics of the active disks is obtained from the following overdamped equation of motion:

$$\eta \frac{d\mathbf{r}_i}{dt} = \mathbf{F}_{s,a}^i + \mathbf{F}_m^i + \mathbf{F}_{s,\text{obs}}^i + \mathbf{F}_D. \quad (1)$$

Here $\eta = 1.0$ is the damping constant and the interaction with other active disks $\mathbf{F}_{s,a}$ has the form of the stiff spring repulsion \mathbf{F}_s . Each active disk has a motor force \mathbf{F}_m^i that is constant in magnitude and oriented in a randomly chosen direction. In run-and-tumble systems, the motor force orientation is held fixed during the run time τ_r , after which a new random orientation is chosen for the next running time. In the ballistic limit, we set τ_r to infinity so that the running direction never changes. The forces from the obstacles $\mathbf{F}_{s,\text{obs}}$ are also given by \mathbf{F}_s , and the external driving force $\mathbf{F}_D = F_D \hat{\mathbf{x}}$ is applied uniformly to all active disks. We also consider the passive particle limit in which $\mathbf{F}_m = 0$ and the only driving force is the externally applied \mathbf{F}_D . To initialize the system, after establishing the location of the obstacles we place mobile disks with artificially reduced radii in nonoverlapping locations and allow the mobile disks to rearrange while gradually expanding the radii to the final value of $R = 0.5$. With this method we can reach disk densities up to $\phi_{\text{tot}} = 0.86$.

To characterize the system we measure the average drift velocity of the disks $\langle V \rangle = \langle N_a^{-1} \sum_{i=0}^{N_a} \mathbf{v}_i \cdot \hat{\mathbf{x}} \rangle$ in the direction of the drive. We wait 1×10^6 simulation time steps for the system to settle into a steady state and then average over an additional 9×10^6 simulation time steps to obtain $\langle V \rangle$. We have found that increasing the waiting time produces negligible changes in the results. In this work, unless otherwise noted we set $F_D = 0.05$, $|\mathbf{F}_m| = 0.5$, and $\tau_r = \infty$; however, we also consider varied F_D and finite τ_r .

III. PHASES OF ACTIVE BALLISTIC DISKS

In Fig. 1(a) we plot the fraction C_{max}/N_a of active disks in the largest cluster versus time in simulation time steps for a system with $F_D = 0.05$, $N_a = 4450$, and $N_{\text{obs}} = 50$, giving an active disk density of $\phi_a = 0.3495$, an obstacle density of $\phi_{\text{obs}} = 0.00393$, and an overall density of $\phi_{\text{tot}} = 0.3534$. The largest cluster size C_{max} is defined as the largest number of disks in direct contact with each other as determined using the cluster identification algorithm described in Ref. [73]. Figure 1(b) shows the corresponding disk velocity $V = N_a^{-1} \sum_{i=0}^{N_a} \mathbf{v}_i \cdot \hat{\mathbf{x}}$ versus time. Both C_{max}/N_a and V exhibit strong fluctuations during the first 2.5×10^6 time steps, indicating that the system is in a dynamic unpinned fluctuating state. The disks then become trapped in a single pinned

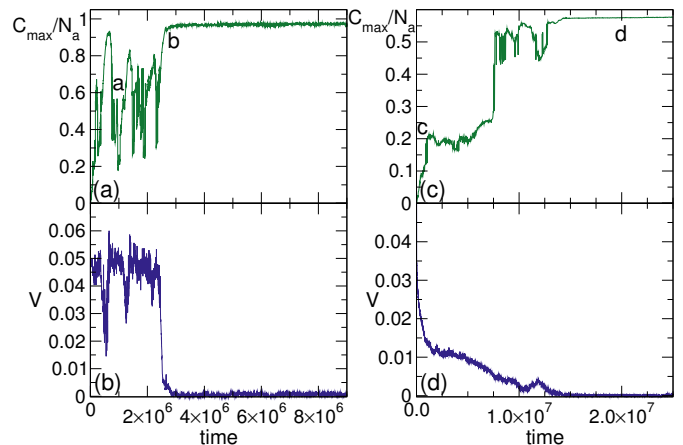


FIG. 1: (a,b) An active ballistic system with an area coverage of $\phi_a = 0.3495$ active disks and $\phi_{\text{obs}} = 0.00393$ obstacles for a total $\phi_{\text{tot}} = 0.3534$. An external drift force of $F_D = 0.05$ is applied in the positive x -direction. (a) The fraction of disks in the largest cluster, C_{max}/N_a , versus time in simulation time steps. (b) The drift velocity V of the active disks vs time. There is a transition from a fluctuating cluster state that is drifting in the direction of drive, illustrated in Fig. 2(a), to a pinned single cluster, shown in Fig. 2(b). At the transition, C_{max}/N_a abruptly increases to a value close to one and V simultaneously drops nearly to zero. The letters **a** and **b** indicate the times corresponding to the images in Fig. 2(a,b). (c) C_{max}/N_a and (d) V vs time for the same system in the passive $|\mathbf{F}_m| = 0$ limit at $\phi_{\text{tot}} = 0.3534$ and $\phi_{\text{obs}} = 0.1178$. For this value of ϕ_{tot} , the system can reach a pinned or clogged state only when $\phi_{\text{obs}} \geq 0.098$. The system evolves over time into a pinned state, with a gradual drop in V accompanied by a gradual increase in C_{max}/N_a . The initial unclogged state at the time marked **c** is illustrated in Fig. 2(c), while the $V = 0$ pinned state at the time marked **d** is shown in Fig. 2(d).

cluster, as shown by the sudden jump in C_{max}/N_a to $C_{\text{max}}/N_a \approx 1.0$ which is accompanied by a drop in V to nearly zero. There are still some small fluctuations in both C_{max}/N_a and V due to the presence of a small number of freely running disks that do not join the cluster. In Fig. 2(a) we show a snapshot of the depinned fluctuating clusters at time 1.0×10^6 , where the active disks form temporary clusters. Here $V = 0.046$, which is close to the expected obstacle-free value of $V = 0.05$. A pinned single cluster state appears at time 3×10^6 , as illustrated in Fig. 2(b), where the active disks form a single large immobile cluster. We find that even a small number of obstacles ($N_{\text{obs}}/N_a = 0.011$) can produce a pinned state for active ballistic disks. In contrast, for passive disks at the same total density of $\phi_{\text{tot}} = 0.3534$, the system does not reach a pinned or clogged state until $N_{\text{obs}}/N_a \geq 0.384$, indicating that nearly 35 times more obstacles are required to pin the passive disks compared to the active ballistic disks.

In Fig. 1(c,d) we plot C_{max}/N_a and V versus time for a passive $|\mathbf{F}_m| = 0$ system with $\phi_{\text{tot}} = 0.3534$ and $\phi_{\text{obs}} = 0.1178$, where the disks reach a pinned or

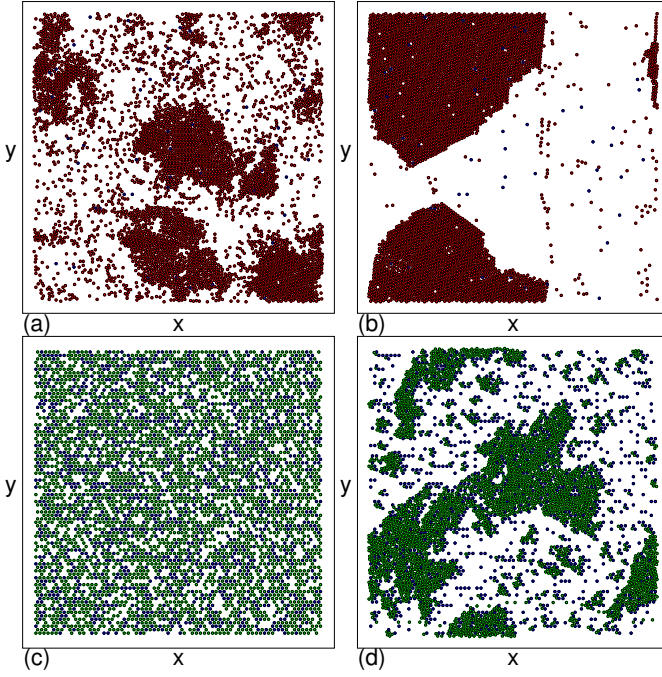


FIG. 2: (a,b) The active ballistic disk positions (red circles) and obstacle locations (blue circles) for the system in Fig. 1(a,b) with $\phi_{\text{tot}} = 0.3534$, $\phi_{\text{obs}} = 0.00393$, and a drift force $F_D = 0.05$ applied in the positive x direction. (a) Depinned fluctuating clusters appear at time 1×10^6 , marked **a** in Fig. 1(a). (b) The pinned single cluster state at time 3×10^6 , marked **b** in Fig. 1(a). (c,d) Passive disk positions (green circles) and obstacle locations (blue circles) for the system from Fig. 1(c,d) with $\phi_{\text{tot}} = 0.3534$ and $\phi_{\text{obs}} = 0.1178$. (c) The initial flowing state at the time marked **c** in Fig. 1(c). (d) The clogged or pinned state at time 3×10^6 , marked **d** in Fig. 1(c).

clogged state. In contrast to the active ballistic system in Fig. 1(a,b), the pinned state does not appear abruptly; instead, the passive disks continuously evolve toward the pinned state over time, with a growing number of pinned clusters gradually emerging as indicated by the steady increase in C_{max}/N_a and the gradual decrease in V . In Fig. 2(c) we illustrate the initial uniform spatial distribution of the passive disks, while in Fig. 2(d) we show a snapshot of the disk positions in the clogged state at a time of 2×10^6 . Unlike the active ballistic disks in Fig. 2(b), the passive disks in Fig. 2(d) do not form a single clump but instead assemble into a number of smaller clumps. This indicates that the passive and active clogged or pinned states are very different in nature.

In Fig. 3(a,b) we plot $C_L = \langle C_{\text{max}}/N_a \rangle$ and $\langle V \rangle$ versus ϕ_{obs} for the active ballistic system from Fig. 1(a) with $\phi_{\text{tot}} = 0.3534$ at $F_D = 0.05$. For $\phi_{\text{obs}} < 0.0039$ the system is in a depinned fluctuating cluster state, labeled phase I_{fc}, where $\langle V \rangle$ is finite and the long-time average of C_{max}/N_a is $C_L \approx 0.5$. For $0.0039 \leq \phi_{\text{obs}} < 0.025$, we find a pinned single cluster state, denoted phase II, with $C_L > 0.9$ and $\langle V \rangle \approx 0.0$. In Fig. 4(a) we illustrate

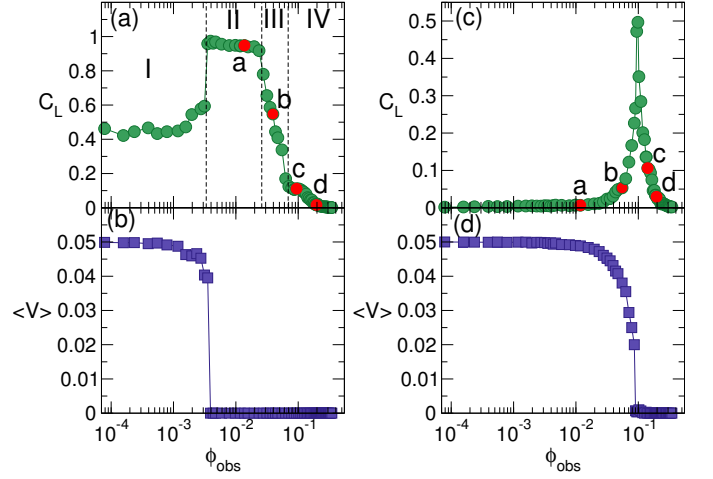


FIG. 3: (a) C_L and (b) $\langle V \rangle$ vs obstacle density ϕ_{obs} for active ballistic disks with $F_D = 0.05$ and $\phi_{\text{tot}} = 0.3534$. For $\phi_{\text{obs}} < 0.0039$, $\langle V \rangle$ is finite and the system is in phase I_{fc}, the depinned fluctuating cluster state, while at the transition to phase II, the pinned single cluster state, $\langle V \rangle$ drops to zero. Phase II extends from $0.0039 \leq \phi_{\text{obs}} < 0.025$ and is illustrated in Fig. 4(a). For $0.025 \leq \phi_{\text{obs}} < 0.065$, the system is in phase III, a pinned multicluster state, illustrated in Fig. 4(b), while for $\phi_{\text{obs}} \geq 0.065$ the system is in phase IV, the pinned disordered phase illustrated in Fig. 4(c,d) at $\phi_{\text{obs}} = 0.0942$ and $\phi_{\text{obs}} = 0.1963$. The labels **a** to **d** in (a) indicate the values of ϕ_{obs} at which the images in Fig. 4 are obtained. (c) C_L and (d) $\langle V \rangle$ vs ϕ_{obs} for passive disks at the same F_D and ϕ_{tot} . Here there are only two phases: a plastic flow state $\phi_{\text{obs}} < 0.098$, and a completely pinned or clogged state for $\phi_{\text{obs}} \geq 0.098$. There is a peak in C_L at $\phi_{\text{obs}} = 0.098$ where $\langle V \rangle$ drops to zero. The labels **a** to **d** in (c) indicate the values of ϕ_{obs} at which the images in Fig. 5 are obtained.

phase II at $\phi_{\text{obs}} = 0.01178$. Phase II can be viewed as a pinned jammed state in which a single clump has nucleated around the obstacles and acts as a rigid solid. Active disks that are not adjacent to obstacles are pinned or prevented from moving by other active disks through contact interactions, so the collective pinning of the clump is dominated by disk-disk interactions rather than by direct disk-obstacle interactions. Since we are using monodisperse disks rather than the bidisperse disk mixture commonly studied in jammed systems, the particles forming the cluster have a substantial amount of hexagonal or crystalline ordering, whereas typical 2D jammed systems form amorphous rather than polycrystalline packings [69, 71]; however, in both our system and the jamming systems, it is the disk-disk contact interactions that cause the system to act like a solid that can be pinned by a small number of obstacles. Previous work [52] on active ballistic systems revealed similar large-scale frozen cluster states, but did not include an external drift force. In the present study, the formation of the single frozen cluster in the presence of a drift force results in a pinned state.

For $0.025 \leq \phi_{\text{obs}} < 0.065$ in Fig. 3(a,b), we observe

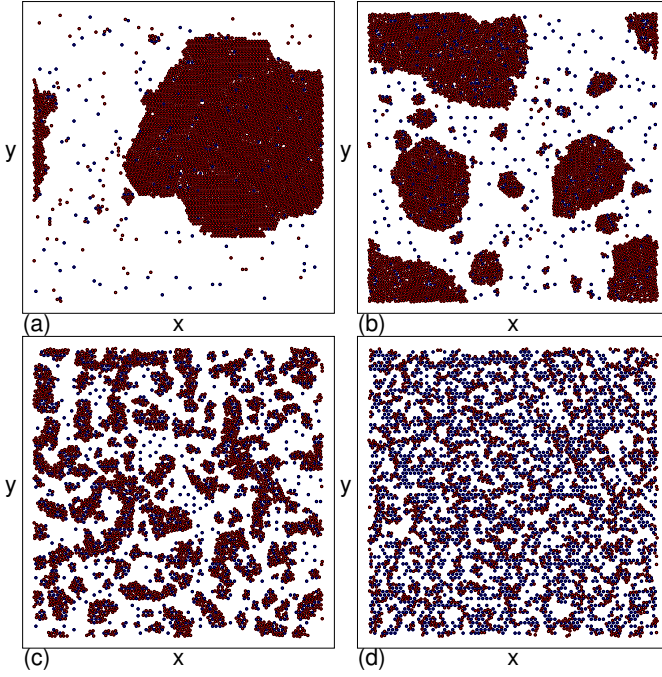


FIG. 4: The active ballistic disk positions (red circles) and obstacle locations (blue circles) for the system in Fig. 3(a,b) with $F_D = 0.05$ and $\phi_{\text{tot}} = 0.3534$ obtained at the values of ϕ_{obs} marked by the letters **a** to **d** in Fig. 3(a). (a) The pinned single cluster phase II at $\phi_{\text{obs}} = 0.01178$. (b) The pinned multicluster phase III at $\phi_{\text{obs}} = 0.039$. (c) The pinned disordered phase IV at $\phi_{\text{obs}} = 0.0942$ consists of a group of small clusters. (d) The pinned disordered phase IV at $\phi_{\text{obs}} = 0.1963$ is composed of even smaller clusters.

a pinned multicluster state termed phase III in which $\langle V \rangle = 0$ where C_L decreases from $C_L = 0.9$ to $C_L = 0.12$ with increasing ϕ_{obs} . A snapshot of the disk positions in phase III at $\phi_{\text{obs}} = 0.039$ appears in Fig. 4(b). For $\phi_{\text{obs}} \geq 0.065$, the system is in phase IV, a pinned disordered state with $C_L < 0.15$ in which the disks form numerous small clumps that gradually decrease in size with increasing ϕ_{obs} , as illustrated in Fig. 4(c) at $\phi_{\text{obs}} = 0.0942$ and in Fig. 4(d) at $\phi_{\text{obs}} = 0.1963$.

We plot C_L and $\langle V \rangle$ versus ϕ_{obs} for passive disks with $\phi_{\text{tot}} = 0.3534$ and $F_D = 0.05$ in Fig. 3(c,d). The passive disks do not reach a pinned state with $\langle V \rangle \approx 0$ until $\phi_{\text{obs}} > \phi_c = 0.098$. Figure 3(c) shows that C_L is small for low ϕ_{obs} and increases to a peak value of $C_L = 0.5$ just below ϕ_c . For $\phi_{\text{obs}} > \phi_c$, C_L decreases with increasing obstacle density. In Fig. 5(a) we show a snapshot of the flowing state at $\phi_{\text{obs}} = 0.01178$. Although no clusters appear, the disks tend to form one-dimensional (1D) flowing chains. In the flowing state at $\phi_{\text{obs}} = 0.055$, illustrated in Fig. 5(b), small clusters are beginning to appear. The pinned cluster state near the peak value of C_L at $\phi_{\text{obs}} = 0.1178$ is shown in Fig. 2(d). Above the peak in C_L , the size of the clusters decreases with increasing obstacle density and a clogged state forms as shown in Fig. 5(c) for $\phi_{\text{obs}} = 0.14137$ and in Fig. 5(d)

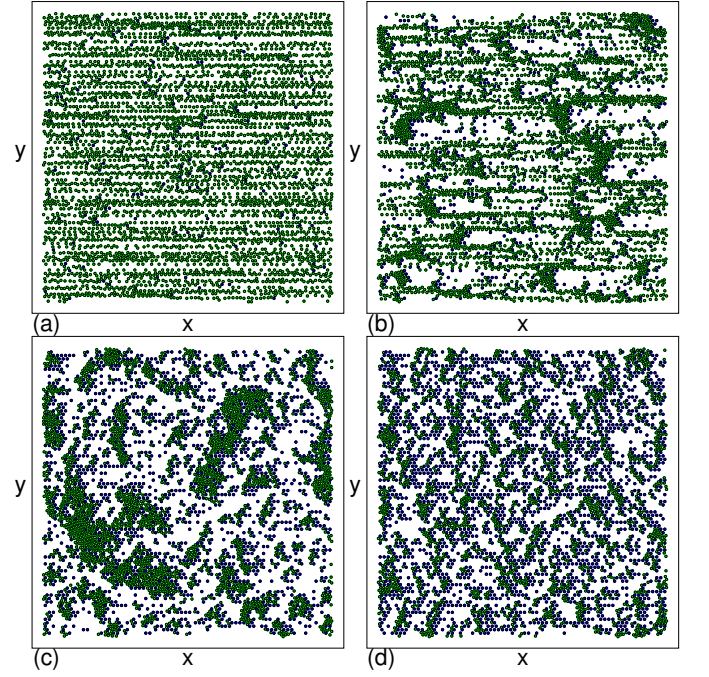


FIG. 5: Passive disk positions (green circles) and obstacle locations (blue circles) for the system in Fig. 4(c,d) with $F_D = 0.05$ and $\phi_{\text{tot}} = 0.3534$ obtained at the values of ϕ_{obs} marked by the letters **a** to **d** in Fig. 3(c). (a) The flowing state at $\phi_{\text{obs}} = 0.01178$. (b) At $\phi_{\text{obs}} = 0.055$, clusters begin to form. (c) The clogged state at $\phi_{\text{obs}} = 0.14137$. (d) The clogged state at $\phi_{\text{obs}} = 0.1963$.

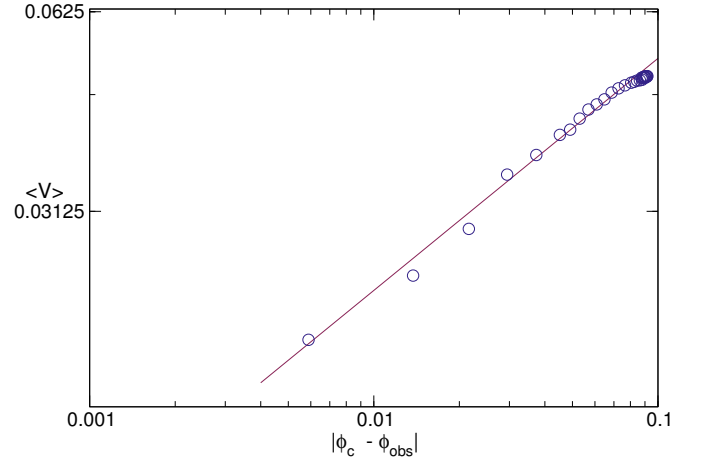


FIG. 6: The scaling of $V \propto |\phi_c - \phi_{\text{obs}}|^\beta$ for the passive disks in Fig. 3(d) with $\phi_c = 0.098$ and $\beta = 0.35$.

for $\phi_{\text{obs}} = 0.1963$, where the clusters have become quite small.

The peak or divergence in C_L for the passive disk system in Fig. 3(c) suggests that the onset of complete clogging at ϕ_c occurs at a critical point. We have tried performing a power law fit $C_L \propto |\phi_c - \phi_{\text{obs}}|^\nu$ on either side of the divergence; however, we find only a limited range for

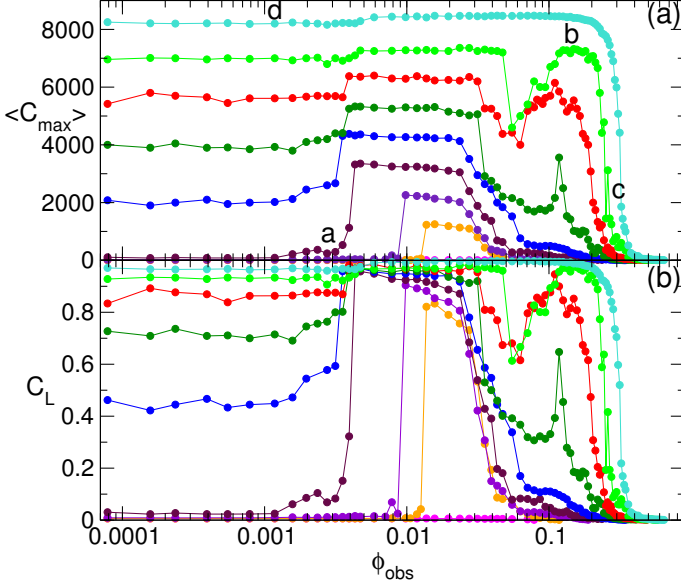


FIG. 7: (a) $\langle C_{\max} \rangle$, the average number of disks in the largest cluster, vs ϕ_{obs} at $F_D = 0.05$ for $\phi_{\text{tot}} = 0.668$ ($N_{\text{tot}} = 8500$, turquoise), 0.589 (7500, light green), 0.511 (6500, red), 0.432 (5500, dark green), 0.354 (4500, blue), 0.275 (3500, maroon), 0.196 (2500, violet), 0.118 (1500, orange), and 0.0786 (1000, magenta). (b) The corresponding C_L vs ϕ_{obs} curves. The I-II transition is associated with a jump or increase in $\langle C_{\max} \rangle$ and C_L , while at large ϕ_{obs} , the system enters a pinned disordered phase as indicated by the drop in C_{\max} and C_L to nearly zero. The labels **a** to **d** in (a) indicate the values of ϕ_{obs} at which the images in Fig. 8 were obtained.

the fit resulting in strong variations in the exponent. We find more consistent scaling of the average drift velocity as ϕ_c is approached, with $\langle V \rangle \propto |\phi_c - \phi_{\text{obs}}|^\beta$, as shown in Fig. 6 where $\beta = 0.35$. We have studied the critical clogging behavior of passive disks in more depth in Ref. [14], where we find a robust power law divergence in the transient times near ϕ_c consistent with an absorbing phase transition. The focus of the present work is active ballistic jamming and we measure the passive disks for comparison. Our results indicate that the onset of pinned or clogged states for the active ballistic disks is very different in nature from the clogging of passive disks. In particular, we find only two phases for the passive disks and four phases for the active ballistic disks. The I_{fc}-II transition marking the onset of a pinned state for the active ballistic disks produces discontinuities in both $\langle V \rangle$ and C_L , consistent with a first order phase transition, while for the passive disks, the onset of pinning or clogging has the character of a second order phase transition or a crossover phenomenon.

In Fig. 7(a) we plot $\langle C_{\max} \rangle$, the average number of disks in the largest cluster, versus ϕ_{obs} for the active ballistic system at varied ϕ_{tot} to highlight the evolution of phases I through IV. Figure 7(b) shows the collapse of the curves when the same results are plotted in terms of C_L , the average fraction of disks in the largest cluster,

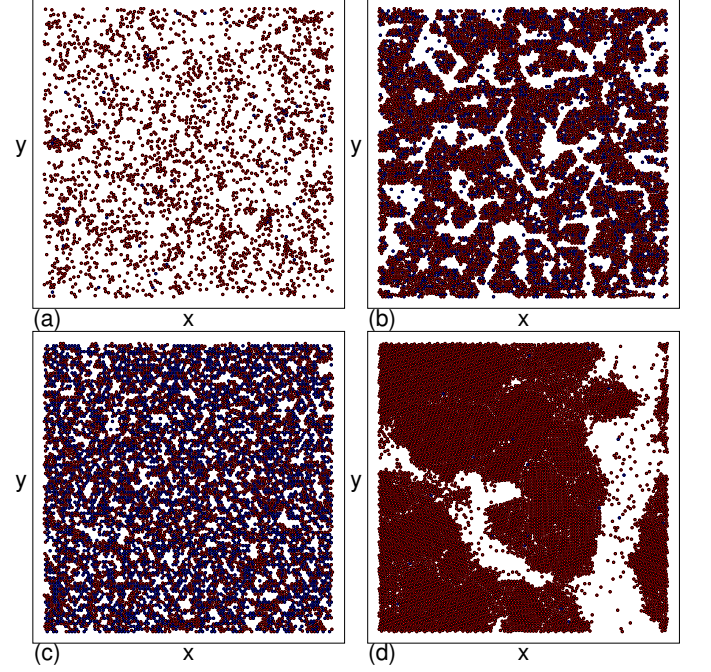


FIG. 8: The active ballistic disk positions (red circles) and obstacle locations (blue circles) for the system in Fig. 7 obtained at the values of ϕ_{tot} and ϕ_{obs} marked by the letters **a** to **d** in Fig. 7. (a) The drifting liquid for $\phi_{\text{tot}} = 0.196$ and $\phi_{\text{obs}} = 0.003926$. (b) The pinned gel phase containing a large-scale percolating cluster at $\phi_{\text{tot}} = 0.589$ and $\phi_{\text{obs}} = 0.1256$. (c) The pinned disordered phase at $\phi_{\text{tot}} = 0.589$ and $\phi_{\text{obs}} = 0.283$. (d) The high density depinned fluctuating cluster state at $\phi_{\text{tot}} = 0.668$ and $\phi_{\text{obs}} = 0.000156$.

ter, versus ϕ_{obs} . For $\phi_{\text{tot}} \leq 0.275$, phase I is a uniform drifting liquid with $C_L < 0.04$, as illustrated in Fig. 8(a) for $\phi_{\text{tot}} = 0.196$ at $\phi_{\text{obs}} = 0.003926$. As ϕ_{obs} increases, there is a transition from the drifting liquid phase I to the pinned single cluster phase II, as indicated by the large increase in C_L to a value of $C_L = 0.8$ or higher. The obstacle density ϕ_{obs} at which the I-II transition occurs shifts upward as ϕ_{tot} decreases, and at the lowest values of ϕ_{tot} that we consider, the system always remains in phase I, as shown for $\phi_{\text{tot}} = 0.0786$ in Fig. 7. We note that the maximum allowed value of ϕ_{obs} decreases with decreasing ϕ_{tot} since it is bounded by the total disk density. For $\phi_{\text{tot}} \geq 0.35$, instead of the drifting liquid phase I, we find the depinned fluctuating cluster state I_{fc} as described previously, and in all cases the I-II and I_{fc}-II transitions are associated with a jump or increase in C_L . The II-III transition also shifts to higher values of ϕ_{obs} with increasing ϕ_{tot} .

Within the pinned disordered phase IV in Fig. 7, an additional feature emerges for $\phi_{\text{tot}} = 0.432$ in the form of a peak in $\langle C_{\max} \rangle$ and C_L near $\phi_{\text{obs}} = 0.115$. This peak grows in both height and extent with increasing ϕ_{tot} . At the onset of the pinned disordered phase IV, C_L is low and the disks form a small number of isolated clumps. As ϕ_{obs} increases, these clumps break apart and

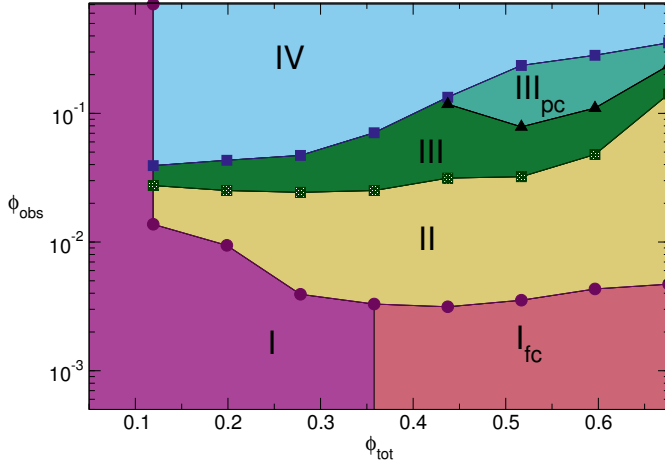


FIG. 9: The locations of the phases in Fig. 7 as a function of ϕ_{obs} vs ϕ_{tot} . Phase I (magenta) is the drifting liquid state; phase I_{fc} (pink) is the depinned fluctuating clump state; phase II (yellow) is the pinned single cluster state; phase III (dark green) is the pinned multicluster state; phase III_{pc} (light green) is the pinned gel state; and phase IV (blue) is the pinned disordered state.

the disks are spread more evenly over the substrate. At higher overall disk densities $\phi_{\text{tot}} \geq 0.432$, this produces a percolation transition in which the broken clumps merge to form a pinned gel or labyrinth state of the type illustrated in Fig. 8(b) at the peak in $\langle C_{\text{max}} \rangle$ and C_L for the $\phi_{\text{tot}} = 0.589$ and $\phi_{\text{obs}} = 0.1256$ system in Fig. 7. For higher ϕ_{obs} , the active ballistic disks spread further apart and the gel transforms to a pinned disordered state, as shown in Fig. 8(c) for the $\phi_{\text{tot}} = 0.589$ system from Fig. 7 at $\phi_{\text{obs}} = 0.283$. The emergence of the intermediate pinned gel state is responsible for the additional peak in $\langle C_{\text{max}} \rangle$ and C_L . At the highest value $\phi_{\text{tot}} = 0.668$ in Fig. 7, the II-III transition is lost and the $\langle C_{\text{max}} \rangle$ and C_L curves become nearly featureless below the transition to the pinned disordered phase IV. When the total disk density is high, motion in the depinned fluctuating cluster phase I_{fc} becomes less intermittent and the steady state value of C_L increases to $C_L > 0.95$. The high density depinned fluctuating cluster state is illustrated in Fig. 8(d) for $\phi_{\text{tot}} = 0.668$ and $\phi_{\text{obs}} = 0.00156$.

Using the results in Fig. 7, we can construct a phase diagram showing the evolution of the different phases as a function of ϕ_{obs} versus ϕ_{tot} , as shown in Fig. 9. The unpinned state is a drifting liquid (phase I) for low ϕ_{tot} and a depinned fluctuating clump state (phase I_{fc}) for high ϕ_{tot} . Phase II is the pinned single clump state and phase III is the pinned multiclump state. For large ϕ_{tot} we find a window of phase III_{pc} , the pinned gel state, at ϕ_{obs} values above phase III. Phase IV is the pinned disordered state. The features in the C_L and $\langle V \rangle$ curves indicate that the I-II and I_{fc} -II transitions are first order in nature, while the II-III and III-IV transitions are continuous or show crossover behavior. In Ref. [14] we

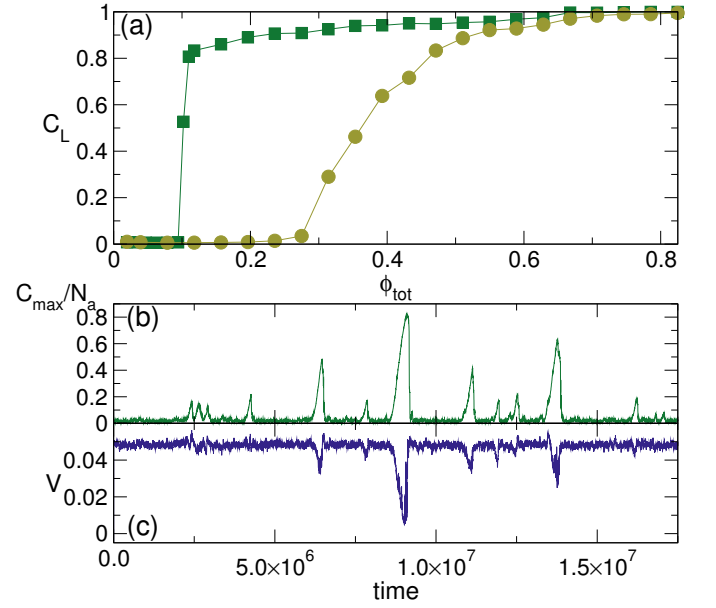


FIG. 10: (a) C_L vs ϕ_{tot} for active ballistic disks with $F_D = 0.05$. Light green circles: In the obstacle-free system $\phi_{\text{obs}} = 0$, clustering begins near $\phi_{\text{tot}} = 0.28$. Dark green squares: A system with $\phi_{\text{obs}} = 0.00157$, showing that inclusion of a small number of obstacles can stabilize a cluster state at much lower densities $\phi_{\text{tot}} \approx 0.1$. (c) C_{max}/N_a and (d) instantaneous velocity V versus time for a system with $\phi_{\text{tot}} = 0.196$ and $\phi_{\text{obs}} = 0.00785$ that remains in phase I but is near the I-II transition. Drops in V indicate the temporary formation of a pinned cluster as shown by the corresponding jumps in C_{max}/N_a .

perform a detailed study of the behavior of passive disks as a function of ϕ_{obs} versus ϕ_{tot} , where we find that the pinned phase appears at much higher values of ϕ_{obs} than in the active ballistic system. We note that there may be additional phases in the active ballistic system at values of ϕ_{tot} higher than those shown in Fig. 9, particularly upon approaching $\phi_{\text{tot}} = 0.9$ where the system crystallizes into a close-packed lattice.

In nonballistic active particle systems that undergo driven diffusion or have run-and-tumble motion with finite τ_r , a phase separated state appears in the absence of quenched disorder as a function of disk density ϕ_{tot} and activity. Typically there is a density $\phi_{\text{tot}}^{\text{min}}$ below which phase separation does not occur. For the ballistic active matter system we consider, in the absence of obstacles a phase separated state appears only for $\phi_{\text{tot}} \geq 0.35$. The introduction of obstacles makes it possible for a cluster state to nucleate at much smaller values of ϕ_{tot} . For $F_D = 0.05$, we find clustering at densities as small as $\phi_{\text{tot}} \approx 0.1$. To illustrate this more clearly, in Fig. 10 we plot C_L versus ϕ_{tot} for systems with $\phi_{\text{obs}} = 0$ and $\phi_{\text{obs}} = 0.00157$. In the obstacle-free system, the cluster size begins to increase near $\phi_{\text{tot}} = 0.28$, reaching a value of $C_L = 0.8$ at $\phi_{\text{tot}} = 0.47$. In contrast, adding a small number of obstacles shifts the onset of clustering down

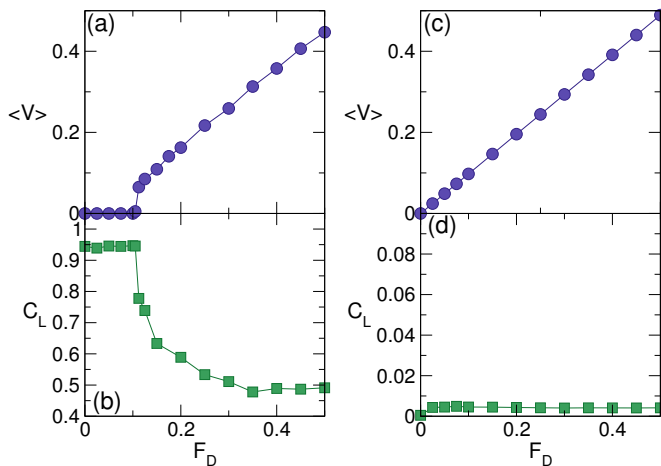


FIG. 11: (a) $\langle V \rangle$ and (b) C_L vs F_D for active ballistic disks with $\phi_{\text{tot}} = 0.3534$ and $\phi_{\text{obs}} = 0.00178$. At $F_D = 0.05$ the system is in phase II, the pinned single cluster state, and at $F_D = F_c = 0.1$ it depins and enters phase I_{fc}. (c) $\langle V \rangle$ and (d) C_L vs F_D for passive ballistic disks at the same values of ϕ_{tot} and ϕ_{obs} . There is no depinning threshold and clustering does not occur.

to $\phi_{\text{tot}} = 0.095$, and C_L reaches a value of $C_L = 0.8$ at $\phi_{\text{tot}} = 0.12$. This indicates that the obstacles are responsible for nucleating stable clusters over the range $0.12 < \phi_{\text{tot}} < 0.28$. The disorder-induced cluster state can be viewed in terms of a wetting phenomenon where the active particles accumulate not along walls [74] but next to the obstacles.

The ability of a cluster to form at low obstacle densities also depends on F_D . As F_D decreases, the transition from the drifting liquid phase I to the pinned single cluster phase II occurs at lower ϕ_{obs} . In general, even when F_D is too large to stabilize phase II for a given value of ϕ_{obs} , a transient pinned cluster can still form on a temporary basis. An example of this appears in Fig. 10(b,c) where we plot C_{max}/N_a and instantaneous velocity V versus time for a system with $\phi_{\text{tot}} = 0.196$ and $\phi_{\text{obs}} = 0.00785$, which is close to the I-II transition on the phase I side. There are a series of dips in V correlated with jumps in C_{max}/N_a which arise when a pinned cluster forms, reducing the velocity temporarily. The cluster quickly breaks apart, restoring V and C_{max}/N_a to their steady state average values.

IV. DEPINNING AND DRIVE DEPENDENCE

We next consider the effect of changing F_D in order to construct velocity-force ($v-f$) curves and measure the depinning threshold F_c . We compare the depinning of the active ballistic disks to that of passive disks. In Fig. 11(a,b) we plot $\langle V \rangle$ and C_L versus F_D for an active ballistic system with $\phi_{\text{tot}} = 0.3534$ and $\phi_{\text{obs}} = 0.00178$, which is in the pinned single cluster phase II at $F_D = 0.05$. We find a drive-induced depinning transition at

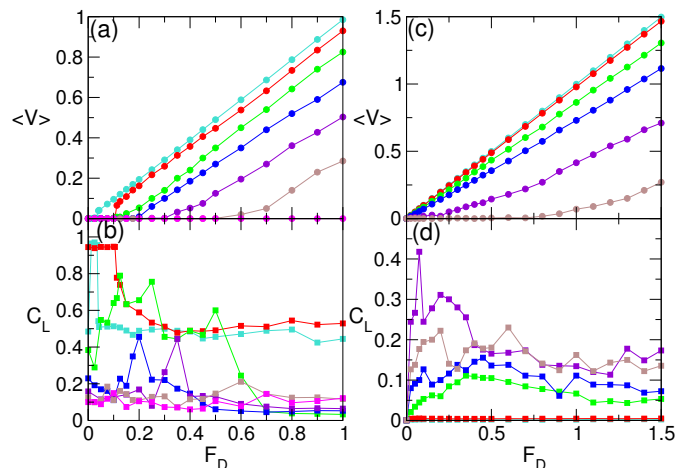


FIG. 12: (a) $\langle V \rangle$ and (b) C_L vs F_D for active ballistic disks with $\phi_{\text{tot}} = 0.3534$ at $\phi_{\text{obs}} = 0.00235$ (light blue), 0.01178 (red), 0.03927 (green), 0.0628 (dark blue), 0.0785 (violet), 0.0942 (brown), and 0.1256 (magenta). (c) $\langle V \rangle$ and (d) C_L vs F_D for passive disks with $\phi_{\text{tot}} = 0.3534$ at $\phi_{\text{obs}} = 0.00235$ (light blue), 0.01178 (red), 0.03927 (green), 0.0628 (dark blue), 0.094274 (violet), and 0.1267 (brown), showing that a finite depinning threshold does not appear until $\phi_{\text{obs}} > 0.094274$.

$F_D = F_c = 0.1$ from phase II to phase I_{fc}, as shown by the sharp drop in C_L that coincides with the onset of a linear increase in $\langle V \rangle$ with increasing F_D . In Fig. 11(c,d) we show $\langle V \rangle$ and C_L versus F_D for a passive disk system with the same values of ϕ_{tot} and ϕ_{obs} . There is no depinning threshold and $C_L < 0.005$ for all values of F_D , indicating a complete lack of clustering.

In Fig. 12(a,b) we plot $\langle V \rangle$ and C_L versus F_D for the active ballistic disks with $\phi_{\text{tot}} = 0.3534$ at $\phi_{\text{obs}} = 0.00235$ to 0.1256. We find that the II-I_{fc} transition, corresponding to the depinning threshold, drops to lower values of F_D as ϕ_{obs} decreases, as indicated most clearly by the $\phi_{\text{obs}} = 0.00235$ curve in Fig. 12(a) which has $F_c = 0.035$. The II-I_{fc} depinning transition is generally quite sharp, and the system goes directly from the pinned state to a fully flowing state without passing through a regime in which moving and pinned active particles coexist. In contrast, the depinning transition separating phases III and IV for $\phi_{\text{obs}} \geq 0.03927$ is smooth or continuous and is plastic in nature, so that above depinning only a portion of the active disks are flowing while the other portion remains pinned. In studies of depinning in other systems such as superconducting vortices or colloidal particles moving over quenched disorder, elastic depinning is associated with a sharp transition in the $v-f$ curve and a scaling of $V \propto (F_D - F_c)^\alpha$ with $\alpha < 1.0$. Plastic depinning is accompanied by an extensive nonlinear regime in the $v-f$ curves with $\alpha > 1.0$. In our active ballistic disk system, the resolution of the $v-f$ curves is not high enough to perform a scaling analysis; however, the qualitative change in the $v-f$ curve at the I_{fc}-II transition is

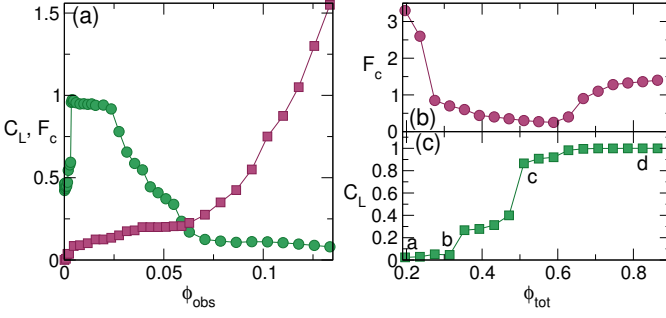


FIG. 13: (a) F_c (red squares) and the value of C_L at $F_D = 0.05$ (green circles) vs ϕ_{obs} for the active ballistic disks at $\phi_{\text{tot}} = 0.3534$. The depinning threshold remains finite down to $\phi_{\text{obs}} = 0.0008$, and there is an increase in F_c at the onset of phase IV. (b) F_c and (c) the value of C_L at $F_D = 0.05$ vs ϕ_{tot} for $\phi_{\text{obs}} = 0.09427$. The behavior of F_c is nonmonotonic, and F_c increases with increasing ϕ_{tot} for $\phi_{\text{tot}} > 0.6$ when $C_L = 1$, indicating the formation of a jammed state. The labels **a** to **d** in (c) indicate the values of ϕ_{tot} at which the images in Fig. 14 were obtained.

consistent with an elastic or collective depinning at low ϕ_{obs} crossing over to plastic depinning for higher ϕ_{obs} . For the III-IV depinning transition, there is generally a small peak in C_L at F_c produced when the disks start to accumulate behind the obstacles, and in all cases there is an overall drop in C_L at higher values of F_D in the moving phase. For $\phi_{\text{obs}} = 0.1256$, depinning does not occur until $F_D = F_c = 1.35$.

In Fig. 12(c,d) we plot $\langle V \rangle$ and C_L versus F_D for the passive disks with $\phi_{\text{tot}} = 0.3534$ at $\phi_{\text{obs}} = 0.00235$ to 0.1267. There is no finite depinning threshold for $\phi_{\text{obs}} \leq 0.094274$. We find an extended regime of nonlinear flow for $\phi_{\text{obs}} > 0.0628$ associated with a coexistence of flowing and clogged disks. In Fig. 12(d), $C_L < 0.01$ at all F_D for $\phi_{\text{obs}} = 0.00235$ and $\phi_{\text{obs}} = 0.01178$, while for higher ϕ_{obs} there is generally a decrease in C_L with increasing F_D for $F_D > 0.3$. The maximum value of C_L occurs for $\phi_{\text{obs}} = 0.094274$, which is just below the obstacle density at which a finite depinning threshold first appears. These results show that for varied F_D , the active ballistic disks have a much higher susceptibility to becoming pinned than the passive disks.

In Fig. 13(a) we plot the evolution of the depinning threshold F_c along with the value of C_L at $F_D = 0.05$ versus ϕ_{obs} for the active ballistic disks at $\phi_{\text{tot}} = 0.3534$. There is a sharp increase from $F_c = 0$ at $\phi_{\text{obs}} = 0$ to $F_c = 0.007$ at $\phi_{\text{obs}} = 0.0008$, the lowest nonzero obstacle density we considered, for which the ratio of obstacles to active particles is $N_a/N_{\text{obs}} = 440$. For $0.0045 < \phi_{\text{obs}} < 0.039$, there is a more gradual linear increase of F_c with increasing ϕ_{obs} over the range of phase II depinning through half of the phase III depinning. This is followed by a regime of roughly constant F_c for $0.039 < \phi_{\text{obs}} < 0.065$ in the second half of phase III depinning, while in phase IV for $\phi_{\text{obs}} > 0.65$, there is a rapid increase in F_c which coincides with a drop in C_L . Plastic

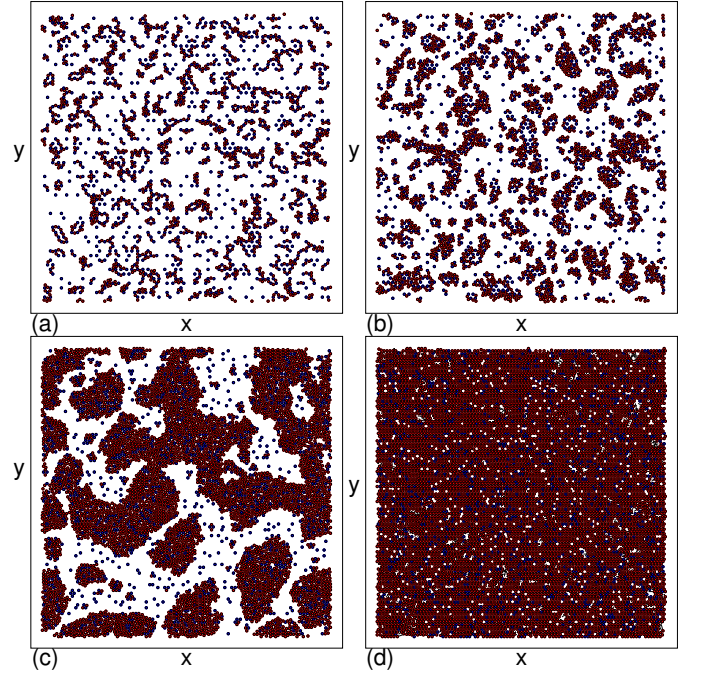


FIG. 14: The active ballistic disk positions (red circles) and obstacle locations (blue circles) for the system in Fig. 13(b,c) with $\phi_{\text{obs}} = 0.09427$ obtained at the values of ϕ_{tot} marked by the letters **a** to **d** in Fig. 13(c). (a) The pinned liquid at $\phi_{\text{tot}} = 0.1963$. (b) A pinned weakly clustered state at $\phi_{\text{obs}} = 0.275$ (phase IV). (c) A pinned gel state at $\phi_{\text{tot}} = 0.51$ (phase III_{pc}). (d) A jammed solid state at $\phi_{\text{tot}} = 0.8246$.

depinning appears in phase IV, and the rapid increase in F_c in this regime is reminiscent of the “peak effect” phenomenon observed for the depinning of superconducting vortices [2, 3, 75]. At low disorder strength, the vortices form a crystal that can be collectively pinned by a small number of pinning sites; however, when the pinning strength is increased, the crystalline structure breaks apart, the vortex structure becomes amorphous, and there is a pronounced increase in the depinning force that is much larger than what would be expected from the increase in the pinning strength [2, 3]. In the active ballistic disk system, the clusters have considerable crystalline order, and when the amount of disorder is increased by raising the number of obstacles, the large clusters break up into smaller clusters that can be pinned more easily, leading to the increase in F_c .

In Fig. 13(b,c) we plot F_c and the value of C_L at $F_D = 0.05$ versus ϕ_{tot} for the active ballistic disks at $\phi_{\text{obs}} = 0.09427$. For low $\phi_{\text{tot}} < 0.35$, a disordered but strongly pinned phase appears, as indicated by the large F_c and the low $C_L < 0.1$. In Fig. 14(a) we illustrate the disk configuration at $\phi_{\text{tot}} = 0.1963$ where a pinned liquid with small local disk clusters forms. As ϕ_{tot} increases, F_c decreases and a pinned weakly clustered state emerges, as shown in Fig. 14(b) at $\phi_{\text{tot}} = 0.275$. Since the number of obstacles is fixed, as ϕ_{tot} increases, each obstacle must restrain a larger number of mobile disks, causing

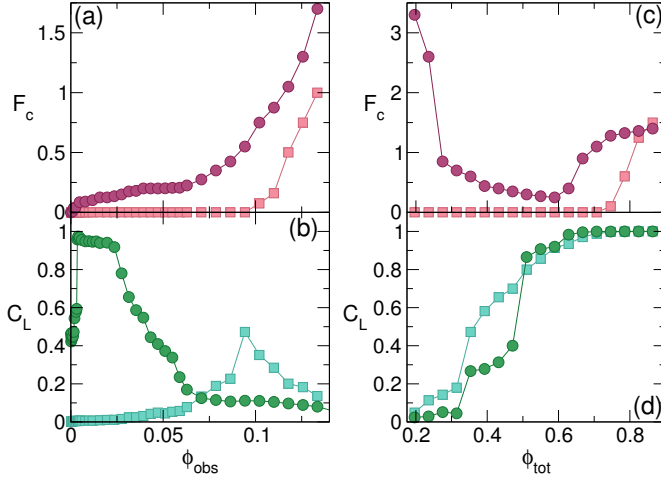


FIG. 15: (a) F_c and (b) the value of C_L at $F_D = 0.05$ vs ϕ_{obs} at $\phi_{\text{tot}} = 0.3534$ for the active ballistic disks (circles) and passive disks (squares). (c) F_c and (d) the value of C_L at $F_D = 0.05$ vs ϕ_{tot} at $\phi_{\text{obs}} = 0.094$ for the active ballistic disks (circles) and passive disks (squares). At high ϕ_{tot} , both the active and passive disks undergo a transition into a jammed state.

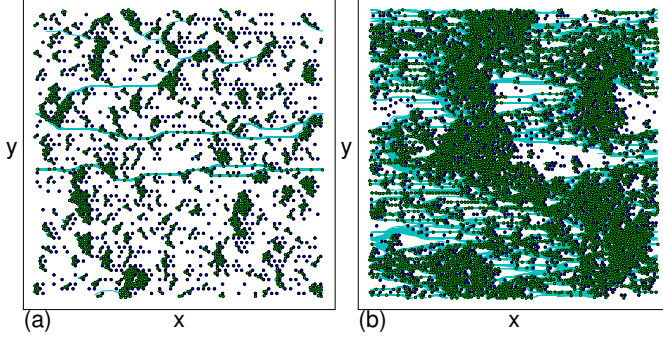


FIG. 16: The passive disk positions (green circles), obstacle locations (blue circles), and disk trajectories for the system in Fig. 15(c,d) (a) At $\phi_{\text{tot}} = 0.196$, 1D flow channels coexist with pinned disks. (b) At $\phi_{\text{tot}} = 0.4712$, collective interactions between disks cause a larger fraction of the disks to flow.

F_c to decrease with increasing ϕ_{tot} . In Fig. 14(c) we plot the disk configuration at $\phi_{\text{tot}} = 0.51$, where the system forms a pinned gel phase with low F_c . Figure 13(c,d) shows a local minimum in F_c near $\phi_{\text{tot}} = 0.628$, where $C_L = 0.92$. For $\phi_{\text{tot}} > 0.628$, F_c begins increasing with increasing ϕ_{tot} and C_L approaches $C_L = 1$ as the disks assemble into a single jammed solid packing, as shown in Fig. 14(d) at $\phi_{\text{tot}} = 0.8246$.

In Fig. 15(a,b) we plot F_c and the value of C_L at $F_D = 0.05$ versus ϕ_{obs} at $\phi_{\text{tot}} = 0.3534$ for active ballistic disks and passive disks. The depinning threshold remains finite in the active system for $\phi_{\text{obs}} \geq 0.0008$, whereas in the passive system the depinning threshold drops to zero when $\phi_{\text{obs}} \leq 0.094$. This indicates that 100 times fewer obstacles are needed to pin the active system compared to the passive system. In addition, the depinning thresh-

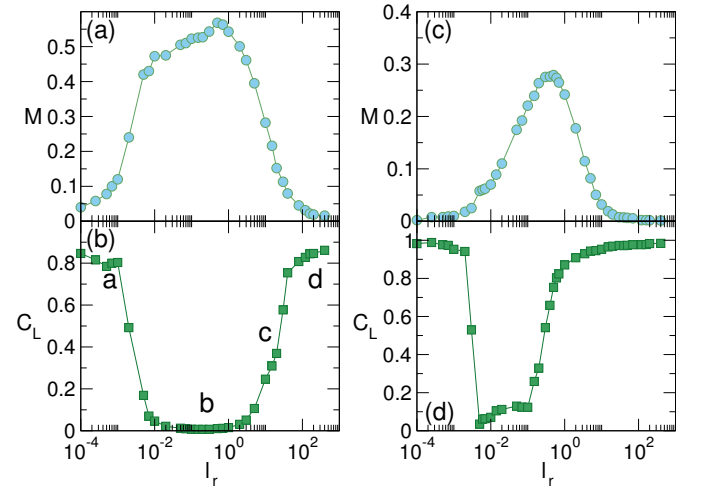


FIG. 17: (a) The disk mobility M and (b) C_L vs run length l_r for finite run time active disks at $F_D = 0.05$, $\phi_{\text{tot}} = 0.51$ and $\phi_{\text{obs}} = 0.14137$. The low l_r behavior is similar to that found in the passive disk limit while the high l_r behavior is similar to that found in the active ballistic limit. Between these two limits the disks form a uniform density, highly mobile liquid state. The labels **a** to **d** in (b) indicate the values of l_r at which the images in Fig. 18 were obtained. (c) M and (d) C_L vs l_r for finite run time active disks at $\phi_{\text{tot}} = 0.667$ and $\phi_{\text{obs}} = 0.14137$.

old for the active system is always higher than that of the passive system. In Fig. 15(c,d) we show F_c and the value of C_L at $F_D = 0.05$ versus ϕ_{tot} at $\phi_{\text{obs}} = 0.094$ for the active ballistic and passive disks. Here the depinning threshold for the passive disks does not become finite until $\phi_{\text{tot}} > 0.7$, the density above which C_L increases to $C_L = 1.0$. The high density active ballistic and passive disk states are similar in nature and are dominated by the formation of a jammed solid state. Even when the depinning threshold in the passive disk system is zero, the $v - f$ curves can show strongly nonlinear behavior, and a large fraction of the passive disks remain pinned or immobile for $\phi_{\text{tot}} < 0.35$, the same total disk density at which the active ballistic disks exhibit a large increase in F_c . In Fig. 16(a) we show the passive disk locations and trajectories at $\phi_{\text{tot}} = 0.196$ and $F_D = 0.05$. A portion of the disks move in 1D filamentary channels while the remaining disks are pinned. These filamentary channels persist down to $F_D = 0.0$ in the passive disks, but are generally absent for active ballistic disks. In the passive disk system, as ϕ_{tot} increases, cooperative interactions between the mobile disks reduce the overall trapping and lead to a higher fraction of flowing disks, as illustrated in Fig. 16(b) at $F_D = 0.05$ and $\phi_{\text{tot}} = 0.4712$. As ϕ_{tot} further increases, the system approaches a jammed solid with a finite depinning threshold.

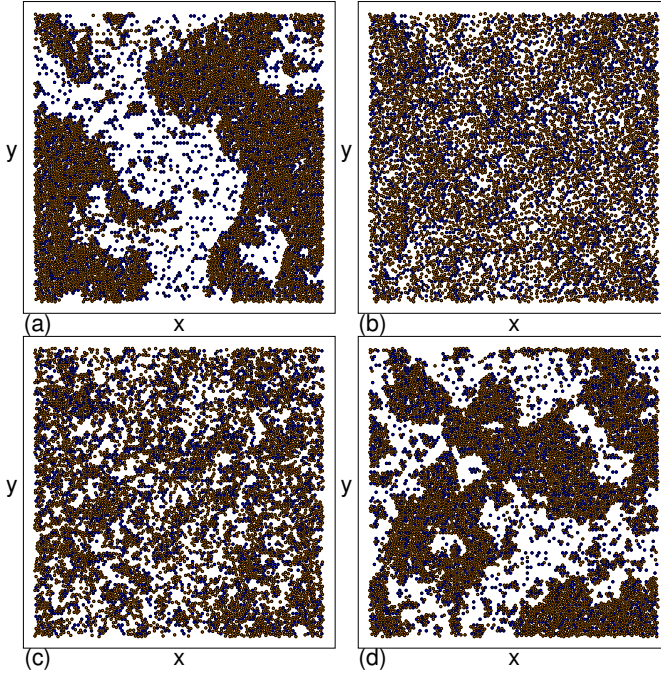


FIG. 18: The finite run time active disk locations (orange circles) and obstacle locations (blue circles) for a system with $\phi_{\text{tot}} = 0.51$ and $\phi_{\text{obs}} = 0.14137$ obtained at the values of l_r marked by the letters **a** to **d** in Fig. 17(b). (a) The low mobility clogged state at $l_r = 0.001$. (b) The high mobility uniform liquid at $l_r = 2.0$. (c) The actively clogged state at $l_r = 10$. (d) The actively clogged state at $l_r = 200$.

V. FINITE RUN TIME ACTIVE DISKS

We next show how the passive disk pinning and active ballistic disk pinning limits can be connected to each other by considering run-and-tumble particles where we gradually increase the running time from close to zero, which is the passive limit, to large values which approach the ballistic limit. In Fig. 17(a,b) we plot the mobility $M = \langle V \rangle / \langle V_0 \rangle$ per disk and C_L versus run length l_r for a finite run time active disk system at $\phi_{\text{tot}} = 0.51$, $\phi_{\text{obs}} = 0.14137$, and $F_D = 0.05$. Here $\langle V_0 \rangle$ is the average drift velocity of an individual disk in the absence of obstacles or other disks, so for $F_D = 0.05$, $\langle V_0 \rangle = 0.05$, and in the obstacle-free limit, $M = 1.0$. The disk dynamics are the same as those described in Eq. 1 except the running time τ_r is now finite. For convenience, we characterize the activity level in the system at a fixed $|\mathbf{F}_m|$ in terms of the run length $l_r = |\mathbf{F}_m| \tau_r$, so that large l_r corresponds to large τ_r . In the passive limit, $l_r = 0$, while in the ballistic limit, l_r is infinite. For the parameters we consider in this section, the system reaches a completely pinned state in both the passive and ballistic limits. In Fig. 17(a) at low l_r , $\langle M \rangle$ is small, indicating that a clogged state has formed that is similar to the passive disk clogged state which appears at $l_r = 0$. At the same time, $C_L > 0.8$ in Fig. 17(b), indicating the formation of

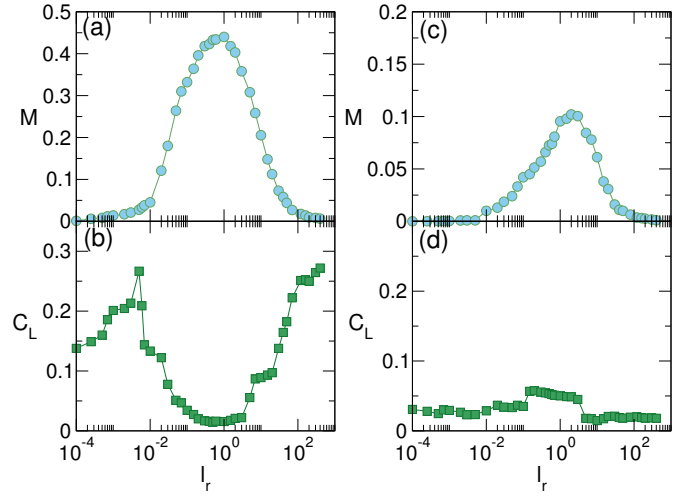


FIG. 19: (a) The disk mobility M and (b) C_L vs run length l_r at $F_D = 0.05$, $\phi_{\text{tot}} = 0.3543$ and $\phi_{\text{obs}} = 0.14137$ for finite run time active disks. (c) M and (d) C_L vs l_r at $\phi_{\text{tot}} = 0.2356$ and $\phi_{\text{obs}} = 0.14137$ for finite run time active disks, where the system is always in the disordered regime.

a large cluster. In Fig. 18(a) we plot the disk configurations for $l_r = 0.001$, where the clogged state has highly heterogeneous local disk density. For $0.01 < l_r < 5$, an easily flowing liquid appears, as indicated by the increase in M and the drop in C_L to $C_L < 0.01$. We illustrate the flowing liquid at $l_r = 2.0$ in Fig. 18(b), where the disk density is uniform and clustering behavior is lost. As l_r increases, M decreases once self-clustering of the disks begins to occur, as shown in Fig. 18(c) for $l_r = 10$. At large l_r , C_L approaches $C_L = 0.9$ and M drops to a low value. In this regime, the image of the $l_r = 200$ system in Fig. 18(d) indicates that clustering similar to that found in the active ballistic pinned state is present. We note that $M > 0$ for any finite l_r since there is always a chance that the activity can unpin a fraction of the disks even when l_r becomes large. The motion in the finite but large l_r regime becomes highly intermittent and exhibits avalanche-like fluctuations, as has been described in detail elsewhere [65]. In Fig. 17(c,d) we plot M and C_L versus l_r for the run-and-tumble disks at a higher $\phi_{\text{tot}} = 0.667$. We find the same trend in which clustering occurs at both small and large l_r , with a correspondingly low value of M , while for intermediate l_r , the clustering disappears, C_L is low, and M is high. The maximum value of M is lower at $\phi_{\text{tot}} = 0.667$ than at $\phi_{\text{tot}} = 0.51$ due to the crowding effect that appears at higher disk densities.

In Fig. 19(a,b) we plot M and C_L for finite run time active disks at $\phi_{\text{tot}} = 0.3534$ and $\phi_{\text{obs}} = 0.1413$. We find the same trend as in Fig. 17 where high values of C_L are associated with low values of M ; however, at this lower total disk density ϕ_{tot} , the maximum value of C_L is reduced. A peak in C_L near $l_r = 0.005$ indicates the appearance of additional clustering just before the activ-

ity becomes large enough to liquefy the system and increase M . This suggests that the activity becomes strong enough to cause untrapping of individual disks at lower l_r but that the untrapped disks then pool into clusters that are too large to be broken apart by individual disks until l_r increases, at which point the clumps break apart and the system flows in a liquid state. In Fig. 19(c,d) we plot M and C_L versus l_r for finite run length disks at $\phi_{\text{tot}} = 0.2356$ and $\phi_{\text{obs}} = 0.1413$. At this low total disk density, little clustering occurs and C_L is always small. We still observe a peak in M at intermediate l_r values; however, M is relatively low overall, reaching a maximum value of only $M = 0.1$. This result emphasizes the role of collective disk-disk interactions in liquefying the system and increasing the mobility for intermediate l_r .

Although we consider only monodisperse disks in this work, the introduction of obstacles causes the clogged states to exhibit a considerable amount of structural disorder, similar to that found in jammed states for bidisperse or completely amorphous systems [68, 70, 71]. For passive disks at low obstacle densities, previous studies have shown that jamming occurs near the obstacle-free jamming density of ϕ_j or point J , but that the jamming transition shifts to lower disk densities as the obstacle density increases [70, 72]. When the obstacle density is high enough, the behavior of the system changes and instead of a uniform jammed state, the passive disks assemble into a clogged state with strongly heterogeneous local disk density [14]. For active disks we find three limiting regimes of behavior. At high disk densities, the effect of the activity becomes negligible and the behavior is similar to that found in the passive high disk density limit, which is controlled by jamming near point J in amorphous systems or crystallization at a density $\phi_{\text{tot}} = 0.9$ for monodisperse disks. The second limiting regime appears for low activity and intermediate disk densities, where the active disks form a clogged configuration similar to that found for the clogging of passive disks. The third limiting regime, consisting of the actively pinned or clogged states that appear for active ballistic disks or for disks with finite but large l_r , is unique to the active disks and does not appear in passive disk systems. This regime extends over a wide range of obstacle densities and can assume the form of phase II, III, or IV, as described in this work. Our results suggest that in addition to the density, temperature, and load axes on the jamming phase diagram [68, 69], there could be two additional axes, the activity level and the obstacle density or disorder level, that produce jammed states.

In future work for both the passive and active disk systems, it would be interesting to investigate how different the pinned phases are. For example, at high disk densities near the jamming transition, it is likely that the system is highly stable to perturbations since there are fewer available degrees of freedom. Similarly, in a clogged state with high obstacle densities, a perturbed system is likely to fall back into the same or a similar clogged state. On the other hand, in phases II or III, a small perturba-

tion could readily break up one or more of the clusters, permitting the system to flow again, so that although the active systems are more susceptible to clogging, the clogged state they reach may be more fragile than that formed by passive disks.

VI. SUMMARY

We have examined the pinning and clogging behaviors of active run-and-tumble disks in the ballistic limit driven through an array of obstacles. As a function of increasing obstacle density, we find four generic phases: an unpinned fluctuating cluster phase, a pinned single cluster phase in which a small number of obstacles can pin a large number of active disks, a pinned multiclump phase, and a pinned disordered phase. We find that in contrast to passive disks, the active ballistic disks can reach a pinned state at relatively low obstacle densities. Within the pinned disordered phase, as the density of active ballistic disks increases, a pinned gel state or labyrinth pattern appears. By constructing velocity-force curves we find that the active ballistic disks exhibit a finite depinning threshold. As the obstacle density increases, there is a transition from collective depinning of the pinned clusters to plastic depinning of the disordered pinned states that is associated with a large increase in the depinning threshold. As a function of total disk density, the depinning threshold for the active ballistic disks is non-monotonic, dropping at intermediate disk densities when collective disk-disk interactions reduce the threshold, but rising at high disk densities as the system approaches a jammed or crystalline state. In contrast, for passive disks the depinning threshold is only finite at high disk densities, while at lower disk densities filamentary channels of disk flow form that are absent in the active disk system. For both the active and passive disks, the pinned or clogged states are phase separated; however, the phase separation appears at a much lower obstacle density for the active disks. Finite run time active disks provide a connection between the active ballistic and passive disk systems, and exhibit a low mobility phase separated state for short run times in the passive limit as well as for long run times in the active ballistic limit. At intermediate run times, the active disks form an easily flowing uniform liquid with reduced clustering, and there is an optimal level of activity that maximizes the flux through the system. We describe how our results can be related to systems that exhibit jamming. Since a self-jamming state appears in the limit of high activity or sufficiently large obstacle density, both activity and disorder strength act as two additional parameters that can produce jamming in particle assemblies.

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