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Parametric pulse amplification by acoustic quasi-modes in electron-positron plasma

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Abstract

In a recent paper, M.R. Edwards, N.J. Fisch, and J.M. Mikhailova (Phys. Rev. Lett. **116**, 015004 (2016)) reported that in electron-positron plasma stimulated Brillouin scattering is drastically enhanced, while stimulated Raman scattering is completely absent. However, when theory was compared to PIC (particle in cell) simulations, a discrepancy by at least a factor four appeared. Authors correctly argued that the disparity might be due to the fluid approximation of the low-frequency mode. They noted that a more precise analytic description of the acoustic resonance requires a kinetic approach, which was beyond the scope of the mentioned paper. Here we deliver the so far missing kinetic calculation. It shows quite good agreement with the PIC simulations presented in the above mentioned paper by Edwards et al. The principal result of enhancement of Brillouin scattering and absence of Raman scattering remains valid. The Brillouin enhancement factors depend on electron temperature and background particle density. These dependencies as well as the transition to the well-known behavior of electron-ion plasma are discussed. It is also shown that pulse amplification in electron-positron plasma crosses over to the strong-coupling regime when the pump amplitude becomes large. Then, the fluid approximation becomes acceptable again.

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I. INTRODUCTION

The present work is stimulated by a recent paper [1] on electron-positron-plasma interaction with electromagnetic fields. Leptons (such as electrons) and anti-leptons (such as positrons) dominated the mass of the early universe during the so called lepton era (approx. 10^{-4} up to a few seconds after the big bang, at temperatures up to 10^{12} K) [2, 3]. When electrons and positrons collide, they can annihilate each other and energy is released in the form of photons. In the lepton era, however, colliding photons in turn created more electron-positron pairs. It is believed that electron-positron (e-p) plasmas still appear in the environment of many astrophysical objects (e.g. quasars, pulsars, and black holes) as well as in the accreation discs of young galaxies [1, 4–6]. Tsytovich and Wharton [7] launched the idea of laboratory e-p plasma as a new research object. Shortly after, plasma based techniques for science with positrons were discussed by several authors; see, e.g., the review by Danielson et al. [8]. In the article on antimatter plasmas and antihydrogen, Greaves and Surko [9, 10] summarized the work by a number of groups on trapping antimatter plasmas and presented an overview of the promises and challenges in this field. First successful experiments were reported by Jørgensen et al. [11], Cassidy et al. [12], as well as Chen et al. [13]. Most recently, Sarri et al. [14] succeeded in generating neutral and high-density e-p pair plasma in the laboratory. More experiments are planned, e.g. the PAX/APEX experiment by the Max-Planck-Society in Germany [15]. However, the laboratory production of e-p plasmas with the requisite of Ref. [1] in density, size and temperature can be expected only in a rather distant future.

In principle, an e-p plasma is a quite exotic state of matter. However, it could exist in laboratory experiment (with sufficient vacuum) for minutes or even hours before being destroyed by annihilation. Pair annihilation is comparable to recombination in an "ordinary" electron-ion (e-i) plasma. The latter, e.g., consists of electrons and protons (hadrons). Electron-positron plasma allows the propagation of electromagnetic and electrostatic waves, but the situation is quite different from that in e-i plasmas. While e-p plasmas show perfect mass symmetry and perfect charge anti-symmetry, that will not be true in e-i plasmas. Most of the fundamental features of an e-i plasma are due to the large mass asymmetry between the negative and positive species. Thus, e-p plasmas will differ considerably.

With respect to the modes occurring in an e-p plasma quite controversial arguments appear. Within a fluid description, Zank and Greaves [16] calculated acoustic and Langmuir branches. On the other hand, Liu and Liu [17] argued that, on the contrary to e-i plasmas, no longer electrostatic acoustic-like waves in e-p plasma exist because of the absence of mass difference. A similar argument was presented by Pedersen et al. [15]. They stated that in an equal temperature e-p plasma no pressure-generated electric space charge field should develop. Because of the same masses, the two species are expected to escape a high-pressure region at the same rate, and no regular acoustic wave should appear. The conclusion that the e-p plasma simply relaxes and eliminates the pressure perturbation through free streaming of the particles contradicts, in principle, some acoustic mode results [1, 16] in an equal temperature e-p plasma. Here we will show that the so called acoustic mode is actually a (heavily damped) quasi-mode. The acoustic quasi-mode does not require a net charge difference.

This situation becomes especially interesting when in e-p plasma pulse amplification via scattering instabilities is considered. Three-wave interaction theory [18, 19] was first developed for e-i plasma where it was frequently applied to practice [20–24]. Now, with respect to three-wave interaction in e-p plasma, a similar controversial situation occurs as for the already mentioned linear modes. Early papers [25–27] showed already that in an equal-temperature e-p plasma coupling cannot occur with a Langmuir wave. The low-frequency ponderomotive force arising from the beating of two high-frequency waves cannot create the charge separation that is required for the existence of non-thermal Langmuir modes. Edwards et al. [1] complemented that such an argument does not apply to stimulated Brillouin scattering because the acoustic mode does not require a net charge difference. On the other hand, Tinakiche et al. [28] calculated that, when increasing the positron density to electron density ratio from 0 to 1 (maintaining quasi-neutrality of the plasma through additional ions), the growth rates of stimulated Raman, Brillouin, and Compton scattering processes in isothermal plasma tend to zero. Lopez et al. [29] also reported that some of the couplings generally appearing in relativistic magnetized e-i plasmas are suppressed in the e-p configuration. The most recent paper by Edwards et al. [1] clearly demonstrated by PIC simulations that stimulated Brillouin scattering in an e-p plasma is strongly enhanced when compared with an e-i plasma. The only weak point in their interpretation of the PIC results is the application of fluid theory. They stated already their expectation that a kinetic treatment will remove the discrepancy. Kinetic formulation of the three-wave interaction processes already exists in literature. For e-i plasma it has been applied to Raman [30] and Brillouin processes [31]. Relativistic Eulerian Vlasov codes are available [32, 33]. Landau damping in e-p plasma was considered in, e.g., Refs. [34] and [35].

We conclude from the discussion that a fluid description is not appropriate to fully understand the enhanced stimulated backscattering in an e-p plasma. In Ref. [1] this appeared already in the disparity between theoretical fluid prediction and PIC simulation results. Here we will show that the authors of Ref. [1] pointed out correctly the reason for the significant difference. It stems from the heavy damping of the acoustic quasi-mode in an e-p plasma. Indeed, a kinetic treatment confirms this probable cause. As a consequence of damping, the parametric kinetic dispersion relation shows less Brillouin growth than a fluid description. The question which we address here is whether, as a function of $\beta = \frac{m_e}{m} \leq 1$ where m_e is the electron mass and m is the mass of the positively charged plasma species (positron or ion), the Brillouin scattering off heavily damped acoustic quasi-modes is still drastically enhanced in comparison to the stimulated Raman scattering.

In general, we shall consider plasmas consisting of electrons (index $\kappa = e$, mass m_e, charge $q_e = -e$, and temperature T_e) and positively charged species (with mass m, charge $q = e$, and temperature T). The latter species might be positrons (index $\kappa = p$, $m = m_p \equiv m_e$, $q = q_p = e$, and $T = T_p$) or ions (index $\kappa = i$, $m = m_i$, $q = q_i = e$, and $T = T_i$). As characteristic parameters we introduce

$$\alpha = \frac{T_e}{T} \,, \qquad \beta = \frac{m_e}{m} = \begin{cases} 1 & \text{for e-p plasma,} \\ \frac{m_e}{m_i} \ll 1 & \text{for e-i plasma,} \end{cases} \tag{1}$$

Furthermore, we define the plasma frequency $\omega_{p\kappa} = \sqrt{\frac{4\pi n_0 e^2}{m_\kappa}}$ and the thermal velocity $v_{th,\kappa} = \sqrt{\frac{T_\kappa}{m_\kappa}}$ of species κ. Here, n_0 is the constant background particle density $n_0 = n_{\kappa 0}$. The Debye lengths are $\lambda_{D\kappa} = \frac{v_{th,\kappa}}{\omega_{p\kappa}}$. The index κ runs over all particle species, i.e. $\kappa = e, p, i$. Later we shall use another index σ which only runs over the positively charged species, i.e. $\sigma = p, i$. The index $\sigma = p$ always indicates positron while the index $\sigma = i$ should indicate ion (e.g. proton), but we take the freedom to vary m continuously in order to demonstrate the transition from e-p to e-i plasma. In that sense the parameter β will be considered in the region $0 < \beta \leq 1$. The main emphasis, however, will be on e-p plasma. As just mentioned, the additional freedom in the mass m will allow the straightforward comparison with known results for e-i plasmas.

The paper is organized as follows. Next, in Sec. II we briefly summarize what is known for electrostatic modes in e-p plasma. The kinetic formulation of Sec. III for parametric processes in e-p plasma leads to a dispersion relation. Its solution can be compared to the results from a fluid approximation. In Sec. IV we show that the kinetic treatment supports the enhancement of Brillouin amplification in e-p plasma, although at a lower level than predicted by fluid theory. We conclude by a short summary in Sec. V.

II. ELECTROSTATIC (QUASI-)MODES IN AN ELECTRON-POSITRON PLASMA

A. Fluid model predictions

Fluid models for non-relativistic e-p plasma were discussed by several authors. With respect to the corresponding longitudinal modes, one might follow, e.g., Zank and Greaves [16] or the supplementary material of Ref. [1] for details. A question arises with respect to the equation of state, whether the adiabatic or isothermal approximation should be used [36]. Since for the acoustic mode in an e-p plasma the phase velocity might not be small with respect to the electron thermal velocity, as a compromise an effective adiabatic coefficient Γ_κ (which should not be confused with the relativistic factor γ or later on with the growth rate) has been introduced. For 1D propagation it varies between 1 and 3. When one combines the pressure P obtained from the ideal gas law $P = nT$ (with temperature T in eV such that no Boltzmann constant appears) with the adiabatic relation $P \sim n^\Gamma$, one easily finds

$$\frac{dP}{dx} = \Gamma T \frac{dn}{dx} \; . \tag{2}$$

That has been used in Ref. [1] with appropriate values for Γ_e and Γ_σ in the region $1 \leq \Gamma_{e,\sigma} \leq 3$. Of course, a kinetic treatment will not suffer from such an indecisiveness.

A fluid treatment [1, 16] leads for an e-p plasma with $\beta = 1$ to the result

$$\omega^2 = \omega_{pe}^2 + V_e^2 k^2 \pm \omega_{pe}^2 \; , \tag{3}$$

where the upper sign is for the Langmuir mode

$$\omega^2 = 2\omega_{pe}^2 + V_e^2 k^2 \; , \tag{4}$$

and the lower sign for the acoustic mode

$$\omega^2 = V_e^2 k^2 \; . \tag{5}$$

Here, $V_e = \sqrt{\Gamma_e} v_{th,e}$. Nothing is said about the damping. The damping can only be determined by a kinetic theory.

B. Kinetic results

The Landau damping of longitudinal waves in isotropic pair plasmas is well-known, even in the relativistic regime [34]. For the simple(r) non-relativistic case, standard procedures can be applied. Using, e.g., Eq. (4.214) together with Eq. (4.192) of Ref. [36] we have for arbitrary β

$$k^2\lambda_{De}^2 = -[1+\zeta_e Z(\zeta_e)] - \frac{\lambda_{De}^2}{\lambda_{D\sigma}^2}[1+\zeta_\sigma Z(\zeta_\sigma)] \ , \quad \sigma = p, i \ , \tag{6}$$

where

$$\zeta_e = \frac{\omega}{\sqrt{2}kv_{th,e}} \ , \quad \zeta_\sigma = \frac{\omega}{\sqrt{2}kv_{th,\sigma}} = \sqrt{\frac{\alpha}{\beta}}\zeta_e \ , \tag{7}$$

and $Z(\zeta)$ is the plasma dispersion function. We may rewrite (6) in the form

$$1+\alpha+k^2\lambda_D^2+\zeta Z(\zeta)+\alpha\sqrt{\frac{\alpha}{\beta}}\zeta Z\left(\sqrt{\frac{\alpha}{\beta}}\zeta\right) = 0 \ , \qquad \zeta \equiv \zeta_e \ , \quad \lambda_D \equiv \lambda_{De} \ , \tag{8}$$

with $\alpha = T_e/T_p$ and $\beta = 1$ for an e-p plasma. In the appendix of Ref. [34], for $\alpha = \beta = 1$ the dispersion relation is given in the form

$$\Delta = \Delta(\zeta) \equiv 1 + \frac{1}{2}k^2\lambda_D^2 + \zeta Z(\zeta) = 0 \ , \tag{9}$$

in complete agreement with the above formulation.

Having in mind that $\zeta \equiv x - iy$ is a complex quantity, one must ensure the simultaneous solution of $\Re(\Delta) = \Im(\Delta) = 0$, as discussed in Ref [34]. Figure 1 shows the areas $\Re(\Delta) \geq 0$ with the borderlines of $\Re(\Delta) = 0$ on the one hand and the areas $\Im(\Delta) \geq 0$ with the borderlines of $\Im(\Delta) = 0$ on the other hand. The intersection points of the borderlines yield the solutions of the dispersion relation in terms of x and y. The case shown in Fig. 1 is for $S = k\lambda_{De} = 0.1$ and $\alpha = \beta = 1$. One finds several low-frequency roots. For the following discussion, only the branch with smallest damping y will be of relevance. In any case, since at the low-frequency points of intersection $y \sim \mathcal{O}(x)$, damping is significant. Actually, significant damping occurs over a few cycles. That explains why one does not expect such an oscillation to exist for a long time. It makes plausible why many authors object to the existence of acoustic modes in equal temperature e-p plasma. The above

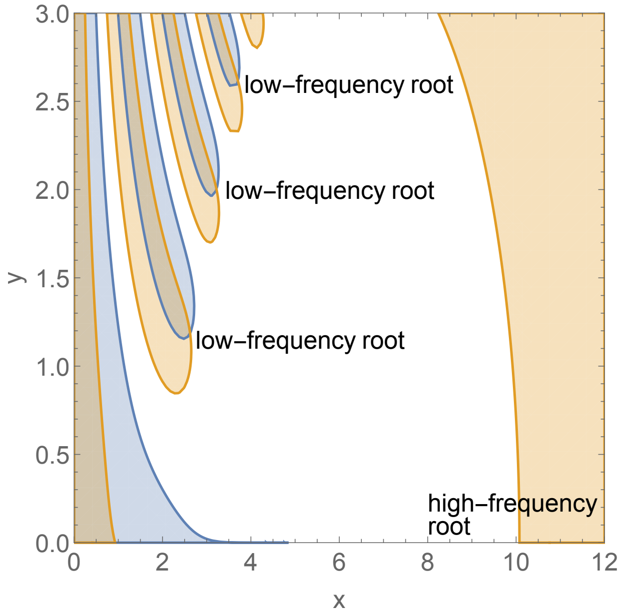

FIG. 1. Numerical (graphical) procedure to solve Eq. (9). Shown are the areas $\Re(\Delta) \geq 0$ (yellow) with the borderlines of $\Re(\Delta) = 0$ on the one hand and the areas $\Im(\Delta) \geq 0$ (blue) with the borderlines of $\Im(\Delta) = 0$ on the other hand, for $S = k\lambda_{De} = 0.1$ and $\alpha = \beta = 1$. The function $\Delta(\zeta)$ is defined in (9), and $x = \Re(\zeta)$, $y = -\Im(\zeta)$.

discussion shows that only heavily damped acoustic quasi-modes do exist. Nevertheless, via parametric processes such a quasi-mode can be driven. Often, this process is called Compton scattering or quasi-mode scattering instead of Brillouin amplification [25].

Besides the heavily damped acoustic quasi-modes also a high-frequency Langmuir mode exists. It occurs at the lower right corner in Fig. 1. For the parameters used for the figure, Landau damping is extremely small ($y \to 0$). Whether this always holds for the Langmuir modes being involved in Raman scattering is still open. It depends on the wave-number (or parameter $S = k\lambda_{De}$). We will come back to this point when evaluating the Raman growth in dependence on β.

III. PARAMETRIC EXCITATION IN KINETIC FORMULATION

In this section we calculate dependence of the Raman and Brillouin growth rates on β. Although our main interest is for $\beta = 1$ (e-p plasma), the argument for Brillouin enhancement and Raman suppression [1] needs a consideration of the wider β-range. Similar to Ref.

[37] we start with Maxwell's equations and the Vlasov equation,

$$\frac{\partial}{\partial t}f_\kappa + \mathbf{v}\cdot\nabla f_\kappa + q_\kappa\left(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\nabla_{\mathbf{p}}f_\kappa = 0\ ,\quad \kappa = e,p,i\ , \tag{10}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A}_\perp = -\frac{4\pi}{c}\mathbf{j}_\perp\ , \tag{11}$$

$$\nabla^2\phi = -4\pi e\left(n_\sigma - n_e\right)\ ,\quad \sigma = p,i\ . \tag{12}$$

We introduced the vector potential $\mathbf{A}$, the electrostatic potential ϕ, the distribution function f_κ for species $\kappa = e,p,i$, speed of light c, perpendicular electric current density $\mathbf{j}_\perp$, particle density n_κ, velocity coordinate $\mathbf{v}$, electric field $\mathbf{E}$, and magnetic field $\mathbf{B} = \nabla\times\mathbf{A}$. We start with a fully relativistic formulation, and simplify later to the non-relativistic limit.

For 1D applications, we assume that the distribution function f_κ only depends on the pulse propagation direction z, and make the ansatz

$$f_\kappa\left(z,\mathbf{p},t\right) = n_{\kappa 0}g_\kappa\left(z,p_z,t\right)\delta\left(p_x + \frac{q_\kappa A_x}{c}\right)\delta\left(p_y + \frac{q_\kappa A_y}{c}\right), \tag{13}$$

where δ denotes the Dirac-delta-distribution. With $n_{\kappa 0}\equiv n_0$ we obtain

$$n_\kappa\left(z,t\right) = n_0\int_{-\infty}^{+\infty} dp_z\left[g_\kappa\left(z,p_z,t\right)\right], \tag{14}$$

$$\mathbf{j}_\perp\left(z,t\right) = -\frac{e^2 n_0}{m_e c}\mathbf{A}\left(z,t\right)\int_{-\infty}^{+\infty} dp_z\left[\beta\frac{g_\sigma}{\gamma_\sigma}+\frac{g_e}{\gamma_e}\right]\ ,\quad \sigma = p,i\ . \tag{15}$$

The Lorentz factor γ_κ is given as

$$\gamma_\kappa = \sqrt{1+\left(\frac{p_z}{m_\kappa c}\right)^2+\left(\frac{e\mathbf{A}\left(z,t\right)}{m_\kappa c^2}\right)^2}\ ,\quad \kappa = e,p,i\ . \tag{16}$$

Inserting Eqs. (13)-(16) into the initial system (10) - (12) leads to

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A} = \frac{\omega_{pe}^2}{c^2}\mathbf{A}\int_{-\infty}^{+\infty} dp_z\left[\beta\frac{g_\sigma}{\gamma_\sigma}+\frac{g_e}{\gamma_e}\right], \tag{17}$$

$$\frac{\partial^2}{\partial z^2}\phi = -4\pi e n_0\int_{-\infty}^{+\infty} dp_z\left[g_\sigma - g_e\right]\ ,\quad \sigma = p,i\ , \tag{18}$$

$$\frac{\partial}{\partial t}g_\kappa + \frac{p_z}{m_\kappa\gamma_\kappa}\frac{\partial}{\partial z}g_\kappa + \left[-q_\kappa\frac{\partial}{\partial z}\phi - \frac{m_\kappa}{2}\left(\frac{e}{m_\kappa c}\right)^2\frac{1}{\gamma_\kappa}\frac{\partial}{\partial z}\mathbf{A}^2\right]\frac{\partial}{\partial p_z}g_\kappa = 0. \tag{19}$$

In order to determine the parametric dispersion relation for the system (17) - (19), we linearize the equations with respect to a consistent stationary solution. We assume to zeroth order circularly polarized light with

$$\mathbf{A}_0 = \frac{1}{2}\left(A_{0\perp}\mathbf{e} + c.c.\right)\ ,\quad A_{0\perp} = A_0 e^{\mathrm{i}\left(k_0 z - \omega_0 t\right)}\ , \tag{20}$$

that propagates in a homogeneous plasma $[g_{\kappa 0}(z,p_z)\equiv g_{\kappa 0}(p_z)]$. Here $\mathbf{e}=\mathbf{e}_x+\mathrm{i}\mathbf{e}_y$ is the polarization vector. With this definition we have $|\mathbf{A}_0|=A_0$. The ponderomotive pressure due to $\mathbf{A}_0$ will not cause any charge separation, and if we require additionally that $g_{\kappa 0}$ is symmetric in v_z, we have $\phi_0=0$.

Let the perturbations $\mathbf{A}_1,\phi_1$, and $g_{\kappa 1}$ be such that

$$\begin{aligned}\mathbf{A}_1 &= \frac{1}{2}\left(A_{1\perp}\mathbf{e}+c.c.\right)\ ,\\ A_{1\perp} &= A_+e^{\mathrm{i}(k_+z-\omega_+t)}+A_-e^{\mathrm{i}(k_-z-\omega_-t)}\ ,\\ \phi_1 &= \tilde{\phi}e^{\mathrm{i}(kz-\omega t)}+\tilde{\phi}^*e^{-\mathrm{i}(k^*z-\omega^*t)}\ ,\\ g_{\kappa 1} &= \tilde{g}_\kappa e^{\mathrm{i}(kz-\omega t)}+\tilde{g}_\kappa^*e^{-\mathrm{i}(k^*z-\omega^*t)}\ ,\end{aligned}$$

where $k_+=k_0+k$, $k_-=k_0-k^*$ (and $\omega_\pm$ analogously). The asterisk indicates complex conjugate (c.c.). Introducing the dimensionless vector potential as $a=eA/m_ec^2$, and selecting the resonant terms that are first order in the perturbations, leads to the relativistic kinetic dispersion relation for arbitrary positively charged species ($\sigma=p$ for e-p plasma and $\sigma=i$ for e-i plasma)

$$D_+D_-=\frac{\omega_{pe}^2a_0^2}{4}\left(D_++D_-\right)\left[I_4-m_ec^2k\left(F+I_3\right)\right]\ , \tag{21}$$

where

$$I_n=\beta^{n-1}I_{\sigma n}+I_{en}\ ,\quad \sigma=p,i\ , \tag{22}$$

$$I_{\kappa n}=\int_{-\infty}^{+\infty}dp_z\left[\frac{1}{\gamma_{\kappa 0}^{n-1}}\frac{\partial g_{\kappa 0}/\partial p_z}{v_zk-\omega}\right],\quad n=1,2,3,\quad I_{\kappa 4}=\int_{-\infty}^{+\infty}dp_z\left[\frac{g_{\kappa 0}}{\gamma_{\kappa 0}^3}\right]\ , \tag{23}$$

$$D_\pm=-\omega_\pm^2+c^2k_\pm^2+\omega_{pe}^2\int_{-\infty}^{+\infty}dp_z\left[\frac{\beta}{\gamma_{\sigma 0}}g_{\sigma 0}+\frac{1}{\gamma_{e0}}g_{e0}\right]\ , \tag{24}$$

$$F=\frac{4\pi e^2n_0I_2^2}{k-4\pi e^2n_0I_1}\ . \tag{25}$$

In the non-relativistic limit we have $\gamma_{\kappa 0}\approx 1$, $I_{\kappa 4}\approx 0$, and we take the 1D-distribution function to be Maxwellian

$$g_{\kappa 0}=\frac{1}{m_\kappa v_{th,\kappa}\sqrt{\pi}}e^{-v_z^2/v_{th,\kappa}^2}\ . \tag{26}$$

Furthermore, since then $I_{\kappa 1}=I_{\kappa 2}=I_{\kappa 3}$ we may identify the integrals $I_{\kappa n}$ with the electric susceptibilities χ_κ (see e.g. [36]) via

$$I_{\kappa n}=-\frac{1}{km_\kappa v_{th,\kappa}^2}\left[1+\frac{\omega}{\sqrt{2}m_\kappa v_{th,\kappa}}Z\left(\frac{\omega}{\sqrt{2}m_\kappa v_{th,\kappa}}\right)\right]=-\frac{k}{m_e\omega_{pe}^2}\chi_\kappa\ . \tag{27}$$

For predominant resonance with the downshifted light wave we have $D_- \approx 0$. Inserting into Eq. (21) yields

$$\frac{a_0^2 c^2 k^2}{4D_-} = \frac{1+\chi_\sigma+\chi_e}{\chi_e\left[1+(1+\beta)^2\chi_\sigma\right]+\beta^2\chi_\sigma}\,, \tag{28}$$

with $D_- = -{\omega_-^*}^2 + c^2 {k_-^*}^2 + (1+\beta)\,\omega_{pe}^2$. This form of the kinetic dispersion relation incorporates the susceptibilities

$$\chi_e \equiv \chi_e^{\rm kin} = \frac{1+\zeta_e Z(\zeta_e)}{k^2\lambda_{De}^2}\,, \qquad \chi_\sigma \equiv \chi_\sigma^{\rm kin} = \alpha\frac{1+\sqrt{\alpha/\beta}\zeta_e Z(\sqrt{\alpha/\beta}\zeta_e)}{k^2\lambda_{De}^2}\,. \tag{29}$$

The definition of a_0 is the same as in Ref. [19]. When the pump intensity I is measured in $\frac{\rm W}{\rm cm^2}$ and the laser wavelength λ_0 in μm we have [36]

$$a_0 = \sqrt{7.3\times 10^{-19}\,\lambda_0^2\, I}\,. \tag{30}$$

In the following we will exemplify all results for $\lambda_0 = 800$ nm. Then, a laser intensity $I = 10^{14}\ \frac{\rm W}{\rm cm^2}$ will lead to $a_0 = 0.0068352$.

Equation (28) is the basis for the following discussion. It contains both, Raman and Brillouin amplification. From here we can recover the dispersion relation in fluid approximation by substituting

$$\text{Raman:} \qquad \chi_e \to \chi_e^{\rm flu,R} = -\frac{\omega_{pe}^2}{\omega^2}, \qquad \chi_\sigma \to \chi_\sigma^{\rm flu,R} = 0, \tag{31}$$

$$\text{Brillouin:} \qquad \chi_e \to \chi_e^{\rm flu,B} = \frac{1}{k^2\lambda_{De}^2}, \qquad \chi_\sigma \to \chi_\sigma^{\rm flu,B} = -\frac{\omega_{p\sigma}^2}{\omega^2}\,, \tag{32}$$

where $\chi_\kappa^{\rm kin}$ denotes the kinetic and $\chi_\kappa^{\rm flu,R/B}$ the fluid susceptibilities for Raman (R) or Brilloun (B), respectively. Obviously, in the Brillouin case (with $\alpha \gg 1$), the fluid description is based on the assumption that a low-frequency mode is involved whose phase velocity is large compared to the ion thermal velocity and small compared to the electron thermal velocity. However, for an e-p plasma, the limiting thermal velocities (of electrons and ions) approach each other. Then, phase and thermal velocities become of the same order, resulting into scattering off a heavily damped low-frequency quasi-mode.

The growth rate Γ will be calculated from

$$\Gamma = \Im\omega\,, \tag{33}$$

where ω is the solution of the dispersion relation with the largest imaginary part.

Summarizing so far, Eq. (28) is the non-relativistic kinetic dispersion relation for arbitrary positively charged species. Note that in the limit $\beta \to 1$, thus for an e-p plasma, we arrive at the dispersion relation presented already by Shukla *et al.* [25] (where we have to replace a_0 by $a_0/2$ because of different amplitude notation).

IV. KINETIC EFFECTS VS. FLUID PREDICTION

When we compare the results of the fluid approach [1] with the present kinetic treatment, several parameters may play an important role: mass ratio β, temperature T, temperature ratio α, background density n_0, and pump amplitude a_0. We will work out the different dependencies in the following.

A. Typical e-p plasma ($\beta = 1$) and comparison with $\beta \ll 1$ plasmas

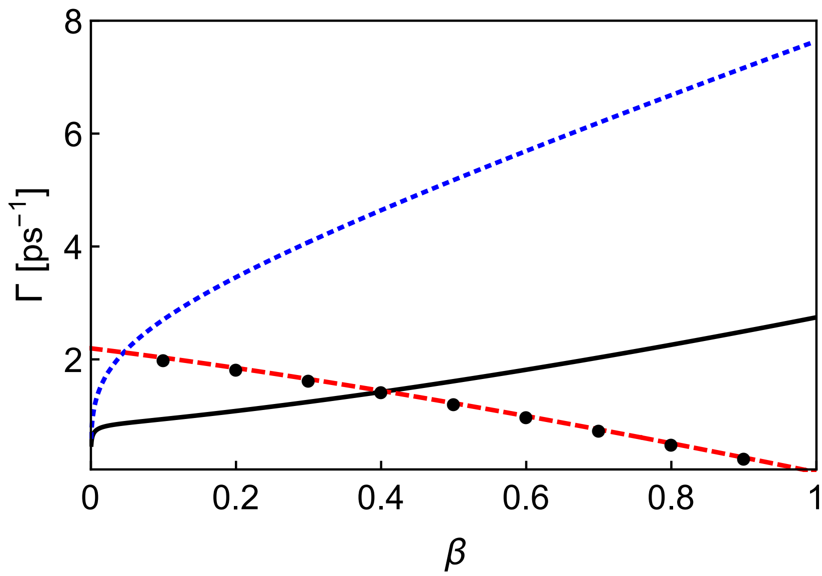

FIG. 2. Growth rate Γ vs. mass ratio β obtained by solving different dispersion relations. In dashed red and dashed blue *fluid* predictions are shown for Raman and Brillouin instability, respectively. The solutions of Eq. (28) with *kinetic* susceptibilities (29) are depicted by black dots for Raman and the solid black line for Brillouin, respectively. Parameters are $\alpha = 1$, $T_e = 20$ eV , $n_0 = 10^{19}$ cm^{-3}, and laser intensity $I = 10^{14}$ W/cm^2.

We start with a consideration of the same parameter regime as in Ref. [1]. Figure 2 summarizes the main part of the new findings which may be appreciated in the best manner when comparing to Fig. 4 of Ref. [1]. First, let us concentrate on an e-p plasma with $\beta = 1$ and $\alpha = 1$. Within a fluid calculation we completely agree with the fluid results of Ref. [1]. The *fluid* growth rates are shown in Fig. 2 by the dashed red line for Raman and the broken blue line for Brillouin, respectively. However, the full *kinetic* treatment with $\chi_\kappa^{\mathrm{kin}}$ leads to significantly different results in the case of Brillouin amplification. The latter is shown in Fig. 2 by the solid black line. The kinetic prediction for Raman amplification (black dots in Fig. 2), on the other hand, completely agrees with the fluid results. Note, however, the relatively small electron temperature $T_e = 20$ eV. Raman amplification is still suppressed in an e-p plasma. Brillouin amplification is still enhanced, however less than predicted by the fluid description, but in agreement with the PIC simulations of Ref. [1].

Besides the e-p plasma, Fig. 2 shows the maximum growth rate Γ in the whole region $0 < \beta \leq 1$ for Raman and Brillouin scattering, respectively. For Raman, fluid and kinetic results agree, but for Brillouin the correct kinetic description shows a reduction in a broad region. The cross-over between Raman and Brillouin at $\beta \approx 0.4$ in Fig. 2 still means that for an e-i plasma the Raman process dominates while for an e-p plasma only Brillouin appears with significant enhancement [1]. An e-i plasma with hydrogen ions (protons) has $\beta = 1/1836$, and usually [19] it is considered for $\alpha = \frac{T_e}{T_i} \gg 1$. Then the kinetic prediction agrees with the results obtained by solving the fluid dispersion relation [19].

B. Temperature dependence

The growth rates depend on temperature. This can be recognized from Fig. 3 where we show results for three temperature values, $T = 20$, 70, and 120 eV, respectively. In general, the smaller the temperature, the larger are the growth rates. Note that in Fig. 3 we have chosen the temperature ratio $\alpha = 1$ such that always $T_e = T$.

For Brillouin amplification in the regime $0 < \beta \leq 1$, it is the ion temperature which can cause a considerable reduction due to ion Landau damping. For an e-i plasma, Andreev et al. [38] and Lehmann et al. [39] have shown that for the fixed temperature ratio $\alpha \geq 10$ a variation of the electron temperature between 10 and 1000 eV has little influence on the growth rate. Here, we have decided to concentrate on the equal temperature $\alpha = 1$ case since for an e-p plasma this is the most plausible scenario. On the other hand, for e-i plasmas,

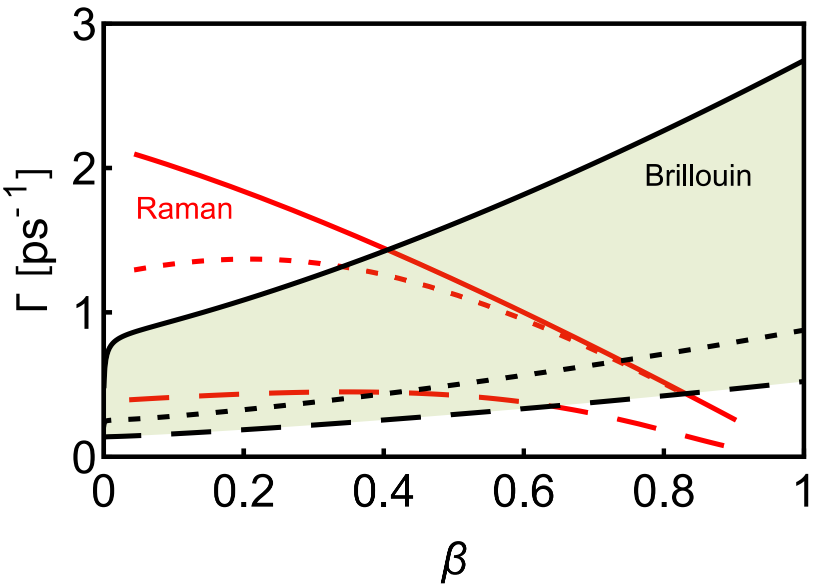

FIG. 3. Growth rate Γ vs. mass ratio β in kinetic description (28) for different temperatures T, namely 20 eV (solid lines), 70 eV (dotted lines), and 120 eV (dashed lines), respectively. The temperature ratio is fixed to $\alpha = 1$. Shown are the Brillouin growth rates in black (within the shaded area) as well as the Raman growth rates in red. Other parameters are $n_0 = 10^{19}$ cm^{-3} and $I = 10^{14}$ W/cm^2.

the α-value will depend strongly on the plasma production method.

Raman amplification depends on the electron temperature. Following an estimate presented in Ref. [28] we may predict an upper limit for the electron temperature, above which Landau will prevent significant Raman amplification. For the density value $n_0 = 10^{19}$ cm^{-3} and the critical density $n_c \approx 1.74 \times 10^{21}$ cm^{-3} used in Fig. 3, one expects [28] that only for $T_e \leq 73$ eV Landau damping will be less important. Thus, for the three temperatures shown in Fig. 3, the first (20 eV) is far below this limit, but the second one (70 eV) is close and the third (120 eV) is considerably above. In agreement with the theoretical prediction [28], in Fig. 3 the two latter cases show a considerably lower growth rate than the first one, at least for low values of β, for which the cited theory applies.

C. Density dependence

Increasing the particle density leads to an increase of both, Brillouin as well as Raman amplification. Figure 4 shows the corresponding results for two density values. Although both, Raman as well as Brillouin growth, increase with increasing density, the effect is more pronounced for Brillouin amplification. Thus in dense e-p plasmas the Brillouin enhancement will be very large, even much larger than the Raman process in e-i plasma.

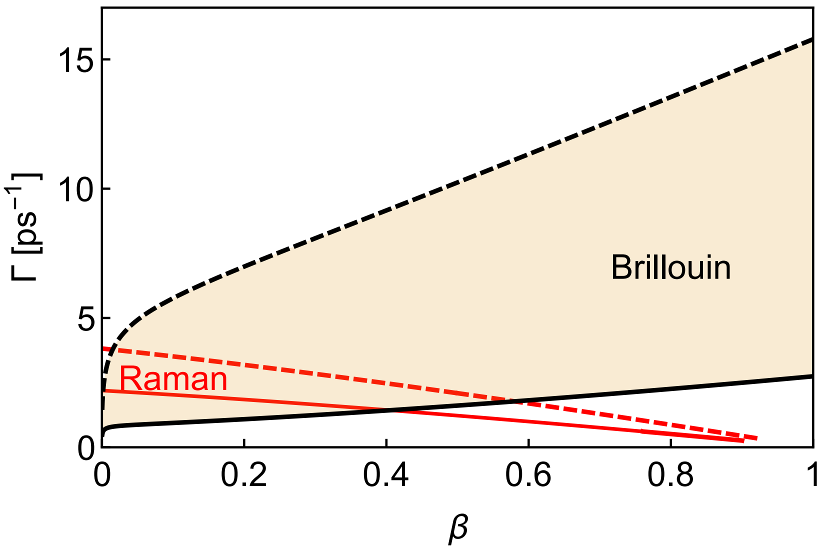

FIG. 4. Growth rate Γ vs. mass ratio β in kinetic description (28) for different background densities. Shown is the Brillouin growth rate (black lines) in comparison to the Raman growth rate (red lines). The solid lines are for $n_0 = 10^{19}$ cm^{-3} while the broken lines are for $n_0 = 10^{20}$ cm^{-3}. Other parameters are $\alpha = 1$, $T = 20$ eV, and $I = 10^{14}$ W/cm^2.

D. Transition to the strong-coupling regime in an e-p plasma

So far we fixed the pump amplitude a_0 to a reasonably small value. Figure 5 now shows for an e-p plasma the variation of Γ with a_0. We recognize two curves which generalize for an e-p plasma ($\beta = 1$) the Brillouin result of Fig. 2, which was obtained in there for $a_0 = 0.0068352$, to a broader range of the pump amplitude a_0. The graph is for $\lambda_0 = 800$ nm, leading to a critical density $n_c = \frac{m_e \omega_0^2}{4\pi e^2} \approx 1.74 \times 10^{21}$ cm$^{-3} \gg n_0 = 10^{19}$ cm^{-3}.

The *kinetic* Brillouin result (solid curve) is compared to *fluid* prediction (dashed curve) [1]. We recognize the already reported reduction due to significant kinetic effects (damping) in an e-p plasma. The discrepancy is large for small pump amplitude and almost disappears for strong pump amplitudes. For small a_0, both lines in Fig. 5 show a linear increase with

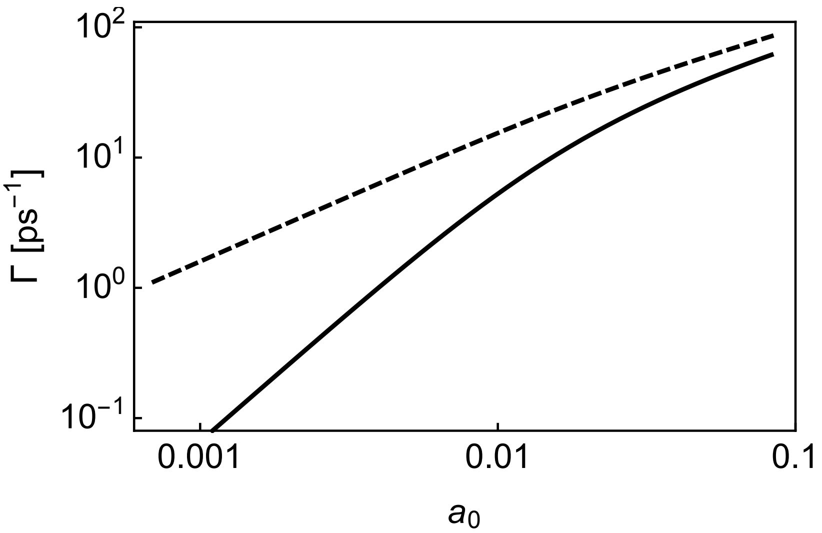

FIG. 5. Growth rate Γ in dependence of pump amplitude a_0 for an e-p plasma ($\beta = 1$). The solid line represents the *kinetic* Brillouin growth rate Γ as obtained from Eq. (28). The broken line follows from the *fluid* description with susceptibilities (32) Parameters are $\alpha = 1$, $T = 20$ eV, and $n_0 = 10^{19}$ cm^{-3}.

a_0. By fluid theory the prediction is [1]

$$\Gamma = \frac{cka_0}{2\sqrt{2}}\sqrt{\frac{2\omega_{pe}^2}{kV_e\left(\omega_0 - kV_e\right)}} \propto a_0 \ , \tag{34}$$

were $\omega_0 = \frac{2\pi c}{\lambda_0}$ is the pump frequency. These predicted values agree with the left part of the broken line in Fig. 5. However, the correct curve (solid line) has much smaller values, but a larger slope than predicted by fluid theory. That means that with increasing a_0 the discrepancy between fluid and kinetic results becomes less pronounced. For larger a_0, a cross-over to a different a_0-dependence occurs (for $a_0 \gtrsim 0.02$ in Fig. 5). This is true for both curves in Fig. 5. Indeed, when we solve the full fluid dispersion relation we obtain the dashed curve which also shows the mentioned cross-over. Such a cross-over is known as transition from weak to strong coupling. In the strong-coupling regime, the fluid and kinetic descriptions converge towards each other. At larger a_0, growth occurs with $\Gamma \propto a_0^{2/3}$. This is typical for strong coupling [18].

V. SUMMARY AND CONCLUSIONS

In the present paper we reassured that in an unmagnetized e-p plasma the resonant three-wave interaction between a finite pump electromagnetic wave and a *Langmuir wave* cannot occur. The ponderomotive force, arising from the beating of the pump and scattered wave, does not create the charge separation that is required for the existence of non-thermal Langmuir waves [25–27]. That argument does not apply to Brillouin scattering because the acoustic mode does not require a net charge difference [1]. However, as is shown here, the low-frequency mode is a heavily damped *acoustic quasi-mode*. Nevertheless it can be driven by an external pump. The scattering off an acoustic quasi-mode is enhanced in an e-p plasma when compared with the situation of "traditional" e-i plasmas. In dense e-p plasmas the Brillouin enhancement will be very large, even much larger than the Raman process in e-i plasmas. However, a fully kinetic treatment becomes necessary. It is shown here that the latter provides a completely consistent scenario with PIC simulations of Ref. [1]. Additionally, we have conducted Vlasov simulations. So far the simulations were carried out in the linear regime only and allowed us to determine the growth rate numerically. We found excellent agreement with the analytic predictions. At present we upgrade the Vlasov simulations to the nonlinear regime. All Vlasov simulation results and their interpretation will be the object of a forthcoming publication.

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