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# Collective Transport for Active Matter Run and Tumble Disk Systems on a Traveling Wave Substrate

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We numerically examine the transport of an assembly of active run-and-tumble disks interacting with a traveling wave substrate. We show that as a function of substrate strength, wave speed, disk activity, and disk density, a variety of dynamical phases arise that are correlated with the structure and net flux of disks. We find that there is a sharp transition into a state where the disks are only partially coupled to the substrate and form a phase separated cluster state. This transition is associated with a drop in the net disk flux and can occur as a function of the substrate speed, maximum substrate force, disk run time, and disk density. Since variation of the disk activity parameters produces different disk drift rates for a fixed traveling wave speed on the substrate, the system we consider could be used as an efficient method for active matter species separation. Within the cluster phase, we find that in some regimes the motion of the cluster center of mass is in the opposite direction to that of the traveling wave, while when the maximum substrate force is increased, the cluster drifts in the direction of the traveling wave. This suggests that swarming or clustering motion can serve as a method by which an active system can collectively move against an external drift.

## I. INTRODUCTION

Collections of interacting self-motile objects fall into the class of systems known as active matter<sup>1,2</sup>, which can be biological in nature such as swimming bacteria<sup>3</sup> or animal herds<sup>4</sup>, a social system such as pedestrian or traffic flow<sup>5</sup>, or a robotic swarm<sup>6,7</sup>. There are also a wide range of artificial active matter systems such as self-propelled colloidal particles<sup>8–10</sup>. Studies of these systems have generally focused on the case where the motile objects interact with either a smooth or a static substrate; however, the field is now advancing to a point where it is possible to ask how such systems behave in more complex static or dynamic environments.

One subclass of active systems is a collection of interacting disks that undergo either run-and-tumble<sup>11,12</sup> or driven diffusive<sup>13–15</sup> motion. Such systems have been shown to exhibit a transition from a uniform density liquid state to a motility-induced phase separated state in which the disks form dense clusters surrounded by a low density gas phase<sup>9–18</sup>. Recently it was shown that when phase-separated run-and-tumble disks are coupled to a random pinning substrate, a transition to a uniform density liquid state occurs as a function of the maximum force exerted by the substrate<sup>19</sup>. In other studies of run-and-tumble disks driven over an obstacle array by a dc driving force, the onset of clustering coincides with a drop in the net disk transport since a large cluster acts like a rigid object that can only move through the obstacle array with difficulty; in addition, it was shown that the disk transport was maximized at an optimal activity level or disk running time<sup>20</sup>. Studies of flocking or swarming disks that obey modified Vicsek models of self-propulsion<sup>21</sup> interacting with obstacle arrays indicate that there is an optimal intrinsic noise level at which

collective swarming occurs<sup>22,23</sup>, and that transitions between swarming and non-swarming states can occur as a function of increasing substrate disorder<sup>24</sup>. The dynamics in such swarming models differ from those of the active disk systems, so it is not clear whether the same behaviors will occur across the two different systems.

A number of studies have already considered overdamped active matter such as bacteria or run-and-tumble disks interacting with periodic obstacle arrays<sup>25</sup> or asymmetric arrays<sup>26–29</sup>. Self-ratcheting behavior occurs for the asymmetric arrays when the combination of broken detailed balance and the substrate asymmetry produces directed or ratcheting motion of the active matter particles<sup>30,31</sup>, and it is even possible to couple passive particles to the active matter particles in such arrays in order to shuttle cargo across the sample<sup>29</sup>. In the studies described above, the substrate is static, and external driving is introduced via fluid flow or chemotactic effects; however, it is also possible for the substrate itself to be dynamic, such as in the case of time dependent optical traps<sup>32,33</sup> or a traveling wave substrate. Theoretical and experimental studies of colloids or cells in traveling wave potentials reveal a rich variety of dynamical phases, self-assembly behaviors, and directed transport<sup>34–41</sup>.

Here we examine a two-dimensional system of run and tumble active matter disks that can exhibit motility induced phase separation interacting with a periodic quasi-one dimensional (q1D) traveling wave substrate. In the low activity limit, the substrate-free system forms a uniform liquid state, while in the presence of a substrate, the disks are readily trapped by the substrate minima and swept through the system by the traveling wave. As the activity increases, a partial decoupling transition of the disks and the substrate occurs, producing a drop in the net effective transport. This transition is correlated

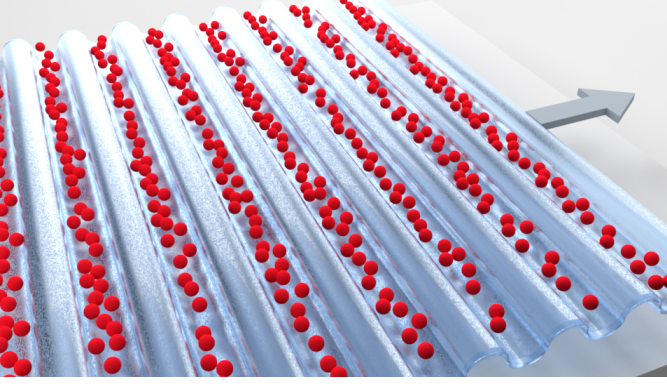


FIG. 1: Schematic of the system. Red spheres represent the active run and tumble disks in a two-dimensional system interacting with a periodic q1D traveling wave potential which is moving in the positive  $x$ -direction (arrow) with a wave speed of  $v_w$ .

with the onset of the phase separated state, in which the clusters act as large scale composite objects that cannot be transported as easily as individual disks by the traveling wave. We also find that the net disk transport is optimized at particular traveling wave speeds, disk run length, and substrate strength. In the phase separated state we observe an interesting effect where the center of mass of each cluster moves in the direction opposite to that in which the traveling wave is moving, and we also find reversals to states in which the clusters and the traveling wave move in the same direction. The reversed motion of the clusters arises due to asymmetric growth and shrinking rates on different sides of the cluster. The appearance of backward motion of the cluster center of mass suggests that certain biological or social active systems can move against biasing drifts by forming large collective objects or swarms.

## II. SIMULATION

We model a two-dimensional system of  $N$  run and tumble disks interacting with a q1D traveling wave periodic substrate, as shown in the schematic in Fig. 1 where the substrate moves to the right at a constant velocity  $v_w$ . The dynamics of each disk is governed by the following overdamped equation of motion:

$$\eta \frac{d\mathbf{r}_i}{dt} = \mathbf{F}_i^{\text{inter}} + \mathbf{F}_i^m + \mathbf{F}_i^s, \quad (1)$$

where the damping constant is  $\eta = 1.0$ . Here the net velocity of disk  $i$  produced by self-propulsion, disk-disk interaction, and substrate forces is  $d\mathbf{r}_i/dt$ . The disk-disk repulsive interaction force  $\mathbf{F}_i^{\text{inter}}$  is modeled as a harmonic spring,  $\mathbf{F}_i^{\text{inter}} = \sum_{j \neq i}^N \Theta(d_{ij} - 2R)k(d_{ij} - 2R)\hat{\mathbf{d}}_{ij}$ , where  $R = 1.0$  is the disk radius,  $d_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$  is the distance between disk centers,  $\hat{\mathbf{d}}_{ij} = (\mathbf{r}_i - \mathbf{r}_j)/d_{ij}$ , and the spring constant  $k = 20.0$  is large enough to prevent significant

disk-disk overlap under the conditions we study yet small enough to permit a computationally efficient time step of  $\delta t = 0.001$  to be used. We consider a sample of size  $L \times L$  with  $L = 300$ , and describe the disk density in terms of the area coverage  $\phi = N\pi R^2/L^2$ . The run and tumble self-propulsion is modeled with a motor force  $\mathbf{F}_i^m$  of fixed magnitude  $F^m = 1.0$  that acts in a randomly chosen direction during a run time of  $\tilde{t}_r$ . After this run time, the motor force instantly reorients into a new randomly chosen direction for the next run time. We take  $\tilde{t}_r$  to be uniformly distributed over the range  $[t_r, 2t_r]$ , using run times ranging from  $t_r = 1 \times 10^3$  to  $t_r = 3 \times 10^5$ . For convenience we describe the activity in terms of the run length  $r_l = F^m t_r \delta t$ , which is the distance a disk would move during a single run time in the absence of a substrate or other disks. The substrate is modeled as a time-dependent sinusoidal force  $F_i^s(t) = A_s \sin(2\pi x_i/a - v_w t)$  where  $A_s$  is the substrate strength and  $x_i$  is the  $x$  position of disk  $i$ . We take a substrate periodicity of  $a = 15$  so that the system contains 20 minima. The substrate travels at a constant velocity of  $v_w$  in the positive  $x$ -direction. We measure the average drift velocity of the disks in the direction of the traveling wave,  $\langle V \rangle = N^{-1} \sum_i^N \mathbf{v}_i \cdot \hat{\mathbf{x}}$ . We vary the run length, substrate strength, disk density, and wave speed. In each case we wait for a fixed time of  $5 \times 10^6$  simulation time steps before taking measurements to avoid any transient effects.

## III. RESULTS

In Fig. 2(a) we plot the average velocity per disk  $\langle V \rangle$  versus wave speed  $v_w$  at different substrate strengths  $A_s$  for a system containing  $N = 13000$  active disks, corresponding to  $\phi = 0.45376$ , at  $r_l = 300$ , a running length at which the substrate-free system forms a phase separated state. The number of disks that are in the largest cluster,  $\tilde{C}_L$ , serves as an effective measure of whether the system is in a phase separated state or not. We measure  $\tilde{C}_L$  using the cluster identification algorithm described in Ref.<sup>42</sup>, which identifies all disks that are in force contact with a given disk, a well defined quantity. Once all force contact clusters have been identified, it is straightforward to pick out the largest cluster by simple disk counting. We call the system phase separated when  $\tilde{C}_L/N > 0.55$ . In Fig. 2(b) we plot  $\tilde{C}_L$  versus  $v_w$  at varied  $A_s$ , and in Fig. 2(c) we show the corresponding number of sixfold-coordinated disks,  $\tilde{P}_6 = \sum_i^N \delta(z_i - 6)$ , where  $z_i$  is the coordination number of disk  $i$  determined from a Voronoi construction<sup>43</sup>. In phase separated states, most of the disks within a cluster have  $z_i = 6$  due to the triangular ordering of the densely packed state. In Fig. 2(a), the linearly increasing dashed line denotes the limit in which all the disks move with the substrate so that  $\langle V \rangle = v_w$ . At  $A_s = 3.0$ ,  $\langle V \rangle$  initially increases linearly, following the dashed line, up to  $v_w = 1.25$ , indicating that there is a complete locking of the disks to the substrate. For  $v_w > 1.25$ , there is a slipping process in which the disks

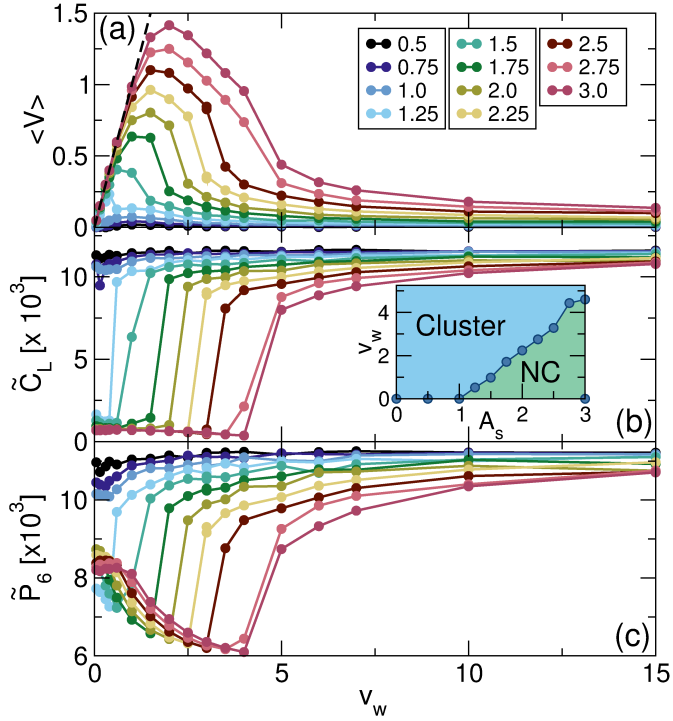


FIG. 2: (a) The average velocity per disk  $\langle V \rangle$  vs wave speed  $v_w$  for a system with  $N = 13000$  disks,  $\phi = 0.45376$ , and  $r_l = 300$  for varied substrate strengths of  $A_s = 0.5$  to  $3.0$ . The dashed line indicates the limit in which all the disks move at the wave speed,  $\langle V \rangle = v_w$ . (b) The corresponding number  $\tilde{C}_L$  of disks that are in a cluster vs wave speed. The inset shows the regions of cluster and non-cluster (NC) states as a function of  $v_w$  vs  $A_s$ . (c) The number  $\tilde{P}_6$  of sixfold-coordinated disks vs wave speed  $v_w$ .

cannot keep up with the traveling wave and jump to the next well. A maximum in  $\langle V \rangle$  appears near  $v_w = 2.0$ , and there is a sharp drop in  $\langle V \rangle$  near  $v_w = 5.0$ , which also coincides with a sharp increase in  $\tilde{C}_L$  and  $\tilde{P}_6$ . The  $\langle V \rangle$  versus  $v_w$  curves for  $A_s > 1.0$  all show similar trends, with a sharp drop in  $\langle V \rangle$  accompanied by an increase in  $\tilde{C}_L$  and  $\tilde{P}_6$ , showing that the onset of clustering results in a sharp decrease in  $\langle V \rangle$ . For  $A_s \leq 1.0$ , the substrate is weak enough that the system remains in a cluster state even at  $v_w = 0$ , indicating that a transition from a cluster to a non-cluster state can also occur as a function of substrate strength. In the inset of Fig. 2(b) we show the regions in which clustering and non-clustering states appear as a function of  $v_w$  versus  $A_s$ . At  $v_w = 0$ , there is a substrate-induced transition from a cluster to a non-cluster state near  $A_s = 1.0$ , while for higher  $A_s$ , the location of the transition shifts linearly to higher  $v_w$  with increasing  $A_s$ . Since the motor force is  $F^m = 1.0$ , when  $A_s < 1.0$  individual disks can escape from the substrate minima, so provided that  $r_l$  is large enough, the disks can freely move throughout the entire system and form a cluster state. For  $A_s > 1.0$ , the disks are confined by the substrate minima, but when  $v_w$  becomes large enough,

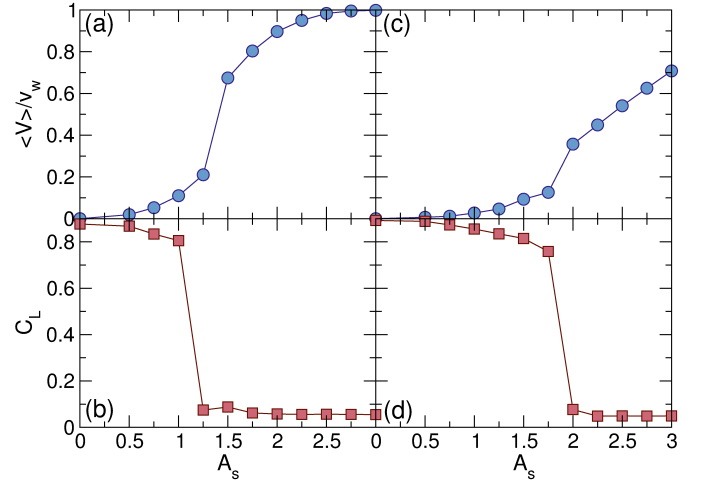


FIG. 3: (a)  $\langle V \rangle / v_w$  vs  $A_s$  for the system in Fig. 2 at  $v_w = 0.6$ . (b) The corresponding normalized  $C_L$  showing that the transition from a cluster to a non-cluster state coincides with an increase in  $\langle V \rangle / v_w$ . (c)  $\langle V \rangle / v_w$  vs  $A_s$  for the same system with  $v_w = 2.0$  where the cluster to non-cluster transition occurs at a higher value of  $A_s$ . (d) The corresponding normalized  $C_L$  vs  $A_s$ .

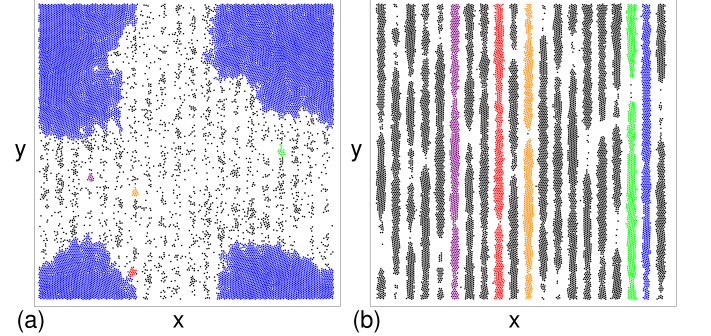


FIG. 4: The real space positions of the active disks for the system in Fig. 3(a,b) with  $v_w = 0.6$ . (a) At  $A_s = 0.75$ , a phase separated state appears. (b) At  $A_s = 2.5$ , the disks are strongly localized in the substrate minima and move with the substrate.

the disks can readily escape the minima and again form a cluster state.

To highlight the correlation between the changes in the transport and the onset of clustering, in Fig. 3(a,b) we plot  $\langle V \rangle / v_w$  and the normalized  $C_L = \tilde{C}_L / N$  versus  $A_s$  at a fixed value of  $v_w = 0.6$  from the system in Fig. 2. Here the cluster to non-cluster transition occurs at  $A_s = 1.25$ , as indicated by the drop in  $C_L$  which also coincides with a jump in  $\langle V \rangle / v_w$ . For this value of  $v_w$ , a complete locking between the disks and the traveling wave occurs for  $A_s \geq 3.0$ , where  $\langle V \rangle / v_w = 1.0$ . In Fig. 3(c,d) we plot  $\langle V \rangle / v_w$  and  $C_L$  versus  $A_s$  for the same system at  $v_w = 2.0$ , where the cluster to non-cluster transition occurs at a higher value of  $A_s = 2.0$ . This transition again coincides with a sharp increase in  $\langle V \rangle / v_w$ . In Fig. 4(a) we show images of

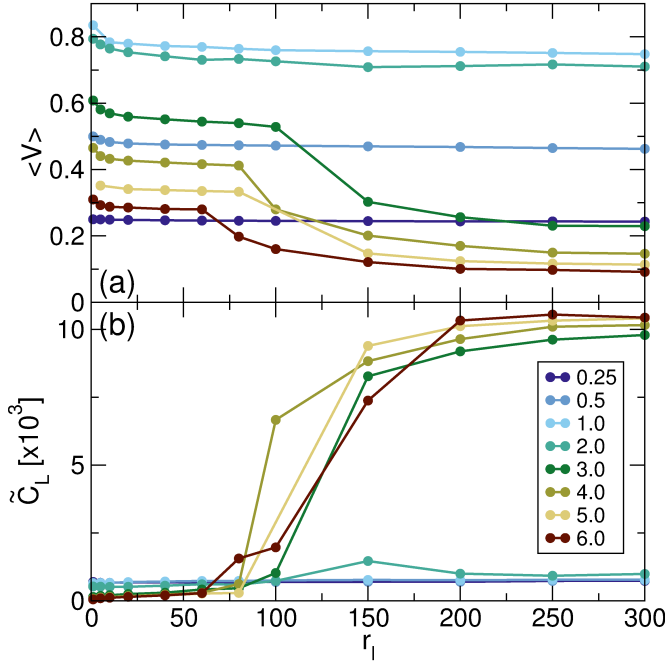


FIG. 5: (a)  $\langle V \rangle$  vs  $r_l$  in samples with  $A_s = 2.0$  and  $\phi = 0.453$  for varied  $v_w$  from  $v_w = 0.25$  to  $v_w = 6.0$ . (b) The corresponding  $\tilde{C}_L$  vs  $r_l$ .

the disk configurations for the system in Fig. 3(a,b) with  $v_w = 0.6$  at  $A_s = 0.75$ , where the disks form a cluster state, while in Fig. 4(b), at  $A_s = 2.5$  in the same system, the clustering is lost and the disks are strongly trapped in the substrate minima, forming chain like states that move with the substrate. These results indicate that the clusters act as composite objects that only weakly couple to the substrate.

We next examine the case with a fixed substrate strength of  $A_s = 2.0$  and varied  $r_l$ . Figure 5(a) shows  $\langle V \rangle$  versus  $r_l$  for  $v_w$  values ranging from  $v_w = 0.25$  to  $v_w = 6.0$ , and Fig. 5(b) shows the corresponding  $\tilde{C}_L$  versus  $r_l$ . For  $v_w < 3.0$  the system remains in a non-cluster state for all values of  $r_l$ , while for  $v_w \geq 3.0$  there is a transition from a non-cluster to a cluster state with increasing  $r_l$  as indicated by the simultaneous drop in  $\langle V \rangle$  and increase in  $\tilde{C}_L$ . In Fig. 6(a,b) we plot  $\langle V \rangle$  and  $C_L$  versus  $v_w$  at  $A_s = 2.0$  for  $r_l = 200$  and  $r_l = 5.0$ . The system is in a non-cluster state for all  $v_w$  when  $r_l = 5.0$ , and there is a peak in  $\langle V \rangle$  near  $v_w = 1.0$ , while for  $r_l = 200$  there is a transition to a cluster state close to  $v_l = 3.0$  which coincides with a drop in  $\langle V \rangle$  that is much sharper than the decrease in  $\langle V \rangle$  with increasing  $v_w$  for the  $r_l = 5$  system. In general, when  $r_l$  is small, the net transport of disks through the sample is greater than in samples with larger  $r_l$ . The fact that the net disk transport varies with varying  $r_l$  suggests that traveling wave substrates could be used as a method for separating different types of active matter, such as clustering and non-clustering species.

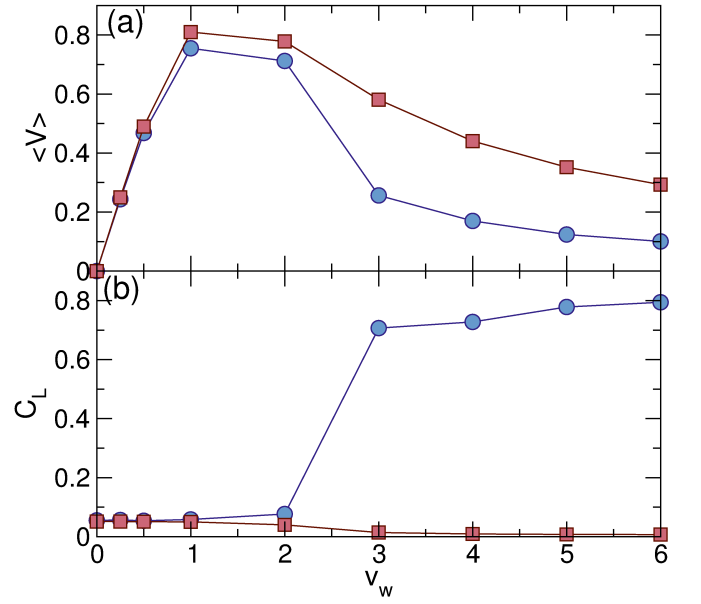


FIG. 6: A sample with  $A_s = 2.0$  and  $\phi = 0.453$  for  $r_l = 5$  (red squares) and  $r_l = 200$  (blue circles). (a)  $\langle V \rangle$  vs  $v_w$ . (b)  $C_L$  vs  $v_w$ .

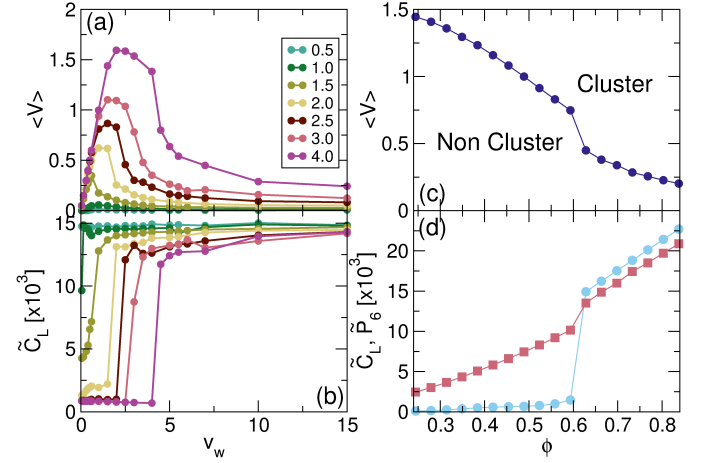


FIG. 7: (a)  $\langle V \rangle$  vs  $v_w$  at  $\phi = 0.56$  and  $r_l = 300$  for varied  $A_s = 0.5$  to  $A_s = 4.0$ . (b) The corresponding  $\tilde{C}_L$  vs  $v_w$ . (c)  $\langle V \rangle$  vs  $\phi$  for  $A_s = 2.5$ ,  $r_l = 300$  and  $v_w = 2.0$ . (d) The corresponding  $\tilde{C}_L$  (blue circles) and  $\tilde{P}_6$  (red squares) vs  $\phi$  where the onset of clustering occurs near  $\phi = 0.6$  at the same point for which there is a drop in  $\langle V \rangle$  in panel (c).

When we vary the disk density  $\phi$  while holding  $r_l$  fixed, we find results similar to those described above. In Fig. 7(a) we plot  $\langle V \rangle$  versus  $v_w$  at  $\phi = 0.56$  for varied  $A_s$  from  $A_s = 0.5$  to  $A_s = 4.0$ , where we find a similar trend in which  $\langle V \rangle$  increases with increasing wave speed when the disks are strongly coupled to the substrate. A transition to a cluster state occurs at higher  $v_w$  as shown in Fig. 7(b) where we plot  $\tilde{C}_L$  versus  $v_w$  for the same samples. The increase in  $\tilde{C}_L$  at the cluster state onset



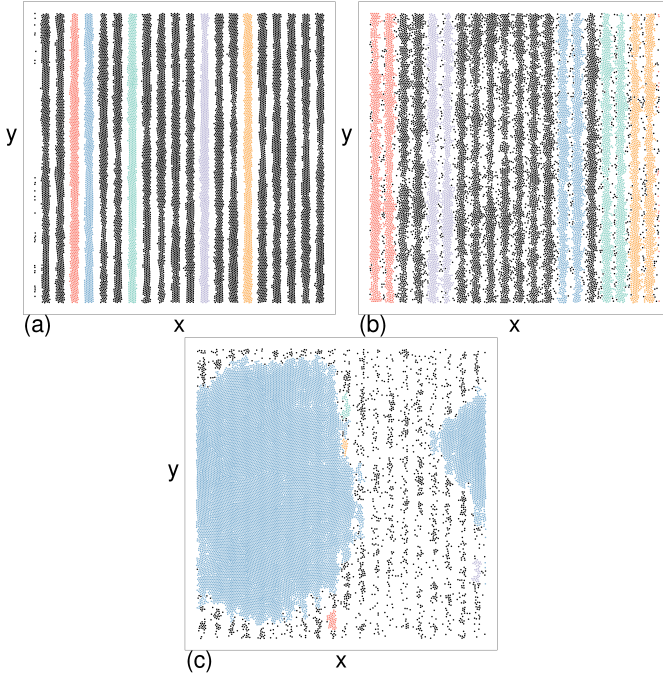


FIG. 8: The disk positions on the traveling wave substrate for the system in Fig. 7(a) at  $\phi = 0.56$ . Colors indicate disks belonging to the five largest clusters. (a) Complete locking at  $A_s = 4.0$  and  $v_w = 1.0$ , where the transport efficiency is  $\langle V \rangle / v_w = 0.998$ . (b) Partial locking at  $A_s = 2.0$  and  $v_w = 1.5$ , where  $\langle V \rangle / v_w = 0.41$ . (c) Weak locking at  $A_s = 1.0$  and  $v_w = 0.6$  with  $\langle V \rangle / v_w = 0.078$ .

coincides with a drop in  $\langle V \rangle$ . In Fig. 7(c) we plot  $\langle V \rangle$  versus  $\phi$  for a system with fixed  $v_w = 2.0$ ,  $A_s = 2.5$ , and  $r_l = 300$ , while in Fig. 7(d) we show the corresponding  $\tilde{C}_L$  and  $\tilde{P}_6$  versus  $\phi$ . A transition from the non-cluster to the cluster state occurs near  $\phi = 0.6$ , which correlates with a sharp drop in  $\langle V \rangle$  and a corresponding increase in  $\tilde{C}_L$  and  $\tilde{P}_6$ .

In Fig. 8(a) we show the disk configurations from the system in Fig. 7(a) at  $A_s = 4.0$  and  $v_w = 1.0$ . Here  $\langle V \rangle / v_w = 0.998$ , indicating that the disks are almost completely locked with the traveling wave motion and there is little to no slipping of the disks out of the substrate minima. In Fig. 8(b), the same system at  $A_s = 2.0$  and  $v_w = 1.5$  has a transport efficiency of  $\langle V \rangle / v_w = 0.41$ . No clustering occurs but there are numerous disks that slip as the traveling wave moves. At  $A_s = 1.0$  and  $v_w = 0.6$  in Fig. 8(c) there is a low transport efficiency of  $\langle V \rangle / v_w = 0.078$ . The system forms a cluster state and smaller numbers of individual disks outside of the cluster are transported by the traveling wave.

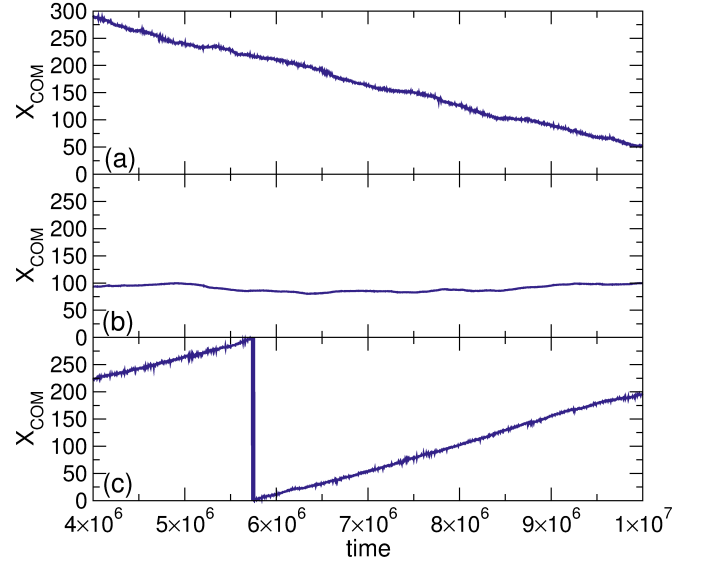


FIG. 9: The center of mass  $X_{\text{COM}}$  location of a cluster vs time in simulation time steps for a system with  $\phi = 0.454$  and  $r_l = 300$ . (a) At  $A_s = 1.25$  and  $v_w = 0.6$ , the cluster moves in the negative  $x$ -direction, against the direction of the traveling wave. (b) At  $A_s = 0.5$  and  $v_w = 4.0$ , the cluster is stationary. (c) At  $A_s = 3.0$  and  $v_w = 7.0$ , the cluster moves in the positive  $x$ -direction, with the traveling wave. The dip indicates the point at which the center of mass passes through the periodic boundary conditions.

#### IV. FORWARD AND BACKWARD CLUSTER MOTION

In general, we find that when the traveling wave is moving in the positive  $x$ -direction,  $\langle V \rangle > 0$ ; however, within the cluster phase, the center of mass motion of a cluster can be in the positive or negative  $x$  direction or the cluster can be almost stationary. By using the cluster algorithm to identify the location of the cluster center of mass, we can track the  $x$ -direction motion of the cluster center of mass  $X_{\text{COM}}$  over fixed time periods, as shown in Fig. 9(a) for a system with  $\phi = 0.454$ ,  $r_l = 300$ ,  $A_s = 1.25$ , and  $v_w = 0.6$ . During the course of  $6 \times 10^6$  simulation time steps the cluster moves in the negative  $x$ -direction a distance of 235 units, corresponding to a space containing 16 potential minima. Even though the net disk flow is in the positive  $x$  direction, the cluster itself drifts in the negative  $x$  direction. In Fig. 9(b) at  $A_s = 0.5$  and  $v_w = 4.0$ , the disks are weakly coupled to the substrate and the cluster is almost completely stationary. Figure 9(c) shows that at  $A_s = 3.0$  and  $v_w = 7.0$ , the cluster center of mass motion is now in the positive  $x$  direction, and the cluster translates a distance equal to almost 20 substrate minima during the time period shown. The apparent dip in the center of mass motion is due to the periodic boundary conditions.

We have conducted a series of simulations and measured the direction and amplitude  $V_{\text{COM}} = dX_{\text{COM}}/dt$  of the

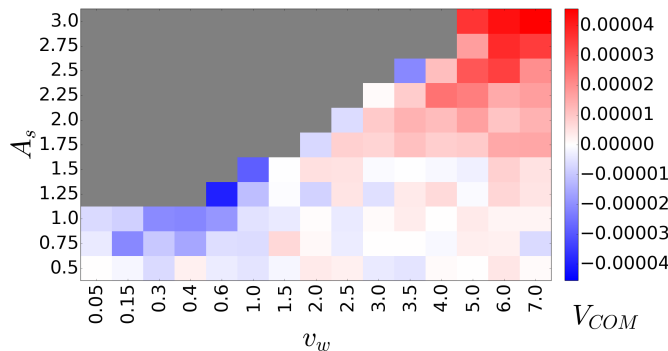


FIG. 10: Height field of the direction and magnitude of the center of mass motion in the  $x$  direction,  $V_{COM}$ , as a function of  $A_s$  vs  $v_w$  for the cluster obtained after  $4 \times 10^6$  simulation steps. The gray area indicates a regime in which there is no cluster formation.

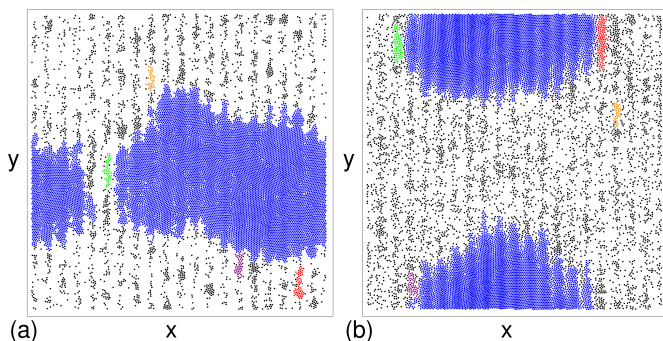


FIG. 11: The disk positions for the system in Figs. 9 and 10. (a) At  $A_s = 1.0$  and  $v_w = 0.4$ , the cluster drifts in the negative  $x$ -direction. (b) At  $A_s = 3.0$  and  $v_w = 5.0$ , the cluster drifts in the positive  $x$ -direction.

center of mass motion, as plotted in Fig. 10 as a function of  $A_s$  versus wave speed for the system in Fig. 7. The gray area indicates a region in which clusters do not occur, and in general we find that the negative cluster motion occurs at lower wave speeds while the positive motion occurs for stronger substrates and higher wave speeds. There are two mechanisms that control the cluster center of mass motion. The first is the motion of the substrate itself, which drags the cluster in the positive  $x$  direction, and the second is the manner in which the cluster grows or shrinks on its positive  $x$  and negative  $x$  sides. At lower substrate strengths and low wave speeds, the disks in the cluster are weakly coupled to the substrate so the cluster does not move with the substrate. In this case the disks can leave or join the cluster anywhere around its edge; however, disks tend to join the cluster at a higher rate on its negative  $x$  side since individual disks, driven by the moving substrate, collide with the negative  $x$  side of the cluster and can become trapped in this higher density area. The positive  $x$  side of the cluster tends to shed disks at a higher rate since the disks can be carried away by the moving substrate

into the low density gas region. The resulting asymmetric growth rate causes the cluster to drift in the negative  $x$  direction. There is a net overall transport of disks in the positive  $x$  direction due to the large number of gas phase disks outside of the cluster region which follow the motion of the substrate. Figure 11(a) shows the disk positions at  $A_s = 1.0$  and  $v_w = 0.4$ , where the cluster is drifting in the negative  $x$  direction. For strong substrate strengths, all the disks that are outside of the cluster become strongly confined in the q1D substrate minima, and the disk density inside the cluster itself starts to become modulated by the substrate. Under these conditions, the cluster is dragged along with the traveling substrate in the positive  $x$  direction, as illustrated in Fig. 11(b) for  $A_s = 3.0$  and  $v_w = 5.0$ . These results suggest that it may be possible for certain active matter systems to collectively form a cluster state in order to move against an external bias even when isolated individual particles on average move with the bias.

## V. SUMMARY

We have examined run and tumble active matter disks interacting with traveling wave periodic substrates. We find that in the non-phase separated state, the disks couple to the traveling waves, and that at the transition to the cluster state, there is a partial decoupling from the substrate and the net transport of disks by the traveling wave is strongly reduced. We also find a transition from a cluster state to a periodic quasi-1D liquid state for increasing substrate strength, as well as a transition back to a cluster state for increasing traveling wave speed. We show that there is a transition from a non-cluster to a cluster state as a function of increasing disk density which is correlated with a drop in the net disk transport. Since disks with different run times drift with different velocities, our results indicate that traveling wave substrates could be an effective method for separating active matter particles with different mobilities. Within the regime in which the system forms a cluster state, we find that as a function of wave speed and substrate strength, there are weak substrate regimes where the center of mass of the cluster moves in the opposite direction from that of the traveling wave, while for stronger substrates, the cluster center of mass moves in the same direction as the traveling wave. The reversed cluster motion occurs due to the spatial asymmetry of the rate at which disks leave or join the cluster. This suggests that collective clustering could be an effective method for forming an emergent object that can move against gradients or drifts even when individual disks on average move with the drift.

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