Erratum: Nonlinear Dirac equation solitary waves in external fields [Phys. Rev. E 86, 046602 (2012)]

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Erratum: Nonlinear Dirac equation solitary waves in external fields [Phys. Rev. E 86, 046602 (2012)]

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In section IV of our original paper [1] we assumed a particular conservation law Eq. (4.6), which was true in the absence of external potentials, to derive some particular potentials for which we obtained solutions to the nonlinear Dirac equation (NLDE). Because the conservation law of Eq. (4.6) for the component T^{11} of the energy-momentum tensor is not true in the presence of these external potentials, the solutions we found do not satisfy the NLDEs in the presence of these potentials. Thus all the equations from Eq (4.6) through Eq. (4.44) are not correct, since the exact solutions that followed in that section presumed Eq. (4.6) was true. Also the equations Eq. (A4) and Eq. (A5) are a restatement of Eq. (4.6) and also are not correct. These latter equations are also not used in section V and beyond. The rest of our original paper (starting with section V) was not concerned with exact solutions, but instead was concerned with how the exact solitary wave solutions to the NLDE in the *absence* of an external potential responded to being in the presence of various external potentials.

In this erratum, we correct this mistake and show how to directly find exact solutions of the NLDE in a particular class of external potentials. That is, we show how to directly solve the equations for the two components of the NLDE in 1+1 dimension with scalar-scalar self interaction $\frac{g^2}{\kappa+1}(\bar{\Psi}\Psi)^\kappa$ in the presence of an external electromagnetic potential in the Axial Gauge $eA^0(x) = V(x), A^1(x) = 0$ without resorting to the conservation law of Eq. (4.6).

Writing the two components of the bound state solution of the NLDE as $\Psi = e^{-i\omega t}\{R\cos\theta, iR\sin\theta\}$ and assuming that $V(x)$ depends on x only through its dependence on $R^2 = y$, we are able to find new exact solutions of the NLDE for arbitrary κ in these potentials.

We start with the NLDE in the presence of an external electromagnetic potential:

$$(i\gamma^\mu\partial_\mu - m)\Psi - e\gamma^\mu A_\mu\Psi + g^2(\bar{\Psi}\Psi)^\kappa\Psi = 0\,. \tag{0.1}$$

Using the freedom of gauge invariance, one can choose the axial gauge $A^1 = 0$, $eA_0 = V(x)$. One can also rescale the fields so that we can set the coupling constant $g = 1$. In the axial gauge the NLDE becomes

$$i\gamma^\mu\partial_\mu\Psi - m\Psi + (\bar{\Psi}\Psi)^\kappa\Psi - \gamma^0 V(x)\Psi = 0. \tag{0.2}$$

Going into the rest frame and choosing $\Psi(x,t) = e^{-i\omega t}\psi(x)$, and for $\psi(x)$ the representation

$$\psi(x) \quad = \begin{pmatrix} A(x) \\ i\ B(x) \end{pmatrix} = R(x)\begin{pmatrix} \cos\theta \\ i\sin\theta \end{pmatrix}, \tag{0.3}$$

we find that the NLDE becomes

$$\begin{aligned} &\partial_x A + (m+\omega)B - g^2[A^2 - B^2]^\kappa B - V(x)B = 0, \\ &\partial_x B + (m-\omega)A - g^2[A^2 - B^2]^\kappa A + V(x)A = 0. \end{aligned} \tag{0.4}$$

These two equations can also be written if we let $y = R^2(x)$ as:

$$\frac{dy}{dx} = 2y^{\kappa+1}(\cos 2\theta)^\kappa \sin 2\theta - 2ym\sin 2\theta, \tag{0.5}$$

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and

$$\frac{d\theta}{dx} = y^{\kappa} \cos^{\kappa+1} 2\theta - m \cos 2\theta + (\omega - V(x)). \tag{0.6}$$

We can now follow the approach that Chang, Ellis, and Lee [2] used in obtaining exact solutions when $V = 0$. Dividing equation Eq. (0.5) by Eq. (0.6) and assuming that V is just a function of $y = R^2$ and furthermore setting $V[y] = df[y]/dy$ then we obtain:

$$\frac{d}{d\theta}\left[\frac{y^{\kappa+1}}{\kappa+1} \cos^{\kappa+1} 2\theta + [\omega - m\cos 2\theta]y - f[y]\right] = 0. \tag{0.7}$$

Integrating with respect to θ and assuming that we have a no node solution going to zero at large $|x|$ so that the constant of integration is zero we obtain

$$y^{\kappa} \cos^{\kappa+1} 2\theta = -(\kappa+1)\left[\omega - m\cos 2\theta - \frac{f[y]}{y}\right]. \tag{0.8}$$

Substituting this result into Eq. (0.6), one obtains the equation

$$\frac{d\theta}{dx} = -\kappa(\omega - m\cos 2\theta) + \left(\frac{df}{dy} - (\kappa+1)\frac{f}{y}\right). \tag{0.9}$$

We notice that if we choose f to be a solution of

$$\frac{df}{dy} - (\kappa+1)\frac{f}{y} = 0. \tag{0.10}$$

i.e.

$$f = v_0 \frac{y^{\kappa+1}}{\kappa+1}, \tag{0.11}$$

so that

$$V[y] = v_0 y^{\kappa}, \tag{0.12}$$

then we obtain the *same* equation for θ as when $V[y] = 0$. Namely,

$$\frac{d\theta}{dx} = -\kappa(\omega - m\cos 2\theta), \tag{0.13}$$

whose solution is

$$\theta(x) = \tan^{-1}(\alpha \tanh \beta_{\kappa}(x), \tag{0.14}$$

where $\alpha = \sqrt{\frac{m-\omega}{m+\omega}}$ and $\beta_{\kappa} = \kappa\sqrt{m^2 - \omega^2}$.

When $\kappa = 1$, this solution was obtained by different means by Nogami and Toyama [3]. Now we can solve for $y = R^2$ by using Eq. (0.8) to obtain

$$R^2 = \left[\frac{(\kappa+1)(m\cos 2\theta - \omega)}{(\cos^{\kappa+1} 2\theta - v_0)}\right]^{1/\kappa}. \tag{0.15}$$

This can also be written using the fact that

$$\cos 2\theta = \frac{m + \omega\cosh 2\beta_{\kappa} x}{\omega + m\cosh 2\beta_{\kappa} x} \tag{0.16}$$

as

$$R^2 = \left(\frac{(\kappa+1)(m-\omega)(m+\omega)}{(m\cosh(2\beta_{\kappa} x) + \omega)\left(\left(\frac{m+\omega\cosh(2\beta_{\kappa} x)}{m\cosh(2\beta_{\kappa} x)+\omega}\right)^{\kappa+1} - v_0\right)}\right)^{\frac{1}{\kappa}}. \tag{0.17}$$

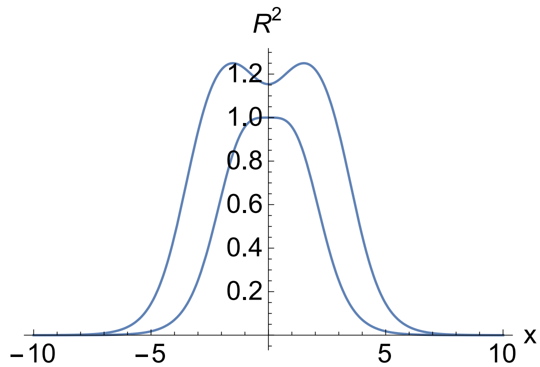

FIG. 1: R^2 vs. x when $\omega = 7/10$, $v_0 = 0.4$ (lower curve), 0.48 (upper curve) for the bound state solution in the external potential $V = v_0 R^2$,

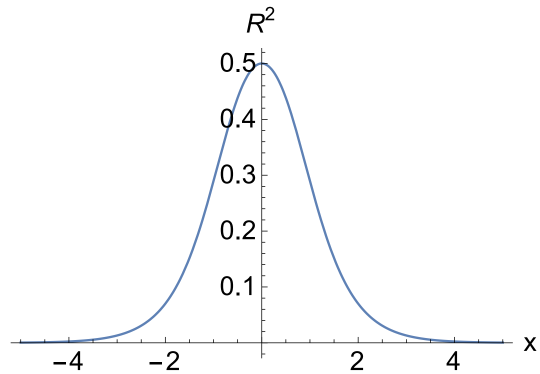

FIG. 2: R^2 vs. x when $\omega = 1/2$, $v_0 = -1$ for the bound state solution in the external Potential $V = -R^2$,

This reduces to our previous result when $v_0 \to 0.$, i.e.

$$R^2 = \left(\frac{m \cosh(2\beta_\kappa x) + \omega}{m + \omega \cosh(2\beta_\kappa x)} \right) \left[\frac{(\kappa+1)\beta_\kappa^2}{\kappa^2 (m + \omega \cosh 2\beta_\kappa x)} \right]^{1/\kappa} . \tag{0.18}$$

Now R^2 has to be positive and vanish when $|x| \to \infty$ which means that $v_0 < \omega^2/m^2$. When $v_0 < 0$ one has an attractive potential and this type of solution always exists. As v_0 approaches ω^2/m^2, R^2 can start becoming double humped. We show some results for $\kappa = 1$ in the figures. For example when $\omega = 7/10$ and we go from $v_0 = 0.4$ to $v_0 = 0.48$ the shape of R^2 shifts from single humped to double humped as seen in Fig. (1). For $v_0 \geq 0.49$ there are no solutions which vanish when $|x| \to \infty$. Choosing $v_0 = -1$ and $\omega = 1/2$ we obtain instead the results of Fig. (2) for the Bound State solution.

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