Reply to “Comment on ‘Nonparametric forecasting of low-dimensional dynamical systems' ”

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Response to Comment

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In this response we provide additional results which allow a better comparison of the diffusion forecast and the PNF approach for the El Niño index. We remark on some qualitative differences between the diffusion forecast and PNF, and we suggest an alternative use of the diffusion forecast for the purposes of forecasting the probabilities of extreme events.

First, we would like to apologize for misinterpreting the results in [1] which caused the comparison in [2] to be misleading, and we thank the authors of [1] for clarifying their results. In particular, the authors of [1] are completely correct that we mistakenly cited the RMS error of the incorrect curve in Fig 2 of [1] (RMS 1.4 listed in our manuscript should have been 0.99). Second, it was misleading of us to compare our results on the El Niño 3.4 index to the results of [1] on the El Niño 3 index. This oversight was caused by our inability to find a El Niño 3 index time series which matched Figure 3 of [1] and we mistakenly used a El Niño 3.4 index which looked similar to Figure 3 of [1] (see Figure 1 below). We would like to note that the caption for Figure 3 of [1], which contains the plot of RMS error for the PNF forecast, refers to the metric simply as "RMS" and the phrase "normalized by the variability" is not attached to RMS specifically and is not explained except in the Comment [3]. While the choice of metric is up to the authors, we would note that this normalization makes comparisons ambiguous since it is sensitive to the choice of dates over which the variability is estimated empirically (e.g. the entire data set, the training data set, or the validation data set). We mention these points only to explain how such significant oversights were made, not to excuse them.

In order to allow a better comparison we have computed the metric of [1] using variability estimated over the entire data set. For the El Niño 3.4 index [4], at 14-month lead time our forecast has a normalized RMS of 0.77 and a correlation of 0.64. We have also applied the diffusion forecast (using the same choices for parameters) to the El Niño 3 index [4] to allow a valid comparison to [1], using the normalized RMS metric. At the 14-month lead time our forecast has a normalized RMS of 0.80 and a correlation of 0.41. This shows a less biased forecast (reduced RMS error compared to 0.99 in [1]) and a com-

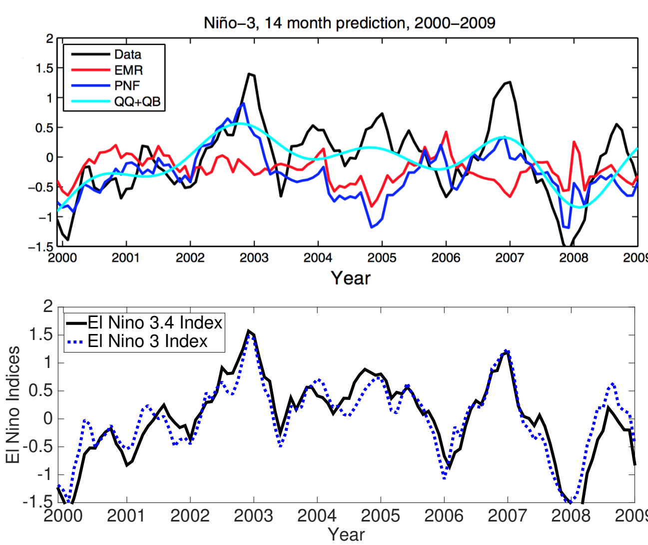

FIG. 1. Result copied from [1] (top) and identically scaled plots of El Niño 3 and El Niño 3.4 indices from NOAA [4] (bottom).

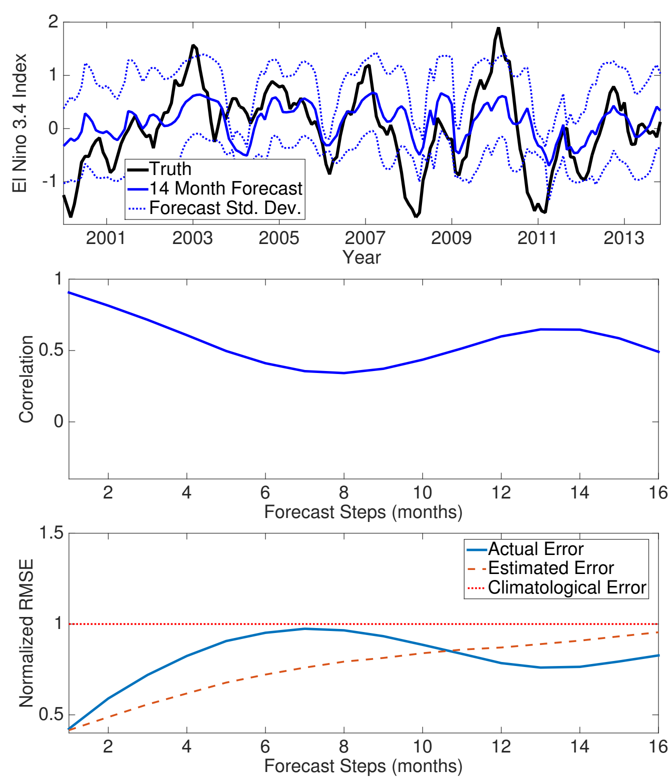

FIG. 2. El Niño 3.4 index [4] 14-month lead forecast (top) Correlation (middle) and Normalized RMS (bottom, RMS divided by the climatological standard deviation)

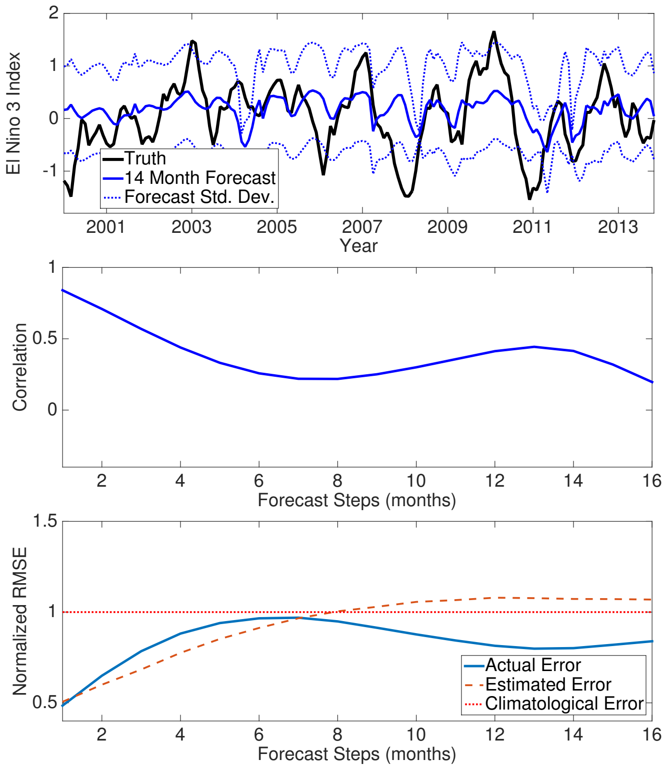

FIG. 3. El Niño 3 index [4] 14-month lead forecast (top) Correlation (middle) and Normalized RMS (bottom, RMS divided by the climatological standard deviation)

parable correlation.

With these corrections, and for such a short validation time series, we agree with the conclusion of the Comment [3] that we cannot claim a significant improvement over the method of [1]. However, we disagree with claim that the PNF forecast provides better prediction of 'extreme' episodes. While the PNF forecast does correctly predict the extreme episodes of late 2002 and 2008, it also erroneously predicts an extreme episode in late 2004. This suggests that the PNF forecast, does not have superior skill in extreme episode prediction, but is merely a more biased forecast which more often makes extreme predictions. This is also shown by the error metric which shows that the diffusion forecast has a reduced RMS error due to the more conservative mean forecasts as pointed out in the Comment [3]. While a more biased forecast model may be preferable in some applications, we would suggest that this is not an objectively preferable property. Moreover, we should emphasize that the method of [2] is a generic "black-box" forecast which used only the 1-dimensional time series of the El-Niño index, as opposed to PNF which uses a significantly larger training data set incorporating SST fields. As pointed out in [2], the diffusion forecast can be a valuable diagnostic tool for minimum acceptable model skill. In this case, the diffusion forecast performance suggests that PNF is not able to gain any additional information from the SST field data that was not already present in the 1-dimensional

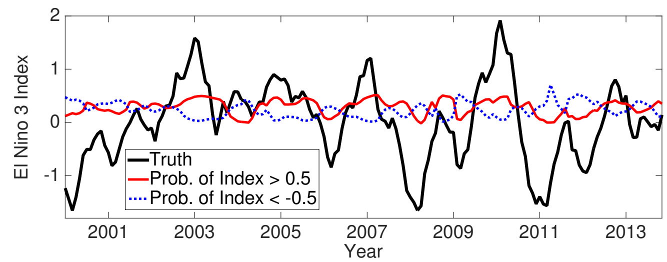

FIG. 4. El Niño 3.4 index [4] 14-month lead forecast probability of the index being great than 0.5 (solid) and probability of the index being less than -0.5 (dashed)

El Niño index. If one has access to the full field rather than just the index, then the corresponding Laplacian eigenfunctions (which are used as data-driven basis functions in [2]) correspond to physically meaningful patterns of SST variability including ENSO, the Pacific Decadal Oscillation, the North Pacific Gyre Oscillation, and other modes [5]. The diffusion forecast is likely to be successful for predicting the spatiotemporal patterns associated with these modes, but if one is interested in forecasting the full field at small lengthscales and short timescales, the diffusion forecast is unlikely to be successful for predicting such a high-dimensional data set as mentioned in [2].

Finally, we would like to point out that the diffusion forecast actually forecasts a distribution, which enables one to estimate the variance of the forecast and produce the error bars shown in Figures 2 and 3. This allows additional tests for the statistical validity of the diffusion forecast approach, for example in Figure 2 the truth falls within the forecast error bars 78% of the time and 81% of the time in Figure 3. Moreover, the forecast distribution can also be used to compute the probability of an extreme event as shown in Figure 4, meaning that the diffusion forecast does not have to rely on a biased mean forecast to indicate extreme episodes as advocated in the Comment [3].

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