Peer pressure: Enhancement of cooperation through mutual punishment

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Peer pressure: enhancement of cooperation through mutual punishment

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An open problem in evolutionary game dynamics is to understand the effect of peer pressure on cooperation in a quantitative manner. Peer pressure can be modeled by punishment, which has been proved to be an effective mechanism to sustain cooperation among selfish individuals. We investigate a symmetric punishment strategy, in which an individual will punish each neighbor if their strategies are different, and vice versa. Because of the symmetry in imposing the punishment, one might expect intuitively the strategy to have little effect on cooperation. Utilizing the prisoner's dilemma game as a prototypical model of interactions at the individual level, we find, through simulation and theoretical analysis, that proper punishment, when even symmetrically imposed on individuals, can enhance cooperation. Besides, we find that the initial density of cooperators plays an important role in the evolution of cooperation driven by mutual punishment.

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I. INTRODUCTION

Cooperation is ubiquitous in biological, social and economical systems [1]. Understanding and searching for mechanisms that can generate and sustain cooperation among selfish individuals remains to be an interesting problem. Evolutionary game theory represents a powerful mathematical framework to address this problem [2, 3]. Previous theoretical [4–11] and experimental [12–19] studies showed that, for evolutionary game dynamics in spatially extended systems, punishment is an effective approach to enforcing the cooperative behavior, where the punishment can be imposed on either cooperators or defectors. The agents that get punished bear a fine while the punisher pays for the cost of imposing the punishment [20, 21]. In existing studies, individuals who hold a specific strategy (usually defection) are punished.

In realistic situations, punishment can be mutual and the strategy would typically depends on the surrounding environment, e.g., on neighbors' strategies. An example is "peer pressure." Previous psychological experiments demonstrated that, an individual tends to conglomerate (fit in) with others in terms of behaviors or opinions [22]. Dissent often leads to punishment either psychologically or financially, or both, as human individuals attempt to attain social conformity modulated by peer pressure [22–24]. To understand *quantitatively* the effect of peer pressure on cooperation through developing and analyzing an evolutionary game model is the main goal of this paper. In particular, we propose a mechanism of punishment in which an individual will punish neighbors who hold the opposite strategy, regardless of whether they are cooperators or defectors.

Differing from previous models where additional strategies of punishment were introduced, in our model there are only two strategies (pure cooperators and pure defectors). More importantly, the punishment is mutual in our model, i.e., individual i who punishes individual j is also punished by j, so the cost of punishment can be absorbed into the punishment fine. Because of this symmetry at the individual or "microscopic" level, intuitively one may expect the punishment not to have any effect on cooperation. Surprisingly, we find that symmetric punishment can lead to enhancement of cooperation. We provide computational and heuristic arguments to establish this finding.

II. MODEL

Without loss of generality, we use and modify the classic prisoner's dilemma game (PDG) [25] to construct a model to gain quantitative understanding of the effect of peer pressure on cooperation by incorporating our symmetric punishment mechanism. In the original PDG, two players simultaneously decide whether to cooperate or defect. They both receive payoff R upon mutual cooperation and payoff P upon mutual defection. If one cooperates but the other defects, the defector gets payoff T while the cooperator gains payoff S. The payoff rank for the PDG is $T > R > P > S$. As a result, in a single round of PDG, mutual defection is the best strategy for both players, generating the well-known social dilemma. There are different settings of payoff parameters [26, 27]. For computational convenience [28], the parameters are often rescaled as $T = b > 1$, $R = 1$, and $P = S = 0$, where b denotes the temptation to defect.

In their pioneering work, Nowak and May included spatial structure into the PDG [28], in which individuals play games only with their immediate neighbors. In the spatial PDG, cooperators can survive by forming clusters in which mutual cooperation outweigh the loss against defectors [29–32]. In the past decade, the PDG has been extensively studied for populations on various types of network configurations [33–35], including regular lattices [36–39], small-world networks [40, 41], scale-free networks [42–45], dynamic networks [46–49], and interdependent networks [50].

Our model is constructed, as follows. Player x can take

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one of two strategies: cooperation or defection, which are described by

$$s_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{1}$$

respectively. At each time step, each individual plays the PDG with its neighbors. An individual will punish the neighbors that hold different strategies. The accumulated payoff of player x can thus be expressed as

$$P_x = \sum_{y \in \Omega_x} [s_x^T M s_y - \alpha(1 - s_x^T s_y)], \tag{2}$$

where the sum runs over the nearest neighbor set Ω_x of player x, α is the punishment fine, and M is the rescaled payoff matrix given by

$$M = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}. \tag{3}$$

Initially, the cooperation and the defection strategies are randomly assigned to all individuals in terms of some probabilities: the initial densities of cooperators and defectors are set to be ρ_0 and $1 - \rho_0$ respectively. The update of strategies is based on the replicator equation [51] for well-mixed populations and the Fermi rule [52] for structured populations.

III. RESULTS FOR WELL-MIXED POPULATIONS

In the case of well-mixed populations, i.e., a population with no structure, where each individual plays with every other, the evolutionary dynamics is determined by the replication equation of the fraction of the cooperators ρ in the population [51]:

$$\frac{d\rho}{dt} = \rho(1-\rho)(P_c - P_d), \tag{4}$$

where $P_c = \rho - (1-\rho)\alpha$ is the rescaled payoff of a cooperator and $P_d = \rho b - \rho\alpha$ is the rescaled payoff of a defector. The equilibria of ρ can be obtained by setting $d\rho/dt = 0$. There exists a mixed equilibrium

$$\rho_e = \frac{\alpha}{2\alpha + 1 - b}, \tag{5}$$

which is unstable. Provided that the initial density of cooperators ρ_0 is different from 0 and 1, the asymptotic density of cooperators $\rho_c = 1$ if $\rho_0 > \rho_e$, and $\rho_c = 0$ if $\rho_0 < \rho_e$.

Figure 1 shows the asymptotic density of cooperators ρ_c as a function of the punishment fine α for different values of the initial density of cooperators ρ_0 when the temptation to defect $b = 1.5$. From Eq. (5), we note that the mixed equilibrium ρ_e definitely exceeds 0.5. As a result, for $\rho_0 \leq 0.5$, ρ_c is always zero regardless of the values of the temptation to defect and the punishment fine. However, for $0.5 < \rho_0 < 1$, there exist a critical value of the punishment fine (denoted by α_c),

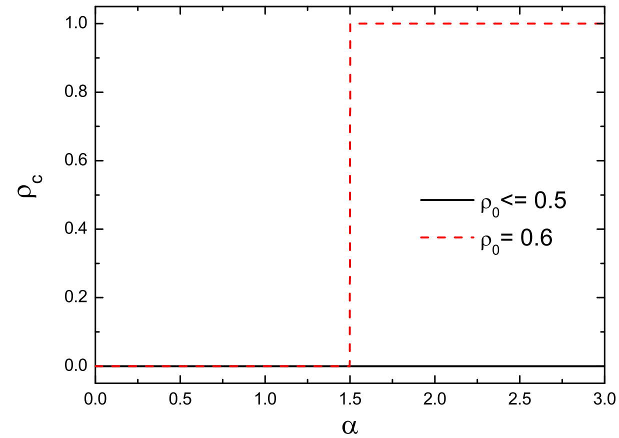

FIG. 1: (Color online) Asymptotic density of cooperators ρ_c as a function of the punishment fine α for different values of the initial density of cooperators ρ_0. The temptation to defect $b = 1.5$.

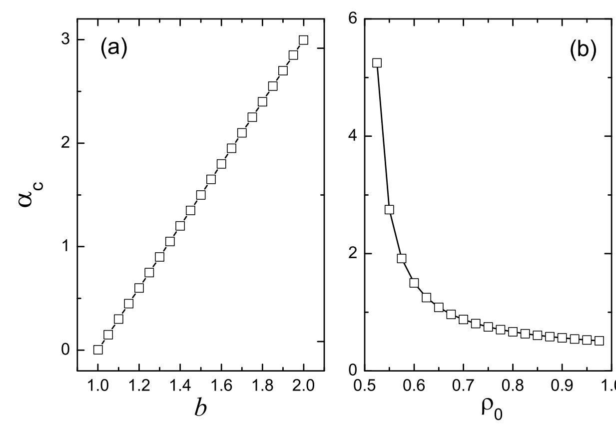

FIG. 2: (a) The critical value of the punishment fine α_c as a function of the temptation to defect b. The initial density of cooperators $\rho_0 = 0.6$. (b) The dependence of α_c on ρ_0. The temptation to defect $b = 1.5$.

below which cooperators die out while above which defectors become extinct. According to Eq. (5), we obtain α_c as

$$\alpha_c = \frac{(b-1)\rho_0}{2\rho_0 - 1}. \tag{6}$$

For example, $\alpha_c = 1.5$ when $\rho_0 = 0.6$ and $b = 1.5$. From Eq. (6), one can find that α_c increases as the temptation to defect b increases but it decreases as the initial density of cooperators ρ_0 increases, as shown in Fig. 2.

IV. RESULTS FOR STRUCTURED POPULATIONS

In a structured population, each individual plays the game only with its immediate neighbors. Without loss of generality, we study the evolution of cooperation on a square lattice, which is the simple and widely used spatial structure. In the

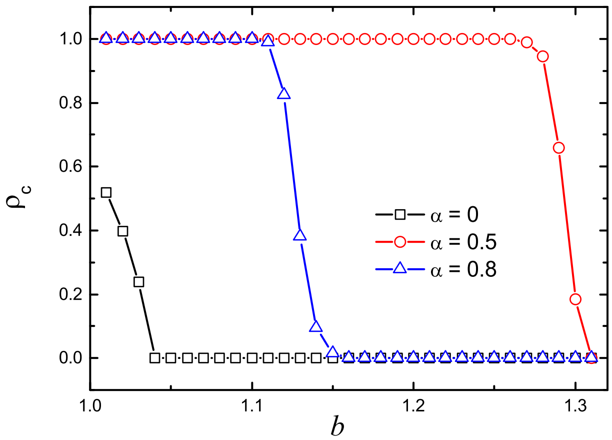

FIG. 3: (Color online) Fraction of cooperators ρ_c as a function of b, the temptation to defect, for different values of the punishment fine α.

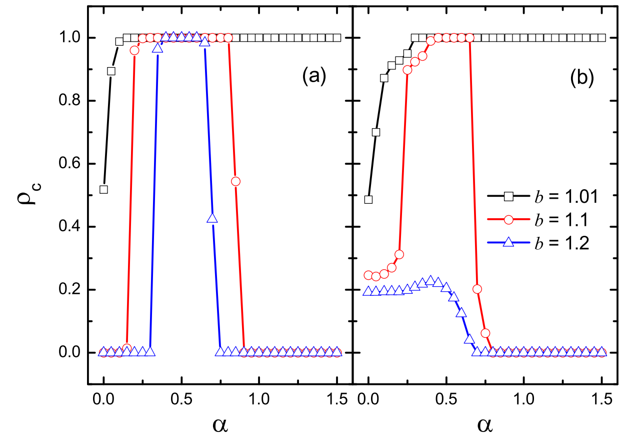

FIG. 4: (Color online) Fraction of cooperators ρ_c as a function of the punishment fine α for different values of b. The results in (a) and (b) from simulation and theoretical analysis, respectively.

following, we use a 100×100 square lattice with periodic boundary conditions. We find that the results are qualitatively unchanged for larger system size, e.g., 200×200 lattice.

In the following studies, we set the initial density of cooperators $\rho_0 = 0.5$ without special mention. Players asynchronously update their strategies in a random sequential order [52–54]. Firstly, player x is randomly selected who obtains the payoff P_x according to Eq. (2). Next, player x chooses one of its nearest neighbors at random, and the chosen neighbor y also acquires its payoff P_y by the same rule. Finally, player x adopts the neighbor's strategy with the probability [52]:

$$W(s_x \leftarrow s_y) = \frac{1}{1 + \exp[-(P_y - P_x)/K]}, \tag{7}$$

where parameter K characterizes noise or stochastic factors to permit irrational choices. Following previous studies [52–54], we set the noise level to be $K = 0.1$. (Different choices of K, e.g., $K = 0.01$ and $K = 1$, do not affect the main results.)

The key quantity to characterize the cooperative behavior of the system is the fraction of cooperators ρ_c in some steady state. All simulations are run for 30000 time steps to ensure that the system reaches a steady state, and ρ_c is obtained by averaging over the last 2,000 time steps. Each time step consists of on average one strategy-updating event for all players. Each data point is obtained by averaging the fraction over 200 different realizations.

Figure 3 shows the fraction of cooperators ρ_c as a function of b, the temptation to defect, for different values of the punishment fine α. We observe, for any given value of α, a monotonic decrease in ρ_c as b is increased. In addition, we find that ρ_c can never reach unity in the whole range of b when the punishment fine is zero. However, for certain values of α, e.g., $\alpha = 0.5$ and $\alpha = 0.8$, cooperators can dominate the whole system for b below some critical value.

Figure 4 shows ρ_c as a function of α for different values of b. We see that, for relatively small values of b (e.g., $b = 1.01$), ρ_c increases with α. However, for larger values of b (e.g., $b = 1.1$ or $b = 1.2$), there exists an optimal region of α in which full cooperation ($\rho_c = 1$) is achieved. For example, the optimal region in α is approximately $[0.3, 0.8]$ and $[0.4, 0.6]$ for $b = 1.1$ and $b = 1.2$ respectively. The optimal value of α is moderate, indicating that either minor or harsh punishment does not promote cooperation. The dependence of ρ_c on α can be qualitatively predicted analytically through a pair-approximation analysis [52, 55], the results from which are shown in Fig. 4(b).

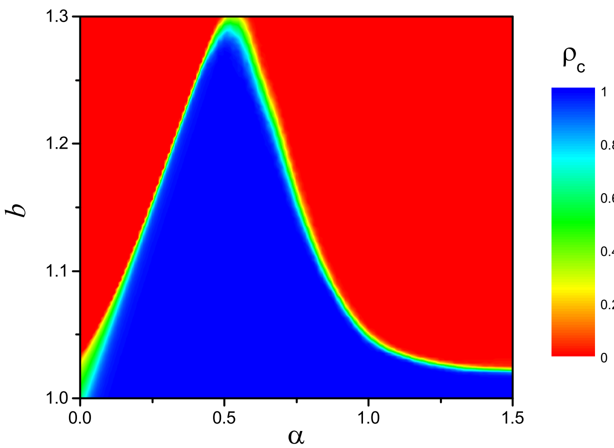

FIG. 5: (Color online) Color coded map of the fraction of cooperators ρ_c in the parameter plane (α,b).

To quantify the ability of punishment fine α to promote cooperation for various values of b more precisely, we compute the behavior of ρ_c in the parameter plane (α, b), as shown in Fig. 5. We see that, for $b < 1.02$, ρ_c increases to unity as α is increased. For $1.02 < b < 1.27$, there exists an optimal region of α in which complete extinction of defectors occurs ($\rho_c = 1$). The optimal region of α becomes narrow as b is increased. For $b > 1.27$, there also exists an optimal value of

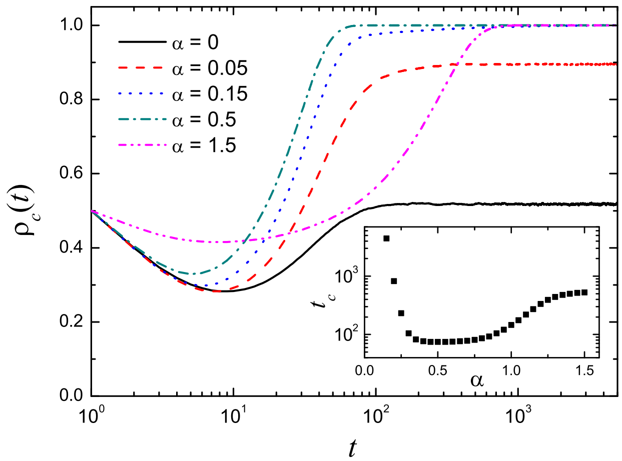

FIG. 6: (Color online) For $b = 1.01$, time series of the fraction of cooperators, $\rho_c(t)$, for different values of α. The inset presents the convergence time t_c versus α.

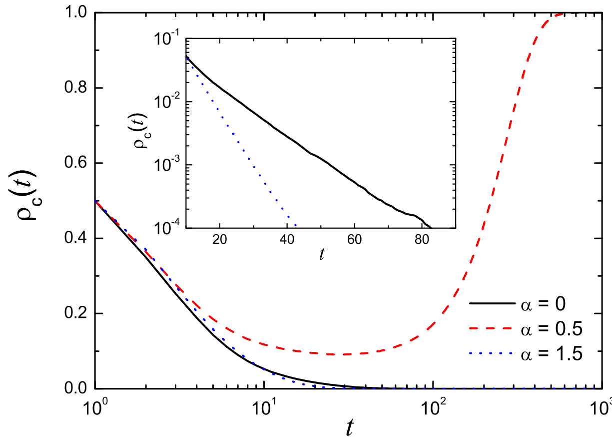

FIG. 7: (Color online) For $b = 1.2$, time series $\rho_c(t)$ for different values of α. Inset shows that the fraction of cooperators decays exponentially for $\alpha = 0$ and $\alpha = 1.5$.

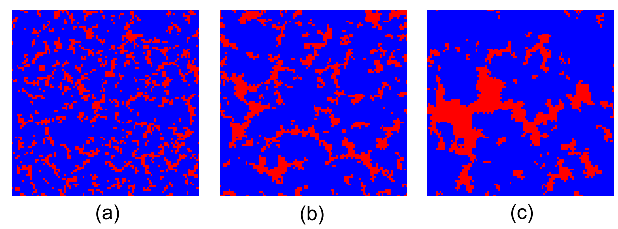

FIG. 8: (Color online) For a number of values of α, snapshots of typical distributions of cooperators (blue) and defectors (red) in the steady state. The fraction of cooperators in the equilibrium state is set to be $\rho_c = 0.8$ for different values of α. The values of α and b are (a) $\alpha = 0.02$, $b = 1.001$; (b) $\alpha = 0.2$, $b = 1.116$ and (c) $\alpha = 0.4$, $b = 1.245$.

α that results in the highest possible level of cooperation for the corresponding b values, albeit $\rho_c < 1$.

To gain insights into the mechanism of cooperation enhancement through punishment, we examine the time evolution of ρ_c for a number of combinations of the parameters α and b. Figure 6 shows the time series $\rho_c(t)$ for different values of α and a relatively small value of b (e.g., $b = 1.01$). In every case, $\rho_c(t)$ decreases initially but then increases to a constant value. The similar phenomenon was also observed in Refs. [56, 57]. For small values of α (e.g., $\alpha = 0$ or $\alpha = 0.05$), $\rho_c(t)$ cannot reach unity. For relatively large values of α (e.g., $\alpha = 0.15$, $\alpha = 0.5$ or $\alpha = 1.5$), at the end defectors are extinct and all individuals are cooperators. We define the convergence time t_c as the number of time steps required for complete extinction of defectors. In the inset of Fig. 6, we show t_c as a function of α and observe that t_c is minimized for $\alpha \approx 0.5$.

Figure 7 shows the time series $\rho_c(t)$ for different values of α when there is strong temptation to defect (e.g., $b = 1.2$). We observe that cooperators gradually die out for either small (e.g., $\alpha = 0$) or large (e.g., $\alpha = 1.5$) α values. A remarkable phenomenon is that, asymptotically, the fraction of cooperators decreases exponentially over time for small or large α values: $\rho_c(t) \propto e^{-t/\tau}$, where the value of τ depends on α, as shown in the inset of Fig. 7. For moderate values of α (e.g., $\alpha = 0.5$), $\rho_c(t)$ decreases initially and then increases to unity.

How the cooperators and defectors are distributed in the physical space when a steady state is reached? Figure 8 shows spatial strategy distributions for different values of the punishment fine α in the equilibrium state. By varying the value of b, we produce the same fraction of cooperators ($\rho_c = 0.8$) for each value of α. We see that, defectors spread homogeneously in the whole space when α is small (e.g., $\alpha = 0.02$), while the same amount of defectors are more condensed for the higher value of α (e.g., $\alpha = 0.4$). Such condensation of defectors prevents them to reach competitive payoffs.

How does the distribution of cooperators and defectors evolve with time? Figure 9 shows the distribution of cooperators and defectors at different time steps for a large value of b (e.g., $b = 1.2$) and a moderate value of α (e.g., $\alpha = 0.5$). Initially, cooperators and defectors are randomly distributed with equal probability [Fig. 9(a)]. After a few time steps, cooperators and defectors are clustered, and the density of cooperators is lower than that associated with the initial state [Fig. 9(b)]. With time the cooperator clusters continue to expand and the defector clusters shrink [Fig. 9(c)]. Finally, the whole population is cooperators [Fig. 9(d)]. From Fig. 9, one can also observe that interfaces separating domains of cooperators and defectors become smooth as time evolves. As illustrated in Refs. [58, 59], noisy borders are beneficial for defectors, while straight domain walls help cooperators to spread.

In the above studies, we set the initial density of cooperators ρ_0 to be 0.5. Now we study how different values of ρ_0 affect the evolution of cooperation. From Fig. 10(a), one can find that for the small value of ρ_0 (e.g., $\rho_0 = 0.2$), the cooperation level reaches maximum at moderate punishment fine when the temptation to defect b is fixed. However, for the large value of

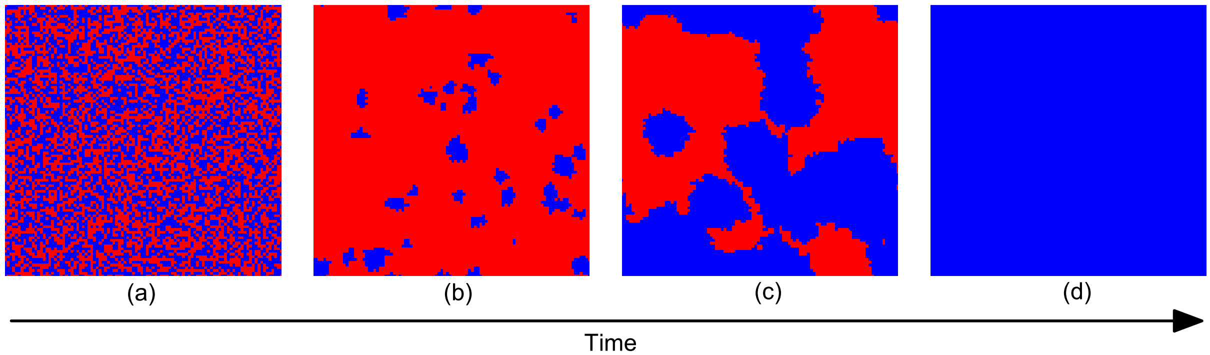

FIG. 9: (Color online) For $\alpha = 0.5$ and $b = 1.2$, snapshots of typical distributions of cooperators (blue) and defectors (red) at different time steps t.

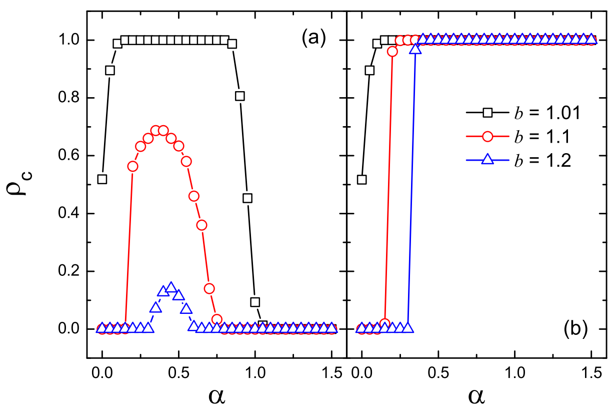

FIG. 10: (Color online) Fraction of cooperators ρ_c as a function of the punishment fine α for different values of the temptation to defect b. The initial density of cooperators ρ_0 is (a) 0.2 and (b) 0.8, respectively.

ρ_0 (e.g., $\rho_0 = 0.8$), the cooperation level increases to 1 as the punishment fine increases [Fig. 10(b)].

V. CONCLUSIONS AND DISCUSSIONS

To obtain quantitative understanding of the role of peer pressure on cooperation, we study evolutionary game dynamics and propose the natural mechanism of mutual punishment in which an individual will punish a neighbor with a fine if their strategies are different, and vice versa. The mutual punishment can be interpreted as a term modifying the strength of coordination type interaction [60]. Because of the symmetry in imposing the punishment between the individuals, one might expect that it would have little effect on cooperation. However, we find a number of counterintuitive phenomena.

In a well-mixed population, if the initial density of cooperators is no more than 0.5, cooperators die out regardless of the values of the punishment fine and the temptation to defect. If the initial density of cooperators exceeds 0.5, for each value of the temptation to defect, there exists a critical value of the punishment fine, below (above) which is the full defection (cooperation). The critical value of the punishment fine increases as the temptation to defect increases but it decreases as the initial density of cooperators increases.

For structured population, our main findings are as follows. (i) If the initial density of cooperators is small (e.g., 0.2), there exists an optimal value of the punishment fine, leading to the highest cooperation. Too weak or too harsh punishment will suppress cooperation. Similar phenomenon was also observed in Refs. [9, 61]. (ii) If the initial density of cooperators is moderate (e.g., 0.5), for weak temptation to defect, the final fraction of cooperators increases to 1 as the punishment fine increases. For strong temptation to defect, the cooperation level can be maximized for moderate punishment fine. (iii) If the initial density of cooperators is large (e.g., 0.8), for each value of the temptation to defect, the final fraction of cooperators increases to 1 as the punishment fine increases.

In the present studies, we use the prisoner's dilemma game to understand the role of peer pressure in cooperation. It would be interesting to explore the effect of mutual punishment on other types of evolutionary games (e.g., the snowdrift game and the public goods game) in future work. By our mechanism, an individual can be punished least by taking the local majority strategy. In fact, following the majority is an important mechanism for the formation of public opinion [62]. As a side result, our work provides a connection between the evolutionary games and opinion dynamics.

Acknowledgments

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[1] R. Axelrod, *The Evolution of Cooperation* (Basic Books, New York, 1984)
[2] A. M. Colma, *Game Theory and Its Applications in the Social and Biological Sciences* (Butterworth-Heinemann, Oxford, 1995).
[3] M. A. Nowak, *Evolutionary Dynamics* (Harvard University, Cambridge, MA, 2006).
[4] J. Henrich and R. Boyd, J. Theor. Biol. **208**, 79 (2001).
[5] C. Hauert, S. De Monte, J. Hofbauer, and K. Sigmund, Science **296**, 1129 (2002).
[6] H. Brandt and K. Sigmund, Proc. Natl. Acad. Sci. USA **102**, 2666 (2005).
[7] A. Traulsen, C. Hauert, H. D. Silva, M. A. Nowak, and K. Sigmund, Proc. Natl. Acad. Sci. USA **106**, 709 (2009).
[8] H. Ohtsuki, Y. Iwasa, and M. A. Nowak, Nature (London) **457**, 79 (2009).
[9] D. Helbing, A. Szolnoki, M. Perc, and G. Szabó, New J. Phys. **12**, 083005 (2010).
[10] D. G. Rand and M. A. Nowak, Nat. Commun. **2**, 434 (2011).
[11] A. Szolnoki, G. Szabó, and M. Perc, Phys. Rev. E **83**, 036101 (2011).
[12] T. Clutton-Brock and G. A. Parker, Nature (London) **373**, 209 (1995).
[13] E. Fehr and S. Gächter, Nature (London) **415**, 137 (2002).
[14] E. Fehr and B. Rockenbach, Nature (London) **422**, 137 (2003).
[15] D. Semmann, H.-J. Krambeck, and M. Milinski, Nature (London) **425**, 390 (2003).
[16] D. J.-F. de Quervain, U. Fischbacher, V. Treyer, M. Schellhammer, U. Schnyder, A. Buck, and E. Fehr, Science **305**, 1254 (2004).
[17] J. H. Fowler, Proc. Natl. Acad. Sci. USA **102**, 7047 (2005).
[18] J. Henrich, Science **312**, 60 (2006).
[19] T. Sasaki, I. Okada, and T. Unemi, Proc. R. Soc. London B **274**, 2639 (2007).
[20] C. Hauert, A. Traulsen, H. Brandt, M. A. Nowak, K. Sigmund, Science **316**, 1905 (2007).
[21] M. Egas and A. Riedl, Proc. R. Soc. London B **275**, 871 (2008).
[22] S. Asch, *Social Psychology* (Prentice-Hall, Englewood Cliffs, NJ,1952).
[23] N. Eisenberger, M. Lieberman, and K. Williams, Science **302**, 290 (2003).
[24] L. Somerville, T. Heatherton, and W. Kelley, Nat. Neurosci. **9**, 1007 (2006).
[25] R. Axelrod R and W. D. Hamilton, Science **211**, 1390 (1981).
[26] C. P. Roca, J. A. Cuesta, and A. Sánchez, Phys. Rev. E **80**, 046106 (2009).
[27] Z.-X. Wu and H.-X. Yang, Phys. Rev. E **89**, 012109 (2014).
[28] M. A. Nowak and R. M. May, Nature (London) **359**, 826 (1992).
[29] C. Hauert, Proc. R. Soc. Lond. B **268**, 761 (2001).
[30] C. Hauert and M. Doebeli, Nature (London) **428**, 643 (2004).
[31] J. Gómez-Gardeñes, M. Campillo, L. M. Floría, and Y. Moreno, Phys. Rev. Lett. **98**, 108103 (2007).
[32] H.-X. Yang, Z. Rong, and W.-X. Wang, New J. Phys. **16**, 013010 (2014).
[33] G. Szabó and G. Fáth, Phys. Rep. **446**, 97 (2007).
[34] M. Perc, J. Gómez-Gardeñes, A. Szolnoki, L. M. Floría, and Y. Moreno, J. R. Soc. Interface **10**, 20120997 (2013).
[35] A. Szolnoki and M. Perc, J. R. Soc. Interface **12**, 20141299 (2015).
[36] A. Traulsen and J. C. Claussen, Phys. Rev. E **70**, 046128 (2004).
[37] M. Perc, A. Szolnoki, and G. Szabó, Phys. Rev. E **78**, 066101 (2008)
[38] A. Szolnoki, M. Perc, and G. Szabó, Phys. Rev. E **80**, 056104 (2009).
[39] Z. Rong, Z.-X. Wu, and G. Chen, EPL **102**, 68005 (2013).
[40] F. Fu, L. Liu, and L. Wang, Eur. Phys. J. B **56**, 367 (2007).
[41] X. Chen and L. Wang, Phys. Rev. E 77, **017103** (2008).
[42] F. C. Santos and J. M. Pacheco, Phys. Rev. Lett. **95**, 098104 (2005).
[43] W.-B. Du, X.-B. Cao, M.-B. Hu, and W.-X. Wang, EPL **87**, 60004 (2009).
[44] Z. Rong, H.-X. Yang, and W.-X. Wang, Phys. Rev. E **82**, 047101 (2010).
[45] H.-X. Yang, Z.-X. Wu, and W.-B. Du, EPL **99**, 10006 (2012).
[46] F. C. Santos, J. M. Pacheco, T. Lenaerts, PLOS Comp. Biol. **2**, 1284 (2006).
[47] J. M. Pacheco, A. Traulsen, and M. A. Nowak, Phys. Rev. Lett. **97**, 258103 (2006).
[48] S. Meloni, A. Buscarino, L. Fortuna, M. Frasca, J. Gómez-Gardeñes, V. Latora, and Y. Moreno, Phys. Rev. E **79**, 067101 (2009).
[49] D. G. Randa, S. Arbesmanc, and N. A. Christakisc, Proc. Natl. Acad. Sci. USA **108**, 19193 (2011).
[50] Z. Wang, A. Szolnoki, and M. Perc, Sci. Rep. **3**, 2470 (2013).
[51] J. Hofbauer and K. Sigmund, *Evolutionary Games and Population Dynamics* (Cambridge University Press, Cambridge, England, 1998).
[52] G. Szabó and C. Tőke, Phys. Rev. E **58**, 69 (1998).
[53] M. H. Vainsteina, A. T. C. Silvab, and J. J. Arenzon, J. Theor. Biol. **244**, 722 (2007).
[54] D. Helbing and W. Yu, Advs. Complex Syst. **11**, 641 (2008).
[55] C. Hauert and G. Szabó, Am. J. Phys. **73**, 405 (2005).
[56] A. Szolnoki and M. Perc, Eur. Phys. J. B **67**, 337 (2009).
[57] J. Tanimoto, Phys. Rev. E **89**, 012106 (2014).
[58] A. Szolnoki, Z. Wang, and M. Perc, Sci. Rep. **2**, 576 (2012).
[59] M. Perc and A. Szolnoki, New J. Phys. **14**, 043013 (2012).
[60] G. Szabó, K. S. Bodó, B. Allen, and M. A. Nowak, Phys. Rev. E **90**, 042811 (2014).
[61] L.-L. Jiang, M. Perc, A. Szolnoki, PLoS ONE **8**, e64677 (2013).
[62] P. L. Krapivsky and S. Redner, Phys. Rev. Lett. **90**, 238701 (2003).