Enhanced efficiency of internal combustion engines by employing spinning gas

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Enhanced efficiency of internal combustion engines by employing spinning gas

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The efficiency of the internal combustion engine might be enhanced by employing spinning gas. A gas spinning at near sonic velocities has an effectively higher heat capacity, which allows practical fuel cycles, which are far from the Carnot efficiency, to approach more closely the Carnot efficiency. A remarkable gain in fuel efficiency is shown to be theoretically possible for the Otto and Diesel cycles. The use of a flywheel, in principle, could produce even greater increases in efficiency.

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I. INTRODUCTION

Optimizing the internal combustion engine to achieve the highest possible fuel efficiency can be approached both from a theoretical perspective and from a practical perspective [1]. From the practical perspective, which has attracted the most attention, research has focused on the optimization of the irreversible processes that occur in the working engine, by considering finite time thermodynamics [2] and irreversible thermodynamics [3]. These processes include friction losses [4], inhomogeneous combustion and heat transfer to the wall [5, 6], optimal piston trajectory [7], and other non-ideal effects in combusting gas [8]. From a theoretical perspective, equilibrium thermodynamics places upper bounds on efficiencies, which in practice are not nearly reached.

Recently, it was shown that spinning a gas equips it with an effectively higher, spin-dependent heat capacity, with sonic speeds giving the largest heat capacity [9]. It is shown here that this effect may be exploited in the internal combustion engine. Specifically, in practical fuel cycles, like the Otto and Diesel cycles, by spinning the working gas, the theoretical limit in fuel efficiency may be increased by as much as 10 to 40 percent. The new theoretical limits rely only on the equilibrium thermodynamics of spinning gases.

It is assumed here that the working fluids are ideal Boltzmann gases and that the chemical reactions of combustion do not change the gas constituents significantly. The gases may be compressed axially in a cylindrical container, like in a typical engine cycle. The only difference is that the gases may be spinning around the axis of the cylinder. For simplicity, the cylinder is considered in the large aspect ratio limit, where end effects can be neglected. The gas angular momentum is assumed to be conserved on the time scale of the compression; in other words, the cylinder is assumed to be frictionless. The compression or expansion cycles, however, are assumed to be slow enough that equilibrium thermodynamics prevails, under the constraint imposed by the conservation of the angular momentum. Note that the enhanced heat capacity utilized here arises from this constraint on the collective motion of the gas constituents, rather than the spin properties of individual atoms.

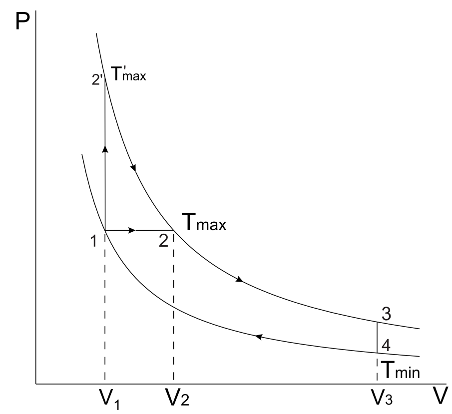

FIG. 1: (Color online) Otto cycle $1 \to 2' \to 3 \to 4$ and Diesel cycle $1 \to 2 \to 3 \to 4$. Heat is transferred at $1 \to 2'$ or $1 \to 2$ phase.

II. ENGINE EFFICIENCY

Consider two practical cycles for engines, namely the Otto cycle and the Diesel cycle. The P-V diagrams of these cycles are shown in Fig. 1. The Otto cycle consists of adiabatic compression and expansion processes, separated by ignition and rejection of heat processes at constant volume. The Otto cycle efficiency depends only on the volumetric compression ratio $n = V_{\mathtt{max}}/V_{\mathtt{min}}$, and is given by

$$\eta_o = 1 - n^{1-\gamma}, \tag{1}$$

where $\gamma = c_p/c_v$ is the specific heat ratio. In the Diesel cycle, the heating occurs at constant pressure, rather than constant volume, as in the Otto cycle. If the ratio of the volumes after heating and before heating is $p = V_2/V_1$ and the total volume compression ratio is $n = V_{\mathtt{max}}/V_{\mathtt{min}}$, the Diesel cycle efficiency may be written as

$$\eta_d = 1 - \frac{1}{\gamma n^{\gamma-1}} \frac{p^\gamma - 1}{p - 1}. \tag{2}$$

Depending on what constraints are imposed, any of these types of engines might be the most efficient. Given say maximum and minimum volumes, the Otto cycle would be most fuel efficient, even more so than the Carnot cycle.

However, constraints on the temperature appear to be the most fundamental from a practical viewpoint. Constraints on the volume are likely less important, since usually there is ample room for the engine. Constraints on pressure might be circumvented by the use of compressors. On the other hand, material properties limit temperature on the high side; while the ambient temperature marks the low temperature limit. Given two limiting temperatures, namely a maximum temperature $T_{\mathtt{max}}$ and a minimum temperature $T_{\mathtt{min}}$, the Carnot cycle gives, of course, the optimum fuel efficiency, $\eta_c = 1 - T_{\mathtt{min}}/T_{\mathtt{max}}$.

In practice though, the Carnot cycle is impossible, because, first, it contains isothermal processes that are not implementable, and second, it requires a heat reservoir at $T_{\mathtt{max}}$ that is not present in real engines. Thus, consider instead the Otto and Diesel cycles, but constrained by maximum and minimum temperatures. To render the efficiency of these cycles, Eq. (1) and Eq. (2), in terms of the temperature extrema, introduce the ratio of minimum and maximum temperatures, $\delta = T_{\mathtt{min}}/T_{\mathtt{max}}$, and the ratio of total heat per particle and maximum temperature, $q = Q/NT_{\mathtt{max}}$, so that the efficiencies of the Otto and Diesel cycles can be put as

$$\eta_o = 1 - \frac{\delta}{1 - q/c_v}, \tag{3}$$

$$\eta_d = 1 - \frac{\delta c_v}{q}\left(\frac{1}{(1 - q/c_p)^\gamma} - 1\right), \tag{4}$$

respectively. Note that, for q large enough, a singularity appears in the denominators, indicating that such processes are not feasible, namely, that more heat is introduced than can be accommodated by the temperature difference. For small q, the Otto cycle efficiency can be approximated as $\eta_o \approx 1 - \delta - q\delta/c_v$ and the Diesel cycle efficiency as $\eta_d \approx 1 - \delta - (q\delta/c_v)(\gamma + 1)/2\gamma$, so that it can be seen that, as $q \to 0$, the Diesel cycle is more efficient than the Otto cycle for all temperature ratios.

III. SPINNING GAS

Consider now the effect of spinning the working gas in each of these thermal cycles. Two types of compression may now be distinguished, axial and perpendicular, since a centrifugal force now acts on the gas. However, here only the longitudinal compression (along the axis of the spinning) will be considered, since radial compression is very hard to realize practically.

The thermodynamic properties of spinning gas are captured entirely by one parameter, what we call the *spinning parameter* $\varphi = m\omega^2 r_0^2/2T$, which measures the spinning energy compared to the thermal energy [9]. Here m is the mass of gas molecule, r_0 is radius of the cylinder, ω is angular frequency, and T is the gas temperature. For negligible friction losses over a thermal cycle, the angular momentum of the gas, given by

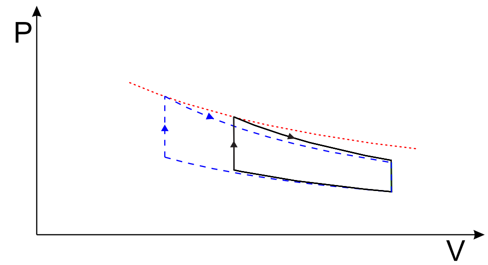

FIG. 2: (Color online) Modification of Otto cycle for spinning gas. Dotted red curve is the temperature constraint. Black curve denotes a non-spinning case; dashed blue curve denotes spinning case.

$$M_g = Nmr_0^2\omega A(\varphi), \tag{5}$$

is conserved, where the function $A(\varphi)$,

$$A(\varphi) = \frac{e^\varphi(\varphi - 1) + 1}{\varphi(e^\varphi - 1)} \tag{6}$$

is the normalized moment of inertia of the gas, changing from $1/2$ to 1 as φ goes from 0 to ∞. The gas energy is

$$E = c_v NT + \omega M_g/2, \tag{7}$$

where the second term denotes the energy of rotation.

The physical picture is as follows: Rotation flings the gas molecules to the cylinder walls, an effect counteracted by high temperature. Under compression, the gas adiabatically heats up, forcing molecules away from the walls, thereby decreasing the moment of inertia $A(\varphi)$. Since angular momentum is conserved, the angular velocity must increase, as does the energy of rotation. This effect can be described as a rotation-dependent heat capacity that now goes from c_v to $c_v + B(\varphi)$, where $B(\varphi)$ is a smooth *compression function* that goes from 0 to 1 as φ goes from 0 to ∞ [9]. For small φ, the system behavior is very close to the non-spinning case; only for $\varphi \geq 1$ is the difference noticeable. The parameter φ changes under compression or heating of the gas, but the change is modest. Thus, under axial compression, in the limit of high φ, the effect of rotation is to increase the specific heat c_v by 1.

Note that, if constrained by a fixed compression ratio, it is inefficient to compress rotating gas axially in the Otto cycle, where the efficiency increases with γ. For example, for a monatomic gas with $c_v = 3/2$ and $\gamma = 5/3$, the specific heat increases to $5/2$, meaning that $\gamma = 7/5$ in the limit of supersonic spinning. For compression ratio $n = 2$, the efficiency $\eta \approx 0.24$ for the spinning gas is less than the efficiency $\eta \approx 0.37$ for non-spinning gas. In contrast, if constrained by a fixed temperature ratio, as seen from Eq. (3), the Otto cycle is more efficient under spinning by

$$\hat{\eta}_o - \eta_o \approx q\delta/c_v c_{\mathtt{p}}. \tag{8}$$

This difference can lead to remarkable increases in efficiency. Fig. 2 demonstrates how the Otto cycle, under spinning, traces a modified, larger area, P-V curve. The larger heat capacity accommodates the maximum temperature constraint, while the volume ratio increases in order to increase the cycle efficiency.

Fig. 3 shows how the efficiency increases with the spinning parameter φ for parameters pertinent to actual modern vehicle engines, namely, with diatomic buffer gas (nitrogen and oxygen) with $\gamma = 7/5$, temperature ratio $\delta = 300/2000$, and a compression ratio of about 10:1, which corresponds to $q \approx 1.5$. Since the Diesel cycle begins with somewhat better efficiency, particularly for q small, there is somewhat less room for improvement. However, as can be seen from Eq. (4), like for the Otto cycle, the efficiency grows with φ up to saturation. For either cycle, strong gas rotation $\varphi \geq 1$ achieves the greatest improvement in the efficiency.

Note, however, the efficiency increase from the baseline efficiency can be remarkably on the order of 10-40 percent, and that the efficiency increase can be very large even for sonic or near sonic velocities. For example, with $q = 1.75$, the increase in the theoretical efficiency for the Otto cycle rises about 24% with $\varphi = 2$, but also as much as 14% for sonic speeds $\varphi = 1$.

Note from Eq. (8) that greater efficiency improvement through spinning occurs specifically for larger heat transfer q and higher δ (smaller temperature differences). While the efficiency increases with φ for all q, lower q signifies a cycle closer to the ideal Carnot cycle, so increasing the efficiency is difficult. However, to overcome fixed inefficiencies in actual devices, q is generally designed to be finite, like $q = 1.5$, where spinning can be helpful. Moreover, there are niche applications, like low-temperature engines, for which δ might be much higher, and the base efficiency smaller, so that the relative improvements can be greater. Low-temperature engines, although less efficient because of small temperature differences, are attractive because of greatly reduced NO_X emissions [10]. Engines operating at 1500 °K rather than at 2000 °K feature a maximum increase in theoretical efficiency of the Otto cycle engine of 30% rather than 18%; for $q = 1.75$, the maximum increase is 80% rather than 40%. For even lower temperature engines, like 1000 °K, the increase at $q = 1.5$ is already over 100%.

IV. INITIATING THE SPINNING

The key technical issue is how to introduce angular momentum to the system. One possibility is to bleed compressed gas into the cylinder along the cylinder wall in the tangential direction, so that the incoming gas follows the side cylinder wall. This initiation of the spin is similar to techniques used in vortex tubes [11, 12]. The initial gas compression might be done, for example, by use of turbo compressors, similar to that installed in many modern engines.

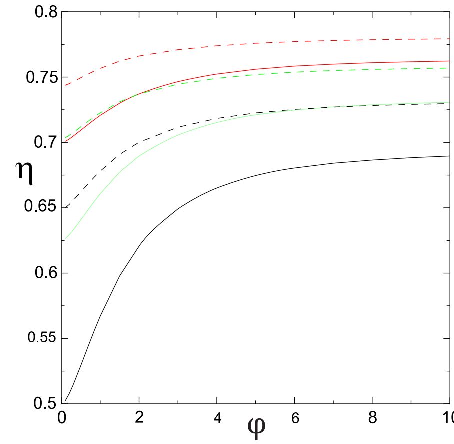

FIG. 3: (Color online) Efficiency of Otto (solid line) and Diesel (dashed line) cycles as function of spinning parameter φ at maximum temperature of the cycle for $c_v = 2.5$ and $\delta = 3/20$. Black $q = 1.75$, green $q = 1.5$, red $q = 1.25$.

V. FLYWHEEL CONTROL

The spinning energy need not, in principle, be refreshed at each thermal cycle, such as, for example, if the spent gases are released at very small radius such that little angular momentum is lost with the gas. Alternatively, the spinning energy might be introduced and recovered through a flywheel, which we now consider in detail. Suppose then an ideal, frictionless flywheel, with blades rotating inside the cylinder all the time, even between thermal cycles, such that the gas and the flywheel equilibrate to the same angular rotation velocity. The flywheel exchanges with the gas the mechanical energy of rotation. Suppose the flywheel has moment of inertia I, so that the total angular momentum becomes

$$M_{\texttt{tot}} = I\omega + M_g, \tag{9}$$

where M_g is given by Eq. (5). The flywheel kinetic energy can then be added to the gas energy given by Eq. (7) to give the total energy,

$$E_{\texttt{tot}} = c_v NT + M_{\texttt{tot}}\omega/2 = \\ I\omega^2/2 + c_v NT + Nm\omega^2 r_0^2 A(\varphi)/2. \tag{10}$$

Using now Eq. (10) together with Eq. (9), and assuming conservation of angular momentum, a generalized compression function $\tilde{B}$ can be found

$$\tilde{B} = \frac{\varphi^2 A(\varphi) H(\varphi)\,(1 + J/A(\varphi))}{J/A(\varphi) + 1 + 2\varphi H(\varphi)}, \tag{11}$$

where the dimensionless parameter, $J = I/Nmr_0^2$, measures the moment of inertia of the flywheel compared to that of the spinning gas. In the limit $J \to 0$, the compression function reduces to the gas-only compression function B, found previously [9]. For finite J, $\tilde{B}(\varphi)$, like $B(\varphi)$, vanishes for $\varphi = 0$ and asymptotes to 1 for

$\varphi \gg 1$, with the inflection point occurring at somewhat lower φ as J grows.

Now consider what happens if the flywheel is given angular velocity ω_1 while the gas has ω_0. Equilibrium is established at the final angular velocity ω, with gas temperature changing from T_0 to T, where ω and T may be found using Eqs. (9) and (10), to get

$$M_{\mathtt{tot}} = I\omega_1 + Nmr_0^2 A(\varphi_0)\omega_0 = \omega(I + Nmr_0^2 A(\varphi)), \quad (12)$$

$$c_v NT_0 + \frac{I\omega_1^2}{2} + \frac{Nmr_0^2 A(\varphi_0)\omega_0^2}{2} = c_v NT + \frac{I\omega^2}{2} + \frac{Nmr_0^2 A(\varphi)\omega^2}{2}. \quad (13)$$

It is more convenient to express quantities in terms of a new spinning parameter φ instead of the frequency ω. After some algebra, φ at equilibrium may be written as

$$\varphi = \varphi_0 \frac{(JR + A(\varphi_0))^2 (J + A(\varphi))^{-2}}{\left[1 + \frac{\varphi_0}{c_v}\left(JR^2 + A(\varphi_0) - \frac{(JR+A(\varphi_0))^2}{J+A(\varphi)}\right)\right]}, \quad (14)$$

where $\varphi_0 = m\omega_0^2 r_0^2/2T_0$ and $R = \omega_1/\omega_0$. For T we have

$$T = \frac{T_0\varphi_0}{\varphi}\left(\frac{JR + A(\varphi_0)}{J + A(\varphi)}\right)^2. \quad (15)$$

Note that the mechanical energy required to change the angular velocity of the flywheel from ω_1 to ω_0 is given by

$$\Delta E = \frac{I\omega_0^2}{2} - \frac{I\omega_1^2}{2} = JNT_0\varphi_0(1 - R^2). \quad (16)$$

Eqs. (14) and (15) describe how gas is spun up or slowed down by the flywheel. In effect, these equations describe removing the flywheel from the gas, changing its angular velocity from ω_0 to ω_1, then again making contact with the gas until a new equilibrium is reached. Note that the gas heats up when it is spun up and cools down when the rotation is slowed. In the limit of differentially small changes, this process can be shown to be reversible, since differential changes in E and T can be put as functions of φ only.

The spinning gas thermal cycle thus can operate as follows. First, the flywheel initiates some rotation. The gas is then compressed and heated. The fuel is then burned and the gas expands. Lastly, the gas is slowed down by the flywheel, which cools it further. The total amount of work done in the cycle is the sum of two adiabatic compressions and two gas rotations with the flywheel. After the first stage of spinning injection, the gas heats up, thereby increasing the minimum temperature from where the adiabatic compression starts. Since the maximum temperature is constrained, the total amount of heat q received from the combustion is also constrained.

The best way to cool is while spinning up, such that temperature remains constant. A completely isothermal process is not feasible because it would take infinitely long, but, to the extent that it can be reached, it gives the highest efficiency. The process of spinning while cooling is not completely infeasible, because it is done at the ambient temperature, for which a thermal reservoir with infinite heat capacity ant $T_{\mathtt{min}}$ is available. Of course, higher efficiency yet would be reached to slow down the spinning also at constant temperature, but for that process there is no readily available heat reservoir.

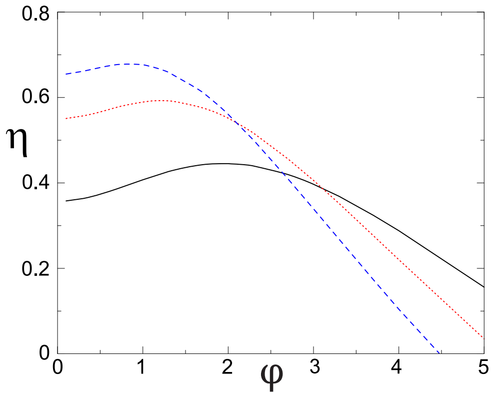

FIG. 4: (Color online) Efficiency of Otto cycle for vs. φ for $J = 1$. Black: $\delta = 4/3$, $q = 1/2$, dotted red: $\delta = 5/3$, $q = 2/5$, dashed blue: $\delta = 2$, $q = 1/3$.

In summary, the Otto cycle modified with spinning gas comprises the processes: 1. isothermal spinning injection; 2. adiabatic compression; 3. isochoric heating; 4. adiabatic expansion; and 5. adiabatic spinning ejection. These processes depend on four dimensionless parameters: δ, q, J, and φ. The efficiency weakly depends on J; there are almost no significant changes in varying J from 0.1 to 10. The efficiency dependence on the other three parameters is shown in Fig. 4, where the dependence of η as a function of φ is plotted for fixed δ and q. Note that the peak of the efficiency is reached at lower values of φ, likely making it easier to reach under real conditions, since spinning injection might be an issue in real devices. Note also that the maximum value of the efficiency is somewhat greater than estimated by Eq. (8).

An important caveat is that the there are likely practical inefficiencies both in the transfer of angular momentum transfer from flywheel to the gas and from the gas to the flywheel. For an ideal flywheel and the nominal case Otto cycle ($c_v = 2.5$, $q = 1.5$, and $\delta = 3/20$), the efficiency for sonic spinning $\varphi = 1$ will be approximately 71.7% while the base efficiency is 62.5%. With 1% energy transfer losses in the flywheel, the efficiency will only decrease to 69.6%, which still represents a relative efficiency of 11.4%. However, in this case, 5% energy transfer losses would eliminate the benefits of using spinning gas. On the other hand, for higher q, higher flywheel losses are tolerable. For example, for $q = 2$ but all the other parameters the same, it would take 16% energy transfer losses to eliminate the benefits of the spinning gas.

VI. CONCLUSION

The equilibrium thermodynamic limits of internal combustion engine efficiency is reconsidered by exploiting the rotation-dependent heat capacity of a spinning working gas. For practical engine cycles, such as the Otto or Diesel cycles, spinning the gas around the axis of the cylinder, while compressing and expanding axially, is shown to give remarkable theoretical efficiency gains, as much as 10 to 40 percent for typical Otto cycle engines, and more for low-temperature engines. As a practical matter, the spinning might be initiated through compressors or though a flywheel. In arriving at the new theoretical limit, many of the important non-ideal effects of real engines were neglected, including friction, insufficient mixing, flywheel efficiency, and heat transfer. However, it is hoped that the remarkable increase in the theoretical maximum efficiencies might be large enough to overcome the neglected inefficiencies in practical settings.

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