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Nonlinear interaction of intense laser pulse and inhomogeneous electron-positron-ion plasma

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The nonlinear interaction of ultra-intense short laser beam and inhomogeneous electron-positron-ion (e-p-i) plasma is investigated. It is found that the presence of positrons and inhomogeneity results in strong modulational and filamentational instabilities, which induce strong nonlinear interaction between the laser beam and the inhomogeneous e-p-i plasma. Light beam focusing, filamentation, trapping and nonlinear interaction between the trapped light spots and inhomogeneous plasma are observed. Interestingly, we find that the inhomogeneity of the plasma can not only boost a mechanism for light beam self-focusing and filamentation, but also provide an effective way to localize and trap the beam in the region where one wanted.

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The nonlinear interaction between ultra-intense short laser beam and plasma is of great interest in various fundamental research and technological applications [1–3]. These include relativistic optical guiding, harmonic excitation, wake-field generation, laser pulse frequency shifting, pulse compression, and particle acceleration. Especially, the nonlinear dynamics of intense laser pulses in electron-positron-ion (e-p-i) plasma have been received a great deal of attention [4–10]. Light beam compression, focusing, and trapping in self-created density holes, light beam filamentation, stable localized solutions and self-modulational instability in e-p-i plasmas are discussed. In fact, propagation of intense short laser beam in plasma can also lead to pair production resulting in a three-component e-p-i plasma [4, 11, 12]. Abundant production of e-p pairs in the collision of a multi-petawatt laser beam and a solid target is predicated and it shows that the positron density can up to $10^{26}\mathtt{m}^{-3}$ [13, 14]. Recently, e-p plasma is produced in laboratory by irradiating a solid gold target with an intense picosecond laser pulse[15–17]. The results show that the positron density can up to 10^{16} cm^{-3}. It also predicts that, with the increasing performance of high-energy ultra-short laser pulses, a high-density, up to 10^{18} cm^{-3}, relativistic pair-plasma is achievable. Thus, from these theoretical and experimental investigations we can expect that, the investigation of nonlinear interaction of pair plasma and high intensity electromagnetic fields is significant for future high-intensity laser experiments. However, the previous investigations about the propagation of laser beam in e-p-i plasma were mainly limited to homogeneous cases. In fact, up to now, few investigations have been devoted to the effect of plasma inhomogeneity on pulse propagation [18–25]. It was shown that axial [18–21] or radial [23–25] inhomogeneity can further boost the self-focusing and compression mechanism and localize the pulse intensity, in comparison with a homogeneous plasma.

In this paper, taking into account the effects of equilibrium density inhomogeneity and positron concentration, we study the nonlinear propagation of the intense laser pulses in an inhomogeneous e-p-i plasma. Starting from the Maxwell equations and Poisson' equation, we show that the system of the governing equations can be reduced to a modified nonlinear Schrödinger equation (NLSE) with density inhomogeneity effect. Linear, Gaussian, and cosine density inhomogeneities along axial direction are discussed. It is shown that the presence of positrons and inhomogeneity results in strong modulational and filamentational instabilities, which induce strong nonlinear interaction between the laser beam and the inhomogeneous e-p-i plasma. Rich and interesting phenomena are observed. We find that the inhomogeneity can not only boost a mechanism for light beam self-focusing and filamentation, but also provide an effective way to localize and trap the pulse in the region where we wanted.

We investigate the propagation of intense laser pluses in a smooth inhomogeneous e-p-i plasma. We assume that the electron and the positron densities are homogenous in radial r direction, but inhomogeneous in axial z direction. The local equilibrium state of the three-component system is characterized by the dimensionless local charge neutrality

$$n_{e0}(z) = \alpha n_{p0}(z) + (1-\alpha) n_{i0}(z), \tag{1}$$

where $n_{e0}(z)$, $n_{p0}(z)$, and $n_{i0}(z)$ are the unperturbed number densities of the electrons, positrons, and ions respectively. The coefficient $\alpha = n_{p0}(0)/n_{e0}(0)$ denotes the ratio of positron density to the electron density at $z = 0$. The electrons and positrons are denoted by the subscripts e and p respectively. The ions do not respond to the dynamics under consideration and provide a neutralizing background due to their relatively large inertia.

In order to describe the nonlinear propagation of intense light in such a plasma, we start with Maxwell equations. The wave equation (in the Coulomb gauge $\nabla \cdot \boldsymbol{A} = 0$) and Poisson equation of the system take the

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forms [1, 5, 7]

$$\frac{\partial^2 \boldsymbol{A}}{\partial t^2} - \nabla^2 \boldsymbol{A} = \alpha n_p \boldsymbol{v}_p - n_e \boldsymbol{v}_e, \tag{2}$$

and

$$\nabla^2 \varphi = n_e - \alpha n_p - (1-\alpha) n_{i0}(z). \tag{3}$$

Here $\boldsymbol{A}$ and φ are the vector and scalar potentials respectively, n_e (n_p) is the electron (positron) number density, $\boldsymbol{v}_e$ ($\boldsymbol{v}_p$) is the electron (positron) velocity in the electromagnetic fields. The system is closed by invoking the equation of motion

$$\frac{\partial \boldsymbol{p}_{e,p}}{\partial t} + (\boldsymbol{v}_{e,p}\cdot\nabla)\boldsymbol{p}_{e,p} = \mp[-\nabla\varphi - \frac{\partial \boldsymbol{A}}{\partial t} + \boldsymbol{v}_{e,p}\times(\nabla\times\boldsymbol{A})] - \frac{T_{e,p}}{n_{e,p}}\nabla n_{e,p} \tag{4}$$

for each of mobile components. Here $\boldsymbol{p}_e$ ($\boldsymbol{p}_p$) is the electron (positron) momentum, T_e (T_p) is the electron (positron) temperature. Equations (1)-(4) are dimensionless with the following scalings:

$$t \sim \omega_e t, \boldsymbol{r} \sim \frac{\omega_e}{c}\boldsymbol{r}, \boldsymbol{v}_{e,p} \sim \frac{\boldsymbol{v}_{e,p}}{c}, \boldsymbol{p}_{e,p} \sim \frac{\boldsymbol{p}_{e,p}}{m_0 c}, T_{e,p} \sim \frac{T_{e,p}}{m_0 c^2},$$
$$n_{i0}(z) \sim \frac{n_{i0}(z)}{n_{i0}(0)}, n_{e,p} \sim \frac{n_{e,p}}{n_{e0,p0}(0)}, \hat{A} \sim \frac{e}{m_0 c^2}\hat{A}, \tag{5}$$

where $\hat{A} \equiv [\boldsymbol{A};\varphi]$, $\omega_e = (4\pi e^2 n_{e0}(0)/m_0)^{1/2}$ is the electron plasma frequency, e is the magnitude of the electron charge, m_0 is the rest mass of electrons.

For the propagation of a circularly polarized electromagnetic with a frequency ω_0 and wave number $\boldsymbol{k}(\boldsymbol{r})$ along the axial direction, the vector potential can be represented as

$$\boldsymbol{A} = \frac{1}{2} a(\boldsymbol{r},t)(\hat{\boldsymbol{x}} + \mathtt{i}\hat{\boldsymbol{y}}) \exp\left(\mathtt{i}\int \boldsymbol{k}(\boldsymbol{r})\cdot d\boldsymbol{r} - \mathtt{i}\omega_0 t\right) + c.c.. \tag{6}$$

Since the dimensionless quiver velocity is given by $\boldsymbol{v}_{e,p} = \boldsymbol{p}_{e,p}/\gamma_{e,p}$, where $\gamma_{e,p} = (1+\boldsymbol{p}_{e,p}^2)^{1/2}$, Eq. (4) is satisfied by[1]

$$\boldsymbol{p}_{e,p} = \pm\boldsymbol{A} \tag{7}$$

and

$$\nabla[\pm\varphi - \gamma_{e,p} - T_{e,p}\ln n_{e,p}] = 0. \tag{8}$$

It is noted that Eq. (7) governs the high-frequency response of the electrons and the positrons with the same frequency ω_0 as that of the incident waves, and $a(\boldsymbol{z},t)$ is slowly varying along the axial z and time t. From Eq. (7), the electron and positron velocities are

$$\boldsymbol{v}_e = \frac{\boldsymbol{A}}{\gamma}, \boldsymbol{v}_p = -\frac{\boldsymbol{A}}{\gamma}, \tag{9}$$

where $\gamma = (1+|\boldsymbol{A}|^2)^{1/2} \equiv \gamma_{e,p}$. Equation (8) describes the electron and positron low-frequency response (neglecting the electron inertia for the slow motion). The second term of Eq. (8) is the usual expression for the relativistic ponderomotive force. The expressions for the electron and positron number densities can be obtained by integrating Eq. (8)

$$n_e = n_{e0}(z)\exp\left[-\frac{\gamma-1}{T_e} + \frac{\varphi}{T_e}\right], \tag{10a}$$

$$n_p = n_{p0}(z)\exp\left[-\frac{\gamma-1}{T_p} - \frac{\varphi}{T_p}\right]. \tag{10b}$$

Using Eq. (9), Eq. (2) becomes

$$\frac{\partial^2 \boldsymbol{A}}{\partial t^2} - \nabla^2\boldsymbol{A} + \frac{\alpha n_p + n_e}{\sqrt{1+|\boldsymbol{A}|^2}}\boldsymbol{A} = 0. \tag{11}$$

Substituting Eq. (6) and the dispersion relation $\omega_0^2 - k^2 = \alpha n_{p0}(z) + n_{e0}(z)$ into Eq. (11), we obtain

$$\mathtt{i}\omega_0\frac{\partial a}{\partial t} + \mathtt{i}(\boldsymbol{k}\cdot\nabla)a + \frac{1}{2}\nabla^2 a + \frac{\alpha n_{p0}(z) + n_{e0}(z)}{2}a - \frac{1}{2}\frac{\alpha n_p + n_e}{\sqrt{1+|a|^2}}a = 0. \tag{12}$$

Noting that all terms in this dynamics vary on a slow time scale, it is now convenient to induce new variables, $t = \tau\omega_0$, $\boldsymbol{r} = \vec{\xi} + \boldsymbol{v}_g\tau\omega_0$, and denoted $\boldsymbol{v}_g = \boldsymbol{k}/\omega_0$ as the group velocity of light. Thus, Eq. (12) takes the form

$$\mathtt{i}\frac{\partial a}{\partial \tau} + \frac{1}{2}\nabla^2 a + \frac{\alpha n_{p0}(z) + n_{e0}(z)}{2}a - \frac{1}{2}\frac{\alpha n_p + n_e}{\sqrt{1+|a|^2}}a = 0. \tag{13}$$

We assume that the unperturbed electrons and positrons obey the same distribution in the axial direction, viz. $n_{e0}(z) = n_{p0}(z) = n_0(z)$. From Eqs. (1), (10), and (3) we have

$$\varphi = \left(1 - \sqrt{1+a^2}\right)(\alpha\beta_p - \beta_e)/(\alpha\beta_p + \beta_e), \tag{14}$$

where $\beta_e = 1/T_e$, $\beta_p = 1/T_p$. Then, Eq. (13) takes the form

$$\mathtt{i}\frac{\partial a}{\partial \tau} + \frac{1}{2}\nabla^2 a + \{\frac{\alpha+1}{2} - \frac{\alpha\exp[\beta(1-\sqrt{1+|a|^2})]}{2\sqrt{1+|a|^2}} - \frac{\exp[\alpha\beta(1-\sqrt{1+|a|^2})]}{2\sqrt{1+|a|^2}}\} n_0(z) a = 0, \tag{15}$$

where $\beta = 2\alpha\beta_p\beta_e/(\alpha\beta_p + \beta_e)$ denotes the temperature parameter. The inhomogeneous is expressed by the term $n_0(z)$. When $n_0(z) = 1$, the system is reduced to the homogeneous case.

The modulational instability of an arbitrary large amplitude electromagnetic pump wave governed by Eq. (15) can be investigated by standard techniques. Accordingly, we let $a = (a_0 + a_1)\exp(\mathtt{i}\delta\tau)$, where a_0 is a real constant, $a_1 (\ll a_0)$ denotes the amplitude of the perturbation and δ is a constant nonlinear frequency shift caused by the nonlinear interaction. Then from Eq. (15) the nonlinear frequency shift δ can be obtained at the lowest order

$$\delta = \frac{\alpha+1}{2} - \frac{\alpha\exp[\beta(1-\sqrt{1+a_0^2})]}{2\sqrt{1+a_0^2}} - \frac{\exp[\alpha\beta(1-\sqrt{1+a_0^2})]}{2\sqrt{1+a_0^2}}\,. \qquad (16)$$

Letting $a_1 = (X + \mathtt{i}\,Y)\exp(\mathtt{i}\int \boldsymbol{K}(\vec{\xi})\cdot d\vec{\xi} - \mathtt{i}\Omega\tau)$, where X and Y are real constants and $\Omega(\boldsymbol{K})$ is the frequency (wavevector) of the low frequency modulations, and linearizing Eq. (15) with respect to X and Y, we obtain the modulational instability growth rate $\Gamma = -\mathtt{i}\Omega$

$$\Gamma = \frac{K}{\sqrt{2}}\{[\alpha(1+\beta\sqrt{1+a_0^2})\exp[\beta(1-\sqrt{1+a_0^2})] + (1+\alpha\beta\sqrt{1+a_0^2})\exp[\alpha\beta(1-\sqrt{1+a_0^2})]] \times \frac{a_0^2}{2(1+a_0^2)^{\frac{3}{2}}}n_0(z) - \frac{K^2}{2}\}^{\frac{1}{2}}. \qquad (17)$$

In Fig. 1, we show the growth rate Γ as a function of K for the position-to-electron density ratio $\alpha = 0.05, 0.10,$ 0.20, 0.50, 0.75, and 1.00 in the homogeneous case, i.e., with $n_0(z) = 1$ in term of Eq. (17). The plasma parameters are $a_0 = 0.1$, $\beta_e = \beta_p = 100$. We see that the growth rate increases with α. In pure e-p plasma ($\alpha = 1$) we obtain a largest instability increment, and the modulational instability reaches a maximum due to the largest positron concentration. On the other hand, the presence of positrons has strong modification on modulational instability. When the positrons are absent ($\alpha = 0$), the system would be stable. The phenomenon can be understood in terms of the ponderomotive force, which appears because of the interaction between the high-frequency laser wave and the background plasma. The force acting on particles is directed along the decreasing of the laser field energy density, and does not depend on the sign of electric charge of particles. Its magnitude is proportional to the gradient of the intensity of laser and is inversely proportional to the plasma particles' mass. Because of the large ion mass, the ponderomotive forces acting on ions are much smaller compared with the forces acting on electrons and positrons, thus it can be neglected here. If the laser wave amplitude has a maximum at some points, then the ponderomotive force tends to push electron away from that points and leaves behind ambipolar field. The ambipolar field prevents the motion of electron. If the positron presents, positrons are pushed away together with electrons, then, the ambipolar field will be decreased. Due to the decreasing of the ambipolar field induced by the increasing of positron population, more electrons will be expulsed away by the ponderomotive force, and then, the modulational instability of the system can be enhanced.

In Fig. 2, we show the growth rate Γ as a function of K and the light propagation direction z for the inhomogeneous cases with $\alpha = 0.5$ in term of Eq. (17). In order to investigate the effects of electrons and positrons distribution on the modulational instability, three types of inhomogeneous cases are discussed: a slow variation of the electron and positron density along axial direction $n_0(z) = 1 + bz$, where b is a characteristic inhomogeneity parameter (Fig. 2(a)); a Gaussian density distribution $n_0(z) = \exp(-z^2/L_z)$, $L_z = 16\pi$, L_z is the simulation box length (Fig. 2 (b)); and a cosine distribution $n_0(z) = 1/2(1 - \cos z)$ (Fig. 2 (c)). It is clear that the inhomogeneity has strong influence on modulational instability. The higher the density is, the stronger the modulational instability becomes. Figures 1-2 indicate that the interaction between the laser beam and the inhomogeneity of e-p-i plasma would induce rich and interesting phenomena.

In order to investigate the effects of positron concentration and the inhomogeneity on the propagation of a large amplitude electromagnetic beams, we assume axial symmetry around the z direction, and solve Eq. (15) numerically for the time evolution of the amplitude of vector potential. As an initial condition, we take a modulated beam of the form $a = 0.1[1 + 0.02\sin(2\pi z/L_z) + 0.02\cos(4\pi z/L_z) + 0.02\cos(6\pi z/L_z)]\exp(-r^2/32)$, where L_z is the simulation box length and r is the radial space coordinate.

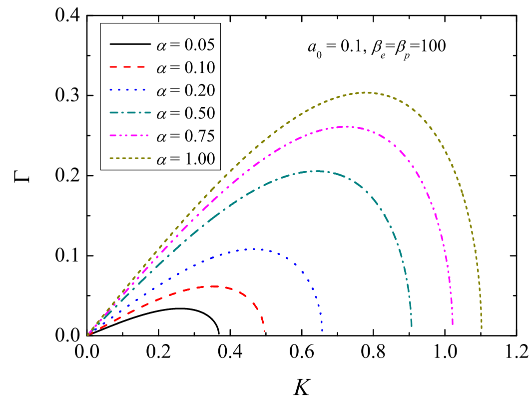

FIG. 1: (color online) The modulational instability growth rate (Γ) versus the wavenumber ($\boldsymbol{K}$), for different value of the positron-to-electron density ratio (α).

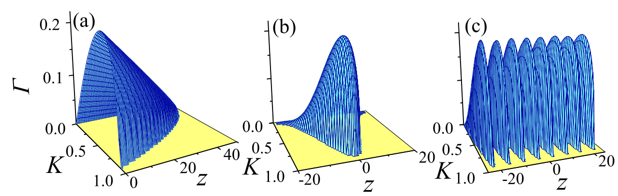

FIG. 2: (color online) The modulational instability growth rate (Γ) versus the wavenumber (K) and the axial direction(z).The initial parameters are $a_0 = 0.1, \beta_e = \beta_p = 100$, $\alpha = 0.5$. (a) $n_0(z) = 1 + bz$, $b = -0.02$; (b) $n_0(z) = \exp(-z^2/L_z)$, $L_z = 16\pi$; (c) $n_0(z) = 0.5(1 - \cos z)$.

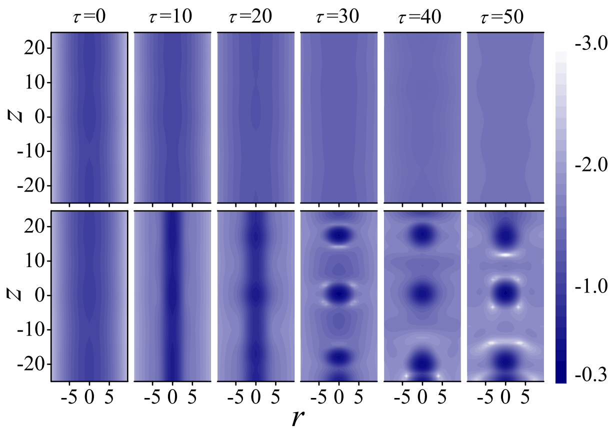

FIG. 3: (color online) The light amplitude a in $\log_{10}$-scale at six different times τ for homogeneous case. The horizontal axis represents the distribution along the radial (r) coordinate, and the axial (z) distribution is on the vertical axis. The upper row for $\alpha = 0$, the lower row for $\alpha = 0.5$.

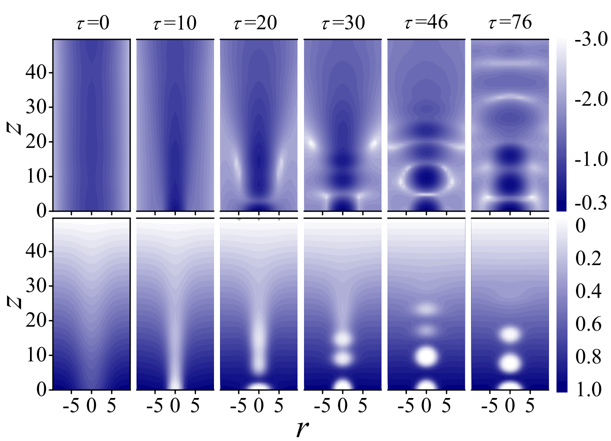

FIG. 4: (color online) The light amplitude a (upper row) and the normalized electron number density (lower row) as functions of r and z at six different times τ for $n_0(z) = 1+bz$, $b = -0.02$, $\alpha = 0.5$.

Figure 3 shows the time evolution of the light amplitude a for the cases of without (the upper row, $\alpha = 0$) and with (the lower row, $\alpha = 0.5$) positrons in homogeneous plasma. It is clear that, when the positrons are absent, the light beam can not be localized and finally defocused. However, when the positrons are present, the light beam firstly self-focus and then breaks up into localized filaments due to modulational instability.

Figures 4-7 show the numerical simulations of time evolution of a and n_e for inhomogeneous cases. For a linear axial density distribution $n_0(z) = 1 + bz$ (see Fig. 4), the self-focusing and the localization of the light and the creating of electron density holes (where the light is trapped) take place in the region with high density ($z \lesssim 30$) where modulational instability is stronger (see Fig. 2(a)). For the Gaussian (Figs. 5-6) and cosine (Fig. 7) types density distributions, however the self-focusing and the light localization and the creation of

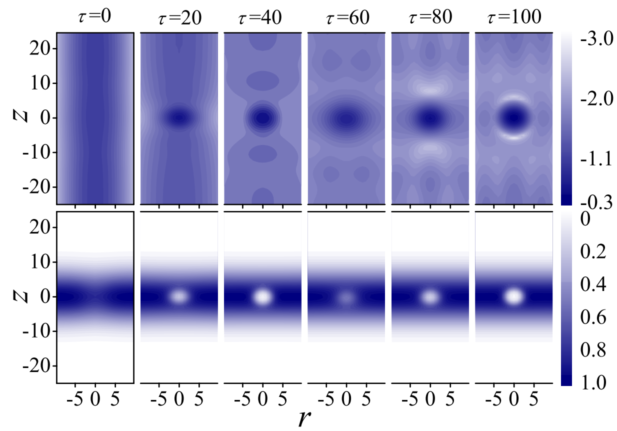

FIG. 5: (color online) The light amplitude a in $\log_{10}$-scale (upper row) and the normalized electron number density (lower row) as functions of r and z at six different times τ for $n_0(z) = \exp(-z^2/L_z)$, $L_z = 16\pi$, $\alpha = 0.2$.

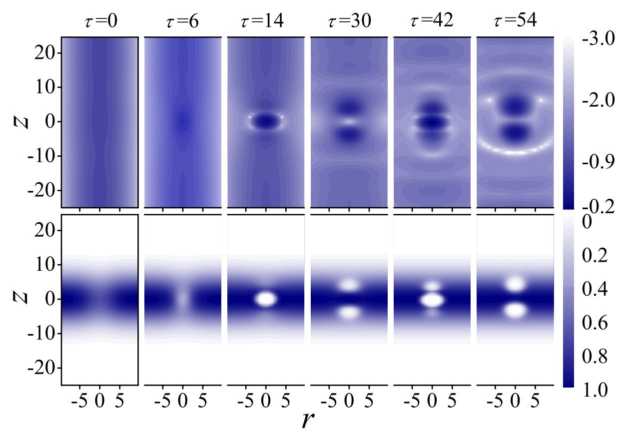

FIG. 6: (color online) The light amplitude a in $\log_{10}$-scale (upper row) and the normalized electron number density (lower row) as functions of r and z at six different times τ for $n_0(z) = \exp(-z^2/L_z)$, $\alpha = 0.5$.

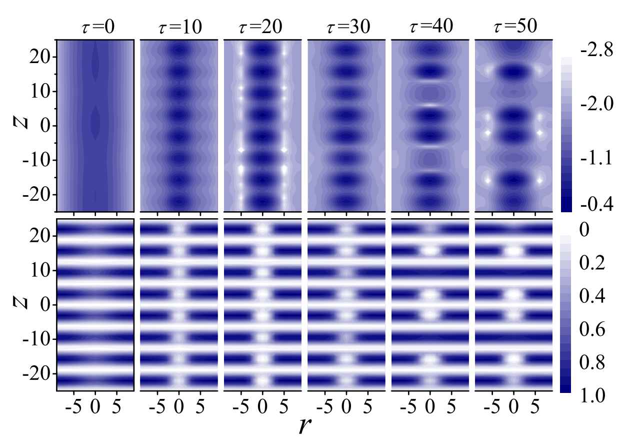

FIG. 7: (color online) The light amplitude a in $\log_{10}$-scale (upper row) and the normalized electron number density (lower row) as functions of r and z at six different times τ for $n_0(z) = 0.5(1 - \cos z)$, $\alpha = 0.5$.

electron density holes only present at the highest density regions where modualtional instability is strongest (see Figs. 2(b),(c)). Interestingly, for the Gaussian density distribution with the larger positron proportion ($\alpha = 0.5$, Fig. 6), the localized light spots experience a process from breaking up into two pieces to merge into one spot alternately; for the cosine density distribution (Fig. 7), merging of the periodically localized eight light spots occur; correspondingly, the electron density holes trapping the light also exhibit the same process. These phenomena indicate the strong nonlinear interaction between light and inhomogeneous plasma can exist.

To summarize, the nonlinear propagation of an arbitrary large amplitude light pulses in an inhomogeneous e-p-i plasma is studied. We find that the positron concentration and plasma inhomogeneity have strong influence on the modulational and filamentational instabilities in e-p-i plasma. Light beam focusing and trapping in self-created density holes induced by modulational and filamentational instabilities depend on the inhomogeneity character of the system. The light beam localization occurs at the highest density regions where the modulational instability is strong. On the other hand, the inhomogeneity of the plasma can not only boost a mechanism for light beam self-focusing and filamentation, but also provide an effective way to localize and trap the pulse in the region where we wanted. The present results should be useful in understanding the dynamics of intense laser pulses in e-p-i plasma.

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