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C. de Tomas, J. M. M. Roco, A. Calvo Hernández, Yang Wang, and Z. C. Tu

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Low-dissipation heat devices: unified trade-off optimization and bounds

C. de Tomas*

Departamento de Física Aplicada, Universidad de Salamanca, 37008 Salamanca, Spain

J.M.M. Roco and A. Calvo Hernández

Departamento de Física Aplicada and Instituto Universitario de Física y Matemáticas (IUFFYM), Universidad de Salamanca, 37008 Salamanca, Spain

Yang Wang and Z. C. Tu

Department of Physics, Beijing Normal University, Beijing 100875, China

Abstract

We apply a unified and trade-off based optimization for low-dissipation models of cyclic heat devices which accounts for both useful energy and losses. The resulting performance regime lies between those of maximum first-law efficiency and maximum χ (a unified figure of merit corresponding to power output of heat engines). The bounds available for both symmetric and extremely asymmetric heat devices are explicitly obtained. The similarities for heat engines and refrigerators and the energetic advantages of the trade-off optimization are specially stressed.

* Present adress: Physics Department; Autonomous University of Barcelona; 08193 Bellaterra, Spain

I. INTRODUCTION

The thermodynamic optimization of heat devices is receiving special attention due to the contemporary growing importance of saving energy resources in relation to any energy converter operation. Along this issue, a number of different performance regimes based on different figures of merit have been considered [1–3] with special emphasis on the analysis of possible universal and unified features.

For any cyclic converter the maximum Carnot efficiency for heat engines, η_{C}, or the maximum Carnot coefficient of performance (COP) for refrigerators and heat pumps, ε_{C}, requires an infinite cycle time with zero-power and zero-entropy generation operation regimes. On the contrary, the maximization of the corresponding useful energy (power output for heat engines [1–17], cooling rate for refrigerators and heating rate for heat pumps [1–3, 18–22]) gives operation regimes which do not necessarily involve either entropy generation constraints or efficiency increase. As a consequence, the optimization of heat devices based on a compromise (trade-off) between energy benefits and unavoidable losses by irreversibilities has been frequently used [2, 3, 23]. Among them, some of us and coworkers proposed the so-called Ω criterion which represents a compromise between energy benefits and losses for a specific job. This criterion is easy to implement for any energy converter (either isothermal or non-isothermal), without the requirement of the explicit evaluation of the entropy generation and it is independent of environmental parameters. Particular details on this unified optimization criterion can be found in [24] and explicit applications for different stochastic and coupled heat engine models have been reported [25–27]. Some unified characteristics were also compared with those obtained under the maximum power regime [28] for heat engines.

In the optimization field a valuable progress was made by Esposito *et al.* [29] by considering a low-dissipation Carnot heat engine model. In this model the entropy generation in the hot (cold) heat exchange process behaves as Σ_h/t_h (Σ_c/t_c) with t_h and t_c denoting the corresponding time duration and Σ_h and Σ_c being coefficients containing information on the irreversibility sources. Thus the reversible regime is approached in the limit of infinite time. The maximum power regime allows recovering the paradigmatic Curzon-Ahlborn value [30] $\eta_{\mathrm{CA}} = 1 - \sqrt{1-\eta_{\mathrm{C}}}$ when symmetric dissipation is considered, but without assuming any specific heat transfer law nor the linear-response regime. These authors also derived the lower and upper bounds for the efficiency at maximum power and found that they can be reached under extremely asymmetric dissipation limits.

Another conceptual insight was reported by de Tomás *et al.* [31] who introduced a unified optimization criterion, χ, for heat devices. The χ figure of merit, independent of any particular model, it is focused in the common characteristic of every energy converter, the cyclic working system, instead of any specific coupling to external heat sources which can vary according to a particular arrangement. This criterion is defined as the product of the converter efficiency z times the heat absorbed by the working system $Q_{\rm in}$, divided by the time duration of cycle $t_{\rm cycle}$: $\chi = z\, Q_{\rm in}/t_{\rm cycle}$. For heat engines χ becomes power output while for refrigerators it allows obtaining an optimized COP $\varepsilon_{\max\chi} = \sqrt{1+\varepsilon_{\rm C}} - 1 \equiv \varepsilon_{\rm CA}$ under symmetric conditions. This result can be seen as the counterpart of Curzon-Ahlborn efficiency for heat engines. It was firstly obtained in the finite-time-thermodynamics (FTT) context for Carnot-like refrigerators by Yan and Chen [32] taking as target function $\varepsilon\dot{Q}_{\rm c}$, where $\dot{Q}_{\rm c}$ is the cooling power of the refrigerator, later and independently by Velasco *et al.* [33, 34] using a maximum per-unit-time COP, and by Allahverdyan *et al.* [35] in the classical limit of a quantum model with two n-level systems interacting via a pulsed external field. Very recently, Wang *et al.* [36] generalized the previous results for refrigerators by obtaining the lower and upper bounds of the COP and shown that these bounds can be approached under extremely asymmetric dissipation limits. All main results obtained with the low-dissipation model have been confirmed within a minimally nonlinear irreversible thermodynamics framework for both heat engines [37] and refrigerators [38].

The goal in this paper is twofold. First, the Ω-figure of merit is applied to the unified low-dissipation model for heat devices and the corresponding efficiency and COP bounds are obtained under general and symmetric conditions. This is done, respectively, in Sects. II and III. Second, in Sect. IV, the results are compared with those obtained under the χ-criterion [36], making special emphasis on possible unified features for both heat engines and refrigerators and on the inherent energetic advantages of the compromise (trade-off) criterion versus the optimization of just the useful energy.

II. HEAT ENGINES AND THE Ω TRADE-OFF.

As it is usual in Carnot-like models, we assume that the adiabatic steps run instantaneously while for the isothermal processes we proceed as follows (more details can be seen in [36]). When the working substance is in contact with the hot reservoir at temperature T_h the constraint on the system is loosened according to some external controlled parameter $\lambda_\mathrm{h}(\tau)$ during the time interval $0 < \tau < t_\mathrm{h}$ with τ being the cycle-time variable. It is in this sense of loosening the constraint that this step is called *isothermal expansion*. A certain amount of heat Q_h is absorbed from the hot reservoir and the corresponding variation of entropy can be expressed as

$$\Delta S_\mathrm{h} = Q_\mathrm{h}/T_\mathrm{h} + \Delta S_\mathrm{h}^{ir}, \tag{1}$$

where $\Delta S_\mathrm{h}^{ir} \geq 0$ is the irreversible entropy production. When the working substance is in contact with the cold reservoir at temperature T_c the constraint on the system is enhanced according to the external controlled parameter $\lambda_\mathrm{c}(\tau)$ during the time interval $t_\mathrm{h} < \tau < t_\mathrm{h}+t_\mathrm{c}$. It is in the sense of enhancing the constraint that this step is called *isothermal compression*. A certain amount of heat Q_c is released to the cold reservoir and the variation of entropy in this process can be expressed as

$$\Delta S_\mathrm{c} = -Q_\mathrm{c}/T_\mathrm{c} + \Delta S_\mathrm{c}^{ir}, \tag{2}$$

where $\Delta S_\mathrm{c}^{ir} \geq 0$ is the irreversible entropy production. Having undergone this Carnot-like cycle, the system comes back to its initial state again. The net energy and variation of entropy changes in the whole cycle are null and then we have $\Delta S_\mathrm{h} = -\Delta S_\mathrm{c} = \Delta S \geq 0$ with a useful work output given by $W = Q_\mathrm{h} - Q_\mathrm{c}$.

The additional assumption of the low-dissipation Carnot heat engine model is that heat transfer accompanying finite-time operation in each isothermal process is inversely proportional to the duration of the processes t_h and t_c, respectively, with dissipation constants $\Sigma_\mathrm{h} > 0$ and $\Sigma_\mathrm{c} > 0$ accounting for irreversibility details [29]. The corresponding entropy production ΔS_i^{ir} is therefore given by $\Delta S_\mathrm{i}^{ir} = \Sigma_\mathrm{i}/t_\mathrm{i}$ $(i = \mathrm{c}, \mathrm{h})$. According to these assumptions and taking into account Eqs. (1) and (2), the heat transfers, Q_h and Q_c, and the efficiency, η, are given by

$$Q_\mathrm{h} = T_\mathrm{h}\left(\Delta S - \frac{\Sigma_\mathrm{h}}{t_\mathrm{h}}\right) \equiv T_\mathrm{h}\left(\Delta S - \Sigma_\mathrm{h} x_\mathrm{h}\right), \tag{3}$$

$$Q_{\rm c} = T_{\rm c}\left(\Delta S + \frac{\Sigma_{\rm c}}{t_{\rm c}}\right) \equiv T_{\rm c}\left(\Delta S + \Sigma_{\rm c} x_{\rm c}\right), \tag{4}$$

and

$$\eta \equiv \frac{W}{Q_{\rm c}} = 1 - \frac{Q_{\rm c}}{Q_{\rm h}} = 1 - \frac{T_{\rm c}\left(\Delta S + \Sigma_{\rm c} x_{\rm c}\right)}{T_{\rm h}\left(\Delta S - \Sigma_{\rm h} x_{\rm h}\right)}, \tag{5}$$

where by convenience $x_{\rm c} \equiv 1/t_{\rm c}$ and $x_{\rm h} \equiv 1/t_{\rm h}$.

Pertinent to our analysis here is that for heat engines the Ω-criterion, a compromise between maximum work performed and minimum work lost, reads as $\Omega = (2\eta - \eta_{\rm max})\, Q_{\rm h}$ [24]. Then we have

$$\dot{\Omega} \equiv \frac{\Omega}{t_{\rm cycle}} = \left[2\left(Q_{\rm h} - Q_{\rm c}\right) - \eta_{\rm C} Q_{\rm h}\right] \frac{x_{\rm c} x_{\rm h}}{x_{\rm c} + x_{\rm h}}, \tag{6}$$

where $\eta_{\rm C} \equiv 1 - T_{\rm c}/T_{\rm h} \leq 1$ is the maximum Carnot efficiency and $t_{\rm cycle} \equiv t_{\rm c} + t_{\rm h} = (x_{\rm c} + x_{\rm h})/x_{\rm c} x_{\rm h}$ is the time for completing the whole cycle. The $\dot{\Omega}$-maximization with respect to $x_{\rm c}$ and $x_{\rm h}$, $\partial\dot{\Omega}/\partial x_{\rm c} = 0$ and $\partial\dot{\Omega}/\partial x_{\rm h} = 0$, leads, respectively, the two following equations:

$$\left[2\left(Q_{\rm h} - Q_{\rm c}\right) - \eta_{\rm C} Q_{\rm h}\right] \frac{x_{\rm h}}{x_{\rm c}} = 2 T_{\rm c} \Sigma_{\rm c}\left(x_{\rm c} + x_{\rm h}\right), \tag{7}$$

$$\left[2\left(Q_{\rm h} - Q_{\rm c}\right) - \eta_{\rm C} Q_{\rm h}\right] \frac{x_{\rm c}}{x_{\rm h}} = T_{\rm h} \Sigma_{\rm h}\left(2 - \eta_{\rm C}\right)\left(x_{\rm c} + x_{\rm h}\right) \tag{8}$$

Dividing Eq. (7) by Eq. (8) we obtain that

$$\frac{x_{\rm c}}{x_{\rm h}} = \sqrt{\frac{\Sigma_{\rm h}(2 - \eta_{\rm C})}{2\Sigma_{\rm c}\left(1 - \eta_{\rm C}\right)}}, \tag{9}$$

while if they are summed up the result is

$$\eta_{\rm C} \Delta S = 2\left(2 - \eta_{\rm C}\right) \Sigma_{\rm h} x_{\rm h} + 4(1 - \eta_{\rm C}) \Sigma_{\rm c} x_{\rm c}. \tag{10}$$

By solving Eqs. (9) and (10), $x_{\rm h}$ and $x_{\rm c}$ are explicitly given by

$$x_{\rm h} = \frac{\eta_{\rm C} \Delta S}{2\left[\left(2 - \eta_{\rm C}\right)\Sigma_{\rm h} + \sqrt{2\left(2 - \eta_{\rm C}\right)\left(1 - \eta_{\rm C}\right)\Sigma_{\rm h}\Sigma_{\rm c}}\right]} \tag{11}$$

and

$$x_{\rm c} = \frac{\eta_{\rm C} \Delta S}{2\left[2\left(1 - \eta_{\rm C}\right)\Sigma_{\rm c} + \sqrt{2\left(2 - \eta_{\rm C}\right)\left(1 - \eta_{\rm C}\right)\Sigma_{\rm h}\Sigma_{\rm c}}\right]} \tag{12}$$

Taking into account Eqs. (5), (11) and (12) we obtain for the efficiency under maximum $\dot{\Omega}$-conditions, η_{Ω}, the following relation in terms of the temperatures (through $\eta_{\rm C}$) and the dissipation constants $\Sigma_{\rm h}$ and $\Sigma_{\rm c}$:

$$\eta_\Omega = \frac{3-2\eta_{\rm C}+3a}{4-3\eta_{\rm C}+4a}\eta_{\rm C} \tag{13}$$

with $a=\sqrt{(1-\eta_{\rm C})(2-\eta_{\rm C})\Sigma_{\rm c}/2\Sigma_{\rm h}}$. This equation monotonically increases with $\Sigma_{\rm h}/\Sigma_{\rm c}$ and it is easy to obtain their lower and upper bounds considering, respectively, the asymmetric limits $\Sigma_{\rm h}/\Sigma_{\rm c}\to 0$ and $\Sigma_{\rm h}/\Sigma_{\rm c}\to\infty$:

$$\eta_\Omega^- \equiv \frac{3}{4}\eta_{\rm C} \leq \eta_\Omega \leq \frac{3-2\eta_{\rm C}}{4-3\eta_{\rm C}}\eta_{\rm C} \equiv \eta_\Omega^+ \tag{14}$$

The particular limit $\Sigma_{\rm c}/\Sigma_{\rm h}\to 1$ in Eq. (13) is also of interest because it represents the symmetric dissipation condition. Under this condition Eq. (13) reduces to

$$\eta_\Omega^{\Sigma_{\rm c}=\Sigma_{\rm h}} \equiv \eta_\Omega^{Sym} = 1-\sqrt{\frac{(1-\eta_{\rm C})\,(2-\eta_{\rm C})}{2}}, \tag{15}$$

a result firstly reported by Angulo *et al.* [39] using the ecological function as figure of merit for the so-called endoreversible Carnot heat engine models.

III. REFRIGERATORS AND THE Ω TRADE-OFF.

As for heat engines, in Carnot-like refrigerator models the adiabatic processes run instantaneously, while for the isothermal processes we proceed as follow [36]. In the *isothermal expansion* the working system is in contact with a cold reservoir at temperature $T_{\rm c}$ and the constraint on the system is loosened according to the external controlled parameter $\lambda_{\rm c}(\tau)$ during the time interval $0<\tau<t_{\rm c}$ with τ being the cycle-time variable. A certain amount of heat $Q_{\rm c}$ is absorbed from the cold reservoir and the the variation of entropy in this process can be expressed as

$$\Delta S_{\rm c} = Q_{\rm c}/T_{\rm c} + \Delta S_{\rm c}^{ir}, \tag{16}$$

where $\Delta S_{\rm c}^{ir}\geq 0$ is the irreversible entropy production. We adopt the convention that the heat absorbed by the refrigerator is positive, so $\Delta S_{\rm c}^{ir}\leq\Delta S_{\rm c}$. During the *isothermal compression* the working substance is in contact with a hot reservoir at temperature $T_{\rm h}$ and the constraint on the system is further enhanced according to the external controlled parameter $\lambda_{\rm h}(\tau)$ during the time interval $t_{\rm c}<\tau<t_{\rm c}+t_{\rm h}$. A certain amount of heat $Q_{\rm h}$ is released to the hot reservoir $T_{\rm h}$. Thus the total variation of entropy in this process is

$$\Delta S_{\rm h} = -Q_{\rm h}/T_{\rm h} + \Delta S_{\rm h}^{ir}, \tag{17}$$

where $\Delta S_{\rm h}^{ir} \geq 0$ is the irreversible entropy production. Having undergone this Carnot-like cycle, the system comes back to its initial state again. The energy change and variation of entropy in the whole cycle are null. Then, we have $\Delta S_{\rm c} = -\Delta S_{\rm h} = \Delta S \geq 0$ and a net work input given by $W = Q_{\rm h} - Q_{\rm c}$.

The low-dissipation Carnot-like refrigerator model additionally assumes that entropy generation accompanying finite-time operation in each isothermal process is inversely proportional to the duration of the processes $t_{\rm h}$ and $t_{\rm c}$ with constant strengths $\Sigma_{\rm h} > 0$ and $\Sigma_{\rm c} > 0$. With these assumptions and according to Eqs. (16) and (17), the heat transfers and the COP, ε, are given by

$$Q_{\rm h} = T_{\rm h}\left(\Delta S + \frac{\Sigma_{\rm h}}{t_{\rm h}}\right), \tag{18}$$

$$Q_{\rm c} = T_{\rm c}\left(\Delta S - \frac{\Sigma_{\rm c}}{t_{\rm c}}\right), \tag{19}$$

$$\varepsilon \equiv \frac{Q_{\rm c}}{W} = \frac{Q_{\rm c}}{Q_{\rm h} - Q_{\rm c}} = \frac{T_{\rm c}\left(\Delta S - \Sigma_{\rm c} x_{\rm c}\right)}{\left(T_{\rm h} - T_{\rm c}\right)\Delta S + T_{\rm c}\Sigma_{\rm c} x_{\rm c} + T_{\rm h}\Sigma_{\rm h} x_{\rm h}} \tag{20}$$

where $x_{\rm c} \equiv 1/t_{\rm c}$, and $x_{\rm h} \equiv 1/t_{\rm h}$.

For refrigerators the Ω-criterion, a trade-off between maximum cooling load and minimum lost cooling load, reads as $\Omega = (2\varepsilon - \varepsilon_{\rm max})\, W$ [24] and then

$$\dot{\Omega} = (2\varepsilon - \varepsilon_{\rm max})\,\frac{W}{t_{\rm cycle}} = \left[2Q_{\rm c} - \varepsilon_{\rm C}(Q_{\rm h} - Q_{\rm c})\right]\frac{x_{\rm c} x_{\rm h}}{x_{\rm c} + x_{\rm h}}, \tag{21}$$

where $\epsilon_{\rm C} \equiv T_{\rm c}/(T_{\rm h} - T_{\rm c})$ is the maximum Carnot COP. The $\dot{\Omega}$-optimization with respect to $x_{\rm c}$ and $x_{\rm h}$ gives, respectively, the two following equations:

$$\left[2Q_{\rm c} - \varepsilon_{\rm C}\left(Q_{\rm h} - Q_{\rm c}\right)\right]\frac{x_{\rm h}}{x_{\rm c}} = T_{\rm c}\Sigma_{\rm c}\left(2 + \varepsilon_{\rm C}\right)\left(x_{\rm c} + x_{\rm h}\right) \tag{22}$$

$$\left[2Q_{\rm c} - \varepsilon_{\rm C}\left(Q_{\rm h} - Q_{\rm c}\right)\right]\frac{x_{\rm c}}{x_{\rm h}} = T_{\rm h}\Sigma_{\rm h}\epsilon_{\rm C}\left(x_{\rm c} + x_{\rm h}\right) \tag{23}$$

Dividing Eq. (22) by Eq. (23) we obtain that

$$\frac{x_{\rm c}}{x_{\rm h}} = \sqrt{\frac{\Sigma_{\rm h}(1 + \epsilon_{\rm C})}{\Sigma_{\rm c}(2 + \epsilon_{\rm C})}} \tag{24}$$

while if they are summed up the result is

$$\Delta S = 2(2 + \epsilon_{\rm C})\Sigma_{\rm c} x_{\rm c} + 2(1 + \epsilon_{\rm C})\Sigma_{\rm h} x_{\rm h}. \tag{25}$$

Substituting Eq. (25) into Eq. (20) and taking into account Eq. (24), we derive the COP under maximum $\dot{\Omega}$-conditions, ε_Ω, in terms of the temperatures and the dissipation constants Σ_h and Σ_c as

$$\varepsilon_\Omega = \frac{3 + 2\varepsilon_C + 2b}{4 + 3\varepsilon_C + 3b}\varepsilon_\mathrm{C}, \tag{26}$$

where $b = \sqrt{(1+\varepsilon_\mathrm{C})(2+\varepsilon_\mathrm{C})\Sigma_\mathrm{h}/\Sigma_\mathrm{c}}$. This equation monotonically increases with $\Sigma_\mathrm{c}/\Sigma_\mathrm{h}$ and it is easy to obtain their lower and upper bounds considering, respectively, the asymmetric limits $\Sigma_\mathrm{c}/\Sigma_\mathrm{h} \to 0$ and $\Sigma_\mathrm{c}/\Sigma_\mathrm{h} \to \infty$:

$$\epsilon_\Omega^- \equiv \frac{2}{3}\epsilon_\mathrm{C} \leq \epsilon_\Omega \leq \frac{3+2\epsilon_\mathrm{C}}{4+3\epsilon_\mathrm{C}}\epsilon_\mathrm{C} \equiv \epsilon_\Omega^+ \tag{27}$$

The particular limit $\Sigma_\mathrm{c}/\Sigma_\mathrm{h} \to 1$ in Eq. (26) represents the symmetric dissipation limit. Under this condition Eq. (26) reduces to

$$\varepsilon_\Omega^{\Sigma_\mathrm{c}=\Sigma_\mathrm{h}} \equiv \varepsilon_\Omega^{Sym} = \frac{\epsilon_\mathrm{C}}{\sqrt{(1+\epsilon_\mathrm{C})\,(2+\epsilon_\mathrm{C})} - \epsilon_\mathrm{C}}, \tag{28}$$

a result first reported for Carnot-like refrigerators with the same optimization criterion but in the FTT-context and under the endorreversible limit [24].

IV. RESULTS, DISCUSSION AND CONCLUSIONS

A. Heat engines

The obtained bounds for the optimized efficiency η_Ω, Eq. (14), are plotted in Fig. 1 together with the symmetric (endoreversible) limit given by Eq. (15). Also we plot in this figure the efficiency bounds obtained by Esposito *et al.* (see Eq. (11) in [29]) under maximum power conditions (i.e., under maximum χ):

$$\eta_\chi^+ = \frac{\eta_\mathrm{C}}{2-\eta_\mathrm{C}},$$

$$\eta_\chi^- = \frac{\eta_\mathrm{C}}{2},$$

and

$$\eta_\chi^{sym} = 1 - \sqrt{1-\eta_\mathrm{C}} \equiv \eta_\mathrm{CA}.$$

By inspection of Fig. 1 it is obvious that the efficiencies under maximum of the $\dot{\Omega}$-function behave qualitatively as the efficiencies under maximum power (i.e. maximum χ) but always each bound at maximum $\dot{\Omega}$ is greater than the corresponding bound at maximum power. Note also as the lower bounds $\eta_\chi^- = \eta_\mathrm{C}/2$ and $\eta_\Omega^- = 3\eta_\mathrm{C}/4$ behave linearly with η_C, while the upper and the symmetric bounds reach the Carnot value as $\eta_\mathrm{C} \to 1$ in both regimes.

Another clear consequence is that in each case the maximum $\dot{\Omega}$ regime yields higher efficiencies, closer to the Carnot values. In fact, it is easy to check numerically (not shown in the figure) that in all cases the efficiency at maximum $\dot{\Omega}$ can be approximated by the semi-sum of the Carnot value and of the efficiency at maximum power: $\eta_\Omega^+ \approx (\eta_\chi^+ + \eta_\mathrm{C})/2$ and $\eta_\Omega^- = (\eta_\chi^- + \eta_\mathrm{C})/2$. These results generalize to asymmetric low-dissipation heat engines the semisum rule obtained by Angulo-Brown et al. [39] in the symmetric, endoreversible limit of the Finite-Time-Thermodynamics framework, $\eta_\Omega^{sym} \approx (\eta_\chi^{sym} + \eta_C)/2$. This semisum rule also holds for isothermal biochemical energy converters as reported in [40].

One of the advantages in the trade-off regime is the saving in the entropy generation, σ_Ω, in comparison with the entropy generation in the χ-regime, σ_χ. In the low-dissipation models the entropy generation for both heat engines and refrigerators is given by $\sigma = \Sigma_\mathrm{h} x_\mathrm{h} + \Sigma_\mathrm{c} x_\mathrm{c}$. For heat engines x_h and x_c are given by Eqs. (11) and (12) in the Ω-regime and by Eq. (7) in [29] for the maximum power (χ) regime. In Fig. 2 one can see how the ratio $\sigma_\Omega/\sigma_\chi$ behaves in terms of η_C in all temperature range and how the saving entropy generation in the $\dot{\Omega}$-regime is specially relevant at high ($\eta_\mathrm{C} \to 0$) and intermediate relative temperatures, getting up to 50%.

B. Refrigerators

The obtained bounds for the optimized COP, ε_Ω, Eq. (27), are plotted in Fig. 3 together with the symmetric (endoreversible) limit given by Eq. (28). Also we plot in this figure the COP bounds obtained by Wang *et al.* (see Eq. 14 in [36]) under maximum$-\chi$:

$$\varepsilon_\chi^+ = \frac{\sqrt{9 + 8\varepsilon_\mathrm{C}} - 3}{2},$$

$$\varepsilon_\chi^- = 0,$$

and

$$\varepsilon_\chi^{sym} = \sqrt{1 + \varepsilon_\mathrm{C}} - 1 \equiv \varepsilon_\mathrm{CA}.$$

In comparison with the χ-optimization the trade-off between maximum cooling power and minimum losses offer a main advantage: both bounds are finite and greater, allowing some guides to design refrigerators more energy-efficient and with entropy saving similar to heat engines showed in Fig. 2. Also for refrigerators it is easy to check numerically that the COP for the $\dot{\Omega}$-regime can be approximated by the semi-sum of the Carnot-COP value and of the COP at maximum χ, $\varepsilon_\Omega \approx (\varepsilon_\chi + \varepsilon_C)/2$, in any analyzed limit: $\varepsilon_\Omega^+ \approx (\varepsilon_\chi^+ + \varepsilon_C)/2$ and $\varepsilon_\Omega^{sym} \approx (\varepsilon_\chi^{sym} + \varepsilon_C)/2$. Then, the semisum rule also applies to refrigerators, a result that to our knowledge, has not been reported before.

In summary, the obtained results show that a unified low-dissipation model for heat devices optimized under unified figures of merit ($\dot{\Omega}$,χ) allow obtaining unified behaviors for the energetic properties, independently of the converter nature: heat engine or refrigerators. Also, the importance of thermodynamic optimizations on the basis of a compromise between useful energy and losses is stressed. Experimental results plotted in Fig. 4 for heat engines and in Fig. 5 for refrigerators show that many real heat devices seem to fit their performance near the maximum χ-region. This fact suggests that at present many of these devices are designed to work at higher velocity or rate, rather than at higher efficiency and entropy saving. Notable exception is the set of experimental values for a nominal 1038 kW screw-compressor chiller which fairly agrees with the theoretical $\dot{\Omega}$-predictions (see upper curve in Fig. 5),*i.e.*, refrigerators performing at higher efficiencies and entropy saving rather that at higher velocity. The serious coming energy shortage we face requires more efficient heat devices than before. Along this line, the trade-off represented by $\dot{\Omega}$ (or another possible compromise) could be a useful guide to design energy converters more involved with the environmental impact and power quality.

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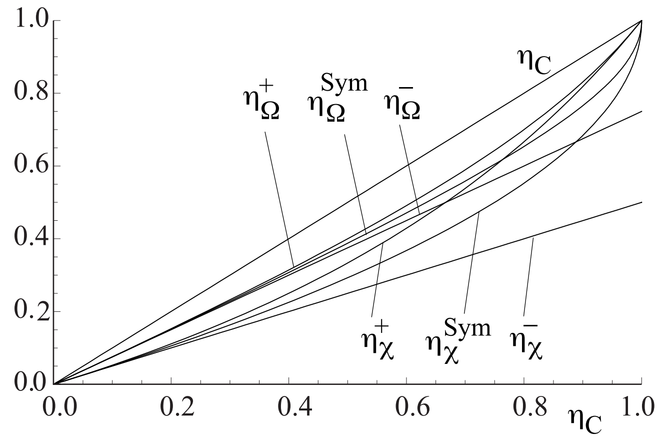

FIG. 1: Comparison between efficiency at maximum power, η_χ, and maximum omega, η_Ω, for the indicated asymmetric and symmetric limits (see text) of low-dissipation heat engines as a function of the Carnot efficiency η_c.

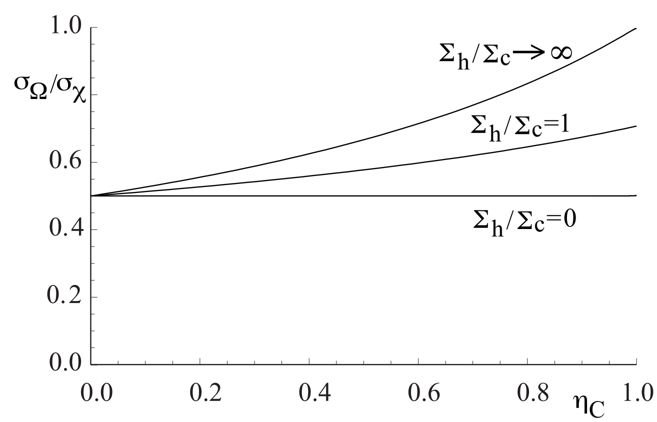

FIG. 2: Ratio of the entropy generation in the Ω and χ performance regimes for the indicated asymmetric and symmetric limits (see text) of low-dissipation heat engines as a function of the Carnot efficiency η_c.

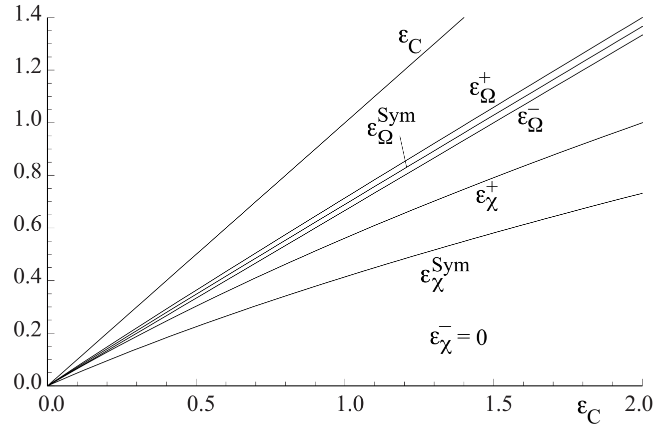

FIG. 3: Comparison between COP at maximum χ, ε_χ, and maximum omega, ε_Ω, for the indicated asymmetric and symmetric limits (see text) of low-dissipation refrigerators as a function of the Carnot COP ε_c

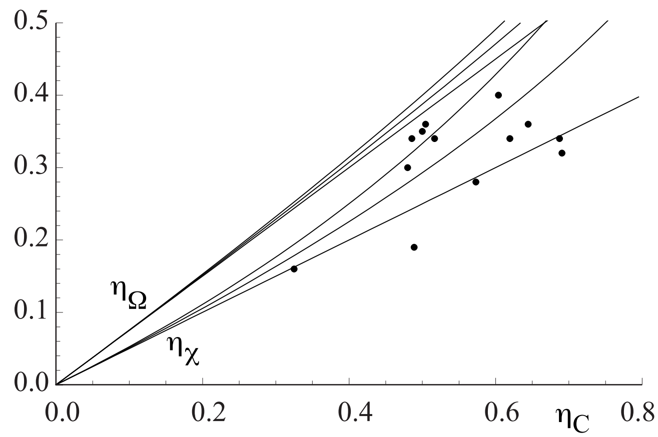

FIG. 4: Comparison between experimental results, dots, [41] and theoretical ones for heat engines. The three curves for η_Ω and η_χ are, respectively, the three limits plotted in Fig. 1

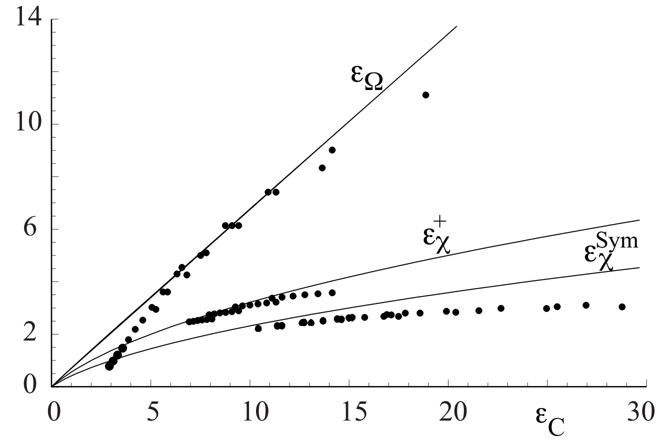

FIG. 5: Comparison between three sets of experimental results, points, [22] (pp. 235, 167, and 111) and theoretical ones for refrigerators. The three limits of ε_Ω, see Fig. 3, are indistinguishable in the plotted scale.