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ABSTRACT

We study the effect of noise recycling on nonequilibrium escape dynamics in a bistable system. For small noise, the non-Markovian problem is reduced to a two-state model with the master equation depending on not only the current state but also the earlier state, based on which we are able to derive the analytical formulae for the switching rate, the autocorrelation function and the power spectrum density (PSD). Both the theoretical and the numerical results show that, with modulating the time delay in noise recycling, a monotonic PSD may switch to a nonmonotonic one and the amplitude of PSD at resonance frequency exhibits a pronounced maximum at a certain noise level, declared the onset of stochastic resonance (SR) in the absence of weak periodic signal. Further, we also demonstrate that the linear response to the external periodic force displays a maximum at a certain level of time delay, displayed the signature of SR.

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I. INTRODUCTION

Noise is ubiquitous in real world, which always plays a destructive role in natural and/or synthetic systems. Consequently, evaluation and suppression of noise destructive impact on dynamical evolutions and signal transmissions have received uninterrupted attention during the last century. The negative perspective on noise stands until 1981, on which physicists observed "the cooperative effect between internal mechanism and the external periodic forcing" in some nonlinear dynamical systems [1], announced the origin of SR, and manifested a constructive role of noise in nonlinear system. SR, in essence, is a nonlinear cooperative effect where a weak periodic stimulus entrains large-scale environmental fluctuations with the result that the periodic component is greatly enhanced. As the first positive example, SR has received tremendous attention and research [2-9] in the past several decades. From then on, lots of active effects of noise have been reported further, for instance, noise-inducing, noise-enhancing or noise-sustaining

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spatiotemporal order [10], noise-inducing synchronization [11-17], etc.

Generally, noise can be injected into a system and couple a variable of interest via different channels, additively or multiplicatively, alike. While being transmitted across the system components, a noise might split into several parts, each one of which is a single noise from the same source and possibly accompanied with a time shift (time delay), due to the combination of diverse propagation or transduction mechanisms. Hence,

$$\varsigma(t) = \sum_{i=0}^{m} \varepsilon_i \xi(t - \tau_i),$$

where ε_i ($i = 0,1,2,\cdots,m$) are coefficients and τ_i ($\geq 0, i = 0,1,2,\cdots,m$) are time shifts (time delays). To simplify, one always sets $\tau_0 = 0$ and $m = 1$ for a stationary noise $\varsigma(t)$, and then reads $\varsigma(t) = \varepsilon_0 \xi(t) + \varepsilon_1 \xi(t - \tau)$ (by setting $\tau_1 \to \tau$). Being coincident with [18,19], $\varepsilon_0 \xi(t)$ is called the master noise and $\varepsilon_1 \xi(t - \tau)$ is called the secondary noise or recycled noise. Such noise recycling can be realized experimentally using a vertical cavity surface emitting laser that exhibits polarization switching as the injection current is varied [20].

As a matter of fact, noise recycling is rather common in physical systems and always impacts a great diversity of nonlinear phenomena [18-28], for example, rectifying an automated Maxwell's daemon of a massless Brownian particle on a symmetric periodic substrate [18], inducing stochastic synchronization in a bistable system [19], controlling resonance dynamics in Brusselator model [21], causing transport of nanoparticles in biological and artificial channels [22,23], driving propagation of charge density waves [24], etc. However, its role has not been fully recognized yet. As a matter of fact, due to the delayed correlation between the master and the secondary noise, the relevant dynamical response is not Markovian, and hence classical tools, e.g., standard Fokker-Planck approach, are not justified any longer. As a result, no theoretical model has been reported on studying the Kramers' escape dynamics related to noise recycling so far.

In the current paper, we focus on the response of a particle trapped in a bistable potential under noise recycling. At variance with the earlier studies [18-20], we demonstrate that, for a moderate time delay, the monotonic PSD may switch to a nonmonotonic one and the amplitude of PSD at resonance frequency exhibits a pronounced peak at a certain noise level. It is also shown that, in

the presence of weak periodic signal, the linear response exhibits a broad peak at a certain level of time delay. To our knowledge, these phenomena have not been reported yet. To quantify these arguments, a theoretical model is suggested by neglecting the small intrawell fluctuations and approximating the aimed bistable model as a two-state system, which shows good agreements with the numerical results.

II. BASIC MODEL AND SETUP

We restrict ourselves to the model that M. Borromeo et al. have studied in ref. [18], which is an over-damped particle trapped in a double-well quartic potential $U(x) = -x^2/2 + x^4/4$ coupled noise recycling,

$$\dot{x} = -U'(x) + \sqrt{D}\xi(t) + \varepsilon\sqrt{D}\xi(t-\tau), \tag{1}$$

where $\xi(t)$ is a Gaussian white noise with $\langle\xi(t)\rangle = 0$ and $\langle\xi(t+T)\xi(t)\rangle = 2\delta(T)$. Set $\varsigma(t) = \eta_1(t) + \eta_2(t)$ with $\eta_1(t) = \sqrt{D}\xi(t)$ and $\eta_2(t) = \varepsilon\sqrt{D}\xi(t-\tau)$, then it yields $\langle\varsigma(t)\rangle = 0$ and $\langle\varsigma(t+T)\varsigma(t)\rangle = 2D[(1+\varepsilon^2)\delta(T) + \varepsilon\delta(T+\tau) + \varepsilon\delta(T-\tau)]$.

Theoretically, if no recycling procedure is performed, viz. $\varepsilon = 0$, the concerned problem degenerates into a classical Kramers' problem with $r_K = (\sqrt{2}\pi)^{-1}\exp(-1/4D)$; if the time delay is sufficiently small, viz. $\tau \to 0$, the concerned problem also reduces to a Kramers' problem with $r_K = (\sqrt{2}\pi)^{-1}\exp(-1/4(1+\varepsilon)^2 D)$; or reversely, if the time delay is large enough, viz. $\tau \to \infty$, the current issue will turns back to a Kramers' problem, too, since $\varsigma(t)$ is a Gaussian white noise with intensity $D(1+\varepsilon^2)$ and the Kramers' rate can be determined by $r_K = (\sqrt{2}\pi)^{-1}\exp(-1/4(1+\varepsilon^2)D)$. These situations were well discussed in [18]. However, for moderate time delay, no result has been reported so far because of the speciality of noise recycling.

III. DYNAMICS: SR-LIKE RESPONSE

Firstly, model (1) is solved numerically for a moderate time delay. The results are exhibited in Figs. 1 and 2. Figure 1 shows the PSD for different time delays: $\tau = 0$, $\tau = 5$, $\tau = 15$ and $\tau = 25$. It reads immediately that without time delay the PSD decays monotonically and no peak

appears in the whole range of frequency, but, as the time delay increases the shape of PSD changes and several peaks appear on the curve (see the curves corresponding to larger time delays: $\tau = 15$ and $\tau = 25$). Set the frequency associated to the highest peak to be ω_0 (resonance frequency, which is near $2\pi/\tau$), then the position of the PSD peaks ω_n might be estimated and determined approximately by $(n+1)\cdot\omega_0$. Figure 2 exhibits the dependence of PSD on noise strength at resonance frequency, wherein a pronounced peak appears at a certain noise level $D \approx 0.25$. To gain an insight into the dynamics, a new indicator, so-called signal-to-noise difference (SND), is introduced here as a measure of the system's response, which is

$$\text{SND} = \int_{\omega_0-\sigma}^{\omega_0+\sigma} S(\omega)d\omega - \int_{\omega_0-\sigma}^{\omega_0+\sigma} S_{back}(\omega)d\omega.$$

Here $S(\omega) = \int_{-\infty}^{+\infty} e^{-iwT}\langle x(t)x(t+T)\rangle dT$ is the total PSD of model (1) obtained by the Fourier transform of the autocorrelation function $\langle x(t)x(t+T)\rangle$ [2-4], wherein the bracket $\langle \bullet \rangle$ indicates the ensemble average over realizations of the noise, 2σ is a small range around the resonant frequency ω_0, and $S_{back}(\omega)$ corresponds to the background PSD (which is the PSD of model (1) when the secondary noise is absent). Evidently, the value of SND represents an extra increment of the output signal power at resonance frequency, and can therefore be regarded as a quantitative measure of signal extraction from background noise. We calculated SND numerically from model (1) in existence of time delay ($\tau = 25$) and displayed it in Fig. 3. At a closer inspection of SND, it is found that SND first increases with increasing noise intensity, reaches its maximum, and then decreases monotonously. A pronounced peak appears at a certain noise level $D \approx 0.25$, which means the output 'signal' at resonance frequency is amplified and optimized by noise recycling (i.e., with the assistance of the secondary noise). It is a striking feature of SR-like response. As is well known, without a weak periodic signal SR could not occur in a classical Langevin model. However, it was well exhibited here that, with the assistance of a reinjected noise (secondary noise), SR-like dynamics was evoked in a classical Langevin model in the absence of weak periodic signal.
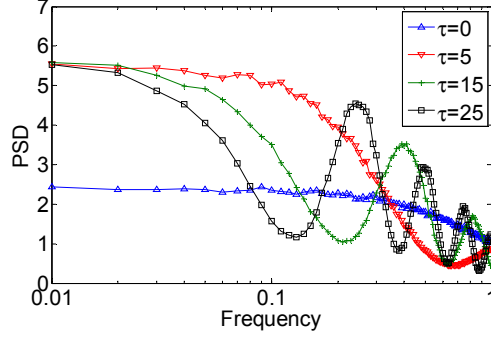
FIG.1. PSD of model (1) for $\tau = 0$, $\tau = 5$, $\tau = 15$ and $\tau = 25$. The other parameters are fixed as $D = 0.5$ and $\varepsilon = 0.5$. Every point is calculated from model (1) numerically by averaging over 2000 realizations of noise hereafter.
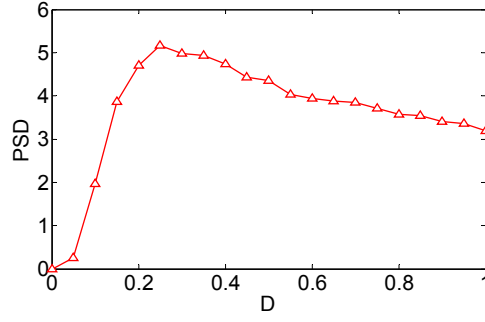

FIG. 2. Dependence of PSD on noise intensity for fixed time delay ($\tau = 25$) at resonance frequency ($\omega_0 \approx 0.23$). The other parameters are fixed the same as Fig. 1.
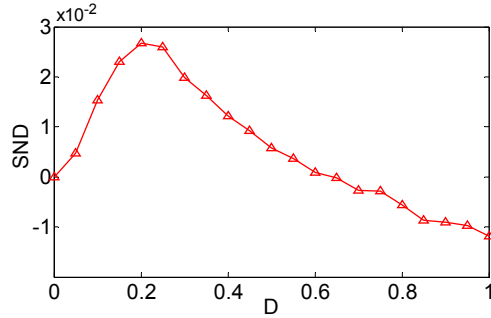

FIG. 3. Dependence of SND on noise intensity at resonance frequency for $2\sigma = 0.02$. The parameters are specified the same as Fig. 2.

IV. THEORETICAL ANALYSIS

A theoretical model is introduced to elucidate and analyze the above observed SR-like dynamics, wherein it will be unveiled why and how a SR-like dynamics evoked only through a noise recycling procedure.

A. Lag synchronization and more

To begin with, let's compare two trajectories of Langevin model perturbed by two special noises,

$$\dot{x}_1 = -U'(x_1) + \eta(t),$$ (2a)

$$\dot{x}_2 = -U'(x_2) + \eta(t-\tau).$$ (2b)

Being coincident with model (1), $\eta(t)$ is specified as Gaussian white noise with $\langle \eta(t) \rangle = 0$, $\langle \eta(t)\eta(t-\tau) \rangle = 2D\delta(\tau)$. Figure 4 shows the time history of $x_1(t)$, $x_2(t)$ and their error $\Delta(t) = x_1(t-\tau) - x_2(t)$ for different noise intensities. One finds that $\Delta(t) = 0$ holds always, and evidently indicates a lag synchronization between $x_1(t)$ and $x_2(t)$, which can be proved theoretically according to Lyapunov's theory. Subtract (2b) from (2a) with a time shift to obtain the error system of $\Delta(t)$ and linearize it at the origin, it yields,

$$\dot{\sigma}(t) = -[3x_1^2(t-\tau) - 1] \cdot \sigma(t).$$ (3)

For small noise intensity, the over-damped particle spends most of its time near the stable equilibriums at $x^\pm = \pm 1$, and only occasionally jumps from one to another under moderate amount of random kicks, which means $x_1^2(t-\tau) \approx 1$ for almost all the time, so $3x_1^2(t-\tau) - 1 > 0$ exists almost always. Therefore, according to synchronization theory lag synchronization achieves between $x_1(t)$ and $x_2(t)$ in a statistical sense.
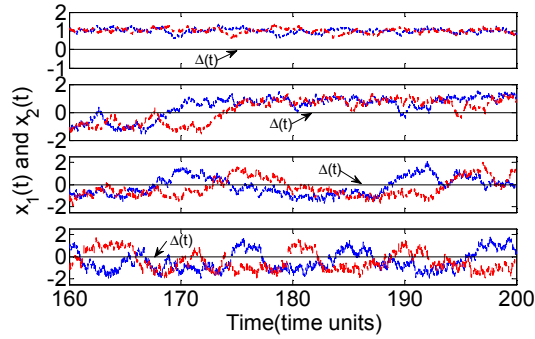

FIG. 4. Evolutions of $x_1(t)$ (dashed line, blue), $x_2(t)$ (dashdot line, red) and their error $\Delta(t)$ (solid line, black) for fixed time delay ($\tau = 10$) and different noise intensity: $D = 0.1$、0.5、

1.0 and 2.0 (from top to bottom).

On the other hand, if a particle, trapped in the same potential, is perturbed by both $\eta(t)$ and

$\eta(t-\tau)$, the position of the particle then can be stated by the following model,

$$\dot{x}_3 = -U'(x_3) + \eta(t) + \eta(t-\tau). \tag{2c}$$

Figure 5 shows the correlation relationships between $x_1(t)$ and $x_3(t)$, $x_2(t)$ and $x_3(t)$,

$x_3(t)$ and $x_3(t)$. From these panels it reads immediately that $x_1(t)$ and $x_3(t)$ are

remarkably correlated at $T = -\tau$ and $T = 0$, $x_2(t)$ and $x_3(t)$ are remarkably correlated at

$T = 0$ and $T = \tau$, so as to $x_3(t)$ correlates with itself not only at $T = 0$ but also at $T = \pm\tau$,

which implies that $x_3(t)$ mainly contains two components, and if possible, it might be split into

an $\eta(t)$ -induced segment and an $\eta(t-\tau)$ -induced segment. Hence, according to the

relationship between $x_1(t)$ and $x_2(t)$, the $\eta(t-\tau)$ -induced behavior will repeat the

$\eta(t)$ -induced one after τ time shift, for example, if a particle hops from one well to another at

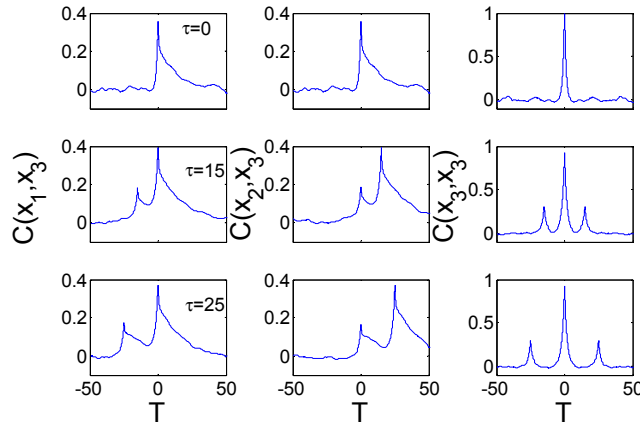time t under $\eta(t)$ then the same jump will occur at time $t + \tau$ under $\eta(t-\tau)$.

FIG. 5. Correlation relationship between $x_1(t)$ and $x_3(t)$, $x_2(t)$ and $x_3(t)$, $x_3(t)$ and

$x_3(t)$ for variety time delays. Here, $C(x_i, x_j)$ stands for the correlation function of $x_i(t)$ and

$x_j(t)$.

B. Two-state model

According to the preceding analysis, apparently, in model (1) $\eta_2(t)$ will tend to assist or hinder an $\eta_1(t)$-caused switch after τ time shift: Without loss of generality, the particle is supposed to be in the right well at time $t-\tau$. Based on the above discussion, if the particle jumped from the right well to the left one at time $t-\tau$ due to the master noise $\eta_1(t)$, and is in the right well again at time t (It doesn't matter whether the particle hops back from the left well to the right well again and then repeats the hopping-action 'right-left-right' many times in time interval $(t-\tau, t)$), the secondary noise $\eta_2(t)$ will tend to assist a jump at time t in the same direction, namely, $\eta_2(t)$ will enhance the possibility (probability) of transition from the right well to the left one at time t. And if the particle jumped from the right well to the left one at time $t-\tau$ caused by the master noise $\eta_1(t)$, and is in the left well at time t, the secondary noise $\eta_2(t)$ will tend to hinder a jump at time t in the opposite direction, viz, $\eta_2(t)$ will reduce the possibility (probability) of transition from the left to the right well at time t. Similar analysis can be carried out for other situations and analogous results will be obtained.

To make these arguments quantitative, we suggest a theoretical model by neglecting the small intrawell fluctuations and approximating model (1) as a two-state model in order to gain insight into the resonance dynamics, in which $s = \pm 1$ stand for $x > 0$ and $x < 0$, respectively. First of all, some notations are introduced: p_m and p_s denote the switching rates caused separately by the master noise and the secondary noise; $W_{\mp}(t)$ denote the total transition rate out of the '$\pm$'wells, and $W_{\mp}^{m,s}(t)$ state the transition rate out of the '$\pm$'wells induced by the master noise and the secondary noise, respectively. For small D and ε, p_m and p_s can be calculated by virtue of Kramers' formula $r_K = (2\pi)^{-1}\sqrt{U''(x^{\pm})U''(x_u)}\exp(-\Delta U/D)$, wherein $x_u = 0$ is the position of the potential barrier. Therefore, one obtains

$$p_m = (\sqrt{2\pi})^{-1}\exp(-1/4D) \quad \text{and} \quad p_s = (\sqrt{2\pi})^{-1}\exp(-1/4\varepsilon^2 D). \tag{4}$$

Since the master-noise-induced transition is adjusted, i.e., enhanced or reduced, by the secondary

noise, the transition probabilities are related to the state of the particle at both times $t - \tau$ and t. Therefore, the master equation is modified as

$$\dot{n}_+ = -\dot{n}_- = (-W_-^m(t) - W_-^s(t - \tau))P(x(t - \tau) = +1, x(t) = +1) + \tag{5}$$

$$(-W_-^m(t) + W_+^s(t - \tau))P(x(t - \tau) = -1, x(t) = +1) +$$

$$(W_+^m(t) + W_-^s(t - \tau))P(x(t - \tau) = -1, x(t) = -1) +$$

$$(W_+^m(t) - W_-^s(t - \tau))P(x(t - \tau) = +1, x(t) = -1),$$

where $n_\pm$ denote the probabilities that the particle occupies "± 1" states at time t, and $P(\bullet, \bullet)$ states the joint probability of $x(t - \tau)$ and $x(t)$. According to probability law, it yields

$$\dot{n}_+ = -\dot{n}_- = -[W_-^m(t) + W_-^s(t - \tau)n_+(t - \tau) - W_+^s(t - \tau)n_-(t - \tau)]n_+(t) \tag{6}$$

$$+ [W_+^m(t) + W_+^s(t - \tau)n_+(t - \tau) - W_-^s(t - \tau)n_+(t - \tau)]n_-(t).$$

Therefore,

$$W_-(t) = W_-^m(t) + W_-^s(t - \tau)n_+(t - \tau) - W_+^s(t - \tau)n_-(t - \tau), \tag{7a}$$

$$W_+(t) = W_+^m(t) + W_+^s(t - \tau)n_-(t - \tau) - W_-^s(t - \tau)n_+(t - \tau). \tag{7b}$$

And the master equation (5) can be recast formally as

$$\dot{n}_+ = -\dot{n}_- = -W_-(t)n_+(t) + W_+(t)n_-(t).$$

From expression (7a), it reads that the switching rate caused by the master noise ($W_-^m(t)$) is enhanced by the secondary noise in the same direction, saying $W_-^s(t - \tau)n_+(t - \tau)$, and is reduced by the secondary noise in the opposite direction, saying $-W_+^s(t - \tau)n_-(t - \tau)$. Thus, the total switching rate from state "$+1$" to "-1" ($W_-(t)$) consists of three parts: $W_-(t)$, $W_-^s(t - \tau)n_+(t - \tau)$ and $-W_+^s(t - \tau)n_-(t - \tau)$. Analogously, the total switching rate $W_+(t)$ consists of $W_+^m(t)$, $W_+^s(t - \tau)n_-(t - \tau)$ and $-W_-^s(t - \tau)n_+(t - \tau)$. These results are well coincident with the preceding analysis.

Without loss of generality, we assume $t_0 = 0$ and $s(0) = +1$, which reads that the particle is

in the right well at the beginning. Utilizing the normalization condition $n_+(t) + n_-(t) = 1$ together with (4), one obtains

$$\dot{n}_+(t) = -\dot{n}_-(t) = q_1 - (q_2 + q_1)n_+(t) + (q_2 - q_1)n_+(t - \tau), \tag{8}$$

with $q_1 = p_m + p_s$ and $q_2 = p_m - p_s$, which can be solved precisely. The initial value of $n_+(t)$ in $[0, \tau)$ can be solved directly from (8),

$$n_+(t) = \frac{1}{2}\exp[-2\bar{r}_K t] + \frac{1}{2},$$

with $\bar{r}_K = (\sqrt{2\pi})^{-1}\exp[-1/4D(1+\varepsilon^2)]$. Then one gets the whole expression of $n_+(t)$,

$$n_+(n\tau + t) = \left\{ n_+(n\tau) + (q_2 - q_1)\int_0^t n_+((n-1)\tau + \theta)\exp[(q_2 + q_1)\theta]d\theta \right\}\exp[-(q_2 + q_1)t]$$
$$+ q_2/(q_2 + q_1)$$

with $t \in [0, \tau)$ and $n = 1, 2, \cdots$. And $n_-(t)$ can be obtained directly from $n_-(t) = 1 - n_+(t)$.

Replacing $n_\pm(t)$ in (7), we acquire $W_\pm(t) = (\sqrt{2\pi})^{-1}\exp[-1/4D(1+\varepsilon^2)]$ and

$$W_\pm(\tau + t) = \pm\frac{q_2 - q_1}{2}\exp[-2\bar{r}_K t] + \frac{q_2 + q_1}{2},$$

$$W_\pm(n\tau + t) = (\frac{q_2 + q_1}{2} \pm \frac{q_2 - q_1}{2})n_+((n-1)\tau - t) + (\frac{q_2 + q_1}{2} \mp \frac{q_2 - q_1}{2})n_-((n-1)\tau - t),$$

with $t \in [0, \tau)$ and $n = 2, 3, \cdots$.

The autocorrelation function $C(T)$ can be calculated from

$$C(T) = \langle s(T)s(0) \rangle = 2n_+(T) - 1, \tag{9}$$

together with its evolution law

$$\frac{dC(T)}{dT} = -(q_2 + q_1)C(T) + (q_2 - q_1)C(T - \tau). \tag{10}$$

According to (9), (10) and complementing with $C(-T) = C(T), C(0) = 1$, we obtain

$$C(T) = \frac{(\sqrt{q_2} + \sqrt{q_1})\exp(-2\sqrt{q_2 q_1}T) + (\sqrt{q_2} - \sqrt{q_1})\exp[2\sqrt{q_2 q_1}(T - \tau)]}{\sqrt{q_2} + \sqrt{q_1} + (\sqrt{q_2} - \sqrt{q_1})\exp(-2\sqrt{q_2 q_1}\tau)},$$

$$C(n\tau + T) = \exp[-(q_2 + q_1)T]C(n\tau) + (q_2 - q_1)\int_0^T C((n-1)\tau + \theta)\exp[(q_2 + q_1)(\theta - T)]d\theta$$

with $T \in [0, \tau)$ and $n = 1, 2, \cdots$.

The power spectral density $S(\omega)$ can be determined through the Fourier transform of $C(T)$.

Letting

$$I(\omega) = \int_0^\tau C(T)\exp(-i\omega T)dT = \operatorname{Re} I(\omega) + i\operatorname{Im} I(\omega),$$

and using (10), we obtain

$$S(\omega) = 2\operatorname{Re}\frac{1 + (q_2 - q_1)\exp(i\omega\tau)[\operatorname{Re} I(\omega) + i\operatorname{Im} I(\omega)]}{q_2 + q_1 - (q_2 - q_1)\exp(i\omega\tau) - i\omega}, \tag{11}$$

where

$$\operatorname{Re} I(\omega) = \frac{\sqrt{q_1} + \sqrt{q_2}}{[\sqrt{q_1} + \sqrt{q_2} + (\sqrt{q_2} - \sqrt{q_1})e^{-\lambda\tau}](\lambda^2 + \omega^2)}[\lambda(1 - e^{-\lambda\tau}\cos(\omega\tau)) + \omega e^{-\lambda\tau}\sin(\omega\tau)]$$

$$+ \frac{\sqrt{q_2} - \sqrt{q_1}}{[\sqrt{q_1} + \sqrt{q_2} + (\sqrt{q_2} - \sqrt{q_1})e^{-\lambda\tau}](\lambda^2 + \omega^2)}[-\lambda\cos(\omega\tau) + \lambda e^{-\lambda\tau} - \omega\sin(\omega\tau)]$$

$$\operatorname{Im} I(\omega) = \frac{\sqrt{q_1} + \sqrt{q_2}}{[\sqrt{q_1} + \sqrt{q_2} + (\sqrt{q_2} - \sqrt{q_1})e^{-\lambda\tau}](\lambda^2 + \omega^2)}[\lambda e^{-\lambda\tau}\sin(\omega\tau) - \omega(1 - e^{-\lambda\tau}\cos(\omega\tau))]$$

$$+ \frac{\sqrt{q_2} - \sqrt{q_1}}{[\sqrt{q_1} + \sqrt{q_2} + (\sqrt{q_2} - \sqrt{q_1})e^{-\lambda\tau}](\lambda^2 + \omega^2)}[\lambda\sin(\omega\tau) + \omega e^{-\lambda\tau} - \omega\cos(\omega\tau)]$$

with $\lambda = 2\sqrt{q_1 q_2}$.

The theoretical values of PSD obtained in (11) are plotted in Fig. 6, and the comparison between the theoretical value and the numerical one is exhibited in Fig. 7. Good agreement can be found between the proposed theory and the simulations.
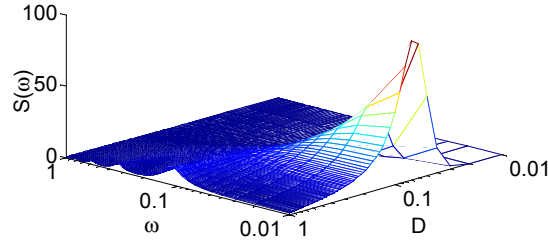

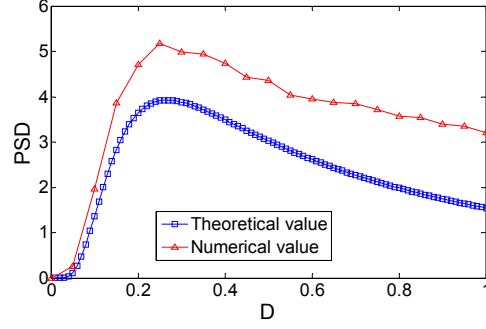FIG. 6. Mesh of PSD of model (1) obtained theoretically from (11) with $\tau = 25$ and $\varepsilon = 0.5$.

FIG. 7. Comparison of PSD obtained numerically from model (1) with theory (11) at resonance frequency with $\tau = 25$.

Based on the above analysis, the physical mechanism of the undergoing SR-like dynamics are: For low level of noise strength, the master-noise-induced Kramers' rate is very small and hence the average waiting time $T_K = 1/r_K$ between two interwell transitions is very large. In this case, if half the time delay is much smaller than the average waiting time, viz., $T_K >> \tau/2$, the secondary noise does not suppose to help the master-noise-induced hopping between the potential wells. Only if half the time delay is comparable with the average waiting time, i.e. $T_K \sim \tau/2$, the secondary noise has the opportunity to assist the master-noise-induced hopping action. This kind of coincidence between the average waiting time and half the time delay signifies a species of statistical synchronization. While statistical synchronization achieves, resonance dynamics occurs. But for very large noise level the statistical synchronization would be destroyed again since $T_K << \tau/2$. Therefore, the mechanism is the 'resonance' between the Kramers' characteristic time and the time delay in noise recycling. As an example, for the parameters of Figs. 2 and 3, the maximum is achieved at $D \approx 0.25$. So, the average waiting time $T_K \approx 12.08$ which is very closed to $\tau/2 = 12.50$.

It is worthy to point out that the current physical mechanism of resonance is intrinsically different from that in [29]. Here, the time delay belongs to the noise recycling, physically, which means an extra probability current (populations) induced by the secondary noise. While the statistical synchronization achieves between the master-noise-caused current and the secondary-noise-caused one, resonance dynamics arises. In [29], differently, the time delay belongs to the feedback term, which implies a modulation of the barrier height according to the

potential $U(x(t), x(t-\tau)) = x^4/4 - x^2/2 - \varepsilon xx(t-\tau)$, and hence causes two different

switching rate depending on the sign of $xx(t-\tau)$ (>0 or <0). While the time delay resonates with

the Kramers' rate, resonance dynamics arises.

V. STOCHASTIC RESONANCE

As another example, let's study the dynamics of the Langevin model coupled with noise

recycling in the presence of weak signal. Similar to the argument stated in [2-4], we assume the

transition rates (7a) and (7b) are modulated with a frequency Ω according to the Arrhenius rate

law. Hence, it yields

$$\overline{W}_\pm(t) = W_\pm(t)e^{\mp\gamma(t)}, \tag{12}$$

where $\gamma(t) = A_0\hat{D}^{-1}\cos(\Omega t + \phi)$ with $\hat{D} = D(1+\varepsilon)^2$. Plugging (7a) and (7b) into (12), one

reads

$$\overline{W}_+(t) = [W_+^m(t) + W_+^s(t-\tau)n_-(t-\tau) - W_-^s(t-\tau)n_+(t-\tau)]e^{-\gamma(t)},$$

$$\overline{W}_-(t) = [W_-^m(t) + W_-^s(t-\tau)n_+(t-\tau) - W_+^s(t-\tau)n_-(t-\tau)]e^{+\gamma(t)}.$$

By introducing two notations $q_1 = p_m + p_s$ and $q_2 = p_m - p_s$, defining

$\sigma(t) = n_+(t) - n_-(t)$, then the time evolution of $\sigma(t)$ reads

$$\frac{d\sigma}{dt} = [-(q_1 + q_2) + (q_2 - q_1)\sigma(t-\tau)]n_+(t)e^{\gamma(t)} + [(q_1 + q_2) + (q_2 - q_1)\sigma(t-\tau)]n_-(t)e^{-\gamma(t)}.$$

Now suppose that $A_0 \ll 1$, and write $\sigma = \sigma_0 + A_0\hat{D}^{-1}\sigma_1$ with σ_0 the solution of (10),

then in the linear approximation, this reduces to

$$\frac{d\sigma_1}{dt} = -(q_1 + q_2)\sigma_1(t) + (q_2 - q_1)\sigma_1(t-\tau)$$
$$+ (q_2 - q_1)\sigma_0(t)\sigma_0(t-\tau)\cos(\Omega t + \phi) - (q_1 + q_2)\cos(\Omega t + \phi) \tag{13}$$

As is known, $\sigma_1(t)$ signifies the periodic component of the response at the frequency Ω,

which can be solved analytically from (13) by using the ansatz $\sigma_1(t) = Ae^{i(\Omega t + \phi)}$, after

neglecting the cross term $(q_2 - q_1)\sigma_0(t)\sigma_0(t-\tau)\cos(\Omega t + \phi)$ based on the fact that

$\sigma_0(t) \to 0$ as $t \to \infty$. Thus the linear response is given by

$$\eta = \frac{1}{2\hat{D}^2} \cdot \frac{(q_1 + q_2)^2}{[(q_2 - q_1)\cos(\Omega\tau) - (q_1 + q_2)]^2 + [(q_2 - q_1)\sin(\Omega\tau) + \Omega]^2}, \quad (14)$$

which coincides with that of [3] for the SR in a two-state model. At a closer inspection of (14), we note that the linear response depends nonlinearly on the time delay, showing the important impact of noise recycling on the periodic response. The relationship between the linear response and the time delay is represented in Fig. 8, from which a noteworthy maximum displays at a certain value of time delay, demonstrating a SR structure. We want to point out that the parameters in Fig. 8 are particularly specified as the values around which the signal-to-noise ratio shows its maximum. Hence, the nonmonotonous dependence of the linear response on the time delay provides one the tool to control the SR dynamics by merely modulating the noise recycling procedure.
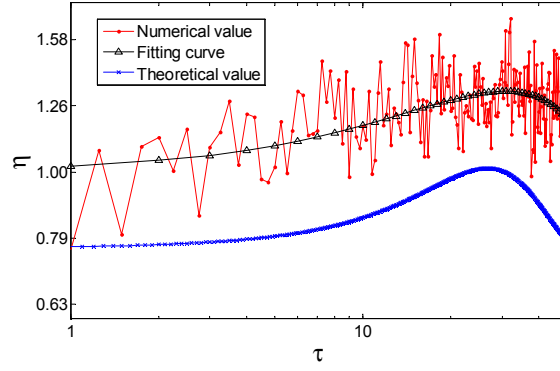

FIG. 8. Linear response of model (1) in the presence of weak signal for $A_0 = 0.2$, $\Omega = 0.1$, $\varepsilon = 0.5$ and $D = 0.3$.

VI. CONCLUSION

In conclusion, the current investigation is dedicated to the problem of how noise recycling affects the resonance dynamics of a particle trapped in a bistable potential. Because of noise recycling, the response process of the model is not Markovian. Therefore, it is quite difficult to deal with such problem because of the lack of analytical tools. In this paper, under the hypothesis of small noise we suggest a theoretical model by neglecting the small intrawell fluctuations and approximating model (1) as a two-state model, based on which we are able to derive the analytical formulae for the switching rate, the autocorrelation function, the PSD and the linear response in a good agreement with numerical simulations. We demonstrate that, with a proper modulating of time delay in noise recycling, the PSD can show several peaks and the amplitude of PSD at

resonance frequency exhibits a pronounced maximum at a certain noise level, announcing the appearance of SR-like response in the absence of weak periodic signal. Furthermore, it is shown that, in the presence of weak periodic signal, the linear response also reaches its maximum at a certain level of time delay, displaying the signature of SR. According to the current research, it reads that one can control, evoke or suppress, resonance dynamics of a bistable system merely via reinjecting a recycled noise, and then modulating the time shift or the noise level in a proper way.

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REFERENCES

[1] R. Benzi, A. Sutera, and A. Vulpiani, J. Phys. A: Math. Gen. 14, L453 (1981).

[2] L. Gammaitoni, P. Hänggi, P. Jung, and F. Marchesoni, Rev. Mod. Phys. 70, 1 (1998).

[3] B. McNamara, and K. Wiesenfeld, Phys. Rev. A 39, 4854 (1989).

[4] T. Wellens, V. Shatokhin, and A. Buchleitner, Rep. Prog. Phys. 67, 45 (2004).

[5] A. Pikovsky, and J. Kurths, Phys. Rev. Lett. 78, 775 (1997).

[6] X. Yang, D. V. Senthilkumar, Z. Sun, and J. Kurths, Chaos 21, 047522 (2011).

[7] M. Perc, Eur. Phys. J. B, 69, 1(2009).

[8] P. K. Ghosh, F. Marchesoni, S. E. Savel'ev, and F. Nori, Phys. Rev. Lett. 104, 020601 (2010).

[9] C. Nicolis, Phys. Rev. E 82, 011139 (2010).

[10] F. Sagués, J. M. Sancho, and J. García-Ojalvo, Rev. Mod. Phys. 79, 1 (2007).

[11] A. Maritan, and J. R. Banavar, Phys. Rev. Lett. 72, 1451 (1994).

[12] H. Herzel, and J. Freund, Phys. Rev. E 52, 3238 (1995).

[13] G. Malescio, Phys. Rev. E 53, 6551 (1996).

[14] R. Toral, C.R. Mirasso, E. Hernandez-Garcia, and O. Piro, Chaos 11, 3 (2001).

[15] W. Lin, and Y. He, Chaos 15, 023705 (2005).

[16] Z. Sun, and X. Yang, Chaos 21, 033114 (2011).

[17] C. Zhou, J. Kurths, E. Allaria, S. Boccaletti, R. Meucci, and F. T. Arecchi, Phys. Rev. E 67, 015205 (2003).

[18] M. Borromeo, S. Giusepponi, and F. Marchesoni, Phys. Rev. E 74, 031121 (2006).

[19] M. Borromeo, and F. Marchesoni, Phys. Rev. E 75, 041106 (2007).

[20] D. Goulding, S. Melnik, D. Curtin, T. Piwonski, J. Houlihan, J. P. Gleeson, and G. Huyet, Phys. Rev. E 76, 031128 (2007).

[21] J. Ma, Z.H. Hou, and H.W. Xin, Eur. Phys. J. B 69, 1 (2009).

[22] S. Matthias, and F. Muller, Nature 424, 53 (2003).

[23] B. Alberts, D. Bray, J. Lewis, M. Raff, K. Roberts, and J.D. Watson, *Molecular Biology of the Cell* (Garland, New York, 1994).

[24] G. Grüner, and A. Zettl, Phys. Rep. 119, 3 (1985).

[25] A. Tonomura, Rev. Mod. Phys. 59, 3 (1987).

[26] J. F. Wambaugh, C. Reichhardt, C. J. Olson, F. Marchesoni, and F. Nori, Phys. Rev. Lett. 83, 5106 (1999).

[27] Q.H. Wei, C. Bechinger, and P. Leiderer, Science 287, 5453 (2000).

[28] C. Lutz, M. Kollmann, and C. Bechinger, Phys. Rev. Lett. 93, 026001 (2004).

[29] L. S. Tsimring and A. Pikovsky, Phys. Rev. Lett. 87, 250602 (2001).