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Control turbulence in heterogeneous excitable media

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Control of turbulence in two kinds of typical heterogeneous excitable media by applying a combined method is investigated. It is found that local-low-amplitude and high-frequency pacing (LHP) is effective to suppress turbulence if the deviation of heterogeneity is minor. However, LHP is invalid when the deviation is large. Studies show that an additional radial electric field can greatly increase the efficiency of LHP. The underlying mechanisms of successful control in two kinds of cases are different and discussed respectively. Since the developed strategy by combining LHP with a radial electric field can terminate turbulence in excitable media with high degree of inhomogeneity, it has potential contribution to promote the practical low-amplitude defibrillation approach.

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I. INTRODUCTION

The excitability is one of the main dynamical principles behind a variety of biological functions. The most well known example of excitable media in biology is the heart tissue. The ventricular fibrillation continues being the leading cause of sudden cardiac death. Investigations over the past decade have suggested that ventricular fibrillation is associated with broken spiral turbulence [1–4]. Consequently, the requirement for defibrillation has attracted much attention on developing methods to terminate spiral turbulence.

The complex structure of cardiac tissue makes it heterogeneous, which results in spatially inhomogeneous distribution of excitability [5–13] and gives rise to complicated dynamic behavior. For example, some investigations have claimed that cardiac fibrillation can be characterized by multiple stable spiral waves with different frequencies resulting from inhomogeneous excitability [12, 13]. Documented studies concentrated on two types of heterogeneities. The first one results from ischemia, fibrosis, or sarcoidosis in cardiac tissue and is treated as non(or sub)-excitable regions. The existence of this type of heterogeneity may induce serious consequences. It may break the excitable planar wave with simultaneous pined reentry spiral [14–17] or turbulence [18, 19]. Therefore, from the view of actual practice, various methods to suppress spiral and turbulence have been put forward [20–22], such as exciting waves on heterogeneities by pulsed electric field [23], high frequency stimulation as anti-tachycardia pacing [24–26], and so on. The second type of heterogeneity contains a "hot" region where excitability is higher than that in the rest region [10, 27–30]. Deleterious results may also be induced by this type of heterogeneity since it negatively affects the stability of propagating waves [8]. However, less concerns are put on this case and corresponding control means.

In clinics, electric shocks with high intensity are applied to surface or directly to cardiac muscle for cardiac defibrillation [31, 32]. These large amplitude shocks have drawbacks: they may damage the cardiac tissue and cause serious pains [33]. As a low field treatment, trains of low amplitude pulses are then widely used as anti-tachycardia pacing. A big challenge of anti-tachycardia pacing is that it usually fails to terminate high-frequency arrhythmias and developed chaos if the media are heterogeneous [26]. The first type of heterogeneity will block (or break) the trains of low amplitude pulses and then protect the existed turbulence behind it. The second kind of heterogeneity may act as another source of pacemaker emitting irregular waves preventing the attempt of anti-tachycardia pacing. For these drawbacks of anti-tachycardia pacing, it is necessary and meaningful to develop strategies to suppress turbulence in heterogeneous media.

In this paper, a radial electric field, in addition to the method of local-low-amplitude and high-frequency pacing(LHP), is proposed as a combined strategy to terminate spiral turbulence in both types of heterogeneous media. It will be found that the radial electric field greatly improves the effectiveness of the local pacing. The mechanisms of suppression of turbulence in two types of heterogeneous media are different and discussed respectively.

II. MODEL

In our simulation, a modified FitzHugh-Nagumo model introduced by M. Bär and Eiswirth is used to describe the electrical activities of cardiac tissue [34],

$$\begin{aligned}\frac{\partial u}{\partial t} &= f(u,v) + \nabla^2 u,\\ \frac{\partial v}{\partial t} &= g(u) - v,\end{aligned} \qquad (1)$$

where the function $f(u,v)$ and $g(u,v)$ takes the following

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form

$$f(u,v) = \frac{1}{\varepsilon(x,y)} u(1-u)(u - \frac{v+b}{a}) \quad (2)$$

$$g(u) = \begin{cases} 0 & 0 \le u < 1/3, \\ 1 - 6.75u(u-1)^2 & 1/3 \le u < 1, \\ 1 & 1 \le u. \end{cases} \quad (3)$$

Here, u and v are the activator and the inhibitor variables, respectively. Parameters a=0.84 and b=0.07 are fixed to ensure the medium is excitable.

The parameter ε can characterize the excitability of the medium which exhibits various distinctive characteristics as ε is varied. The bigger the ε, the smaller the excitability is. When ε is in [0.02,0.07], the medium supports stable or meandering spiral with suitable initial conditions. Due to doppler effect in the range $\varepsilon > 0.071$, spiral waves will break up and the system will quickly fall into a turbulent state. As ε crosses threshold of backfiring at about 0.1, the system can not support propagation of regular waves. In our model, $\varepsilon(x,y)$ characterizes the spatial-distributed excitability of the two-dimensional medium. To simulate a heterogeneous excitable medium, the $\varepsilon(x,y)$ is presented as

$$\varepsilon(x,y) = \begin{cases} \varepsilon_0 & 0 \le x < L_1, \\ \varepsilon_H & L_1 \le x \le L_2, \\ \varepsilon_0 & L_2 < x \le L. \end{cases} \quad (4)$$

The size of the square medium is L=100 (L_1=46 and L_2=58). The stripe with ε_H acts as the heterogeneity. Local pacing $F(t)$=$\tau\cos(\omega t)$ with fixed amplitude $\tau = 2.0$ is imposed on a small square region (3×3 grids) with central point at grid $(x_0, y_0) = (31, 47)$. The radial electric field with the same center point is imposed. At point (x, y), the radial electric field can be divided into two directions along $x-$ and $y-$ axes, that is E_x=$E\cos\theta$=$E(x-x_0)/d$ and E_y=$E\sin\theta$=$E(y-y_0)/d$ where $d = \sqrt{(x-x_0)^2 + (y-y_0)^2}$. One can find the detailed information in the sketches in fig. 1.

Consider the influences of LHP and the radial electric field, the model is then modified as

$$\begin{aligned} \frac{\partial u}{\partial t} &= f(u,v) + \nabla^2 u + F(t) - E_x\frac{\partial u}{\partial x} - E_y\frac{\partial u}{\partial y}, \\ \frac{\partial v}{\partial t} &= g(u) - v, \end{aligned} \quad (5)$$

The simulation is performed on a system 256 × 256 grids with no-flux boundary conditions. Discretizations with $\Delta x = \Delta y = 0.3906$ and $\Delta t = 0.02$ have been used in Euler scheme.

The deviation $\Delta\varepsilon = |\varepsilon_0 - \varepsilon_H|$ is set to evaluate the degree of heterogeneity. The bigger the value $\Delta\varepsilon$, the higher the degree of inhomogeneity of spatial excitability is. When $\Delta\varepsilon = 0$, the medium recovers to homogeneous system.

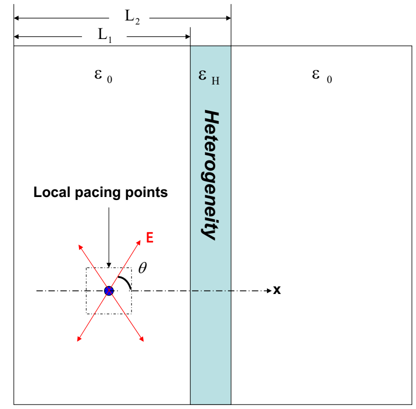

FIG. 1: (Color online) Schematic drawing illustrates the geometry of the system and the combined method with periodic local pacing and radial electric field. The shadow region ($L_2 - L_1$ width) with ε_H denotes the heterogeneity. The arrows indicate the radial electric field. θ is the angle between the radial electric field at one fixed point and the positive x axes. The small square indicated by dot frame represents the region where LHP is injected.

III. RESULTS AND DISCUSSION

1. *Heterogeneity with higher excitability*

Firstly, we study the control of turbulence in the second type of heterogeneous media containing "hot" regions with higher excitability than that in the rest region, that is $\varepsilon_0 > \varepsilon_H$. The value ε_0 is fixed to be 0.075 while ε_H is changed to alter the degree of heterogeneity. Since we focus on the suppression of turbulence, we give chaotic states as initial conditions.

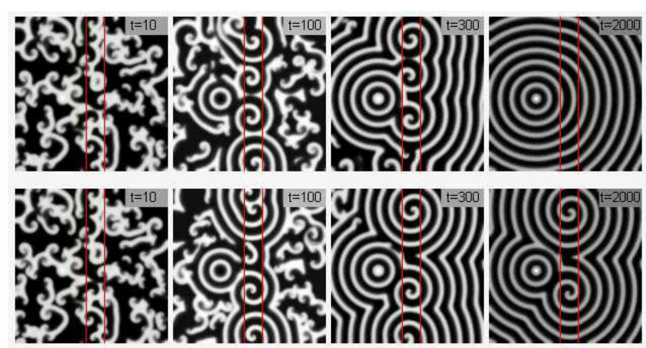

FIG. 2: (Color online) Evolution of turbulence in inhomogeneous medium controlled by LHP with ω_{LHP}=1.45. ε_0 = 0.075 is fixed while ε_H is 0.062 in upper row and 0.06 in lower row.

As an alternative defibrillation technique, the strategy of LHP has been explored in a number of previous experiments [35]. It is found that target waves with high frequency generated by LHP is effective to terminate turbulence when the medium is homogeneous (the value $\Delta\varepsilon$=0) [36]. We present an example with fixed $\varepsilon_0 = 0.075$ and gradually decrease ε_H to increase the excitability of the heterogeneity. With small deviation of excitability $\Delta\varepsilon < 0.015$, the target waves growing from LHP succeed in wiping away turbulence in the system, which can be seen from the evolution in the upper row of fig. 2. In this process, the frequency of LHP is always bigger than $\omega_T(\varepsilon_0)$ and $\omega_T(\varepsilon_H)$ that are the frequencies of turbulence in the ε_0 region and heterogeneity respectively. However, the effort of suppression fails when $\Delta\varepsilon$ crosses a critical value 0.015 (the corresponding critical $\varepsilon_{H1} = 0.06$). It is shown in the lower row of fig. 2 where the excitability of heterogeneity is just the same as the critical value. Spiral waves spontaneously appear in the heterogeneity, which protect against the target waves from LHP. The co-existence of target waves and spiral waves indicates they have identical frequencies. A more obvious example showing the failure of LHP is presented in the upper row of fig. 5 where heterogeneity has much higher excitability ($\varepsilon_H = 0.04$). The target wave can not grow since it is suppressed by the turbulence with high frequency from the heterogeneity.

The condition to ensure the validity of LHP depends on its characteristic, namely, high frequency waves. However, this condition may break down in heterogeneous medium containing "hot" regions where the frequency of turbulence is higher than that of the target waves. To clarify this point and put forward possible control strategy, we present the dispersion relation in fig. 3. To suppress turbulence surrounding the pacing points (in the region with ε_0), the frequency of target waves ω_{LHP} from LHP should be higher than $\omega_T(\varepsilon_0)$. Terminating turbulence in the heterogeneity requires $\omega_{LHP} > \omega_T(\varepsilon_H)$. It is easy to reach these two requirements if the medium is homogeneous or the deviation is small. In the upper row of fig. 2, these two conditions are fulfilled: $\omega_{LHP} > \omega_T(\varepsilon_H) > \omega_T(\varepsilon_0)$, namely, $1.45 > 1.43 > 1.22$. However, there is an additional requirement for heterogeneous medium, that is, ω_{LHP} should also be smaller than $\omega_{max}(\varepsilon_0)$. Otherwise, the pacing is so fast that the next pacing stimulus S2 falls in the refractory tail of the previous wave S1, which is called S1S2 stimulation [37]. Then, the target wave can not be generated until the third stimulus is applied. Consequently, the frequency of target wave will be halved, which will result in failure of suppressing turbulence. In order to make use of the LHP to the most degree, we always select the condition $\omega_{LHP} = \omega_{max}(\varepsilon_0)$. Thus, the first condition is fulfilled automatically. From fig. 3, one can see that the frequency of turbulence in the heterogeneity is increased when ε_H is decreased. Continual increasing deviation of excitability, which results from decrease of ε_H (ε_0 is fixed), the value $\omega_T(\varepsilon_H)$ will inevitably exceed $\omega_{max}(\varepsilon_0)$ and ω_{LHP}. The condition $\omega_T(\varepsilon_H) = \omega_{LHP}$ in the lower row of fig. 2 and $\omega_T(\varepsilon_H) > \omega_{LHP}$ in the upper row of fig. 5 results in failure of suppression. From the dispersion

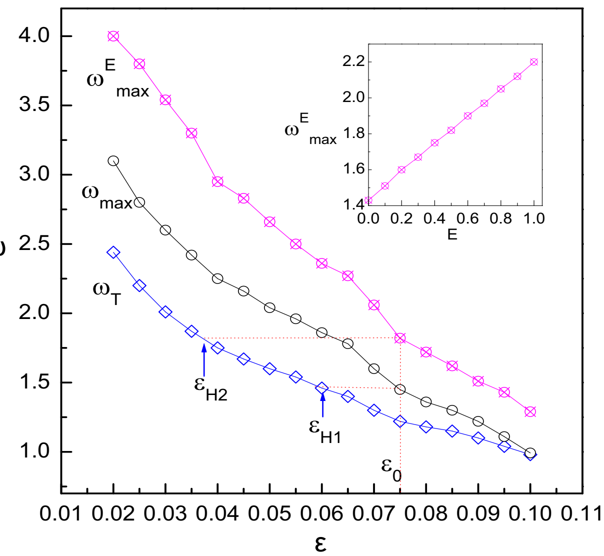

FIG. 3: (Color online) Open squares denote the frequency of existed turbulence $\omega_T(\varepsilon)$ at each ε, which are obtained from the power spectrum by FFT method. Open circles represent maximum frequencies $\omega_{max}(\varepsilon)$ for different value of ε. The values $\omega_{max}(\varepsilon)$ are obtained in the following way: we apply the LHP in homogeneous medium with initial condition $u = v = 0.0$ and measure the frequency of one fixed point when the frequency of LHP is increased. When the measured frequency suddenly becomes half of the LHP instead of identical value, one $\omega_{max}(\varepsilon)$ is obtained. The dot lines are auxiliary lines to show the critical value ε_{H1} and ε_{H2}. The $\omega^E_{max}(\varepsilon)$ curves shows the maximum frequency of target waves by LHP under the effect of radial electric field with $E = 0.5$. Inset: the maximum frequency of target wave generated by LHP vs. intensity of radial electric field. The value of ϵ_0 is 0.075.

relation, one can get the critical value of ε_{H1} (=0.06) (see the auxiliary lines). The analysis is consistent with simulation result in fig. 2 where we exactly find the critical value ε_{H1} is 0.06.

From the discussion mentioned above, one way to suppress the turbulence in heterogeneous media with large deviation of excitability is to increase the $\omega_{max}(\varepsilon_0)$ so that the value ω_{LHP} can be subsequently increased to make the condition $\omega_{LHP} > \omega_T(\varepsilon_H)$ fulfilled. In our strategy, a radial electric field with the center locating on the LHP points is applied to play the role of increasing $\omega_{max}(\varepsilon_0)$. To show the influence of radial electric field, one loop of target wave is plotted in fig. 4. In fig. 4(a), it is evident that the applying of radial electric field results in expanding of refractory tail, which has the same effect as decreasing ε_0 [34]. Note that the decrease of ε_0 will correspondingly decrease deviation $\Delta\varepsilon = (\varepsilon_0 - \varepsilon_H)$. It seems as if the deviation of excitability is moderated once the radial electric field is applied. On the other hand, one can find that the recovery of excitation to the $v = 0$ state becomes faster after radial electric field is applied. This point is distinctive in fig. 4 (b,c) where the evolutions

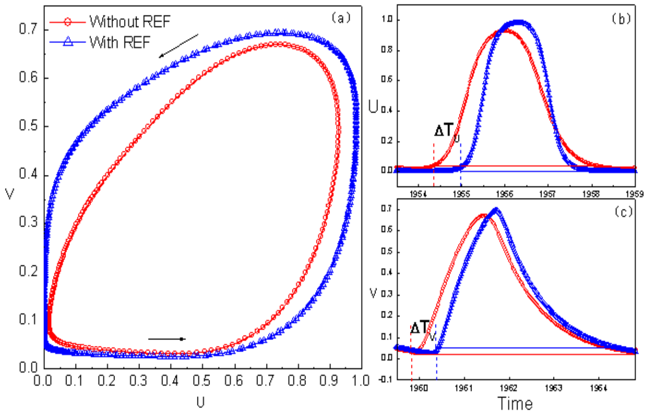

FIG. 4: (Color online) (a) $U-V$ phase portrait of target wave generated by LHP ($\varepsilon_0 = 0.075$) with and without radial electric field. The data are plotted from a point in ε_0 region. The open circle curve is generated by LHP with ω=1.3 while the open triangle curve is also affected by a radial electric field with $E = 1.0$. (b,c) The corresponding evolution of u, and v. The dot auxiliary lines roughly indicate the duration of excitation. The differences of duration are shown by ΔT_U and ΔT_V, respectively.

of u and v are plotted. Compared to the case without radial electric field, the duration of excitation with radial electric field is dramatically decreased (marked by the ΔT_U and ΔT_V). We can take advantage of this effect to increase the $\omega_{max}(\varepsilon_0)$: when trains of pulses are generated from LHP, the front of the second wave will reach the refractory tail of the previous wave if its frequency ω_{LHP} is bigger than $\omega_{max}(\varepsilon_0)$. Now, the radial electric field decreases the duration of the waves, which makes the LHP to generate trains of waves with higher frequencies possible. In the inset of fig. 3, it is found that the maximum frequency of target wave by LHP (identical with $\omega^E_{max}(\varepsilon_0)$) can be greatly increased by radial electric field monotonously. The maximum frequency $\omega^E_{max}(\varepsilon_0)$ at different ε_0 with same intensity of radial electric field is also plotted in fig. 3. Comparing the curve $\omega^E_{max}(\varepsilon_0)$ with $\omega_{max}(\varepsilon_0)$, one can find the enhancement is obvious.

In terms of this mechanism, we can utilize the radial electric field to suppress turbulence in heterogeneous media with big deviation. An example with big $\Delta\varepsilon$ is shown in fig. 5. Without radial electric field in the upper of fig. 5, the target waves can not grow since $\varepsilon_H = 0.04$ is much smaller than the critical value ($\varepsilon_H = 0.06$). Once the radial electric field is applied in the lower row of fig. 5, the target waves with higher frequencies dramatically enter into the heterogeneity and suppress turbulence in the medium. In simulation, we decrease the value of ε_H until the combined method loses its effect when ε_H crosses a critical value which is labeled as ε_{H2}. A surprising thing is that the value of ε_{H2} from simulation is 0.025 which is smaller than that from the analyst in the dispersion relation in fig. 3 where the intersection marked by one

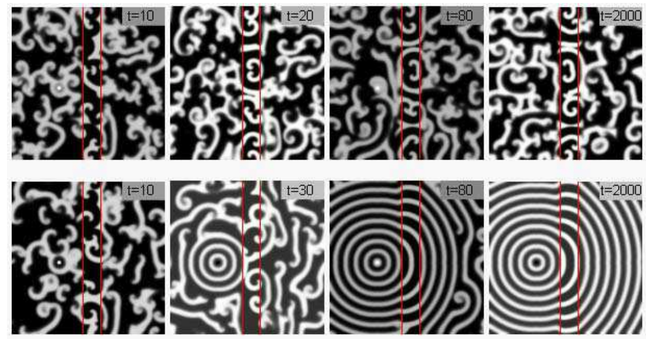

FIG. 5: (Color online) Contour patterns show the evolution of turbulence with LHP (upper row) and with combined method (lower row). Parameters: $\omega_{LHP} = 1.82$, $\varepsilon_0 = 0.075$, $\varepsilon_H = 0.04$.

arrow ε_{H2} shows its value 0.037. To evaluate the efficiency of radial electric field, we plot the value $\Delta\varepsilon$ as a function of the intensity of radial electric field. It is found that the $\Delta\varepsilon$ depends on the amplitude of radial electric field, which has been illustrated in fig. 6 where a monotonically increasing curve is obtained. Compared to the value $\Delta\varepsilon = 0.015$ without radial electric field, the value $\Delta\varepsilon$ is increased to 0.065 under the radial electric field with $E = 1.0$, which means the deviation is greatly enhanced. Thus, we can conclude that the combination of local pacing and radial electric field can suppress turbulence in inhomogeneous medium with very “hot” heterogeneity.

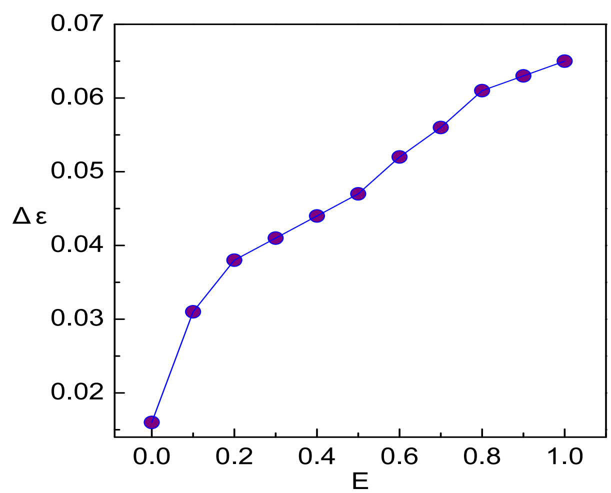

FIG. 6: $\Delta\varepsilon$ *vs.* E. ω_{LHP} is from $\omega^E_{max}(\varepsilon_0)$ which is illustrated in the inset of fig. 3. $\Delta\varepsilon = \epsilon_0 - \epsilon_H$ and $\varepsilon_0 = 0.075$.

2. Heterogeneity with lower excitability

We divert attention to the first type of heterogeneity with excitability lower than that in the rest region ($\varepsilon_0 < \varepsilon_H$, $\varepsilon_0 = 0.04$ is fixed). Traditionally, this kind of heterogeneity is called as "obstacle" since it blocks the propagation of waves. Certainly, turbulence can be suppressed if the deviation $\Delta\varepsilon = (\varepsilon_H - \varepsilon_0)$ is small. In the top row of fig. 7, we show the evolution of target waves by LHP with suitable frequency in medium with $\Delta\varepsilon = 0.023$ ($\varepsilon_H = 0.063$). The growing target waves propagate into the heterogeneity and eliminate the initial existed turbulence. Now, the question is posed again: can the LHP suppress turbulence if the excitability of heterogeneity is very low, namely, the deviation $\Delta\varepsilon$ is very big? In middle row of fig. 7, the $\Delta\varepsilon$ is enhanced from 0.023 to 0.04 (ε_H is increased from 0.063 to 0.08). One can see that the target waves by LHP can not enter into the heterogeneity any longer. The blocking of the heterogeneity makes the effort failed although the frequency of LHP is higher than that of the existed turbulence in any region ($\omega_{LHP} = 1.76$, $\omega_T(\varepsilon_0) = 1.75$, and $\omega_T(\varepsilon_H) = 1.18$). No matter what frequencies of LHP are selected, the target waves can not achieve in suppressing turbulence in heterogeneity. Simulation gives the critical value $\varepsilon_{H1} = 0.065$.

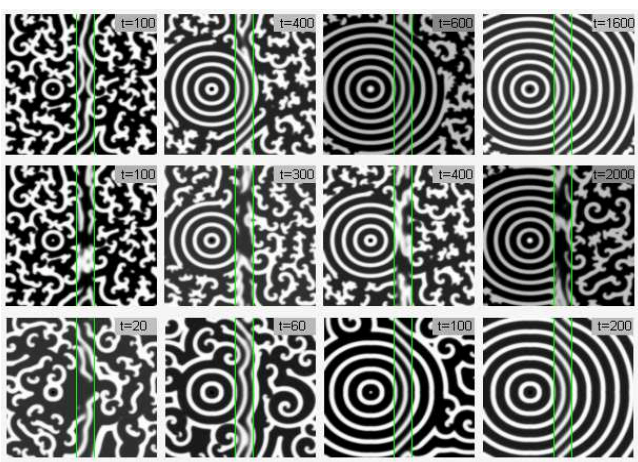

FIG. 7: (Color online) Evolution of turbulence in inhomogeneous medium under the control of LHP and radial electric field. The ε_0 is fixed to be 0.04. Tow row: $\varepsilon_H = 0.063$, $\omega_{LHP} = 1.76$, $E = 0$; Middle row: $\varepsilon_H = 0.08$, $\omega_{LHP} = 1.76$, $E = 0$; Bottom row: $\varepsilon_H = 0.08$, $\omega_{LHP} = 1.73$, $E = 0.5$.

Then we make use of the radial electric field again in heterogeneous media with high deviation of excitability. In the bottom row of fig. 7 where $\Delta\varepsilon = 0.04$, we utilize the radial electric field and decrease the frequency of LHP. Now, the target waves successfully pass through the heterogeneity and wipe away turbulence. Thus, it is interesting that the radial electric field is also effective to this type of heterogeneity.

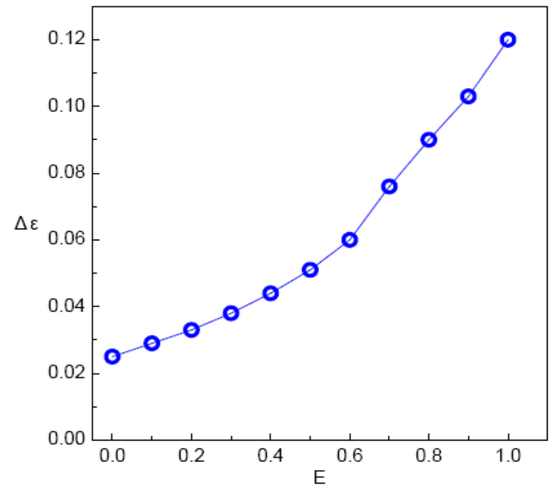

FIG. 8: $\Delta\varepsilon$ *vs.* E. The values of ω_{LHP} are changed according to $\omega_T^E(\varepsilon_0)$ which is illustrated in the inset of fig. 9. $\Delta\varepsilon = \epsilon_H - \epsilon_0$ and $\varepsilon_0 = 0.04$.

The radial electric field increases the critical value ε_{H1} so far. We label the new critical value after radial electric field is imposed as ε_{H2} and $\Delta\varepsilon(E) = (\varepsilon_{H2}(E) - \varepsilon_0)$. From fig. 8, it is found that the $\Delta\varepsilon$ increases with intensity of radial electric field. The deviation parameter $\Delta\varepsilon$ is greatly extended, i.e., $\Delta\varepsilon$=0.12 when amplitude of radial electric field is 1.0. Compared with the efficiency by LHP, $\Delta\varepsilon$ is increased almost 5 times by means of the combined method.

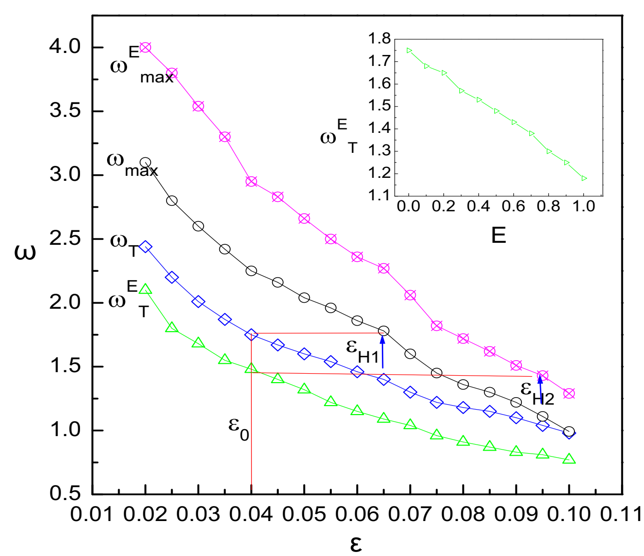

FIG. 9: (Color online) The same dispersion relation as that in fig. 3 except for adding one curve showing $\omega_T^E(\varepsilon_0)$ with $E = 0.5$. Inset: dependence of principal frequency of turbulence on the intensity of radial electric field. The value of ϵ_0 is 0.04.

In order to have an insight into the underlying mechanism of successful control in this type of heterogeneity, we plot the dispersion relation in fig. 9 again. It is clear that suppressing turbulence requires the frequency of LHP ω_{LHP} to be higher than $\omega_T(\varepsilon_0)$ (Since $\omega_T(\varepsilon_0)$ is bigger than $\omega_T(\varepsilon_H)$, the condition $\omega_{LHP} > \omega_T(\varepsilon_H)$ is fulfilled automatically). However, the increase of deviation $\Delta\varepsilon$ will simultaneously decrease the $\omega_{max}(\varepsilon_H)$. Once the ε_H crosses the critical value ε_{H1}, ω_{LHP} will bigger than $\omega_{max}(\varepsilon_H)$. This is not permitted since the frequency of target waves will be halved in the heterogeneity, which results in the failure of terminating turbulence in the heterogeneity by LHP. Thus, ε_{H1} is the maximum value for the method of LHP. For example, ε_{H1} is about 0.065 when ε_0 is fixed to be 0.04, which shows a maximum deviation $\Delta\varepsilon = 0.025$ (see the auxiliary lines). The simulation results illustrated in the top and middle row of fig. 7 confirm this point. In contrast to increase the frequency of LHP in the second type of heterogeneity, in this case, the alternative is to decrease $\omega_T(\varepsilon_0)$ so that we can decrease ω_{LHP} correspondingly to ensure the condition $\omega_{max}(\varepsilon_0) > \omega_{LHP} > \omega_T(\varepsilon_0)$ and $\omega_{max}(\varepsilon_H) > \omega_{LHP} > \omega_T(\varepsilon_H)$ to be both fulfilled. That is what we have done in the bottom row in fig. 7 where the ω_{LHP} is decreased from 1.76 to 1.73 (note $\omega_T(\varepsilon_0)$ is 1.75) with radial electric field and $\omega_T^E(\varepsilon_0)$ is 1.48 with radial electric field. Indeed, the radial electric field can decrease the principal frequency of turbulence in the ε_0 region. The radial electric field acts as a role of radial gradient force, which is indicated in Eq. 5. The essential influence of the gradient force is to make defects and waves move along the gradient direction [38]. It reduces the principal frequencies of turbulent waves of the system due to Doppler effect, which has been illustrated in the fig. 9 (see the curve ω_T^E and curve in the inset). The principal frequency of turbulence decreases when the intensity of radial electric field is increased, which is shown in the inset of fig. 9. We simulate the principal frequencies of turbulence $\omega_T^E(\varepsilon_0)$ at different ε after radial electric field with $E = 0.5$ is imposed and plot them in fig. 9. From the new dispersion relation, we can get the $\varepsilon_{H2} = 0.08$ if $\omega_{LHP} = 1.73$ and ε_{H2}=0.092 if $\omega_{LHP} = \omega_T^E(\varepsilon_0)$, which are also confirmed by the simulation results. If we add the curve $\omega_T^E(\varepsilon_0)$ in fig. 3 and consider the curve $\omega_T^E(\varepsilon_0)$ instead of $\omega_T(\varepsilon_0)$ in the second type of heterogeneity, then the ε_{H2} in terms of dispersion relation is 0.025, which is consistent with simulation.

3. *Suppression turbulence in media with two types of heterogeneities*

From the results and discussion, it is shown that the utilizing of radial electric field can greatly contribute to suppress turbulence in both types of heterogeneous media with large deviation of excitability. Therefore, even the excitability of medium is distributed in a very wild range, the combined method is also proved to be effective. For the complexity of cardiac tissues, the spatial distribution of different excitability is possible, which makes it interesting to develop some efficient ways to suppress turbulence under various conditions. In fig. 10, we consider the coexistence of both types of heterogeneities. The medium is divided into three regions. The excitability is highest in the left region and lowest in the right region. The LHP points are located in the middle region. Simulation shows that the smallest value of ε_l in the left region is 0.025 and the largest value of ε_r in the right region is 0.13. Note that the deviation of (ε_r-ε_l) in this case shows a very big value 0.095. Furthermore, it should be pointed that the turbulence can not be terminated by LHP in the backfiring regime ($\varepsilon > 0.1$) without any auxiliary mean. However, the suppression of turbulence in the right region demonstrates that the radial electric field can contribute to avoid this drawback.

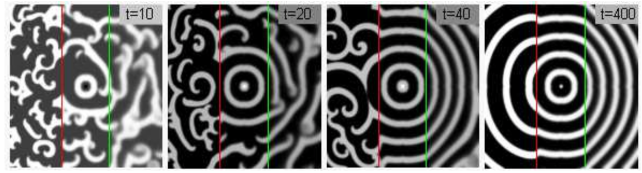

FIG. 10: (Color online) The process of successful control by combined LHP with $\omega_{LHP} = 1.6$ and radial electric field with $E = 1.0$. The excitability is different in three regions. Left region: $\varepsilon_l = 0.025$; middle region: $\varepsilon_m = 0.06$; right region: $\varepsilon_r = 0.13$.

IV. CONCLUSION

In conclusion, we have studied the suppression of turbulence in inhomogeneous media containing two kinds typical of heterogeneities in which the excitability is higher or lower than that in the rest region. The LHP is a general method if the deviation of excitability is small while it fails when deviation is large. Then, a combined strategy, including LHP and radial electric field, is put forward as a method with very high efficiency. Another advantage is that, compared to high intensity electric shocks, the required energy of the combined method to terminate VF is quit small. The underlying mechanism of successful control is different in different heterogeneous medium and discussed in terms of dispersion relation. In the second type of heterogeneity, the utilization of radial electric field increases the frequency of target waves from LHP. In the first one, the radial electric field decreases the frequency of turbulence. The simulation is consistent with the analysis.

We point out that it is not clear whether one can use this combined method for cardiac defibrillation. How to design a device to combine local pacing and radial electric field in experiment is a question. In addition, the model is too simple for simulating actual cardiac systems although it catches the general feature of excitable media.

We hope our studies can offer some useful instruction to experiments and simulations on practical cardiac defibrillation.

Acknowledgements

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