Negative electromagnetic plane-wave force in gain media

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Abstract

It is shown that a uniform electromagnetic plane wave can exert a negative force on a homogeneous medium with gain when there is no component of the electric field in that direction. A physical interpretation for this force is given, along with an estimate of the strength achievable in an experiment.

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Generally, classical electromagnetic forces can be decomposed into a gradient or dipole force associated with a spatially varying field, as used in optical tweezers [1], and that due to radiation pressure involving atomic transitions [2]. Evanescent fields between two surfaces or waveguides can also result in a dipole force [3]. Experimental evidence suggests that radiation pressure is related to a change in momentum (between incident and emitted photons), and that this results in a force in the direction of the incident field momentum (and in the direction of the Poynting vector) [2]. However, it has been proposed that light attraction or a negative force is possible in a negative refractive index medium [4–7].

We consider the various contributions to the electromagnetic force from a fundamental perspective, including the material constitutive parameters and their dispersive properties, and show that the radiation pressure is positive for propagating waves in homogeneous passive media, even when the refractive index is negative. We show, however, that the force can be negative when the medium has gain, and we describe an experiment that should confirm this force in a medium having gain. This work provides a foundation for the treatment of forces in dispersive inhomogeneous material systems, such as objects in some background material, and the further study of the influence of the background material properties.

We arrive at the force for the electromagnetic-kinetic system, starting with Maxwell's equations, written as

$$\begin{aligned}
\nabla \times \mathbf{E} + \mu_0 \frac{\partial \mathbf{H}}{\partial t} &= -\mu_0 \frac{\partial \mathbf{M}}{\partial t} && \text{(1a)}\\
\nabla \times \mathbf{H} - \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} &= \frac{\partial \mathbf{P}}{\partial t} + \mathbf{J} && \text{(1b)}\\
\epsilon_0 \nabla \cdot \mathbf{E} &= -\nabla \cdot \mathbf{P} + \rho && \text{(1c)}\\
\nabla \cdot \mathbf{H} &= -\nabla \cdot \mathbf{M}, && \text{(1d)}
\end{aligned}$$

with $\mathbf{E}$ the electric field intensity, $\mathbf{H}$ the magnetic field intensity, $\mathbf{P}$ the polarization, $\mathbf{M}$ the magnetization, $\mathbf{J}$ the electric current density, ρ the free electric charge density, ϵ_0 the free space permittivity, and μ_0 the free space permeability. Taking the cross product of $\epsilon_0\mathbf{E}$ with (1a) and $\mu_0\mathbf{H}$ with (1b), and adding the resulting equations, gives

$$\begin{aligned}
\epsilon_0 \mathbf{E} \times (\nabla \times \mathbf{E}) + \mu_0 \mathbf{H} \times (\nabla \times \mathbf{H}) \; + \; & \mu_0\epsilon_0 \mathbf{E} \times \frac{\partial \mathbf{H}}{\partial t} - \mu_0\epsilon_0 \mathbf{H} \times \frac{\partial \mathbf{E}}{\partial t} \\
&= -\mu_0\epsilon_0 \mathbf{E} \times \frac{\partial \mathbf{M}}{\partial t} + \mu_0 \mathbf{H} \times \frac{\partial \mathbf{P}}{\partial t} + \mu_0 \mathbf{H} \times \mathbf{J}. \qquad (2)
\end{aligned}$$

In terms of the momentum-flow tensor of the electromagnetic field [8], defined as $\mathbf{T}_e = \frac{1}{2}\left(\epsilon_0 E^2 + \mu_0 H^2\right)\mathbf{I} - \epsilon_0 \mathbf{EE} - \mu_0 \mathbf{HH}$, where $\mathbf{I}$ is the identity matrix, we write

$$\epsilon_0 \mathbf{E} \times (\nabla \times \mathbf{E}) + \mu_0 \mathbf{H} \times (\nabla \times \mathbf{H}) = \nabla \cdot \mathbf{T}_e + \epsilon_0 (\nabla \cdot \mathbf{E}) \mathbf{E} + \mu_0 (\nabla \cdot \mathbf{H}) \mathbf{H}, \tag{3}$$

where $(\nabla \cdot \mathbf{T})_i = \partial T_{ji}/\partial x_j$ and the tensor outer product $\mathbf{ab}$ has components $(\mathbf{ab})_{ij} = a_i b_j$. With the momentum density associated with the electromagnetic field defined as $\mathbf{G}_e = \mu_0 \epsilon_0 \mathbf{E} \times \mathbf{H}$ [8], we obtain

$$\frac{\partial \mathbf{G}_e}{\partial t} = \mu_0 \epsilon_0 \mathbf{E} \times \frac{\partial \mathbf{H}}{\partial t} - \mu_0 \epsilon_0 \mathbf{H} \times \frac{\partial \mathbf{E}}{\partial t}. \tag{4}$$

Using (3) and (4), (2) can be re-written as

$$\nabla \cdot \mathbf{T}_e + \epsilon_0 (\nabla \cdot \mathbf{E}) \mathbf{E} + \mu_0 (\nabla \cdot \mathbf{H}) \mathbf{H} + \frac{\partial \mathbf{G}_e}{\partial t} = -\mu_0 \epsilon_0 \mathbf{E} \times \frac{\partial \mathbf{M}}{\partial t} + \mu_0 \mathbf{H} \times \frac{\partial \mathbf{P}}{\partial t} + \mu_0 \mathbf{H} \times \mathbf{J}. \tag{5}$$

The kinetic force density is $\mathbf{f} = -\nabla \cdot \mathbf{T}_e - \partial \mathbf{G}_e/\partial t$ (N m^{-3} in SI units) [8], and from (5),

$$\mathbf{f} = \mu_0 \epsilon_0 \mathbf{E} \times \frac{\partial \mathbf{M}}{\partial t} - \mu_0 \mathbf{H} \times \frac{\partial \mathbf{P}}{\partial t} + \epsilon_0 (\nabla \cdot \mathbf{E}) \mathbf{E} + \mu_0 (\nabla \cdot \mathbf{H}) \mathbf{H} - \mu_0 \mathbf{H} \times \mathbf{J}. \tag{6}$$

Maxwell's equations (1) were written in the form of fields, on the left, due to all sources, including free and bound charge, on the right. Equation (6) describes the Lorentz force on all charges (free and bound), and on sources, and is a special case of the relativistic treatment of Penfield and Haus [8, 9], based on the principle of virtual power and multiple systems. We note that the form in (6), while consistent with other previous treatments [8–11], has important distinctions from some earlier work on forces in negative index materials [5].

We restrict our attention to plane waves, which results in $\nabla \cdot \mathbf{E} = \nabla \cdot \mathbf{H} = 0$, and to source-free regions ($\mathbf{J} = 0$), so that the force density in (6) becomes

$$\mathbf{f} \; = \; \frac{\partial \mathbf{P}}{\partial t} \times \mu_0 \mathbf{H} - \frac{\partial \mu_0 \mathbf{M}}{\partial t} \times \epsilon_0 \mathbf{E}. \tag{7}$$

Enforcing reality and assuming linear, isotropic materials,

$$\mathbf{P}(t) \; = \; \frac{\epsilon_0}{4\pi} \int \chi_E(\omega) \mathbf{E}(\omega) e^{-i\omega t} d\omega + c.c. \tag{8}$$

$$\mathbf{M}(t) \; = \; \frac{1}{4\pi} \int \chi_H(\omega) \mathbf{H}(\omega) e^{-i\omega t} d\omega + c.c., \tag{9}$$

with χ_E the complex electric susceptibility and χ_H the complex magnetic susceptibility. Note that the influence of the free charge motion on $\mathbf{f}$ is captured by $\mathbf{P}$ in (7), through the imaginary part of $\chi_E(\omega)$.

We consider the modulated field case

$$\mathbf{E}(t) = \hat{e}\ E(t) = \hat{e}\ e(t)cos(\omega_0 t) \tag{10}$$

$$\mathbf{H}(t) = \hat{h}H(t) = \frac{\hat{h}}{4\pi}\int u(\omega)E(\omega)e^{-i\omega t}d\omega + c.c., \tag{11}$$

$$\mathbf{f} = \hat{e}\times\hat{h}f,\ f = \mu_0 H\frac{\partial P}{\partial t} + \mu_0\epsilon_0 E\frac{\partial M}{\partial t} \tag{12}$$

where $e(t)$ is a modulation signal and $u = \eta^{-1} = \left(\eta_0\sqrt{\mu/\epsilon}\right)^{-1}$, with η_0 the free space wave impedance. The scalar form of $\mathbf{f}$ in (12) is used hence forth to describe the instantaneous force density and the time-averaged force density, $\langle f\rangle$, the average value over the carrier period $t_0 = 2\pi/\omega_0$.

Assuming a slowly varying $e(t)$ (small bandwidth) in (10) and a sufficiently slowly varying $\chi_E(\omega)$, a two-term Taylor expansion of $\omega\chi_E(\omega)$, with $\chi_E = \chi_E' + i\chi_E''$, can be used to give [12]

$$\begin{aligned}\frac{\partial P}{\partial t} &\approx \omega_0\epsilon_0 e(t)\left[\chi_E''(\omega_0)\cos(\omega_0 t) - \chi_E'(\omega_0)\sin(\omega_0 t)\right] + \epsilon_0\frac{\partial e(t)}{\partial t}\left[\left.\frac{\partial(\omega\chi_E')}{\partial\omega}\right|_{\omega=\omega_0}\cos(\omega_0 t)\right. \\ &+ \left.\left.\frac{\partial(\omega\chi_E'')}{\partial\omega}\right|_{\omega=\omega_0}\sin(\omega_0 t)\right].\end{aligned} \tag{13}$$

With a Taylor series expansion of $u(\omega)$, from (11),

$$\begin{aligned}H &\approx e(t)\left[u'(\omega_0)\cos(\omega_0 t) + u''(\omega_0)\sin(\omega_0 t)\right] \\ &+ \frac{\partial e(t)}{\partial t}\left[\left.\frac{\partial u'}{\partial\omega}\right|_{\omega_0}\sin(\omega_0 t) - \left.\frac{\partial u''}{\partial\omega}\right|_{\omega_0}\cos(\omega_0 t)\right].\end{aligned} \tag{14}$$

Using (13) and (14),

$$\langle H\frac{\partial P}{\partial t}\rangle \approx \epsilon_0 C_1 e^2(t) + \epsilon_0 C_2 e(t)\frac{\partial e(t)}{\partial t} + \epsilon_0 C_3\left[\frac{\partial e(t)}{\partial t}\right]^2, \tag{15}$$

with

$$\begin{aligned}C_1 &\equiv \frac{\omega_0}{2}\left[u'(\omega_0)\chi_E''(\omega_0) - u''(\omega_0)\chi_E'(\omega_0)\right] \\ C_2 &\equiv \frac{1}{2}\left[u'(\omega_0)\left.\frac{\partial(\omega\chi_E')}{\partial\omega}\right|_{\omega_0} + u''(\omega_0)\left.\frac{\partial(\omega\chi_E'')}{\partial\omega}\right|_{\omega_0} - \omega_0\chi_E'(\omega_0)\left.\frac{\partial u'}{\partial\omega}\right|_{\omega_0} - \omega_0\chi_E''(\omega_0)\left.\frac{\partial u''}{\partial\omega}\right|_{\omega_0}\right] \\ C_3 &\equiv \frac{1}{2}\left[\left.\frac{\partial u'}{\partial\omega}\right|_{\omega_0}\left.\frac{\partial(\omega\chi_E'')}{\partial\omega}\right|_{\omega_0} - \left.\frac{\partial u''}{\partial\omega}\right|_{\omega_0}\left.\frac{\partial(\omega\chi_E')}{\partial\omega}\right|_{\omega_0}\right].\end{aligned} \tag{16}$$

We write the scalar form of the magnetization in terms of the electric field, $M(t) = (4\pi)^{-1}\int v(\omega)E(\omega)\exp(-i\omega t)d\omega + c.c.$, with $v = \chi_H\eta^{-1}$. Noting that $M(t)$ is of the form of $P(t)$ with $\chi_E \to v$, and following a similar procedure as that used to obtain (13), we find

$$\begin{aligned}\frac{\partial M}{\partial t} &\approx \omega_0 e(t)\left[v''(\omega_0)\cos(\omega_0 t) - v'(\omega_0)\sin(\omega_0 t)\right] + \frac{\partial e(t)}{\partial t}\left[\left.\frac{\partial(\omega v')}{\partial \omega}\right|_{\omega=\omega_0}\cos(\omega_0 t)\right. \\ &\quad \left. + \left.\frac{\partial(\omega v'')}{\partial \omega}\right|_{\omega=\omega_0}\sin(\omega_0 t)\right]. \end{aligned} \tag{17}$$

Using (10) and (17), we obtain

$$\langle E\frac{\partial M}{\partial t}\rangle \approx e^2(t)D_1 + e(t)\frac{\partial e(t)}{\partial t}D_2, \tag{18}$$

with

$$\begin{aligned} D_1 &\equiv \frac{\omega_0}{2}v'' = \frac{\omega_0}{2}\left[u'(\omega_0)\chi_H''(\omega_0) + u''(\omega_0)\chi_H'(\omega_0)\right] \\ D_2 &\equiv \frac{1}{2}\left.\frac{\partial(\omega v')}{\partial\omega}\right|_{\omega_0} = \frac{1}{2}\left[u'(\omega_0)\left.\frac{\partial(\omega\chi_H')}{\partial\omega}\right|_{\omega_0} - u''(\omega_0)\left.\frac{\partial(\omega\chi_H'')}{\partial\omega}\right|_{\omega_0} + \omega_0\chi_H'(\omega_0)\left.\frac{\partial u'}{\partial\omega}\right|_{\omega_0}\right. \\ &\quad \left. - \omega_0\chi_H''(\omega_0)\left.\frac{\partial u''}{\partial\omega}\right|_{\omega_0}\right]. \end{aligned} \tag{19}$$

Note that the form of (15) and (18) differ because we use the electric field as the basis. We use the normalized time-averaged force density $\langle f_n\rangle \equiv \langle f\rangle c^2$, with c the speed of light in vacuum. From the (12), and using (15) and (18)

$$\langle f_n\rangle = (C_1 + D_1)\,e^2(t) + (C_2 + D_2)\,e(t)\frac{\partial e(t)}{\partial t} + C_3\left[\frac{\partial e(t)}{\partial t}\right]^2, \tag{20}$$

For sinusoidal steady state, from (20) with $e(t) = E_0$, the exact normalized force density becomes

$$\langle f_n\rangle = \frac{E_0^2\omega_0}{2|\eta|^2}\left[\eta'\left(\epsilon'' + \mu''\right) + \eta''\left(\epsilon' - \mu'\right)\right]. \tag{21}$$

The establishment of the signs for the complex wave impedance, η, is dictated by the phase constant $k = k' + ik'' = \pm(\omega/c)\sqrt{(\mu'\epsilon' - \mu''\epsilon'') + i(\mu''\epsilon' + \mu'\epsilon'')}$, with $k'' > 0$ for a passive, lossy medium (with $\epsilon'' > 0$ and $\mu'' > 0$), and $k'' < 0$ for an active medium having gain (with $\epsilon'' < 0$ and/or $\mu'' < 0$): $\exp(ikz) = \exp(ik'z - k''z)$. Without loss of generality, we choose the TE case with $\eta = \omega\mu_0\mu/k$ to provide the sign choices. From (21),

$$\langle f_n\rangle = \frac{E_0^2}{2\mu_0|\mu|^2}\left[k'\left(\mu''\epsilon' + \mu'\epsilon''\right) + k''\left(\mu'^2 - \mu'\epsilon' + \mu''^2 + \mu''\epsilon''\right)\right]. \tag{22}$$

Enforcing the signs for a doubly passive (lossy dielectric constant and permeability) medium ($\langle f_n \rangle_p$) and a doubly active medium with gain ($\langle f_n \rangle_a$) in (22), we have

$$\langle f_n \rangle_p = \frac{E_0^2 f_n^{rs}}{2\mu_0 |\mu|^2} = -\langle f_n \rangle_a, \tag{23}$$

with $f_n^{rs} = k' \left(|\mu''|\epsilon' + \mu'|\epsilon''| \right) + |k''| \left(|\mu|^2 + |\mu''\epsilon''| - \mu'\epsilon' \right)$, where $\{r, s\} \in \{p, n\}$, with r referring to the sign of μ' (p for positive and n for negative) and s indicating the sign of ϵ'. We are primarily concerned with the sign of the force, and this is dictated by the sign of f_n^{rs}. Expanding k^2 and equating the imaginary parts gives $\operatorname{sgn}(k')\operatorname{sgn}(k'') = \operatorname{sgn}(\mu''\epsilon' + \mu'\epsilon'')$, which provides the sign relationships for k' and k'' for all cases but when $\mu'' = \epsilon'' = 0$ or $\mu' = \epsilon' = 0$. For either a doubly passive medium ($\mu'' > 0$ and $\epsilon'' > 0$) or a doubly active medium ($\mu'' < 0$ and $\epsilon'' < 0$), we have $\operatorname{sgn}(k') = \operatorname{sgn}(|\mu''|\epsilon' + \mu'|\epsilon''|)$. Enforcing the correct signs for the constitutive parameters, we find

$$\begin{aligned}
f_n^{pp} = f_n^{nn} \; &= \; |k'| \left(|\mu''\epsilon'| + |\mu'\epsilon''| \right) + |k''| \left(|\mu|^2 + |\mu''\epsilon''| - |\mu'\epsilon'| \right) \\
f_n^{pn} = f_n^{np} \; &= \; \begin{cases} |k'| \left(|\mu''\epsilon'| - |\mu'\epsilon''| \right) + |k''| \left(|\mu|^2 + |\mu''\epsilon''| + |\mu'\epsilon'| \right) & \text{if } |\mu''\epsilon'| > |\mu'\epsilon''| \\ -|k'| \left(|\mu''\epsilon'| - |\mu'\epsilon''| \right) + |k''| \left(|\mu|^2 + |\mu''\epsilon''| + |\mu'\epsilon'| \right) & \text{if } |\mu''\epsilon'| < |\mu'\epsilon''|. \end{cases}
\end{aligned}$$

Because f_n^{rs} remains positive, we conclude from (23) that for a propagating plane wave in a passive medium, the force is always positive. We also note that this is the case for passive materials with a negative refractive index. On the other hand, in an active medium with net gain, the force is negative.

Consider the special case of the force on a gain medium with $\mu = 1$, which is of practical importance at optical frequencies. The phase constant is then $k = k' + ik'' = \pm\omega\sqrt{\epsilon' + i\epsilon''}/c$, giving $\epsilon' + i\epsilon'' = (k'^2 + k''^2 + 2ik'k'')c^2/\omega^2$. From (22), the normalized force density becomes

$$\langle f_n \rangle_{(\mu=1)} = \frac{E_0^2 k''}{2\mu_0^2} \frac{c^2}{\omega^2} \left[k'^2 + k''^2 + \frac{\omega^2}{c^2} \right]. \tag{24}$$

The sign of the force in (24) is dictated by the sign of k'', and the force is negative for a material offering gain.

To evaluate the significance of the force in a gain medium, consider a Gaussian modulation signal given by $e_g(t) \equiv e(t) = \exp\left[-t^2 \left(2\sigma^2 \right)^{-1} \right]$, where we use $\sigma = 2^8\pi$, $\omega_0 = 1$ in (10), and $N = 2^{20}$ sample points over a temporal support of $2^{13}\pi$. A numerical simulation was carried out by first evaluating the electric field in (10). The magnetic field was obtained from (11) using a discrete Fourier transform and by incorporating the frequency-dependent material parameters. Subsequently, the total instantaneous plane wave force was obtained using (12), and this was numerically integrated over time to form $\langle f \rangle$. We describe this evaluation as exact because the accuracy is subject only to numerical precision. The time-average force was also calculated using the analytical model given in (20). To evaluate the coefficients C_2, C_3, and D_2, we used numerical derivatives of $\mu(\omega)$ and $\epsilon(\omega)$.

Figures 1(a) and (b) show the time-averaged normalized electromagnetic plane wave force density calculated using the exact numerical procedure (solid line) and the analytical expression in (20) (circles) for a non-magnetic material with $\mu = 1$ and having overall (a) loss, with permittivity ϵ_{loss}, and (b) gain, with permittivity ϵ_{gain}. We set $\epsilon_{loss} \equiv \epsilon = 1 + 2\left(0.95^2 - \omega^2 - i0.2\omega\right)^{-1}$ for a passive and lossy two-level system and $\epsilon_{gain} \equiv \epsilon = 1 - 0.6\left(1.05^2 - \omega^2 - i0.2\omega\right)^{-1}$ for an active system having overall gain. From Fig. 1, $\langle f_n \rangle$ is positive for the lossy material and negative for the material with gain, throughout the pulse. Notice that the analytical result is in excellent agreement with the numerical data.

In a physical demonstration of a negative force, the gain medium would be excited to create a population inversion, and this system would be illuminated with a probe electromagnetic wave. The force will be in a direction opposite to the Poynting vector for this probe wave. To evaluate the significance of the negative plane wave force in media with gain, we compare the expected positive force for a metal, which has been measured [2], with the estimated negative force in an excited quantum dot medium. We assume an unmodulated plane wave with E_0 V m^{-1} and silver (Ag) at $\lambda = 633$ nm having $\epsilon_{Ag} = -15.89 + i1.08$ [13]. Using (21), we find $\langle f_n \rangle$ for Ag to be $2.67 \times 10^{14} E_0^2$ N m^{-1} s^{-2}. For the gain medium, we assume a mixture of CdSe quantum dots (dot radius of 1.6 nm and $\hbar\gamma = 0.0469\ eV$) in a SiO_2 background, and that some fraction of the quantum dots are excited to provide gain for a signal applied during the lifetime of the excited exciton state. Using ϵ_{QD} defined in Ref. 14, we find $\epsilon_{QDl} = 17.08 + i1.41$ and $\epsilon_{QDg} = 1.96 - i2.07$ for lossy dots (with $\hbar\omega_{ex} = 2.17$ eV) and excited gain dots ($\hbar\omega_{ex} = 2.13$ eV), respectively. With Maxwell Garnett mixing [14], and fill fractions of $x_g = 0.1$ and $x_l = 0.2$ for gain and loss dots, respectively, we find the homogenized dielectric constant of the CdSe/SiO_2 mixture to be $3.51 - i0.25$ at a wavelength of $\lambda = 633$ nm. This results in $\langle f_n \rangle$ for this gain medium, using (21), of $-1.17 \times 10^{12} E_0^2$ N m^{-1} s^{-2}, two orders of magnitude smaller than that for Ag. We note that the force range for optical tweezers is fN through nN [15], suggesting that a negative force in a medium with gain will be easily measurable with use of common laser sources.

Electromagnetic plane wave forces in homogeneous passive materials, even those having a negative refractive index, appear to be positive and in the direction of the incident Poynting vector. However, we find that the force on a material with gain can be negative and in the direction opposite to the Poynting vector. Although we considered simple material models, there appears no reason to believe that other physical material responses will modify this general understanding. The ensemble gain material we considered, with $\epsilon'' < 0$, could be achieved with either spontaneous or stimulated emission. While spontaneous emission from an atom would occur with random wave vector direction, suggesting that the recoil force [16] from a collection of atoms is zero, it is the gain experienced by a incident wave that results in the negative force on the ensemble.

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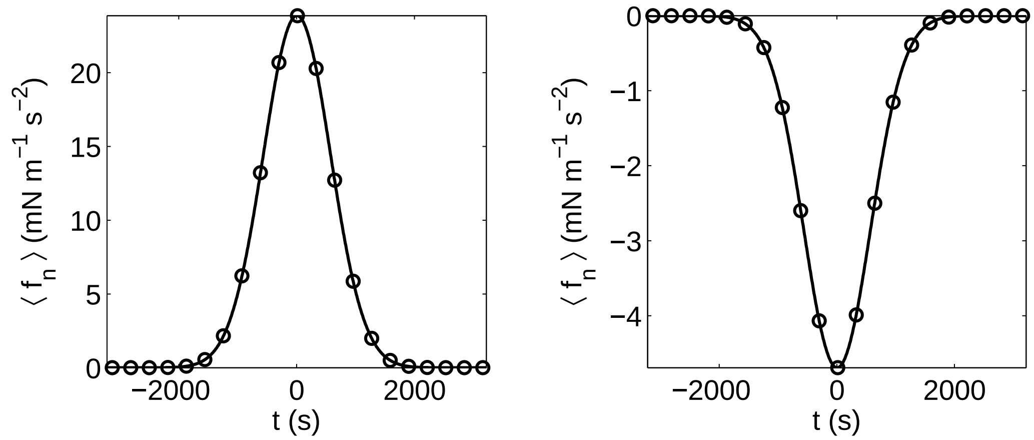

(a) (b)

FIG. 1: The time-averaged normalized electromagnetic plane wave force density calculated using the exact numerical procedure, shown by solid lines, and the analytical expression in (20), shown by circles, for a non-magnetic material with $\mu = 1$ and having overall (a) loss, with permittivity ϵ_{loss} and (b) gain, with permittivity ϵ_{gain}. The electric field modulation is given by $e_g(t)$, and the carrier frequency is $\omega_0 = 1$.