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Emergence of unitary symmetry of microcanonically truncated operators in chaotic quantum systems

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We study statistical properties of matrix elements of observables written in the energy eigenbasis and truncated to small microcanonical windows. We present numerical evidence indicating that for all few body operators in chaotic many-body systems, truncated below certain energy scale, collective statistical properties of matrix elements exhibit emergent unitary symmetry. Namely, we show that below certain scale the spectra of the truncated operators exhibit universal behavior, matching our analytic predictions, which are numerically testable for system sizes beyond exact diagonalization. We discuss operator and system-size dependence of the energy scale of emergent unitary symmetry and put our findings in context of previous works exploring emergence of random-matrix behavior at small energy scales.

Introduction. The eigenstate thermalization hypothesis (ETH) [1, 2] provides a microscopic explanation for the emergence of thermodynamic behavior in isolated quantum systems. In the spirit of random-matrix theory, the ETH asserts individual energy eigenstates with similar energies are physically equivalent. Accordingly, matrix elements of observables O_{mn} written in the energy eigenbasis can be described statistically. Qualitatively this leads to a picture where energy eigenstates confined to a sufficiently narrow microcanonical energy window can be randomly reshuffled without changing the statistics of O_{mn}. This picture of ETH based on typicality was advocated in [3–6].

As a stronger statement, parallel to the Bohigas-Giannoni-Schmit conjecture for the energy spectrum [7], for quantum chaotic systems one may expect that matrix elements O_{mn} at sufficiently small scales exhibit full Random-Matrix-Theory universality. A more careful analysis shows that an *uncorrelated*, i.e. Gaussian RMT description, can only apply at very small scales [8–10], as the correlations between matrix elements are inevitable and important to describe various aspects of quantum dynamics [6, 10–13]. An important step is the evidence that O_{mn} can be described by a rotationally invariant ensemble, i.e. general orthogonal or unitary-invariant RMT, that assumes cross-correlations of O_{mn} [14]. In this Letter, we clarify and further establish this picture by first analytically deriving an infinite set of signatures of unitary/orthogonal invariance, and then by testing them numerically. Crucially, we identify relevant energy scale(s) marking the onset of full or partial unitary symmetry. Our numerical approach is based on quantum typicality, and hence applicable to system sizes substantially beyond the reach of exact diagonalization. We confirm that the behavior is consistent with unitary symmetry for all operators in chaotic quantum systems, while for integrable models these signatures are notably absent [15].

Unitary symmetry of microcanonically-truncated operators. We consider an observable O satisfying ETH, with its smooth diagonal part subtracted $\mathcal{O}_{mn} \equiv O_{mn} - O(E)\delta_{mn}$. It is then projected (microcanonically truncated) onto a narrow energy window of width ΔE,

$$\mathcal{O}_{\Delta E} = P_{\Delta E}\, \mathcal{O}\, P_{\Delta E} \ , \tag{1}$$

where we introduce the projector,

$$P_{\Delta E} = \sum_{|E_m - E_0| < \Delta E/2} |E_m\rangle \langle E_m| \ , \tag{2}$$

and $d = \mathrm{Tr}(P_{\Delta E})$ will denote the number of states within the microcanonical window. We expect that for a sufficiently small scale $\Delta E \leq \Delta E_U$, operator $\mathcal{O}_{\Delta E}$ will exhibits emergent unitary symmetry $U(d)$[16], which will impose constraints on correlations of the matrix elements as well as on the spectrum of $\mathcal{O}_{\Delta E}$ as a function of ΔE. The idea of studying spectral properties of $\mathcal{O}_{\Delta E}$ in conjunction with their ΔE dependence was put forward in [17], with more detailed studies to follow [8–10, 18–20]. A convenient way to probe the spectrum of the microcanonically truncated operator is through its moments

$$\mathcal{M}_k(\Delta E) = \frac{\mathrm{Tr}[(\mathcal{O}_{\Delta E})^k]}{d(\Delta E)} \ , \tag{3}$$

which can be combined into free cumulants $\Delta_k = \Delta_k(\Delta E)$ [5], defined through iterative relation

$$\Delta_k = \mathcal{M}_k - \sum_{j=1}^{k-1} \Delta_j \sum_{a_1 + a_2 + \cdots a_j = k-j} \mathcal{M}_{a_1} \cdots \mathcal{M}_{a_j} \ . \tag{4}$$

We show in the Supplemental Material [15] that unitary symmetry requires $\Delta_k(\Delta E) \propto (d(\Delta E))^{k-1}$. When ΔE_U is much smaller than the effective temperature, which is always the case for spatially-extended systems, density of states within the microcanonical window is approximately constant, $d(\Delta E)/d(\Delta E_U) = \Delta E/\Delta E_U$, leading to

$$\Delta_k(\Delta E) = \lambda_k \, \Delta E^{k-1} \tag{5}$$

for $\Delta E \leq \Delta E_U$. Here, Δ_k is a free cumulant of $\mathcal{O}_{\Delta E}$, and λ_k is an operator and system-specific non-universal coefficient. Relations (5) provide an infinite set of necessary conditions (signatures) of emergent unitary symmetry. We confirm numerically this behavior emerges for all considered operators in a generic nonintegrable quantum spin systems, while also show that in the integrable case this behavior is not present.

In our numerical analysis, we identify ΔE_U with the largest energy window for which (5) holds for all Δ_k that we are able to compute. More generally, one can define a *cascade* of decreasing scales $\Delta E_U^{(\ell)} \geq \Delta E_U^{(\ell+1)}$, where $\Delta E_U^{(\ell)}$ is defined as the maximal window for which (5) holds for all $k \leq \ell$. Thus, ΔE_U is identified with $\Delta E_U^{(\ell)}$ for $\ell \to \infty$.

Emergent unitary symmetry has a clear manifestation at the level of matrix-element correlation functions, captured by the framework of general ETH [5, 21, 22]. The latter proposes that averaged k-th cumulant of $\mathcal{O}_{mn}$ is given by a smooth function f_k of k energies E_i. Emergent unitary symmetry predicts that f_k will be constant [21], which will apply when all E_i are within a narrow energy window of size $\Delta E_U^{(k)}$. For $k = 2$ this is the condition that function $f^2(\omega) \equiv f_2(\omega)$ entering the ETH ansatz for $m \neq n$,

$$\mathcal{O}_{mn} = e^{-S/2} f(\omega) r_{mn}, \; \omega = (E_m - E_n), \; \overline{r^2} = 1,$$

for $|\omega| \leq \Delta E_U^{(2)}$ is constant. We can readily identify $\Delta E_U^{(2)} \equiv \Delta E_T$ with inverse thermalization timescale of $\mathcal{O}$, defined as the size of the short-frequency plateau of $f^2(\omega)$. We provide more technical details connecting our results with the framework of general ETH in Supplemental Material [15].

For any $\Delta E \leq \Delta E_U$ unitary symmetry fixes the spectrum of $\mathcal{O}_{\Delta E}$, and the collective statistics of its matrix elements, in terms of the spectrum of $\mathcal{O}_U \equiv \mathcal{O}_{\Delta E_U}$. In particular, for $\Delta E \leq \Delta E_U$, $\mathcal{O}_{\Delta E}$ should in principle admit a description in terms of a rotational-invariant random-matrix model [14], with the parameters of the model fine-tuned to match the spectrum of $\mathcal{O}_U$. We emphasize, matrix elements within the microcanonical window ΔE_U will show non-trivial correlations constrained only by unitary symmetry. But when truncated to much more narrow windows $\Delta E \ll \Delta E_U$ these correlations will gradually disappear. As follows from (5), for $k > 2$, $\Delta_k/\Delta_2^{k/2} \propto (\Delta E/\Delta E_U)^{(k/2-1)} \to 0$, which implies generic random matrix model reduces to the Gaussian random matrix with uncorrelated $\mathcal{O}_{mn}$ and vanishing cumulants Δ_k. The onset of *Gaussian RMT*, and corresponding scale ΔE_{GUE}, were previously scrutinized numerically in [8–10].

Before embarking on concrete models and numerics, it is natural to ask how ΔE_U would depend on the system size and the type of operator considered. Since the emergent unitary symmetry reshuffles energy *eigenstates*, it is tempting to identify ΔE_U with the Thouless energy scale ΔE_{Th} which marks the onset of random-matrix statistics of energy levels, i.e., emergent unitary symmetry reshuffling energy *eigenvalues*. To complete this proposal, we take into account that Thouless energy controls thermalization time of slowest transport mode present in the system [23–31]. Hence, for a general operator Thouless scale would coincide with inverse thermalization timescale ΔE_T. Given that ω-independence of $\overline{O_{mn}^2}$ is a prerequisite for unitary symmetry, this proposal seems natural. For a generic operator it ties Thouless energy with inverse thermalization time and the scale of unitary symmetry, $\Delta E_{Th} \sim \Delta E_T \equiv \Delta E_U^{(2)} \sim \Delta E_U = \Delta E_U^{(k)}$, $k \to \infty$.

This picture was first outlined in [3] but it can not be correct in general. As we discussed above, Gaussian random-matrix scale ΔE_{GUE} is expected to be smaller, but *not parametrically* smaller than ΔE_U. At the same time, in one-dimensional systems of length L, ΔE_{GUE} has to be *parametrically* smaller than ΔE_T, specifically $\Delta E_{GUE} \propto \Delta E_T/L$ [10], which contradicts $\Delta E_U \sim \Delta E_T$. This contradiction is further elaborated in Supplemental Material [15], where we provide accurate definitions for all scales. The important point is the cascade of scales

$$\Delta E_{Th} \sim \Delta E_U^{(2)} \geq \cdots \geq \Delta E_U^{(k)} \geq \cdots \geq \Delta E_U \geq \Delta E_{GUE},$$

with the *parametric* difference between $\Delta E_{Th} \sim \Delta E_U^{(2)}$ and ΔE_U. We leave the question of systematically understanding system-size dependence of $\Delta E_U^{(k)}$ for the future, but note that ΔE_U evaluated numerically below vary significantly for different operators.

Models and Observables. We proceed to study $\Delta_k(\Delta E)$ numerically in a chaotic many-body quantum system, where we consider a one-dimensional Ising model with transverse and longitudinal fields,

$$H = \sum_{\ell=1}^{L} \left(g\sigma_x^\ell + h\sigma_z^\ell + J\sigma_z^\ell \sigma_z^{\ell+1} \right) . \tag{6}$$

$\sigma_{x,z}^\ell$ are Pauli spin operators at site ℓ and periodic boundary conditions are employed, $\sigma_{x,z}^{L+1} \equiv \sigma_{x,z}^1$. We set $J = 1$ and choose the fields as $g = 1$ and $h = 0.5$ for which H is chaotic and expected to fulfill the ETH [32, 33]. To break the translational and reflection symmetries of H, we further add to H two defect terms $h_d \sigma_z^{\lfloor \frac{L}{3} \rfloor}$ and $-h_d \sigma_z^{\lfloor \frac{2L}{3} \rfloor}$

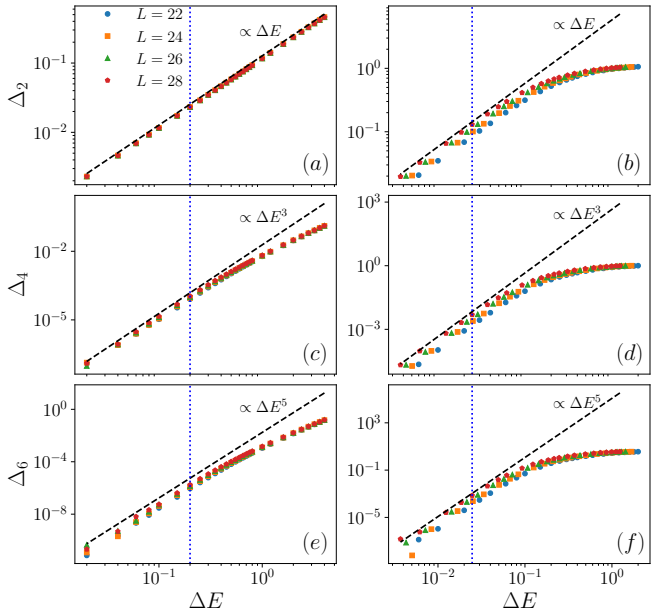


FIG. 1. Even cumulants Δ_k for $k = 2, 4, 6$ as a function of ΔE for the density-wave operator $\mathcal{A}_q$ with the wave-number $q = L/2$ (panels [(a),(c),(e)]) and with $q = 1$ (panels [(b),(d),(f)]). Data are shown for different system sizes $L = 22, 24, 26, 28$. As a guide to the eye, the inclined dashed lines (black) and vertical dotted lines (blue) indicate the theoretically predicted slope $\ln \Delta_k = (k-1) \ln \Delta E + \text{const}$, and an approximate location of ΔE_U, respectively.

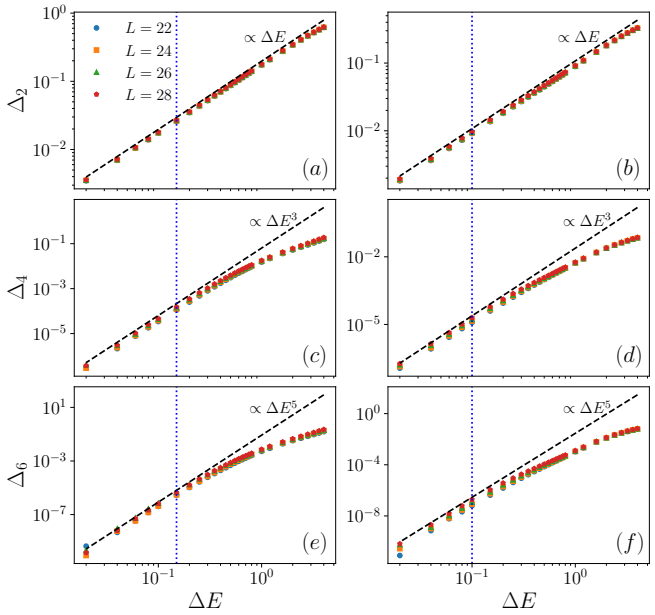


FIG. 2. Similar to Fig. 1, the results for density-wave operator $\mathcal{B}_q$ with $q = \frac{L}{2}$ (panels [(a),(c),(e)]) and $q = 1$ (panels [(b),(d),(f)]).

with $h_d = 0.02775$. Our numerical simulations are thus performed in the full Hilbert space of dimension $D = 2^L$. We study density-wave operators of the form,

$$O_q = \frac{1}{\sqrt{L}} \sum_{\ell=1}^{L} \cos\left(\frac{2\pi}{L} q\ell\right) O^\ell \ , \tag{7}$$

where we consider different momenta $q = 0, 1, L/2$ and two different local O^ℓ, i.e.,

$$\begin{aligned} \mathcal{A}^\ell &= \frac{h}{2}(\sigma_x^\ell + \sigma_x^{\ell+1}) + \frac{g}{2}(\sigma_z^\ell + \sigma_z^{\ell+1}) + J\sigma_z^\ell \sigma_z^{\ell+1} \ , \\ \mathcal{B}^\ell &= \frac{g\sigma_z^\ell - h\sigma_x^\ell}{\sqrt{g^2+h^2}} \ , \end{aligned} \tag{8}$$

with $\mathcal{A}^\ell$ being the energy density and $\mathcal{B}^\ell$ being constructed to have a small overlap with H [17]. Both operators behave in agreement with the usual indicators of the ETH [9, 17] and we denote the corresponding density-wave operators by $\mathcal{A}_q$ and $\mathcal{B}_q$. Numerical data for the local operators $\mathcal{A}^\ell, \mathcal{B}^\ell$ can be found in [15].

Numerical approach. While studying the properties of $\mathcal{O}_{\Delta E}$ [34] would normally require full exact diagonalization (ED), we can go beyond system sizes accessible to standard ED to compute the moments $\mathcal{M}_k(\Delta E)$ (3).

To evaluate moments, we exploit a pure-state technique based on the concept of quantum typicality [35, 36] to compute the moments of $\mathcal{O}_{\Delta E}$ (see also Refs. [9] and [15] for more details). A key idea within this approach is to realize that the energy filter $P_{\Delta E}$ in Eq. (2) can be expanded as [9], $P_{\Delta E} |\psi\rangle \simeq \sum_{i=0}^{N_{\rm tr}} C_i T_i(\frac{H-b}{a}) |\psi\rangle$, where $T_i(x)$ are Chebyshev polynomials of the first kind, C_i are appropriately chosen coefficients that encode the energy window ΔE (see [9, 15]), and $a = (E_{\rm max} - E_{\rm min})/2$, $b = (E_{\rm max} + E_{\rm min})/2$, where $E_{\rm max(min)}$ are the extremal eigenvalues of H. Moreover, the expansion order $N_{\rm tr}$ has to be chosen large enough to yield accurate results. Given $P_{\Delta E}$, the trace in Eq. (3) can then be approximated by expectation values with respect to random pure states, e.g., for $\mathcal{M}_k$ we have $\mathcal{M}_k \approx \frac{\langle\psi|(P_{\Delta E}\mathcal{O}P_{\Delta E})^k|\psi\rangle}{\langle\psi|P_{\Delta E}|\psi\rangle}$, with $|\psi\rangle$ being a Haar-random state constructed in the computational basis. According to quantum typicality, the accuracy of this approximation improves with the number of states d inside the energy window. The accuracy can be further improved by averaging over different realization of random states. By applying $P_{\Delta E}$ efficiently using sparse matrices, we are able to study the free cumulants Δ_k for systems up to $L = 28$.

Results. In Fig. 1, we show cumulants Δ_k as a function of energy window width ΔE, for the energy density-wave operators $\mathcal{A}_q$ for two different wave numbers $q = L/2$ and $q = 1$. Note that for the density-wave operators with $q > 0$ all odd cumulants approximately vanish (for $q = 0$, $\mathcal{A}_q$ is the Hamiltonian apart from the small defect), hence we only show results for even ones $\Delta_{2,4,6}$. In Fig. 1, we observe $\Delta_k \propto \Delta E^{k-1}$ behavior of Eq. (5) at sufficiently small energy scales, $\Delta E \leq \Delta E_U$, indicating the onset of emergent unitary symmetry. The

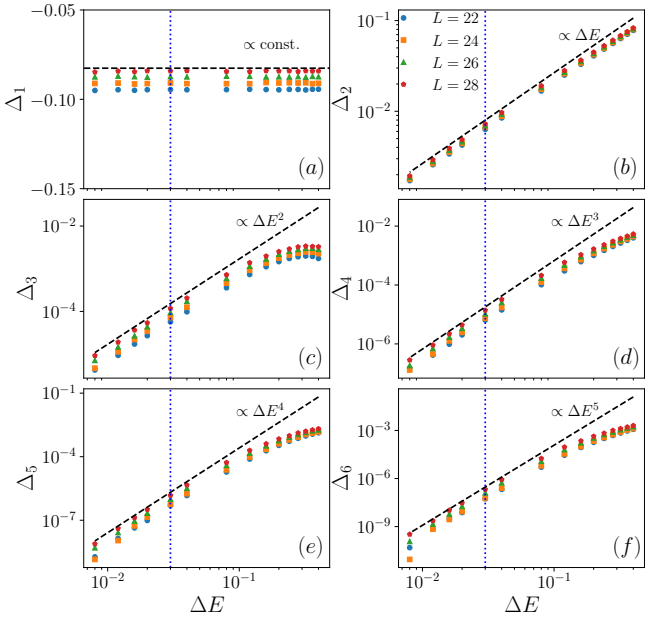


FIG. 3. Similar to Figs. 1, the results for cumulants Δ_k for $k \leq 6$ as a function of ΔE for the density-wave operator $\mathcal{B}_q$ with $q = 0$.

deviations from power-law behavior at extremely small ΔE are due to numerical errors aggravated by small number of states within such energy windows. A very similar picture emerges for the operators $\mathcal{B}_q$ with $q = L/2, 1$ in Fig. 2, with the behavior (5) clearly visible below certain energy scale. We further observe the predicted power-law behavior for $\mathcal{B}_q$ with $q = 0$ in Fig. 3, where we consider $k \leq 6$ including odd orders. Our results clearly support emergence of unitary symmetry for all operators considered. In the Supplemental material [15], we analyze local operators $\mathcal{A}^\ell, \mathcal{B}^\ell$ and show existence of ΔE_U, and expected power-law behavior below this scale, in these cases as well.

Combined with our theoretical results (5), Figs. 1 - 3 represent main contribution of this work. Our results demonstrate existence of scale ΔE_U below which the statistical properties of matrix elements $\mathcal{O}_{mn}$ exhibit RMT universality while still admitting non-trivial correlations. Furthermore, scaling behavior (5), confirmed in Figs. 1 - 3, implies $\Delta_k/(\Delta_2)^{k/2} \to 0$ when $\Delta E \to 0$ (up to statistical fluctuations), which confirms that $\mathcal{O}_{\Delta E}$ approaches a Gaussian random matrix at even smaller scales.

Comparing results of Δ_k for different k, we find that in most cases, the size of the energy window marking the onset of $\Delta_k \propto (\Delta E)^{k-1}$ behavior is similar for higher cumulants with $k > 2$, but typically much smaller than the region of linear growth of Δ_2 (the operator $\mathcal{A}_1$ is an exception, see Fig. 1 (b),(d),(f)). This difference between $\Delta E_U^{(2)}$ and $\Delta E_U^{(k)}$ for $k > 2$ could be related to the fact that correlations between matrix elements have a fingerprint only in higher cumulants $\Delta_{k>2}$. Our data thus suggest that ΔE_U (vertical dashed line in Figs. 1 - 3) in most cases is much smaller than inverse thermalization time $\Delta E_U^{(2)} \equiv \Delta E_T$, tentatively suggesting a scenario when in the thermodynamic limit $\Delta E_U^{(2)} \ll \Delta E_U^{(3)} \sim \cdots \sim \Delta E_U$. A conclusive analysis of L-dependence of $\Delta E_U^{(k)}$ would likely require system sizes much larger than numerically available. Available data in Figs. 1 - 3 (as well as additional data in the Supplemental Material [15]) only indicates that ΔE_U decreases for larger system sizes.

Conclusion & Outlook. In this paper we have introduced a novel energy scale ΔE_U marking the onset of emergent unitary symmetry, i.e., unitary random matrix theory universality of $\mathcal{O}_{mn}$ – matrix elements entering the eigenstate thermalization hypothesis. The scale is operator-specific. For $|E_n - E_m| \leq \Delta E_U$, matrix elements exhibit unitary symmetry, which governs their collective statistical properties. This is to say, below this scale $\mathcal{O}_{mn}$ can be described in terms of a generic RMT. To probe this scale we considered a truncation of an operator $\mathcal{O}$ to microcanonical energy window of size ΔE, defined in (1). We have shown that emergent unitary symmetry for $\Delta E \leq \Delta E_U$ is manifest through a simple power law ΔE dependence of free cumulants Δ_k (4),

$$\Delta_k(\Delta E) \propto (\Delta E)^{k-1} \,. \tag{9}$$

This provides a set of readily testable criteria, which can be accessed numerically beyond exact diagonalization.

We tested this behavior numerically for different operators satisfying ETH in the case of a generic quantum chaotic spin chain, and in each case found explicit evidence of (9). Corresponding values of ΔE_U marking the onset of this behavior vary significantly for different operators, as well as the ratio $\Delta E_U/\Delta E_T$, where ΔE_T is operator's inverse thermalization time. Our finite-size scaling analysis is not conclusive but consistent with theoretical expectation that in the large system size limit energy scales form the hierarchy $\Delta E_{GUE} \leq \Delta E_U \ll \Delta E_T$, see [15] for details. In particular when $L \to \infty$, for the considered one-dimensional system we expect both ΔE_{GUE} and ΔE_U to exhibit the same system-size dependence and be parametrically smaller than ΔE_T.

Our results provide a unifying picture, connecting spectral properties of microcanonically-truncated operators [8–10, 17–20] and correlations between off-diagonal matrix elements [6, 11–14, 37–42]. An implicit question underlying these studies is to identify the degree of universality exhibited by the off-diagonal matrix elements of a typical operator in a generic quantum system. An analogous question about universality of energy spectrum of quantum chaotic systems is answered by the famous Bohigas-Giannoni-Schmit conjecture [7], which postulates random matrix universality (with global symmetries of the matrix model matching those of the original system). For the off-diagonal matrix elements, a conceptually similar proposal is given by a general

random matrix theory. The question of universality was recently investigated for holographic models of quantum gravity [40–42], yielding a multi-matrix model description specific for those cases. Our work, which studies a generic quantum chaotic model, provides evidence that emergent unitary symmetry is indeed the universal description for all operators satisfying ETH, with an operator-specific validity range ΔE_U. This picture constitutes a compelling analog of the Bohigas-Giannoni-Schmit conjecture, and deserves further investigation. A natural question would be to repeat our analysis for different types of quantum chaotic systems, including time-dependent Floquet models without energy conservation, as well as semi-classical few-body systems with a chaotic classical counterpart. We also emphasize the question of understanding relative scaling of ΔE_U, ΔE_T and ΔE_{Th} as a key to unite emergent unitary symmetry of this paper with the RMT universality of energy spectrum in one comprehensive framework of quantum chaos.

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[16] Unitary symmetry can be understood as rotational invariance of the probablity measure $P(\mathcal{O}_{mn}) = P(\mathcal{O}'_{mn})$, $\mathcal{O}' = U\mathcal{O}U^\dagger$. The type of symmetry depends on global symmetries of H and O. When both exhibit time-reversal symmetry, *unitary* symmetry $U(d)$ should be substituted by *orthogonal* symmetry. Nevertheless we use the notations ΔE_U (and ΔE_{GUE} for corresponding Gaussian ensemble) throughout the paper for the scales marking the onset of corresponding regimes.
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[34] Although in principle one needs to consider an operator O with its smooth diagonal part (in the energy eigenbasis) subtracted, for small ΔE, the contribution of the non-trivial part of $O(E)\delta_{mn}$ to the cumulants Δ_k will be suppressed by the system size. Thus, in our numerical analysis we use the original observable O.

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