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# Thermal Conductivity of a Laser Plasma

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We present a model of the electron thermal conductivity of a laser-produced plasma. **The model, supported by Vlasov–Fokker–Planck simulations, predicts that laser absorption** reduces conductivity by forcing electrons out of a **Maxwell–Boltzmann** equilibrium, which results in depletion of both low-velocity bulk electrons and high-velocity tail electrons. **We show that both the bulk and tail electrons approximately follow super-Gaussian distributions, but with distinct exponents that each depend on the laser intensity and wavelength through the parameter  $\alpha = Zv_E^2/v_T^2$ .** For a value of  $\alpha = 0.5$ , **tail depletion reduces the** thermal conductivity to half its zero-intensity value. We present our results as simple analytic fits that can be readily implemented in any radiation-hydrodynamics code **or used to correct the local limit of non-local conduction models.**

Thermal conduction is a key component of the energy balance and transport of laser-produced plasmas, especially those produced in high-energy-density experiments such as high-performance inertial confinement fusion (ICF) implosions[1]. In laser direct drive (LDD) experiments, thermal conduction is responsible for converting laser energy absorbed in the corona into ablation pressure, providing the drive for a spherical implosion[2]. The more conductive the coronal plasma, the better the coupling between the laser and the target, which is beneficial for fusion performance. In laser indirect drive (LID), the laser illuminates the interior wall of a gas-filled hohlraum, which emits x rays that drive the capsule. The radiation transport properties throughout the hohlraum and blowoff plasma are sensitive to the temperature profile and thus to the thermal conductivity of the plasma[3, 4].

The baseline model for thermal conduction in a plasma is that of Spitzer and Härm (SH)[5]. **Not only is this the basic model used in radiation-hydrodynamics simulations, but it is also central to the construction of non-local conduction models[6–8].** The fundamental assumption underpinning this theory is that the electrons are in local thermodynamic equilibrium with a Maxwell–Boltzmann (MB) velocity distribution. However, the inverse bremsstrahlung (IB) absorption of laser light distorts the electron distribution function away from a MB form, a phenomenon sometimes called the “Langdon effect.”[9] In this case, the SH theory no longer applies, **and the local theory of heat conduction as well as non-local models based on SH need to be revised to explicitly account for laser absorption.**

Some theoretical attempts have been made to assess how laser intensity affects heat conduction[10–12], but these have never precipitated a quantitative and practical model to replace the SH theory despite their dramatic predictions: that accounting for laser intensity leads to a reduction in the thermal conductivity by as much as a factor of four to five. The lack of this effect in mainline conduction models means would imply that current

radiation-hydrodynamics simulations of laser-produced plasmas might seriously overestimate the conductivity in regions of high laser intensity, **even before considering possible non-local effects.** In the context of ICF experiments, this region corresponds to the coronal (LDD) or blowoff (LID) plasmas, where changes to the conductivity has compounding effects on the power balance and performance of ICF designs.

In this Letter we resolve this issue by providing an accurate and practical model for the reduced thermal conductivity of a plasma in the presence of laser absorption. Our approach is based on a detailed model of the electron distribution function in the presence of absorption and conduction, supported by Vlasov–Fokker–Planck (VFP) simulations of the relaxation of long-wavelength temperature perturbations in a plasma absorbing laser light with fixed intensity. Electron–ion collisions, electron–electron collisions, and nonlinear absorption (Langdon effect) are all taken into account. The main result is that absorption reduces the thermal conductivity from the SH value according to

$$\frac{\kappa(Z, \alpha)}{\kappa_{SH}(Z)} = \frac{c_0(Z) + c_1(Z)\alpha}{1 + c_2(Z)\alpha} \quad (1)$$

where

$$\kappa_{SH}(Z) = \frac{256\sqrt{2}\pi\epsilon_0^2 k_B^{\frac{7}{2}} T_e^{\frac{5}{2}}}{\sqrt{m_e} Z e^4 \Lambda_{ee}} \frac{Z + 0.24}{Z + 4.20} \quad (2)$$

is the SH conductivity[5],

$$\alpha = Z_{IB} \frac{v_E^2}{v_T^2} \quad (3)$$

is the Langdon parameter, and

$$Z = \frac{\sum_i Z_i^2 n_i \Lambda_{ei}}{\sum_i Z_i n_i \Lambda_{ee}}, \quad Z_{IB} = \frac{\sum_i Z_i^2 n_i \Lambda_{IB,i}}{\sum_i Z_i n_i \Lambda_{ee}} \quad (4)$$

are the effective ion charge numbers for collisions and absorption, respectively. The quantities  $c_0$ ,  $c_1$ , and  $c_2$

are numerical coefficients which depend weakly on  $Z$ . In the above,  $Z_i$ ,  $n_i$ ,  $\Lambda_{ei}$ , and  $\Lambda_{IB,i}$  are the charge state, number density, electron-ion Coulomb logarithm, and IB Coulomb logarithm[13, 14] for ion species  $i$ ,  $\Lambda_{ee}$  is the electron-electron Coulomb logarithm,  $v_T = \sqrt{k_B T_e/m_e}$  is the electron thermal speed for temperature  $T_e$ , and  $v_E = eE_L/(m_e \omega_L)$  is the electron oscillation velocity in a laser with angular frequency  $\omega_L$  and electric field amplitude  $E_L = \sqrt{2I/(c\epsilon_0)}$  for a laser with intensity  $I$ . Fundamental constants appearing in these expressions are the elementary charge  $e$ , the vacuum speed of light  $c$ , the vacuum permittivity  $\epsilon_0$ , and the Boltzmann constant  $k_B$ . The parameter  $Z$  measures the relative strength of electron-ion to electron-electron collisions, which mainly affects the anisotropy of the distribution function responsible for the conduction. The parameter  $\alpha$  measures the relative strength of IB absorption to electron-electron collisions, which mainly affects the shape of the isotropic part of the distribution function. The limit in which  $Z \rightarrow \infty$  at fixed  $\alpha$  corresponds to the Lorentz gas model, on which all prior theoretical work on this problem was based[10–12]. It will be seen that this treatment does capture the overall trend that conductivity reduces as  $\alpha$  increases, but the reduction is significantly overestimated compared to a full kinetic treatment of electron-electron collisions.

The VFP simulations were conducted using the code *K2*, which expands the velocity dependence of the distribution function in spherical harmonics[15]. For the 1-D problems considered here, this is equivalent to a Legendre polynomial expansion,  $f(x, \mathbf{v}, t) = \sum_{\ell=0}^{\infty} f_{\ell}(x, v, t) P_{\ell}(v_x/v)$ , of which we retain only the isotropic component  $f_0$  and flux component  $f_1$ . The two-term truncation is valid provided the temperature perturbation wavelength is much longer than the collision mean free path, otherwise the heat conduction is nonlocal. Convergence to the local limit is straightforward to establish by increasing the background electron density and verifying that the conductivity does not vary.[16] It is also necessary that the laser intensity not be so high that  $v_E > v_T$ , otherwise the laser absorption introduces temperature anisotropy[17, 18].

The coupled VFP equations for the isotropic and anisotropic components of the distribution function are

$$\partial_t f_0 + \frac{v}{3} \partial_x f_1 - \frac{eE}{3m_e v^2} \partial_v (v^2 f_1) = C_0 + C_{IB} \quad (5)$$

$$\partial_t f_1 + v \partial_x f_0 - \frac{eE}{m_e} \partial_v f_0 = -\nu_{ei} f_1 + C_1 \quad (6)$$

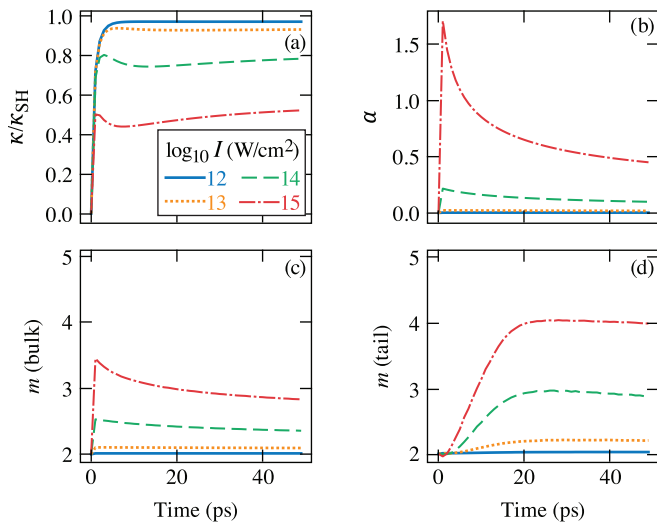
where  $C_0$  and  $C_1$  are respectively the isotropic and anisotropic electron-electron collision operators[19],  $C_{IB}$  is the IB absorption operator[9],  $\nu_{ei}(v) = Ze^4 n_e \Lambda_{ee} / (4\pi \epsilon_0^2 m_e^2 v^3)$  is the velocity-dependent electron-ion collision rate, and  $E$  is the ambipolar

electric field[20]. The Lorentz limit is obtained by excluding  $C_1$  from the simulations. The results shown here were also checked using a more general VFP simulation approach which treats the low- and high-frequency dynamics on equal footing, rather than relying on  $C_{IB}$  to model the absorption[20]. This gives a more accurate account of the IB absorption in principle, but we find for the simple laser fields considered here (uniform and constant) that the results are nearly identical to the conventional approach using  $C_{IB}$ .

The simulations are initialized with a MB distribution at fixed, uniform electron density and a sinusoidally perturbed temperature,  $T_e(x, t=0) = T_0 [1 + 10^{-3} \cos(2\pi x/L)]$ , where  $L = 5\text{mm}$  is the domain size and  $T_0 = 200\text{eV}$  is the initial electron temperature. The electron density ranged from 0.1 to 1 times the critical density for 0.35-micron light as needed to ensure the long-wavelength limit was realized. The choice of a sinusoidal perturbation allows for periodic boundary conditions and facilitates using Fourier analysis to separate the evolution of the mean temperature from the fluctuating one. The laser intensity ramps up from zero to a uniform constant value over the first 100 fs of the simulation. As the simulation progresses, the mean temperature increases monotonically due to IB absorption, while the temperature fluctuation relaxes due to heat conduction. The instantaneous thermal conductivity is extracted from Fourier analysis of the  $k = 2\pi/L$  fundamental mode:  $\kappa = \text{Re}\{iQ_k/(kT_k)\}$ , where  $Q_k$  is the Fourier component of the heat flux,  $Q = 2\pi m_e/3 \int_0^{\infty} v^5 f_1 dv$ , and  $T_k$  is the Fourier component of the temperature fluctuation.

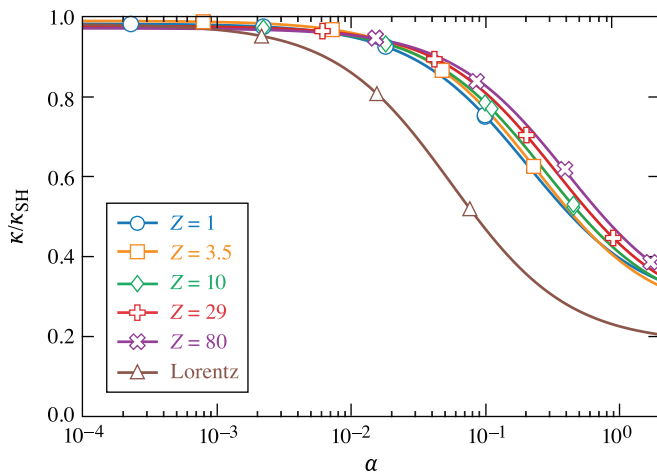
The thermal conductivity obtained in this way varies in time because the mean temperature increases monotonically due to absorption. Consequently, the value of  $\alpha$  also varies in time, as does the shape of  $f_0$ . The time evolution for a typical case is shown in Fig. 1, which shows the instantaneous values of  $\kappa/\kappa_{SH}$ ,  $\alpha$ , as well as super-Gaussian exponents characterizing the shape of  $f_0$  (discussed further below). The thermal conductivity is only formally meaningful if the simulations have reached the hydrodynamic stage of evolution, meaning the time- and space-variation of the distribution function is totally implicit in the time- and space-variation of the temperature[21]. It is difficult to rigorously identify whether or not this is the case in the VFP simulations. Nevertheless, Fig. 1 suggests two clear regimes: a transient relaxation up to about 10 ps and a quasi-steady evolution from about 20 ps onward. Conservatively, we take only the last values of  $\alpha$  and  $\kappa/\kappa_{SH}$  from each simulation to construct our model.

The results are collected in Fig. 2, which shows the reduction in thermal conductivity as a function of  $\alpha$  for various values of  $Z$ . Each data point is the final instantaneous values of  $\kappa/\kappa_{SH}$  and  $\alpha$  obtained from a different value of the laser intensity ( $I = 0, 10^{12}, 10^{13}, 10^{14}, 10^{15}$



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FIG. 1. Time evolution of a  $Z = 10$  plasma at various laser intensities. (a) conductivity reduction factor. (b) Langdon parameter. (c) fitted super-Gaussian exponent for bulk electrons. (d) fitted super-Gaussian exponent of tail electrons.



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FIG. 2. Ratio of the thermal conductivity to the SH value as a function of Langdon parameter  $\alpha$  for various ion charge numbers  $Z$ , as well as for a Lorentz gas. Symbols are the results of VFP simulations and lines are interpolation formulas.

W/cm<sup>2</sup>). For each  $Z$ , the results are fitted to Eq. (1) and shown as solid lines; the resulting fit coefficients are given in Table I. Note that  $c_0$  should in principle be unity, but we find it is systematically a few percent less, indicating that the zero-intensity thermal conductivity from VFP is slightly less than the SH value.

At nonzero intensity, the thermal conductivity is substantially reduced from the SH value. Even at small values of the Langdon parameter,  $\alpha = 0.1$ , the reduction is already about 25%, while at  $\alpha = 1$ , the reduction is almost 60%. This reduction exhibits only a weak dependence on  $Z$ , indicating that the reshaping of  $f_0$  due

	$Z = 1$	$Z = 3.5$	$Z = 10$	$Z = 29$	$Z = 80$	Lorentz
$c_0$	0.982	0.989	0.977	0.976	0.971	0.981
$c_1$	1.332	1.038	0.943	0.739	0.615	3.278
$c_2$	4.862	4.214	3.685	2.984	2.483	17.94

TABLE I. Fit coefficients for the conductivity reduction factor in Eq. (1).

to IB and the reshaping of  $f_1$  due to electron–electron collisions are essentially separable effects. It is mainly the deformation of  $f_0$  that causes the conductivity reduction, while the effect of electron–electron collisions on  $f_1$  is not strongly intensity-dependent. This insensitivity to  $Z$  might lead one to think that the Lorentz gas model should be adequate to describe the thermal conductivity reduction; however, this turns out not to be the case. VFP results for the Lorentz gas severely overestimate the reduction in thermal conductivity. In the absence of a laser, the Lorentz gas model corresponds to the  $Z \rightarrow \infty$  limit of the Spitzer–Härm conductivity, but this relationship no longer holds at nonzero intensity. In fact, when electron–electron collisions are fully accounted for, the trend is that at fixed  $\alpha$ , higher- $Z$  plasmas have a conductivity that is slightly closer to the SH value.

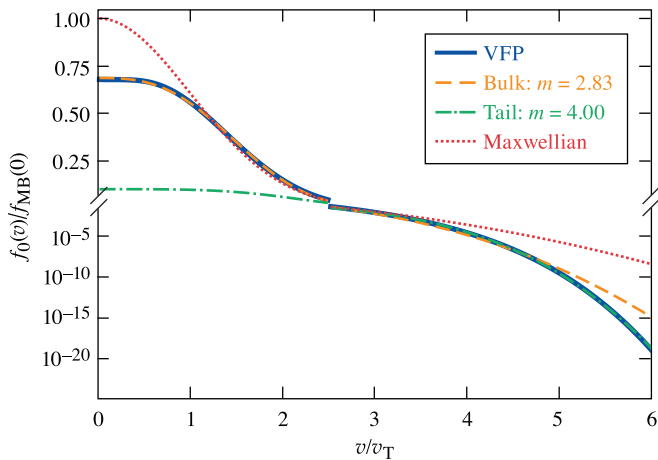
To better understand the failure of the Lorentz gas model, it is instructive to connect with the theory of Mora and Yahi[10]. The Mora–Yahi theory takes  $f_1$  in steady state and neglects  $C_1$  to obtain

$$f_1 = -\lambda_{ei} \left[ \partial_x f_0 - \frac{eE}{mv} \partial_v f_0 \right] \quad (7)$$

where  $\lambda_{ei} = v/\nu_{ei}$  is the mean free path. They consider  $f_0$  to have a super-Gaussian form,  $f_0(v) \propto \exp[-(v/v_m)^m]$ , where the exponent,  $m$ , is assumed to be known. The thermal conductivity may then be determined analytically as a function of  $m$ , with  $m = 2$  recovering the Lorentz-gas limit of SH theory.

To connect with the VFP results, we must determine the value of  $m$  that best characterizes the shape of  $f_0$  in the velocity range relevant to conduction. We emphasize the importance of choosing the appropriate velocity range because this has been a point of confusion in the past[11]. In the literature on IB absorption, there is both simulation and experimental support for relating the exponent to  $\alpha$  as  $m_{IB}(\alpha) = 2 + 3/(1 + 1.66/\alpha^{0.724})$ , as proposed by Matte *et al.*[13, 22–24]. However, IB absorption is sensitive only to the bulk electrons with  $v \lesssim 3v_T$ , and accordingly we find that this formula only describes the low-velocity population in our VFP simulations.

An illustrative example is shown in Fig. 3, which shows  $f_0$  from VFP simulations, as well as super-Gaussian fits to the bulk and tail electron populations. Here, “bulk” refers to electrons with  $v < v_*$  and “tail” to electrons with  $v > v_*$ , where  $v_*$  is the velocity where  $f_1(v_*) = 0$ . That is, the tail electrons are those flowing down the temperature gradient and contributing positively to the thermal



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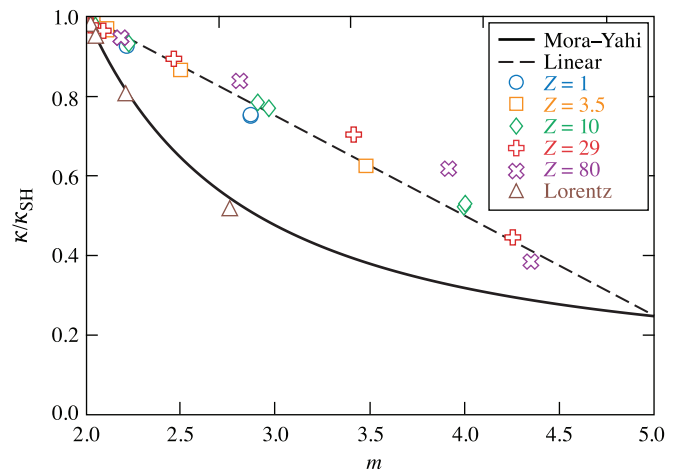
FIG. 3. Isotropic component of the velocity distribution function for a  $Z = 10$  plasma with  $\alpha = 0.45$ . Solid line is the VFP result, dashed line is a super-Gaussian fit to the bulk electrons, dashed-dotted line is a super-Gaussian fit to the tail electrons, and dotted line is a Maxwellian distribution. Note the transition from linear to logarithmic scale at  $v = v_*$ .

conductivity, while the bulk electrons are those which form the neutralizing return current and contribute negatively to the thermal conductivity. The bulk and tail are characterized by different super-Gaussian exponents. The bulk agrees well with the Matte *et al.* model, which gives  $m_{IB}(0.45) = 2.76$  in the case shown in Fig. 3. This implies that the collisional bulk electrons are insensitive to the presence of heat flow, and their energy distribution is described well with standard IB theory. The tail, however, is more sharply non-Maxwellian, with  $m = 4$  being the best-fit value. This implies that the tail electron population is more depleted (and the thermal conductivity reduction more severe) than one would predict using  $m_{IB}$ . Evidently, a separate model for the tail electron distribution is needed.

Using the tail exponent fitted from each VFP simulation, the Mora–Yahi theory is evaluated and compared to VFP simulation result in Fig. 4, which shows the thermal conductivity reduction as a function of the tail exponent. Strikingly, all the VFP simulations which properly account for electron–electron collisions on  $f_1$  collapse in a way that suggests that  $\kappa/\kappa_{SH}$  is a universal function of  $m$ . Though  $m$  generally depends on both  $Z$  and  $\alpha$ , the  $Z$  dependence is weak, and we find that a  $Z$ -independent expression of the same form used for IB,

$$m(\alpha) = 2 + 3/(1 + 0.247/\alpha^{0.972}), \quad (8)$$

approximates the tail exponent of the VFP simulations at any  $Z$  within 3% when  $\alpha < 1$ . The Lorentz-limit VFP results follow a different trend entirely but are in excellent agreement with the Mora–Yahi prediction. An interesting finding is that though the VFP results do not reach  $m = 5$ , they do appear to trend toward the  $m = 5$



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FIG. 4. Ratio of the thermal conductivity to the SH value as a function of super-Gaussian exponent of tail electrons. Symbols are VFP results for various charge numbers  $Z$ , as well as for a Lorentz gas. The solid curve is the Mora–Yahi theory.

Mora–Yahi prediction. A simple linear fit connecting the  $m = 2$  and  $m = 5$  limits

$$\frac{\kappa(m)}{\kappa_{SH}} = 1 - 0.251(m - 2) \quad (9)$$

gives remarkably good agreement with the VFP results, the errors for all but one case being within 8%. [25] Note, however, there is some compounding of errors when Eqs. (8) and (9) are used together, so that applications requiring accuracy better than a few percent might prefer to interpolate the coefficients in Table I for use in Eq. (1).

In the theory of IB,  $m = 5$  is an upper limit obtained when  $C_0$  is neglected [9]. The trend of VFP toward Mora and Yahi’s  $m = 5$  prediction indicates that the Lorentz limit is only realized when *all* electron–electron collisions are neglected, not just those acting on  $f_1$ . One important implication for VFP [15, 26–28] and simplified kinetic models [6, 29–31] of laser plasmas is that one cannot neglect  $C_1$  unless one is also willing to neglect  $C_0$ . This goes against a widespread practice to neglect  $C_1$  and compensate by rescaling  $\nu_{ei}$  to recover the SH conductivity in the local transport limit. This is done because  $C_1$  is cumbersome to invert, making it both a computational bottleneck for VFP codes and inconvenient for analytic theory. This procedure, while tempting, totally misrepresents the effect of IB absorption on the heat flow and produces the wrong local transport limit at non-zero intensity.

In summary, we have determined the thermal conductivity and equilibrium distribution function of a plasma heated by a laser using VFP simulations. In doing so, we have shown the importance of retaining an accurate and fully kinetic model of absorption and electron–electron collisions. This allowed us to identify important short-

comings of previous attempts to determine the thermal conductivity of a laser plasma. Specifically, in this work, the Lorentz approximation was eschewed and no functional form for the isotropic distribution function was assumed. The results can be represented analytically in a simple form which can be straightforwardly implemented in radiation-hydrodynamics codes **and non-local conduction models** as an intensity-dependent correction to the SH conductivity. The impact of this reduced conductivity on coupling efficiency in ICF experimental design will require thorough study and be the subject of future work.

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