Impact of food distribution on lifetime of a forager with or without sense of smell

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Modeling foraging via basic models is a problem that has been recently investigated from several points of view. However, understanding the effect of the spatial distribution of food on the lifetime of a forager has not been achieved yet. We explore here how the distribution of food in space affects the forager's lifetime in several different scenarios. We analyze a random forager and a smelling forager in both one and two dimensions. We first consider a general food distribution, and then analyze in detail specific distributions including constant distance between food, certain probability of existence of food at each site, and power-law distribution of distances between food. For a forager in one dimension without smell we find analytically the lifetime, and for a forager with smell we find the condition for immortality. In two dimensions we find based on analytical considerations that the lifetime (T) scales with the starving time (S) and food density (f) as $T \sim S^4 f^{3/2}$.

I. INTRODUCTION

Optimization of foraging for food spread in space is a problem that has been widely studied [1–4]. Many studies claim that stochastic search yields optimal results [5–7] and that random walks or Lévy flights can be used to model the forager behavior [8–12]. Several models for a forager's movement behavior have been proposed including some that are based on stimuli, memory, and cues from fellow foragers [4, 13–15].

Recent work has suggested a new model where a forager performs a random walk, however the food is explicitly consumed until the forager starves to death [16, 17]. In this model, the forager begins at some point on a lattice where each site contains a unit of food. The forager then moves and eats the food at the discovered site, leaving no remaining food in this site. It continues to move throughout the region either returning to sites without food or eating food at newly visited sites. If the forager walks S steps without finding food and eating, it starves to death. Notably, this process leads to inherent desertification [18, 19], as the forager eventually creates a desert of visited sites among which it could move until starvation. It was shown [16] that the lifetime of a forager, T, in 1D scales linearly with its starving time, S, the number of steps it can walk without food. In 2D, however, it scales approximately as $T \sim S^2$. Later work expanded this model to cases where the food renews after some time [20], where the forager eats only if it is near starvation [21, 22], and where the forager walks preferentially in the direction of a nearby site with food [23, 24].

Another recent study [25] has extended the starving forager models to a forager with an explicit sense of *smell* that extends to potentially longer ranges [26]. The contribution of an individual food site to the overall smell in a given direction decays with its distance d from the forager. While actual patterns of odor diffusion are turbulent and vary in time in highly complex ways [27], it can assumed to be simplified by considering two realistic cases: power-law decay with distance and exponential decay of smell. Power-laws have been suggested in the context of odor regarding

perceptions and concentrations, see for example the Steven's Law [28, 29]. Similarly, in real applications, guidelines for setback distances for odorous agricultural factories, have been applied based on a power-law decay of concentration with distance [30]. We note also that both power-law and exponential decay have been experimentally found for on odor patterns depending on the type of wind, threshold concentration, and other conditions [27]. We therefore consider in this paper both cases in order to better understand the possible behavior of our model. We assume here that the likelihood of the forager to walk in each direction is proportional to the total smell in that direction. In Sanhedrai et al [25] it was shown that when there is a long range smell in 1D, then under some conditions the probability to live forever exists. This study [25] as all above studies, considered the case where initially all the space is full of food.

In contrast to previous studies that analyzed lattices in which food was at every site, here we analyze, for the first time to the best of our knowledge, the more realistic scenario where food is not everywhere. We consider a general food distribution in space and also analyzed in detail several specific distributions, and examine how different distributions impact the life time of the forager. We find that for a smelling forager [25] in 1D, the chance of immortality highly depends on food distribution. Moreover, for a forager walking in two dimensions we find that the existence of long range smell increases the forager's lifetime dramatically from $T \sim S^2$ to $T \sim S^4$. We also find for the lifetime of a random forager in 1D a general scaling function that includes the density of food.

II.   ONE DIMENSION - RANDOM FORAGER

We aim in this Section to find out how the distribution of food in space influences the lifetime of forager in one dimension, see illustration in Fig. 1. First, we consider a forager walking *randomly* on a one dimensional infinite lattice. The forager hops to one of its two neighboring sites at each time step. For getting full analytic solution we analyze the case of a semi-infinite desert. In this scenario, at the beginning there is food only at one side of the forager, while the other side is a semi-infinite desert. We assume that between positions of food there is a distance l distributed according to an arbitrary distance distribution $P(l)$, and the forager walks randomly. If the forager makes S steps without reaching any food it starves and dies.

We are interested in the following quantities, the mean life time of the forager, T, and $\mathcal{N}$, the expectation value of number of meals the forager consumed during its lifetime. We study also τ, the expected time between meals given the next meal occurs. To this end, we should first evaluate $F(t)$, the likelihood of the forager to get food for the first time at step t after it ate. It is well known [31, 32] that the generating function of $F(t, l)$, the first passage time probability to be at $x = 0$ starting at $x = l$, is

$$\mathcal{F}(z, l) = \alpha(z)^l, \tag{1}$$

where

$$\alpha(z) = \frac{1 - \sqrt{1 - z^2}}{z}. \tag{2}$$

Then, we consider $x = 0$ as the site with the closest food and $x = l$ as the site where the last meal happened.

Next, we note that the probability that the closest food is at distance l given that the forager just ate is $P(l)$.
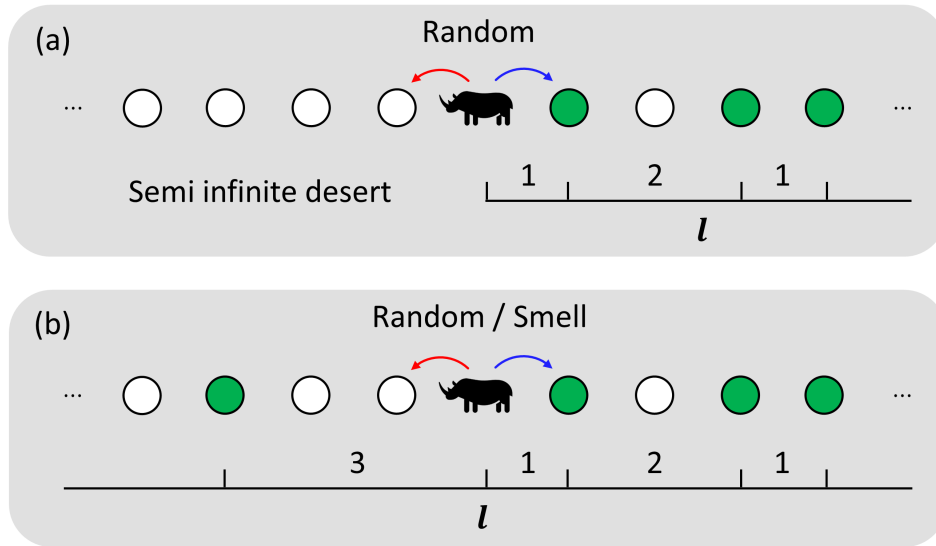
FIG. 1: **Illustration of forager in one dimension** (a) Here all food is in the right of forager (filled circles), whereas in the left there is a semi infinite desert. Between food there are distances, l, distributed according to $P(l)$. The forager walks randomly. We assume that the forager just ate the food in its initial position, hence it can walk S steps without eating. This case is studied in Sec. II. (b) In this scenario food is located in both directions. Here we study in detail the case of forager with long range smell in Sec. III.

Thus, the first passage time probability, $F(t)$, is

$$F(t) = \sum_{l=1}^{\infty} P(l)F(t,l), \tag{3}$$

resulting in,

$$\mathcal{F}(z) = \sum_{l=1}^{\infty} P(l)\mathcal{F}(z,l) = \sum_{l=1}^{\infty} P(l)\alpha(z)^l = G(\alpha(z)), \tag{4}$$

where

$$G(x) = \sum_{l=1}^{\infty} P(l)x^l, \tag{5}$$

is the generating function of the distance distribution $P(l)$.

Note that if the distance l is always one, then $G(x) = x$, and $\mathcal{F}(z) = \alpha(z)$, which converges to the well known result for the scenario where space is filled with food [24].

After having $\mathcal{F}(z)$, following the steps in [24] (see also Appendix A) we obtain for the average number of meals, $\mathcal{N}$, and for the average time between meals, τ,

$$\mathcal{N} = \frac{E(S)}{1 - E(S)}, \tag{6}$$

$$\tau = \frac{\pi(S)}{E(S)}. \tag{7}$$

Thus, the average lifetime is

$$T = \tau \mathcal{N} + S = \frac{\pi(S)}{1 - E(S)} + S, \tag{8}$$

where $E(S)$ is derived from the generating function $\mathcal{E}(z) = \mathcal{F}(z)/(1 - z)$, and the generating function of $\pi(S)$ is $\Pi(z) = z\mathcal{F}'(z)/(1 - z)$, where $\mathcal{F}(z)$ is given in Eq. (4).

To conclude, given the distribution of food in space $P(l)$, we find the lifetime, T, and the number of meals, $\mathcal{N}$. The term which depends directly on $P(l)$ and determines T and $\mathcal{N}$ is $G(x)$. In the next Secs. we discuss three specific cases of food distribution having three different functions for $G(x)$.

A. Asymptotic behavior for large S

For finding the behavior of $\mathcal{N}, \tau, T$ for large S we analyze the asymptotic behavior of Eqs. (6), (7) and (8) by expanding the corresponding generating functions in the limit $z \to 1$ and using the Tauberian Theorems [33–35]. For a more detailed analysis see Appendix B. We show that for the leading term, the only property of food distribution which matters is the mean distance between food, $\langle l \rangle$, in case it is finite. We denote the density of food by $f = 1/\langle l \rangle$. We wish to get $\mathcal{F}(z)$, Eq. (4), which determines all quantities. Because $\mathcal{F}(z) = G(\alpha(z))$ we treat first $\alpha(z)$, and then $G(x)$. An expansion of α where $z \to 1$ gives, $\alpha(z) \sim 1 - \sqrt{2}\sqrt{1 - z}$. Therefore, we analyze $G(x)$ for $x \to 1$. $G(1) = 1$ due to normalization, and if the mean distance, $\langle l \rangle$, is finite, $G'(1) = \langle l \rangle$, and then using Taylor expansion,

$$G(x) \sim 1 - \langle l \rangle (1 - x). \tag{9}$$

Having $G(x)$ we obtain

$$\mathcal{F}(z) = G(\alpha(z)) \sim 1 - \langle l \rangle (1 - \alpha(z)) \sim 1 - \langle l \rangle \sqrt{2}\sqrt{1 - z}.$$

Using this, we can derive all other quantities (see Appendix B) and obtain,

$$\begin{aligned}
\mathcal{N} &\sim \sqrt{\frac{\pi}{2}} \frac{1}{\langle l \rangle} \sqrt{S} \sim \sqrt{\frac{\pi}{2}} f \sqrt{S}, \\
\tau &\sim \sqrt{\frac{2}{\pi}} \langle l \rangle \sqrt{S} \sim \sqrt{\frac{2}{\pi}} \frac{1}{f} \sqrt{S}, \\
T &= \mathcal{N}\tau + S \sim 2S.
\end{aligned} \tag{10}$$

In Appendix B we find that the limit of large S is obtained when S is large relative to the average distance between food units $\langle l \rangle$ such that $\sqrt{S} \gg \langle l \rangle$, or in term of the density $S \gg (1/f)^2$. The effect of f on the limit of large S is shown in Fig. 9d.

Next, we analyze the case in which $\langle l \rangle$ is infinite, and we assume that the distance distribution behaves according to $P(l) \sim l^{-(1+\beta)}$, where $0 < \beta < 1$ such that $\langle l \rangle = \infty$, then unlike Eq. (9),

$$G(x) \sim 1 - A(1 - x)^\beta. \tag{11}$$

This result leads to

$$\mathcal{F}(z) = G(\alpha(z)) \sim 1 - A(1 - \alpha(z))^\beta \sim 1 - 2^{\beta/2} A(1 - z)^{\beta/2},$$

what provides,

$$\mathcal{N} \sim \frac{\Gamma(\beta/2)}{2^{\beta/2}A}S^{\beta/2},$$

$$\tau \sim \frac{\beta 2^{\beta/2}A}{2\Gamma(2-\beta/2)}S^{1-\beta/2}, \tag{12}$$

$$T \sim \left(\frac{\Gamma(1+\beta/2)}{\Gamma(2-\beta/2)}+1\right)S.$$

Next we consider an edge case where $\beta = 1$, which presents an infinite average distance between food as well. In this case we get a logarithmic correction, as follows,

$$G(x) \sim 1 + \frac{1}{\zeta(2)}(1-x)\ln(1-x). \tag{13}$$

Using this we obtain

$$\mathcal{N} \sim \Gamma(1/2)\sqrt{2}\zeta(2)\frac{\sqrt{S}}{\ln S},$$

$$\tau \sim \frac{1}{\Gamma(1/2)\sqrt{2}\zeta(2)}\sqrt{S}\ln S, \tag{14}$$

$$T \sim 2S.$$

The conclusions are that in the asymptotic limit of large S the behavior depends if the mean distance between food locations is finite or infinite. In the finite average case interestingly, while the food distribution does not affect the exponents of scaling relations, it does change the pre-factors, as follows, $\mathcal{N} \sim fS^{1/2}$, $\tau \sim f^{-1}S^{1/2}$ and $T \sim S$ of Eq. (10), where $f = 1/\langle l \rangle$ is the food density.

However, food distribution with power-law tail, $P(l) \sim l^{-(1+\beta)}$, where $\beta < 1$ (where $\langle l \rangle$ diverges), yields exponents which depend on the distribution, i.e., $\mathcal{N} \sim S^{\beta/2}$ and $\tau \sim S^{1-\beta/2}$ rather than $S^{1/2}$, while the scaling of $T \sim S^1$ is conserved. The pre-factor of T, however, depends on β, Eq. (12). Where $\beta = 1$ a logarithmic correction appears, and $\mathcal{N} \sim \sqrt{S}/\ln S$ and $\tau \sim \sqrt{S}\ln S$, Eq. (14).

B. Examples of several distance distributions

I. Uniform distance between food locations

Here we consider the case where l, the distance between food locations, is uniform, $l = L$. Namely,

$$P(l) = \delta_{l,L}. \tag{15}$$

In this case

$$G(x) = \sum_{l=1}^{\infty} \delta_{l,L}x^l = x^L. \tag{16}$$

Note that when $L = 1$, then $G(x) = x$, and we recover the case of food is everywhere [24].

Substituting Eq. (16) in Eq. (4), we have the theory for the constant distance between food, shown in Fig. 2.

The scaling for large S is, according to Eq. (10), $T \sim S$, $\mathcal{N} \sim L^{-1}S^{1/2}$, $\tau \sim LS^{1/2}$.

II. Random spread of food - likelihood f of having food in each site

Here we assume that randomly each site has food with probability f. Hence, the chance that an arbitrary food unit has, at a certain direction, the closest food at distance l, is

$$P(l) = f(1-f)^{l-1}. \tag{17}$$

Thus, Eq. (17) is the normalized distance distribution between food. The average distance is related to the density by $\langle l \rangle = 1/f$. Thus,

$$G(x) = \sum_{l=1}^{\infty} f(1-f)^{l-1}x^l = \frac{fx}{1-(1-f)x}. \tag{18}$$

Note that when $f = 1$, then $G(x) = x$, and we recover the case of food is everywhere [24].

Substituting Eq. (18) in Eq. (4) provides the theory for random spread of food, shown in Figs. 2 and 3.

The scaling for large S is, according to Eq. (10), $T \sim S$, $\mathcal{N} \sim fS^{1/2}$, $\tau \sim f^{-1}S^{1/2}$.

For this food distribution, Eq. (17), we analyze in Appendix C also the behavior of T, τ, N in the limit of small f for given S, and we get $T \sim f$, $\mathcal{N} \sim f$ and $\tau \sim Constant$, see Fig. 9.

In Fig. 2 we also study which way to spread the food in space is better for the forager to live longer. Given the same amount of food we compare the results of life time between random spread of food and a constant distance between food. The black line in the phase diagram, Fig. 2c, distinguishes between the two cases. For parameters below this line it is better to have a constant distance while above this line random distribution of food increases the life time of the forager. One can see that if $S \gg \langle l \rangle$ then constant distance between food leads to a longer life time because random distribution will create at some place a long gap which causes starving. On the other hand, if $\langle l \rangle \approx S$ then random spread is better, because l might be many times lower than $\langle l \rangle \approx S$, and thus the forager will probably get to cross this desert, unlike in constant distance $L \approx S$ where the forager will starve very fast.

III. Power law distribution of distances between food units

If the spread of food is uniformly random, the distances between food are distributed exponentially as shown above, Eq. (17). In reality in many cases food is clustered such that most distances are short but few are long, what can be described by a power law distribution of distances between food locations. Therefore, we assume now that $P(l)$ fulfills

$$P(l) = Al^{-(1+\beta)}, \tag{19}$$

where $A = 1/\zeta(1 + \beta)$, and ζ is Riemann zeta function.

In this case the generating function is,

$$G(x) = A\sum_{l=1}^{\infty} l^{-(1+\beta)}x^l = \frac{\mathrm{Li}_{1+\beta}(x)}{\zeta(1+\beta)}, \tag{20}$$

where $\mathrm{Li}_{1+\beta}(x)$ is the polylogarithm of order $1 + \beta$. Substituting Eq. (20) into Eq. (4) provides the theory for a power law distribution of distances between food units.
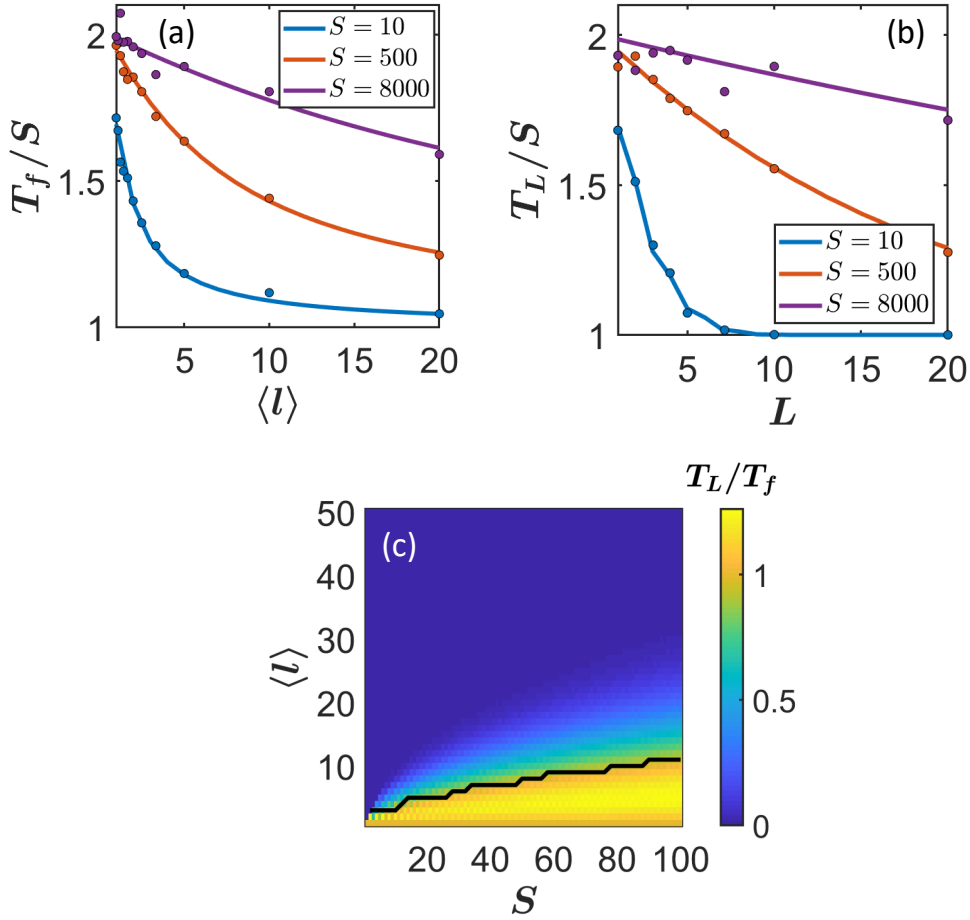
FIG. 2: **Random forager in one dimension. Comparison between constant distance between food and random food distributions.** (a) Results of the forager's lifetime for random spread of food, T_f, from theory (lines), Eqs. (4), (8) and (18), and simulations (symbols) averaged over 10^3 realizations, show good agreement. In our simulations, whenever the forager approaches the edge of the system we add new sites in this direction, therefore we obtain identical behavior as an infinite lattice. (b) Results of forager's lifetime for a constant distance between food units, T_L. Lines represent the theory, Eqs. (4), (8) and (16), and symbols are simulation results. Good agreement can be observed. (c) Phase diagram that compares which food distribution for the same $\langle l \rangle$ (same amount of food) is better and yields longer lifetime of the forager. Above the black line the random spread gives longer life, T_f, while below the line, where S is significantly larger than $\langle l \rangle$, the constant distance enables longer lifetime, T_L. In (c) the life times are calculated excluding the last walk which is always S steps for any food distribution.

For $\beta > 1$ follows $\langle l \rangle < \infty$, and for $\beta \leq 1$ follows $\langle l \rangle = \infty$. We analyzed above both cases for the asymptotic behavior for large S yielding Eqs. (10) and (12). For $\beta > 1$, the average is finite, and the scaling is $\mathcal{N} \sim S^{1/2}, \tau \sim S^{1/2}$ and $T \sim S$, while for $\beta < 1$, $\mathcal{N} \sim S^{\beta/2}$, $\tau \sim S^{1-\beta/2}$, $T \sim S$. For $\beta = 1$, we obtain logarithmic corrections to the scaling, $\mathcal{N} \sim \sqrt{S}/\ln S$, $\tau \sim \sqrt{S} \ln S$ and $T \sim S$, Eq. (14). Results for this power law distribution and the scaling relations are presented in Fig. 3.
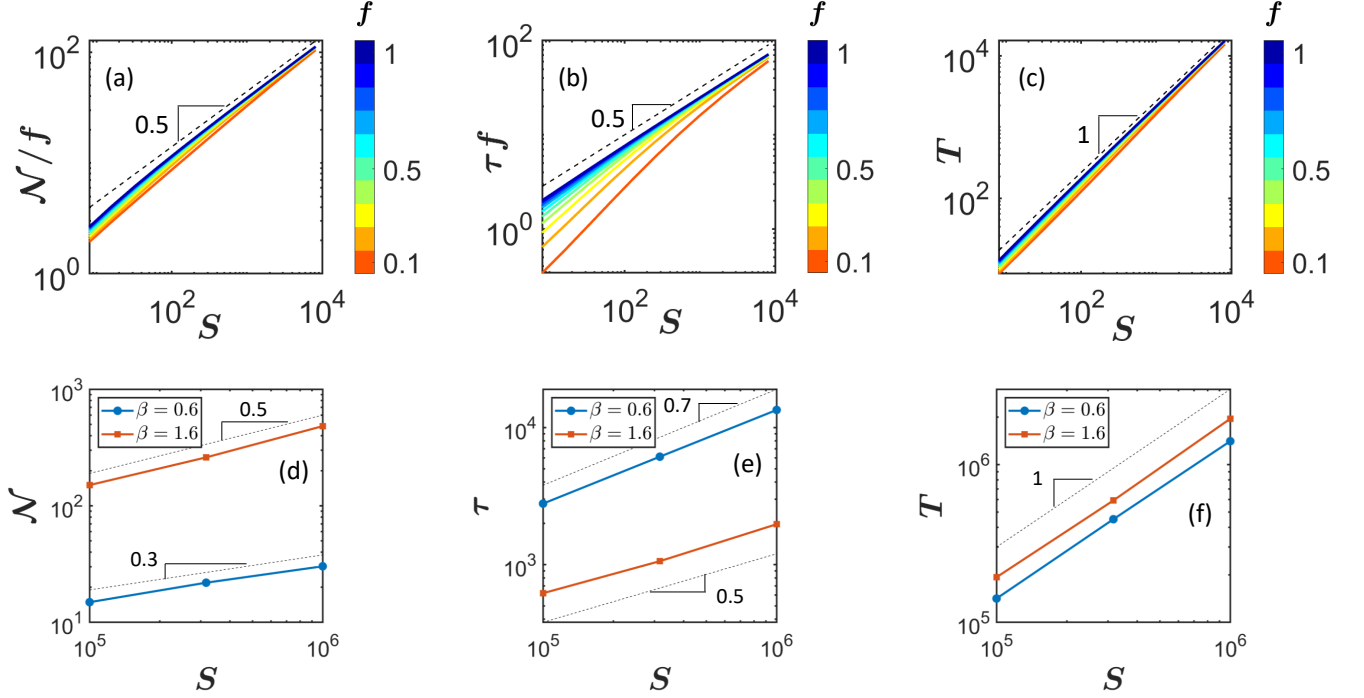
FIG. 3: **Random forager in one dimension with random (uniform) and power-law food distributions.** In the upper panels we show results of theory for random uniform spread of food, Eqs. (4),(6),(7),(8) and (18). One can see that the scaling relations for large S in Eq. (10) are valid, (a) $\mathcal{N} \sim f S^{1/2}$, (b) $\tau \sim f^{-1} S^{1/2}$, and (c) $T \sim S$. Analysis of the behavior for small values of f is presented in the Appendix in Fig. 9. In the lower panels, (d,e,f), we show results of simulations for power-law distribution of distance between food, $P(l) = A l^{-(1+\beta)}$. The results agree well with the theoretical scaling (dashed lines) found in Eq. (12), $\mathcal{N} \sim S^{\beta/2}$, $\tau \sim S^{1-\beta/2}$, $T \sim S$, for $\beta < 1$, and with the scaling in Eq. (10), $\mathcal{N} \sim S^{1/2}$, $\tau \sim S^{1/2}$, $T \sim S$, for $\beta > 1$. The results of simulations have been averaged over 10^3 realizations. Note that in (f) for $\beta = 0.6$ the factor T/S is close to 1.5 instead of 1.99 as found in Eq. (12). This is since the analytical approximation in Eq. (12) should be valid only for much larger S.

III.   ONE DIMENSION - SMELLING FORAGER

In this chapter we study the case where each unit of food generates a smell felt by the forager and direct him towards the food. We assume that the smell decays with the distance from its source. All smell to the forager's right is summed up to F_R, and all smell to the left, to F_L. Then, the probability to go right, p_R, or left, p_L, is determined according to F_R and F_L simply by

$$p_{R,L} = \frac{F_{R,L}}{F_R + F_L}. \tag{21}$$

Note that the the probabilities $p_{R,L}$ are normalized by the total smell, and hence depend only on the ratio between the strength of smell in both directions, and not on their absolute value. Therefore, fast decay of smell causes more significant bias towards the direction with closer food because the strength of smell in the other direction is relatively much weaker.

Food is distributed all over a one dimensional infinite lattice, with some distance distribution $P(l)$ between food

locations. Here, given $P(l)$, we focus on the question whether the forager has a non-zero probability to live forever, p_∞, or it is certainly mortal. To study this question we analyze two decay functions of smell, power law decay and exponential decay.

A. Power Law decay of smell

Here we assume the decay of smell with distance is according to $d^{-\alpha}$, where d is the distance between the locations of the forager and the food units which are the sources of smell. Note that if $\alpha \leq 1$ the total smell to each side diverges, and thus the forager walks completely randomly, a case that has been discussed above. Therefore we consider here only the case $\alpha > 1$ and investigate the impact of smell.

We want to explore whether immortality exists. The reason that immortality might be possible is that as long as the forager propagates in one direction, its bias to this direction gets stronger because of the effect of smell. The question is if and in which conditions, this intensification is significant enough, and forager would live forever.

We define $P(l)$ to be the distribution of distances between food units locations. In order to explore immortality, we treat separately two cases: (i) *limited*: the original distance between food cannot be larger than S according to $P(l)$, and (ii) *unlimited*: the distance between food can be longer than S according to $P(l)$. We first analyze these two types of distributions generally, and then consider the specific examples discussed in the previous section under the corresponding category.

(i) Limited: The case where the distance between food cannot be larger than S

For this case, we find that there is an immortality phase which is dependent on the value of α. There is a critical value α_c below which the forager will die at finite time with probability 1, and above which there is a nonzero chance to live forever. This α_c, as we will show, depends on the distribution of food. In order to find α_c, we follow the steps in [25] and adjust them to our model as follows.

First, we define some useful quantities. p_∞ is the probability to live forever. $\phi(D)$ is the chance to get the next meal, given the forager just ate and left behind a desert of size D without food. p_D is the probability to step towards the desert. We will focus on large D because we are interested here in long time walks, which is needed for determining if the life time can be infinite.

Our goal is to determine if the probability to live forever, p_∞, is zero. Since p_∞ is the probability to always reach the next meal, hence

$$p_\infty = \prod_{n=1}^{\infty} \phi(D_n), \tag{22}$$

where n counts the meals, and D_n is the size of the desert before the nth meal. Note that $D_{n+1} = D_n + l_n$ is satisfied where l_n is distributed according to $P(l)$.

In order to find $\phi(D)$, we study first p_D. After a long time of walking there is a large desert of size D in one direction, thus the likelihood to step towards the desert is small and estimated [25] by

$$p_D \sim D^{1-\alpha}. \tag{23}$$

Next, we denote $\phi(D,l)$ as the likelihood to get a next meal given the next food is at distance l, and the desert on the other side is of size D. We consider long times for which D is very large, hence p_D is small, and thus the chance to starve $1 - \phi(D,l)$ is small. Its leading term comes from the possibility with minimum number of steps, k, towards the desert among S steps, such that the forager does not get the next food. This k, in our model with food distribution, depends on l, thus we denote it by k_l. It was shown in [25] that the chance not to escape a desert is

$$1 - \phi(D,l) \sim p_D^{k_l}. \tag{24}$$

Here, k_l satisfies

$$(S - k_l) - k_l \leq l - 1, \tag{25}$$

or

$$k_l \geq \frac{S - l + 1}{2}. \tag{26}$$

Because k_l is minimal,

$$k_l = \left\lceil \frac{S - l + 1}{2} \right\rceil. \tag{27}$$

The next step is to find $\phi(D)$, the desert escape probability without knowing the distance l from the next food. We denote l^* as the maximal possible distance between food according to $P(l)$. In this section, $l^* \leq S$ because $\Pr(l > S) = 0$.

Then, the likelihood to get a next meal, $\phi(D)$, where l is not given, using Eq. (24), is

$$1 - \phi(D) = \sum_{l=1}^{\infty} [1 - \phi(D,l)] P(l) = \sum_{l=1}^{l^*} [1 - \phi(D,l)] P(l) \sim \sum_{l=1}^{l^*} p_D^{k_l} P(l). \tag{28}$$

Because p_D is very small, the dominant term in the sum is the one with the minimal exponent k_l, which is for the largest l, i.e., l^*. Thus, recalling Eq. (23), we get

$$1 - \phi(D) \sim p_D^{k_{l^*}} \sim D^{(1-\alpha)k_{l^*}}. \tag{29}$$

Now we can evaluate p_∞ using Eqs. (22) and (29),

$$p_\infty = \prod_{n=1}^{\infty} \phi(D_n) = \exp\left(\sum_{n=1}^{\infty} \ln(\phi(D_n)) \right) \sim \exp\left(-\sum_{n=1}^{\infty} D_n^{(1-\alpha)k_{l^*}} \right). \tag{30}$$

Thus, $p_\infty = 0$ if and only if the sum in the exponent diverges. Because the differences between D_n are bounded by S, the sum diverges simply when,

$$(1 - \alpha)k_{l^*} \geq -1 \tag{31}$$

Thus,

$$\alpha_c = 1 + \frac{1}{k_{l^*}} = 1 + \frac{1}{\left\lceil \frac{S - l^* + 1}{2} \right\rceil}, \tag{32}$$

and $p_\infty = 0$ if $\alpha \leq \alpha_c$, i.e., the forager will definitely die at finite time, while for $\alpha > \alpha_c$ there is a non-zero chance to survive forever.

This result is not trivial because the naive guess might be that α_c should be determined by the average distance, $\langle l \rangle$, however we find that the maximal distance, l^*, is the quantity determining α_c.

After treating the case of general distribution we consider the simplest example obeying the condition $l \leq S$ which already was discussed in the previous section.

Constant distance between food units

For a constant distance between food units such that $l = L \leq S$, Eq. (32) simply takes he form

$$\alpha_c = 1 + \frac{1}{\left\lceil \frac{S-L+1}{2} \right\rceil}. \tag{33}$$

Note that while in general critical exponents are not sensitive to microscopic characteristics and change only with dimension or symmetry changes [36, 37], here the critical exponent is governed both by a quantitative feature of the forager (S) and by a quantitative feature of the spread of food in space (L).

In Fig. 4a we show the results for α_c of theory and simulations for a constant distance between food, $L = 3$. Fig. 4b shows the result of Eq. (33). Of course where $L > S$ the forager is mortal, however, for $L \leq S$ each point has a critical value of the exponent α above which the forager is immortal.

In our computer simulations, we summed up the smell until far distance such that increasing the range does not affect the value of F_R or F_L (smell forces) to a precision of 10^{-4}.

(ii) Unlimited: The case where the distance between food units can be larger than S

At this scenario, we show that there is no chance to live forever because after each eating there is a nonzero probability that $l > S$, and when this happens the forager will certainly die. Hence there is no immortality phase, and the lifetime T is finite for any α. Where α is large such that the forager walks almost certainly towards the closest food, we can find the lifetime T.

First, we prove that the forager is mortal in this case. Let us observe the forager after creating a desert larger than S. After each meal the likelihood to eat again is $\phi(D)$. The distance to next food, l, is random and sampled from $P(l)$. If $l > S$ it will not eat again for sure. It is easy to see that $\phi(D) \leq 1 - \Pr(l > S)$, and that $\Pr(l > S)$ is nonzero, and independent on D or on time.

Then, we approach to find p_∞, the chance to live forever, according to Eq. (22),

$$p_\infty = \prod_{n=1}^{\infty} \phi(D_n) \leq \prod_{n=1}^{\infty} [1 - \Pr(l > S)] = 0. \tag{34}$$

Namely, the forager will certainly die in a finite time for any value of S and α.

Since the lifetime is finite, we wish to calculate the average number of meals, $\mathcal{N}$, for large α. We assume that α is large such that the forager steps always towards closest food. Hence, the only chance to die is if $l > S$. Therefore,
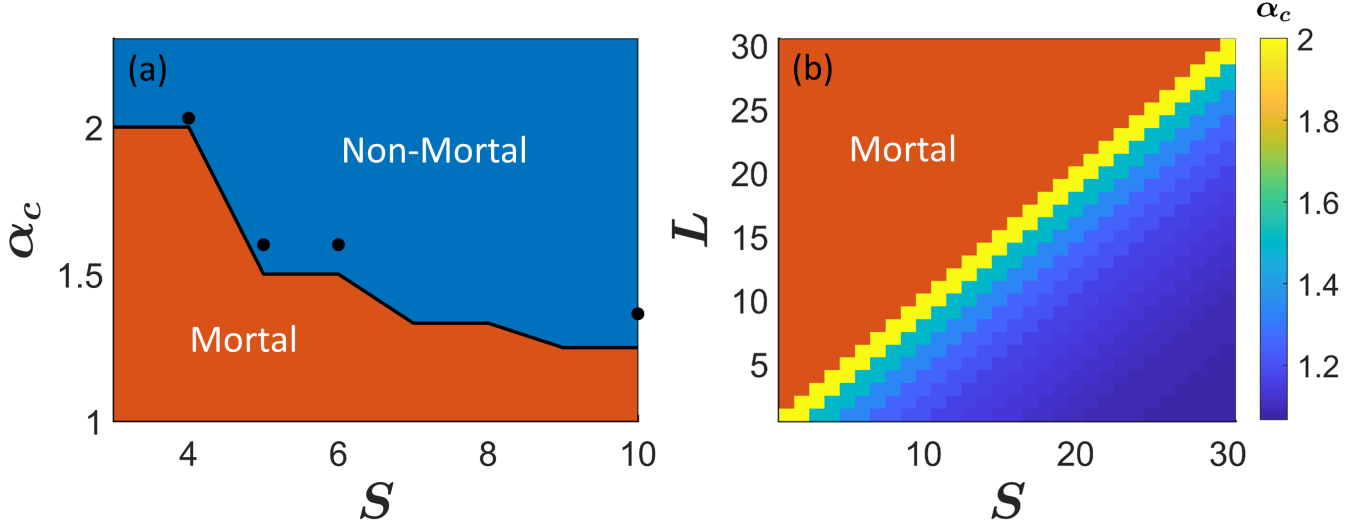
FIG. 4: **Smelling forager with a constant distance between food and power law decay of smell.** (a) Results of theory (line between two phases) and simulations (symbols) for forager in one dimension with power law decay of smell, and food is distributed with a constant distance, $L = 3$, between food units. The theory is taken from Eq. (33), and the simulations have been performed over 10^8 realizations, where α_c is determined by the maximal value of α for which there was no forager which lived forever in any realization, i.e., in all 10^8 realizations the forager died. The deviation between theory and simulations is reasonable because the theory finds when the probability to live forever is completely zero, whereas the simulations find when the probability is small enough such that it does not appear in the finite number of realizations, and thus it happens for a slightly larger value of α. (b) Shows the dependence of α_c on L and S according to Eq. (33). Where $L > S$ of course the forager dies after one walk and it is mortal for any value of α. The color represents the value of α_c required for immortality which changes with L and S.

the chance to reach the next meal is $\phi = 1 - \Pr(l > S)$, and from the average of geometric distribution follows,

$$\mathcal{N} = \frac{1}{\Pr(l > S)}. \tag{35}$$

Next, we study the scaling derived from Eq. (35) for two examples of distance distributions discussed above, random and power law since they fall under the category of the case where l can be larger that S.

I. Random spread of food in space

In this case we assume there is a likelihood f of having food in each site. The result is that the distribution of distance between food is geometrical,

$$P(l) = f(1 - f)^{l-1}. \tag{36}$$

It is clear that $\Pr(l > S) > 0$. Thus, there is no immortality regime. Let us find $\Pr(l > S)$,

$$\Pr(l > S) = \sum_{l=S+1}^{\infty} P(l) = f \sum_{l=S+1}^{\infty} (1 - f)^{l-1} = (1 - f)^S. \tag{37}$$

Therefore, based on Eq. (35), $\mathcal{N}$ for large α is,

$$\mathcal{N} = \left[\frac{1}{(1-f)}\right]^S. \tag{38}$$

The average time between meals is smaller than $\langle l \rangle$ because it is an average given $l \leq S$. However it is in the order of magnitude of $\langle l \rangle$. Therefore, $T = \mathcal{N}\tau + S$ obeys the same scaling as $\mathcal{N}$.

Thus, for large α,

$$T \sim [1/(1-f)]^S, \tag{39}$$

namely the mean lifetime increases exponentially with S.

II. Power law distribution of distances between food units

Here, we assume that $P(l)$ satisfies

$$P(l) = Al^{-(1+\beta)}. \tag{40}$$

In this case,

$$\Pr(l > S) = \sum_{l=S+1}^{\infty} Al^{-(1+\beta)} \approx A \int_{S+1}^{\infty} l^{-(1+\beta)}dl = \frac{A}{\beta}(S+1)^{-\beta}. \tag{41}$$

Therefore, plugging this in Eq. (35),

$$\mathcal{N} \sim S^{\beta}. \tag{42}$$

Here τ might be dependent strongly on S because the tail is not neglected, so it matters where it is cut,

$$\tau = \sum_{l=1}^{S} lAl^{-(1+\beta)} \approx A \int_{1}^{S} l^{-\beta}dl = \frac{A}{1-\beta}\left(S^{1-\beta} - 1\right) \sim \begin{cases} S^{1-\beta}, & 0 < \beta < 1 \\ 1, & \beta > 1 \end{cases}. \tag{43}$$

Therefore,

$$T = \mathcal{N}\tau + S \sim \begin{cases} S, & 0 < \beta < 1 \\ S^{\beta}, & \beta > 1 \end{cases}. \tag{44}$$

Summary of all cases

We denote l^* as the maximal l with non-zero probability. When $l^* \leq S$, then there is α_c above which $p_\infty > 0$, therefore $T = \infty$, and α_c depends on the distance distribution as $\alpha_c = 1 + 1/\lceil(S - l^* + 1)/2\rceil$. Therefore, $T(\alpha)$ is an increasing function that diverges at α_c as illustrated in Fig. 5a.

When $l^* > S$, then $T < \infty$ for any α. Then we get that $T(\alpha)$ is an increasing function with α, starting at the completely random case ($\alpha < 1$) where the scaling is $T \sim S$ as in Eqs. (10) and (12) and approaching a saturation where α is large such that the forager always tends towards the closest food. Then the scaling is a power law or exponential as in Eqs. (39) and (44). See Fig. 5b.
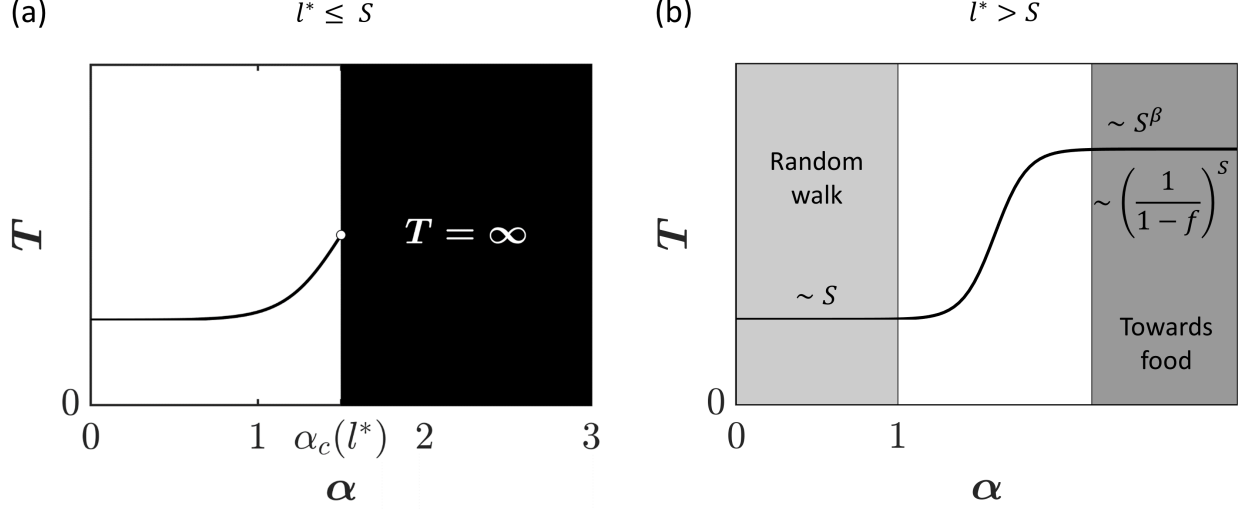
FIG. 5: **Sketches of theory results for lifetime of smelling forager in one dimension** (a) If the maximal possible distance between food (l^*) is not greater than the starving time (S), then there is α_c above which the average lifetime is infinite. Its value depends on l^* according to Eq. (32). (b) If the distance between food can be larger than S, there is no immortality regime. However, the lifetime has a saturation for large α, and in this region the scaling of $T(S)$ is found for power law distribution of distance between food, Eq. (44), and for random spread of food, Eq. (39). For $\alpha < 1$ the total smell diverges, and hence the walk is random, thus the scaling is as in Eqs. (10) and (12).

B. Exponential decay of smell

Here we assume the decay of smell with distance is according to $\exp(-\lambda d)$. The results for this case can be studied using the same formalism as in Sec. III A for power law decay.

For *unlimited* food distribution that allows $l > S$, the analysis to obtain Eq. (39) is the same for exponential and power law decays, hence the forager is mortal. The analysis of Eq. (44) is valid for exponential decay with large λ the same as for power law decay with large α.

For *limited* food distribution where all $l < S$, similar steps as in power law decay can be performed to obtain Eq. (30). Then, for exponential decay of smell, the sum in the exponent is exponential, therefore it converges for any $\lambda > 0$. Thus, in contrast to Eq. (32) where we get critical α_c, in the case of exponential decay of smell there is no critical λ, and $p_\infty > 0$ for any $\lambda > 0$, and the mortal regime vanishes. The intuitive explanation is that since fast decay causes more significant bias towards close food, and for power law function which decays fast enough ($\alpha > \alpha_c$) we get immortality, exponential decay which is faster than any power law decay also provides immortality for any $\lambda > 0$.

IV.   FORAGER IN TWO DIMENSIONS

In this chapter we analyze a forager walking in a two dimensional infinite lattice. We consider several types of walk and compare between them, random walk, short range smell (the forager detects only sites in distance one),

long range smell, and complete bias towards smell. We also consider several distributions of food in space, food is everywhere, food is located in constant distances, and random spread of food with density f.

A. Space is full of food

We define a forager with short range smell as one that if there is food in a site next to it, it steps towards food with probability 1. However the forager does not consider food that are at distances more then 1. Such a forager has been investigated in [23, 24], and it was shown that it dies because of traps it creates to itself, i.e., when the forager closes a loop, it might go inside at the next step, and then eat all food inside, until it finds itself at the middle of a desert without food which it created. Then, since there is no close food it walks randomly. If the loop of the trap is large enough the forager might starve before it reaches the edge of its self made desert, see Fig. 6a.

In contrast, a smelling forager senses also far food. Let us consider a forager that steps with probability 1 to the direction of the closest food. We call this forager *perfect smelling forager*. This forager walks in 2D exactly the same as the short range smell forager we mentioned above, except that if it finds itself in a middle of a desert it does not walk randomly but walks certainly towards the closest food, see Fig. 6a.

We next consider the relation between these two cases, short range smelling and perfect smelling. We argue, using a rigorous mapping, that perfect smelling with starving time S, is similar to a short range smelling forager with starving time S^2. The reason is that the short range smelling forager walks randomly inside the trap, and therefore reaches in S steps a distance of order $\sqrt{S}$, while the perfect smelling forager moves in a straight line, thus reaches a distance S in S steps, see Fig. 6a. The conclusion is that if the function of lifetime of a short range smelling forager is known to be $T_{\text{short}}(S)$, then for the lifetime of the perfect smelling forager,

$$T_{\text{perfect}}(S) \sim T_{\text{short}}(S^2). \tag{45}$$

Computer simulations suggest, as presented in Fig. 7b, that for greedy forager with short range smell the mean life time scales similar to a random forager [16] approximately as

$$T_{\text{short}} \sim S^2. \tag{46}$$

Thus, according to our prediction in Eq. (45), the mean lifetime of a forager with long range smell should scale approximately as

$$T_{\text{perfect}} \sim S^4, \tag{47}$$

and indeed this result is supported in Fig. 7c.

The meaning of Eqs. (45),(46) and (47) is that the difference between short and long range of smell is dramatic, the exponent changes from 2 to 4 and the life time increases tremendously for perfect smelling forager. The result of Eq. (47) will serve us in the next chapter where we study forager with long range smell in two dimensions with food distribution in space.
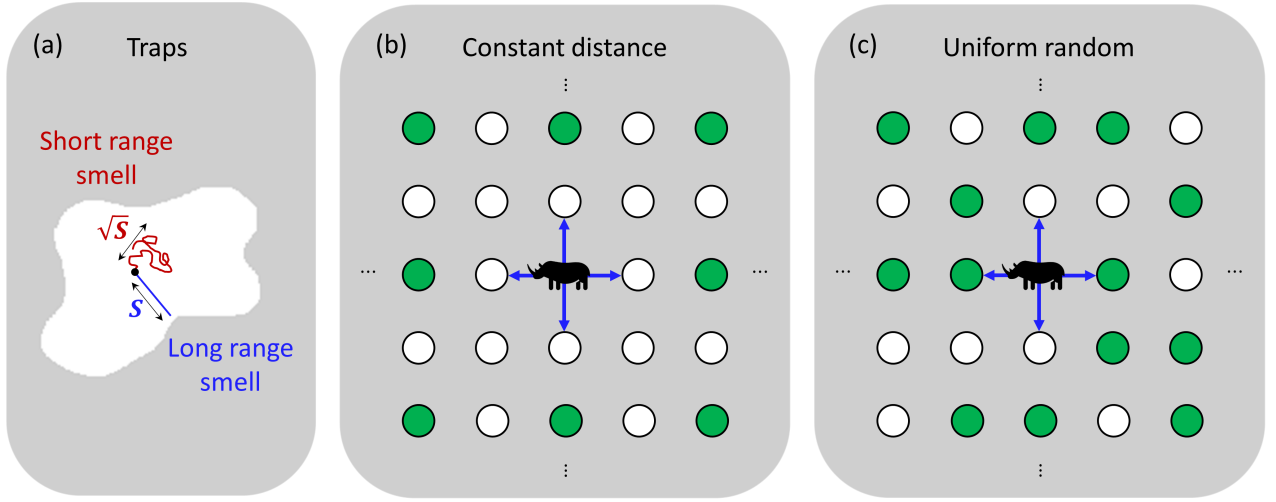
FIG. 6: **Illustrations of forager in two dimensions.** (a) The forager dies when it creates a loop and goes inside. The loop should be large enough such that after it eats most of the food inside and find itself at the middle of desert, it does not succeed escaping the desert in S steps. At this situation, long range smelling forager and one with short range are very different but can be mapped. While the long range smelling forager goes directly towards the closest food (blue straight line), the short range one walks randomly (red random path). Typically random walker reaches distance of $\sim \sqrt{S}$ in S steps, while the direct walk reaches a distance S. Hence the short range smelling forager should have starving time of S^2 to die at the same trap as the long range smeller forager with starving time of S. Therefore, we obtain Eq. (45). (b) Illustration of a constant distance between food in 2D Lattice where $L = 2$. The filled circles represent food, while the white empty ones represent empty sites. Theory for this case is given in Eqs. (48) and (52). Simulation results of this case are shown in Fig. 8. (c) Illustration of uniform random spread of food in two dimensions with density $f \approx 1/2$. Simulation results for this scenario are presented in Fig. 8.

B. Space not full of food

Here we consider a forager in 2D given some distribution of food in space. We focus on forager walking according to its sense of smell. We explore two distributions of food in space, constant distance L between food, Fig. 6b, and random uniform spread of food with density f, Fig. 6c.

1. Constant distance between food locations

Let food located in 2D at points (nL, mL) where m and n are all the integers, and L is the distance between neighboring food units. The forager starts at point $(0, 0)$ and its steps to right/left/up/down have size 1. We call this scenario a constant distance L between food in 2D, see Fig. 6b.

Now, let us consider a smelling forager with a power law decay of smell, $d^{-\alpha}$, with large exponent α, or exponential decay, $e^{-\lambda d}$, with large λ, namely the bias towards the food is very high and the forager walks almost always towards the closest food. We note that in this case, the walk is same as for constant distance $L = 1$ (food is everywhere), except that each step now is replaced by L straight steps. Therefore,

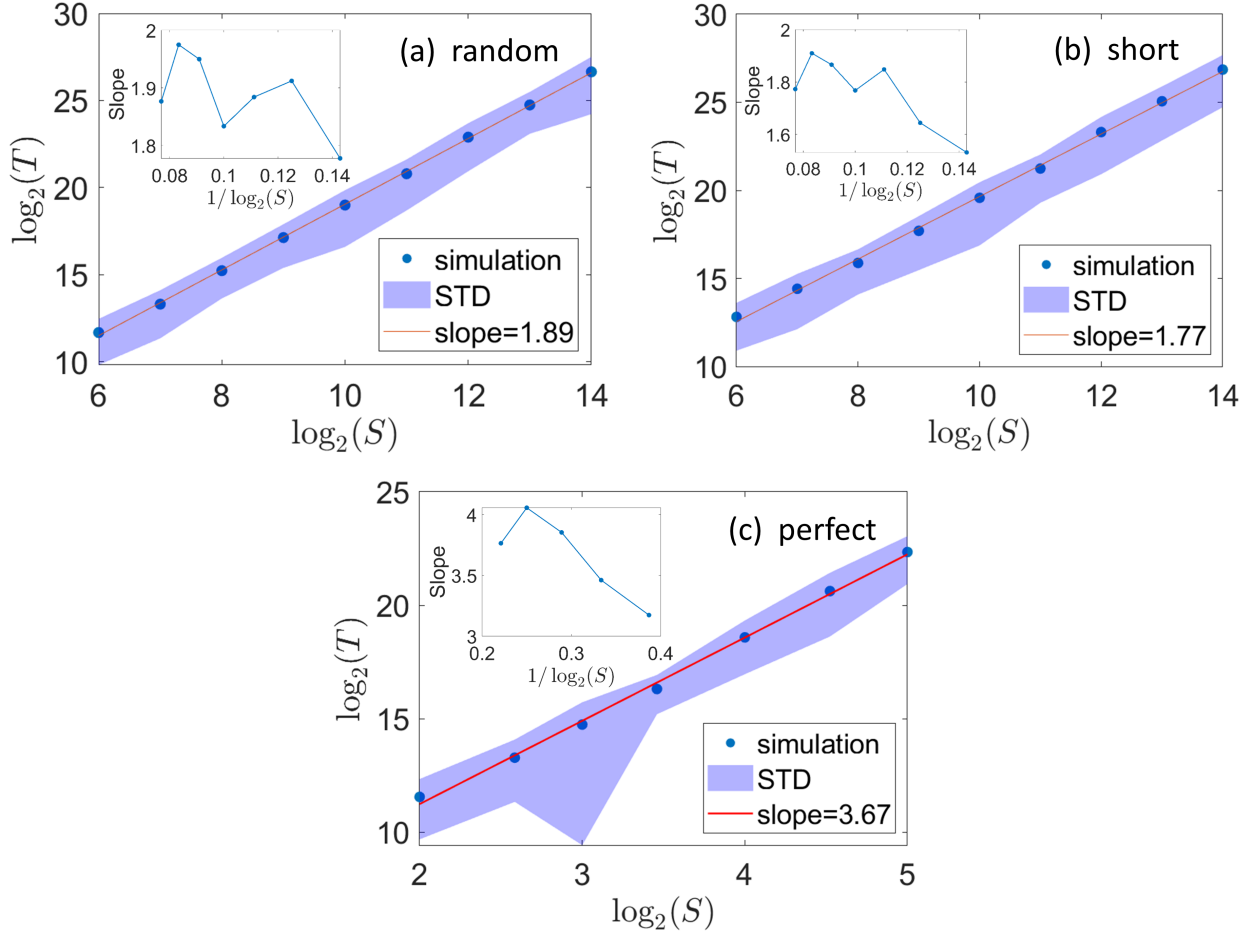$$T_{\text{spread}}(S, L) = L \cdot T_{\text{spread}}(S/L, 1) = LT_{\text{full}}(S/L), \tag{48}$$

FIG. 7: **Lifetime in 2D where the space is full with food.** We show the scaling between lifetime, T, and starving time, S, at three cases: (a) random walk, (b) short range smell (steps towards food only if distance is one), and (c) perfect smelling forager (steps towards the closest food). It can be seen that for random forager and short range smelling forager the scaling is about $T \sim S^2$ when S is getting large, while perfect smelling forager has scaling of about $T \sim S^4$. This confirms our theoretical argument that the transformation from short range smell to perfect smelling should be expressed by $S \to S^2$, see Fig. 6a and Eq. (45). The insets suggest that the exponents smaller than 2 and 4 are due to finite size systems, and when S increases they reach the values 2 and 4 respectively.

where T_{full} is the life time of smelling forager in space full with food. Thus, assuming a smelling forager in full space scales with S as,

$$T_{\text{full}}(S) \sim S^{\gamma}, \tag{49}$$

then

$$T_{\text{spread}}(S, L) = L T_{\text{full}}(S/L) \sim L(S/L)^{\gamma} = L^{1-\gamma} S^{\gamma}, \tag{50}$$

or in different shape, the scaling of T, S and L, for perfect smelling forager in 2D with constant distance between food, is

$$\frac{T}{L} \sim \left(\frac{S}{L}\right)^{\gamma}. \tag{51}$$

This scaling is supported in Fig. 8a where it can be seen that all points of different S and L, where S/L is large, lay on the same curve when plotting T/L vs S/L, what validates the scaling we predicted theoretically in Eq. (48). Using computer simulations we find that $T \sim S^\gamma$ for large S, and $\gamma \approx 4$. Hence, for a large ratio S/L, we expect

$$T_{\text{spread}}(S, L) \sim L^{-3} S^4. \tag{52}$$

In terms of density of food f, rather than the distance between food L, using the simple relation

$$f = 1/L^2,$$

we obtain the scaling

$$T_{\text{spread}}(S, f) \sim f^{\frac{\gamma-1}{2}} S^\gamma \approx f^{3/2} S^4. \tag{53}$$

Note that this scaling is valid for large S/L, or, for large $S\sqrt{f}$, i.e., for $f \gg S^{-2}$.

2. Random uniform spread of food in space

Here we assume that at each site in two dimensional square lattice there is food with likelihood f. This probability f is, therefore, the density of food, see the illustration in Fig. 6c. The forager walks according to a long range smell with high bias, such that it steps towards the closest food. In Fig. 8b we show the results of the lifetime of such a forager for different values of density f. One can see that the approximated scaling we found for a constant distance between food in the previous section, Eq. (53), $T \sim f^{3/2}$, using the combination of simulations results and theoretical considerations, works well also for different cases of food distribution in space.

V.   DISCUSSION

We have studied a forager that walks in space where food is distributed in several fashions, a constant distance between food units, uniform random distribution and power law distribution of distances. We have considered foraging both in one and two dimensions. Moreover, we have treated a few types of forager's walk; random, according to short range smell and according to long range smell. We studied two cases of long-range smell. Smell decaying exponentially and as a power law. We found new scaling relations between forager's lifetime, number of meals, the starving time and the density of food in one and two dimensions. We also found how the immortality of a long range smelling forager in one dimension depends on the distribution of food in space.

Further work could compare these results to experimental measurements, which also might lead to additional extensions to the model such as exploring cases incorporating the fact that food often appears in 'patches' [38]. Likewise, multiple foragers living in the region could be considered with all of them depleting food sources [15]. Another interesting future direction can be to explore how the results here change if the forager performs a Lévy flight instead of constant steps with length 1, since it has been shown that Lévy flights may importantly change foraging efficiency in asymmetric landscapes [39]. Further theoretical studies could investigate how different embedded spaces affect our results, for instance, lattices of higher dimensions and random networks.
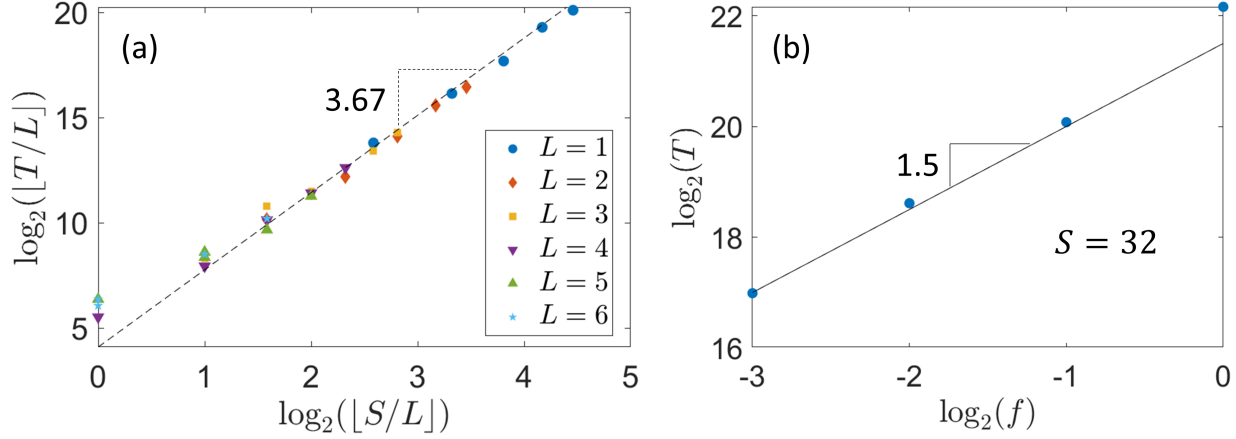
FIG. 8: **Forager in two dimensions where space is not full with food.** (a) Here the distance between food units is constant, L, and the forager walks perfectly according to smell, i.e., towards the closest food. We show that the scaling with the distance L is T/L and S/L when S/L is large, hence all points lay approximately on one curve. This supports Eq. (48) derived from theoretical considerations. This scaling allows us to obtain $T \sim L^{-3}$, see Eq. (52). The dashed line represents the expected approximated slope according to the result of Fig. 7c. (b) Here food is distributed randomly with density f, see Fig 6c. The forager walks perfectly according to smell. We examine in this case the scaling of T vs f. One can see that the approximated scaling in Eq. (53), $T \sim f^{3/2}$, found for constant distances, is approximately valid also for a uniform random food distribution.

VI.   ACKNOWLEDGMENTS

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Appendix A: Derivation of $\mathcal{N}, \tau, T$ using $\mathcal{F}(z)$

In this Appendix we summarize what relevant to us for derivation of N, τ and T based on Ref. [24]. After having the generating function $\mathcal{F}(z)$, we define $E(S)$ as the probability of a random forager to escape the desert, namely to get food before starving, given it starves after S steps without food, and it just ate. One should note that

$$E(S) = \sum_{t=1}^{S} F(t), \tag{A1}$$

what implies regarding the generating functions

$$\mathcal{E}(z) = \frac{\mathcal{F}(z)}{1 - z}. \tag{A2}$$

Next, we find the distribution of N, number of meals which is a geometric distribution,

$$p_N = (1 - E)E^N. \tag{A3}$$

Hence for the average, $\mathcal{N} = \langle N \rangle$,

$$\mathcal{N} = \frac{E}{1 - E} \tag{A4}$$

Then, to evaluate τ, we note that

$$\tau = \frac{\sum_{t=1}^{S} tF(t)}{\sum_{t=1}^{S} F(t)} = \frac{\sum_{t=1}^{S} tF(t)}{E(S)} \equiv \frac{\pi(S)}{E(S)}. \tag{A5}$$

Therefore, we approach to find $\pi(S)$ via its generating function, which obeys

$$\Pi(z) = \frac{z\mathcal{F}'(z)}{1 - z}. \tag{A6}$$

Finally, for the lifetime of the forager we obtain

$$T(S) = \tau\mathcal{N} + S = \frac{\pi(S)}{E(S)}\frac{E(S)}{1 - E(S)} + S = \frac{\pi(S)}{1 - E(S)} + S. \tag{A7}$$

To summarize, given the distribution of food in space $P(l)$, we find $G(x)$, what provides $\mathcal{F}(z)$. Using the last one, we find $\mathcal{N}, \tau, T$.

Appendix B: Asymptotic behavior where S is large in one dimension

We analyze for general distance distribution between food $P(l)$ the asymptotic behavior of $T, \mathcal{N}$ and τ for large S. For this goal we observe the limit $z \to 1$ and use the Tauberian theorems [33–35]. $\alpha(z)$ fulfills $\alpha(1) = 1$. Close to 1 $\alpha(z) \sim 1 - \sqrt{2}\sqrt{1 - z}$. In cases for which the mean distance, $\langle l \rangle$, is finite, $G'(1) = \langle l \rangle$. In addition, $G(1) = 1$. Therefore, for $x \to 1$

$$G(x) \sim G(1) + G'(1)(x - 1) = 1 - \langle l \rangle(1 - x). \tag{B1}$$

Hence, for $z \to 1$

$$\mathcal{F}(z) = G(\alpha(z)) \sim 1 - \langle l \rangle(1 - \alpha(z)) \sim 1 - \langle l \rangle\sqrt{2}\sqrt{1 - z}. \tag{B2}$$

Hence,

$$\mathcal{E}(z) = \frac{\mathcal{F}(z)}{1-z} \sim \frac{1}{1-z} - \frac{\sqrt{2}\langle l \rangle}{\sqrt{1-z}}. \tag{B3}$$

Thus,

$$E(S) \sim 1 - \frac{\sqrt{2}\langle l \rangle}{\Gamma(1/2)} \frac{1}{\sqrt{S}} = 1 - \frac{\sqrt{2}\langle l \rangle}{\sqrt{\pi}} \frac{1}{\sqrt{S}}. \tag{B4}$$

Hence,

$$\mathcal{N} = \frac{E}{1-E} \sim \sqrt{\frac{\pi}{2}} \frac{1}{\langle l \rangle} \sqrt{S}. \tag{B5}$$

In addition,

$$\Pi(z) = \frac{z\mathcal{F}'(z)}{1-z} \sim \frac{\sqrt{2}\langle l \rangle}{2(1-z)^{3/2}}. \tag{B6}$$

Therefore,

$$\pi(S) \sim \frac{\sqrt{2}\langle l \rangle}{2\Gamma(3/2)} \sqrt{S} = \frac{\sqrt{2}\langle l \rangle}{\sqrt{\pi}} \sqrt{S}. \tag{B7}$$

And thus,

$$\tau = \frac{\pi}{E} \sim \frac{\sqrt{2}\langle l \rangle}{\sqrt{\pi}} \sqrt{S}. \tag{B8}$$

Then,

$$T(S) = \frac{\pi(S)}{1-E(S)} + S \sim \frac{(\sqrt{2}\langle l \rangle/\sqrt{\pi})\sqrt{S}}{(\sqrt{2}\langle l \rangle/\sqrt{\pi})/\sqrt{S}} + S = 2S. \tag{B9}$$

The condition for large S

The expansion we performed above is valid for the limit $S \to \infty$, namely much larger than any other quantity of the problem. However, we want to find out what can be regarded as large S. For this purpose, we go back to Eq. (B4), and recognize that we assumed

$$1 \gg \frac{\sqrt{2}\langle l \rangle}{\sqrt{\pi}} \frac{1}{\sqrt{S}}. \tag{B10}$$

Assuming this we can expect the next terms are negligible. Therefore we extract the condition for large S relative to other quantity of the problem,

$$\sqrt{S} \gg \langle l \rangle. \tag{B11}$$

In terms of food density $f = 1/\langle l \rangle$ we obtain

$$S \gg (1/f)^2. \tag{B12}$$

Fig. 9d shows how the limit of large S changes with different densities f, when T/S approaches to a constant value slower for small f.

Power-law distance distribution

Here we consider power law distance distribution $P(l) = Al^{-(1+\beta)}$ between food units, where $A = 1/\zeta(1 + \beta)$, and ζ is Riemann zeta function. The generating function of this distribution is,

$$G(x) = A \sum_{l=1}^{\infty} l^{-(1+\beta)} x^l = \frac{\text{Li}_{1+\beta}(x)}{\zeta(1 + \beta)}, \tag{B13}$$

where $\text{Li}_{1+\beta}(x)$ is the polylogarithm of order $1 + \beta$. Here $\langle l \rangle$ is not finite in all cases, and one should separate the treatment into two cases. The expansion of polylogarithm around 1 is

$$\text{Li}_{1+\beta}(x) \sim \begin{cases} \zeta(1 + \beta) - \zeta(\beta)(1 - x), & \beta > 1 \\ \zeta(1 + \beta) + \Gamma(-\beta)(1 - x)^{\beta}, & 0 < \beta < 1 \\ \zeta(2) + (1 - x)\ln(1 - x), & \beta = 1 \end{cases} \tag{B14}$$

For $\beta > 1$, the mean distance is finite, and we already obtained the asymptotic behavior in this case above.

For $\beta < 1$ we analyze the asymptotic behavior in large S. To this end, we expand the relevant functions in the limit $z \to 1$. At $z \to 1$, $\alpha(z) \sim 1 - \sqrt{2}\sqrt{1 - z}$, and therefore

$$\mathcal{F}(z) = G(\alpha(z)) = \frac{\text{Li}_{1+\beta}(\alpha(z))}{\zeta(1 + \beta)} \sim 1 + \frac{\Gamma(-\beta)}{\zeta(1 + \beta)} (2(1 - z))^{\beta/2} \sim 1 - C(1 - z)^{\beta/2}. \tag{B15}$$

Next, for the generating function of $E(S)$,

$$\mathcal{E}(z) = \frac{\mathcal{F}(z)}{1 - z} \sim \frac{1}{1 - z} - C(1 - z)^{\beta/2-1}. \tag{B16}$$

Thus,

$$E(S) \sim 1 - \frac{C}{\Gamma(\beta/2)} S^{-\beta/2}. \tag{B17}$$

Hence,

$$\mathcal{N} = \frac{E}{1 - E} \sim \frac{\Gamma(\beta/2)}{C} S^{\beta/2}. \tag{B18}$$

In addition, for the generating function of $\pi(S)$

$$\Pi(z) = \frac{z\mathcal{F}'(z)}{1 - z} \sim \frac{C\beta}{2(1 - z)^{2-\beta/2}}. \tag{B19}$$

Therefore,

$$\pi(S) \sim \frac{C\beta}{2\Gamma(2 - \beta/2)} S^{1-\beta/2}. \tag{B20}$$

Then

$$T(S) = \frac{\pi(S)}{1 - E(S)} + S \sim \frac{C\beta}{2\Gamma(2 - \beta/2)} S^{1-\beta/2} \frac{\Gamma(\beta/2)}{C} S^{\beta/2} + S = \left(\frac{\Gamma(1 + \beta/2)}{\Gamma(2 - \beta/2)} + 1\right) S. \tag{B21}$$

In the edge case $\beta = 1$, we get at $z \to 1$

$$\mathcal{F}(z) = G(\alpha(z)) = \frac{\text{Li}_2(\alpha(z))}{\zeta(2)} \sim 1 + \frac{1}{\zeta(2)} \sqrt{2(1 - z)} \ln \sqrt{2(1 - z)} \sim 1 + C\sqrt{1 - z} \ln(1 - z). \tag{B22}$$

Next, for the generating function of $E(S)$,

$$\mathcal{E}(z) = \frac{\mathcal{F}(z)}{1-z} \sim \frac{1}{1-z} + C\frac{\ln(1-z)}{\sqrt{1-z}}. \tag{B23}$$

Thus,

$$E(S) \sim 1 - \frac{C}{\Gamma(1/2)} \frac{\ln S}{\sqrt{S}}. \tag{B24}$$

Hence,

$$\mathcal{N} = \frac{E}{1-E} \sim \frac{\Gamma(1/2)}{C} \frac{\sqrt{S}}{\ln S}. \tag{B25}$$

In addition, for the generating function of $\pi(S)$

$$\Pi(z) = \frac{z\mathcal{F}'(z)}{1-z} \sim -\frac{C\ln(1-z)}{2(1-z)^{3/2}}. \tag{B26}$$

Therefore,

$$\pi(S) \sim \frac{C}{2\Gamma(3/2)}\sqrt{S}\ln S. \tag{B27}$$

Then

$$T(S) = \frac{\pi(S)}{1-E(S)} + S \sim 2S. \tag{B28}$$

Appendix C: Asymptotic behavior for small f for a random forager with random spread of food

Let us analyze Eq. (18) in the limit of small density f,

$$\mathcal{F}(z) = G(\alpha(z)) = f\frac{\alpha}{1-\alpha+f\alpha} = \frac{f\alpha}{1-\alpha}\frac{1}{1+f\alpha/(1-\alpha)}$$
$$\sim \frac{f\alpha}{1-\alpha} - \left(\frac{f\alpha}{1-\alpha}\right)^2 \sim f\frac{\alpha}{1-\alpha}. \tag{C1}$$

Consequently,

$$\mathcal{E}(z) = \frac{\mathcal{F}(z)}{1-z} \sim f\frac{\alpha}{1-\alpha}\frac{1}{1-z}. \tag{C2}$$

Thus,

$$E \sim f. \tag{C3}$$

Hence,

$$\mathcal{N} = \frac{E}{1-E} \sim f. \tag{C4}$$

For $\pi(S)$ we get

$$\Pi(z) = \frac{z\mathcal{F}'(z)}{1-z} \sim f\frac{z}{1-z}\left(\frac{\alpha(z)}{1-\alpha(z)}\right)'. \tag{C5}$$

Therefore,

$$\pi \sim f \tag{C6}$$

As a result,

$$\tau = \frac{\pi}{E} \sim Const. \tag{C7}$$

For the lifetime
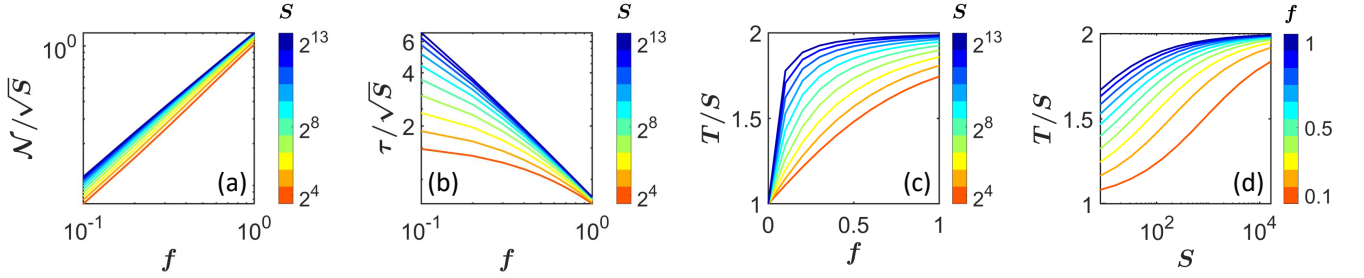
$$T - S = \tau \mathcal{N} \sim f. \tag{C8}$$

FIG. 9: **Random forager in one dimension with uniform random food distribution.** (a-c) We can see the two limits of small densities f and large ones. (a) The quantity $\mathcal{N}$ is linear with f while (b) τ approaches to a constant for small f and then scales as $1/f$ when f tends to 1, and (c) T is linear with f for small f and then approaches to a constant. (d) The linear scaling of T versus S is valid for large S where it is large relative to the average distance between food units such that $\sqrt{S} \gg \langle l \rangle$, which in terms of f becomes $S \gg (1/f)^2$. Therefore T/S is getting constant slower for small values of f. However, this figure shows the non-leading correction while in Fig. 3c one can see only the effect of the leading term.