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Finite-time performance of quantum heat engine with a squeezed thermal bath

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Abstract

We consider the finite-time performance of a quantum Otto engine working between a hot squeezed and a cold thermal bath at inverse temperatures β_h and $\beta_c(>\beta_h)$ with $(k_B \equiv 1)$ $\beta = 1/T$. We derive the analytical expressions for work, efficiency, power, and power fluctuations, in which the squeezing parameter is involved. By optimizing the power output with respect to two frequencies, we derive the efficiency at maximum power as $\eta_{mp} = (\eta_C^{\text{gen}})^2/[\eta_C^{\text{gen}} - (1-\eta_C^{\text{gen}})\ln(1-\eta_C^{\text{gen}})]$, where the generalized Carnot efficiency η_C^{gen} in the high-temperature or small squeezing limit simplifies to an analytic function of squeezing parameter γ: $\eta_C^{\text{gen}} = 1 - \beta_h/[\beta_c \cosh(2\gamma)]$. Within the context of irreversible thermodynamics, we demonstrate that the expression of efficiency at maximum power satisfies a general form derived from nonlinear steady state heat engines. We show that, the power fluctuations are considerably increased, although the engine efficiency is enhanced by squeezing.

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I. INTRODUCTION

An intense effort has been devoted to study on quantum heat engines which began with the seminal work of Scovil and Schulz-DuBois [1], with special emphasis on the thermodynamic optimization and fluctuations for thermodynamic quantities (see, for example, references [1–8] and reviews [9–12]). Because of (quantum) finite-size effects and nonequilibrium nature, the miniaturized engines [2, 13–22] at microscale and nanoscale suggest novel performance behaviors quite different from their classical counterparts in which the thermodynamic limit holds. A typical example is that nanoscale (quantum) heat engines working with nonthermal baths might be beyond the standard Carnot limit, unlike in the classical heat engines where the maximum efficiency must be bounded by the Carnot value. This is because for these quantum heat engines the nonequilibrium nature of non-thermal baths might lead to going beyond the original Carnot theorem. The non-thermal baths may be quantum coherent [13], quantum correlated [15, 16], quantum-measurement-induced [17, 18], and squeezed thermal baths [2, 19–24]. As expected, this result does obey the principles of thermodynamics due to the nonequilibrium nature of these baths. While the efficiency achieves its maximum value, the engine cycle is infinitely slow and thus its power output is vanishing, except in some special cases (when the system evolves dynamically infinitely fast [25] and works at the critical region [5, 26]). Therefore, an investigation into the finite-time performance of quantum heat engines working with small systems and exhibiting non-negligible power fluctuations is of great theoretical and practical interest.

To analyze heat engines operating in finite time, the optimization within context of trade-off between power output and efficiency was usually presented, focusing on the issue of efficiency at maximum power. For standard heat engines energized by a hot and a cold thermal bath at inverse temperatures β_h and β_c, the efficiency at maximum power [27–40] in some certain conditions has the same universality as the Curzon and Ahlborn (CA) efficiency [41]: $\eta_{CA} = 1 - \sqrt{\beta_h/\beta_c}$. Similarly, for diverse cyclic engine models working with quantum squeezed baths, the efficiency at maximum power in the high temperature limit reads [2, 20],

$$\eta_{CA}^{\text{gen}} = 1 - \sqrt{\beta_h/[\beta_c \cosh(2\gamma)]}, \tag{1}$$

which is dependent on the squeezing parameter γ and we call the generalized CA efficiency. The finite-power performance, however, was usually analyzed starting from the high-temperature assumption, and Eq. (1) was obtained from phenomenological heat-transfer laws merely holding in classical thermodynamics. To our knowledge, a unified finite-time thermodynamic description of these quantum engines in contact with squeezed thermal baths is still unavailable.

The present paper employs a harmonic system as the working substance to set up a quantum Otto engine energized by a squeezed thermal bath. We analyze the time evolution of the squeezed quantum engine in a sing cycle, and give an analytical expression of the efficiency at maximum power in the high-temperature limit, which agrees well with the generalized CA formula (1). The physical implication of the expression for efficiency at maximum power is also discussed by introducing the dynamical resistance and dissipative resistance. We show that, compared to the standard heat engine, the efficiency of the squeezed engine is significantly enhanced at the price of increasing power fluctuations.

II. MOTION EQUATION OF THE SYSTEM HAMILTONIAN

The Hamiltonian of a single harmonic oscillator with time-dependent frequency $\omega(t)$ can be given by introducing particle number $\hat{N}(t)$ ($\hbar \equiv 1$),

$$\hat{H}(t) = \omega(t)\hat{N}(t) = \omega(t)\hat{a}^\dagger(t)\hat{a}(t), \tag{2}$$

where $\hat{a}^\dagger$ and $\hat{a}$ are the bosonic creation and annihilation operators, respectively. Here and hereafter we set the the ground state energy to be zero for simplicity. If such an oscillator at time $t = t_0$ is in thermal equilibrium with a heat bath at inverse temperature β, its density operator id in the form of

$$\hat{\rho}(t_0) = \sum_n p_n(t_0)|n(t_0)\rangle\langle n(t_0)| = Z^{-1}\exp(-\beta\hat{H}), \tag{3}$$

where $p_n(t_0) = Z^{-1}\exp[-\beta\varepsilon_n(t_0)]$ is the probability of finding the system in state $\left|n(t_0)\right\rangle$ and $Z = \mathrm{Tr}[\exp(-\beta\hat{H})] = \sum_n \exp[-\beta\varepsilon_n(t_0)]$ is the partition function, with the energy spectrum $\varepsilon_n(t_0) = \langle n(t_0)|\hat{a}^\dagger\hat{a}|n(t_0)\rangle\omega(t_0) = n\omega(t_0)$ $(n = 0, 1, 2, \cdots)$. The excitation number reads $\langle n(t_0)\rangle = \mathrm{Tr}[\hat{\rho}(t_0)\hat{N}(t_0)] = [\exp(\beta\omega(t_0)) - 1]^{-1}$ and the Hamiltonian expectation is $\langle\hat{H}(t_0)\rangle = \mathrm{Tr}[\hat{\rho}(t_0)\hat{H}(t_0)] = \omega(t_0)\langle n(t_0)\rangle$. In what follows the parameter of time t will be omitted for simplicity of notation if not necessary.

A system under consideration is weakly coupled to a squeezed boson bath of the Hamiltonian $\hat{H}_B = \sum_k \Omega_k\hat{b}_k^\dagger b_k$ and inverse temperature β by the interaction $\hat{H}_{\mathrm{int}} = \sum_k ig_k(\hat{a}^\dagger\hat{b}_k -$

$\hat{a}\hat{b}_k^\dagger$), where g_k is the interaction strength at the mode k. The squeezed thermal state of the system with the Hamiltonian $\hat{H}$ can be described by the generalized canonical form [42],

$$\hat{\rho}^{sq} = \hat{\mathcal{S}}(\gamma)\exp(-\beta\hat{H})\hat{\mathcal{S}}^\dagger(\gamma)/Z, \tag{4}$$

where $Z = \mathrm{Tr}[\exp(-\beta\hat{H})]$ is the partition function of the system, and

$$\hat{\mathcal{S}}(\gamma) = \exp[(\gamma^*\hat{a} - \gamma\hat{a}^\dagger)/2] \tag{5}$$

as the squeezing operator on the system mode dependens on the squeezing parameter γ. The excitation number for the squeezed thermal state is determined according to $\langle n\rangle^{sq} = \mathrm{Tr}(\hat{\rho}^{sq}\hat{N})$, where $\hat{N} = \hat{a}^\dagger\hat{a}$ is the particle number operator, leading to [43]

$$\langle n\rangle^{sq} = \langle n\rangle + (2\langle n\rangle + 1)\sinh^2(\gamma), \tag{6}$$

where $\langle n\rangle = 1/(e^{\beta\omega}-1)$ is the excitation number of the system at thermal state. Accordingly, the expectation of system Hamiltonian at squeezed thermal state reads $\langle\hat{H}\rangle^{sq} = \omega\langle n\rangle^{sq} = \omega\mathrm{Tr}(\hat{\rho}^{sq}\hat{N})$. For the system weakly coupled to a thermal (non-thermal) bath, its any instant state $\hat{\rho}(t)$ given in Eq. (3) [or $\hat{\rho}^{sq}(t)$ in Eq. (4)] evolves via a Markovian master equation [14, 35, 36, 44], $\dot{\hat{\rho}} = \mathcal{L}\hat{\rho}$, where $\mathcal{L}$ is a Lindblad operator, such that the system can thermalize to asymptotically achieve the thermal steady state, $\dot{\hat{\rho}}_{ss} = \mathcal{L}\hat{\rho}_{ss} = 0$ (squeezed steady state $\dot{\hat{\rho}}_{ss}^{sq} = \mathcal{L}\hat{\rho}_{ss}^{sq} = 0$).

The quantum dynamics of an operator $\hat{X}$ for a system coupled to a heat bath is determined by the quantum master equation [33, 35, 36]:

$$\dot{\hat{X}} = \frac{i}{\hbar}[\hat{H},\ \hat{X}] + \frac{\partial\hat{X}}{\partial t} + \mathcal{L}_D(\hat{X}), \tag{7}$$

where $\mathcal{L}_D(\hat{X}) = k_u\left(\hat{a}^\dagger\hat{X}\hat{a} - \frac{1}{2}\left[\hat{a}\hat{a}^\dagger, \hat{X}\right]_+\right) + k_d\left(\hat{a}\hat{X}\hat{a}^\dagger - \frac{1}{2}\left[\hat{a}^\dagger\hat{a}, \hat{X}\right]_+\right)$, with anticommutator $[\hat{A}, \hat{B}]_+ = \hat{A}\hat{B} + \hat{B}\hat{A}$, and we have used the dot to denote the differentiation with respect to time t. Here k_u and k_d are phenomenological positive coefficients and they satisfy the detailed balance condition $k_u/k_d = e^{-\beta\omega}$ in order for the system to achieve the equilibrium state in a specific way. Considering Eq. (2) and using $E = \langle\hat{H}\rangle$, we have $\dot{E} = \dot{\mathcal{W}} + \dot{\mathcal{Q}} = \omega\langle\dot{n}\rangle + \langle n\rangle\dot{\omega}$, where the power and heat flux are identified as $\mathcal{P} = \dot{\mathcal{W}} = \langle n\rangle\dot{\omega}$ and $\dot{\mathcal{Q}} = \omega\dot{\langle n\rangle}$, respectively. Physically, while work is produced by the change in frequency (which indicates the system volume)[38], heat occurs with a change in system state. In a quantum adiabatic process the system remains in the same state and work is produced while isolated from a heat reservoir. Substituting $\hat{X} = \hat{H} = \omega \hat{a}^\dagger \hat{a}$ into Eq. (7) and considering its expectation $\langle n \rangle = \langle \hat{a}^\dagger \hat{a} \rangle$, we find that the heat flow (power) is in the form of $\dot{\mathcal{Q}} = \langle \mathcal{L}_D(\hat{H}) \rangle = \omega \dot{\langle n \rangle}$ ($\mathcal{P} = \partial \langle \hat{H} \rangle / \partial t = \langle n \rangle \dot{\omega}$), and that for an isochoric process

$$\dot{\langle n \rangle} = -\Gamma [\langle n \rangle - \langle n \rangle^{eq}], \tag{8}$$

where $\Gamma \equiv k_d - k_u$ indicates the heat conductivity between the system and the heat bath.

The motion of isochoric thermalisation [Eq. (7)] for the system coupled to a squeezed heat bath with inverse temperature β can be modified to [35, 45]

$$\mathcal{L}_D(\hat{X}) = k_u \left(\hat{a}_s^\dagger \hat{X} \hat{a}_s - \frac{1}{2} \left[\hat{a}_s \hat{a}_s^\dagger, \hat{X} \right]_+ \right) + k_d \left(\hat{a}_s \hat{X} \hat{a}_s^\dagger - \frac{1}{2} \left[\hat{a}_s^\dagger \hat{a}_s, \hat{X} \right]_+ \right). \tag{9}$$

Here we use the Bogliubov transformation $\hat{a}_s = \hat{a} \cosh(\gamma) + \hat{a}^\dagger \sinh(\gamma) = \hat{\mathcal{S}} \hat{a} \hat{\mathcal{S}}^\dagger$, where the squeezing operator $\hat{\mathcal{S}}$ was defined in Eq. (5). It follows, substituting Eq. (9) into Eq. (7) and using $\langle n \rangle^{sq} = \mathrm{Tr}(\hat{\rho}^{sq} \hat{N}) = \langle \hat{a}_s^\dagger \hat{a}_s \rangle$, that the motion of the isochoric process under squeezing becomes [46]

$$\dot{\langle n \rangle}^{sq} = -\Gamma [\langle n \rangle^{sq} - \langle n \rangle^{sq,eq}], \tag{10}$$

where $\Gamma = k_d - k_u$ was defined in Eq. (8). The excitation of the harmonic system can be parameterized by introducing an effective inverse temperature β^{eff} which satisfies $\langle n(\beta^{\text{eff}}) \rangle = \langle n(\beta) \rangle + (2\langle n(\beta) \rangle + 1) \sinh^2(\gamma)$. Here $\beta^{\text{eff}} < \beta$ due to $\langle n \rangle^{sq} - \langle n \rangle = (2\langle n \rangle + 1) \sinh^2(\gamma) > 0$. By simple manipulation the detailed balance [23]

$$\beta^{\text{eff}} = \frac{1}{\omega} \ln \frac{\langle n \rangle^{sq,eq} + 1}{\langle n \rangle^{sq,eq}} \tag{11}$$

can be restored in the squeezed bath. The effective inverse temperature β^{eff} is a fictitious, unphysical parameter, since it may depend on the external control parameters (e.g. the frequency ω and squeezing parameter γ). In the high-temperature or small-γ limit, the effective temperature becomes system-independent "thermodynamic" temperature $\beta^{\text{eff}} = \beta \mathrm{sech}(2\gamma)$.

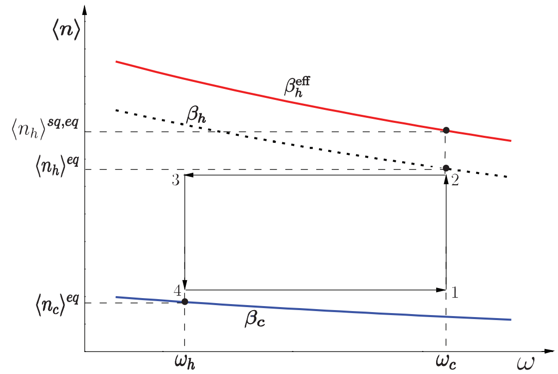

FIG. 1: (Color online) Schematic diagram of a quantum Otto cycle working with a harmonic system in the (ω, n) plane. While $\langle n_h \rangle^{sq,eq} \equiv [\exp(\omega_h \beta_h^{\rm eff}) - 1]^{-1}$, where $\beta_h^{\rm eff}$ is the effective temperature of the hot squeezed bath, is the excitation number for the system after an infinite long time interaction with the bath, $\langle n_h \rangle^{eq}$ ($\langle n_c \rangle^{eq}$) is excitation number of the system at thermal equilibrium with the hot (cold) bath of inverse temperature $\beta_h(\beta_c)$.

III. QUANTUM OTTO CYCLE

A. Work and power fluctuations

A model of quantum Otto engine $1 \to 2 \to 3 \to 4 \to 1$ is sketched in Fig. 1. We assume the time period required for completing the four steps to be $\tau_h, \tau_{hc}, \tau_c, \tau_{ch}$, respectively, and use $\tau_{cyc} \equiv \tau_h + \tau_{hc} + \tau_c + \tau_{ch}$ to denote the total cycle time. The isochoric process $1 \to 2$ ($3 \to 4$) is realized by coupling the working system with fixed frequency ω_h (ω_c) to a hot non-thermal (cold thermal) reservoir at inverse temperature $\beta_h(\beta_c)$, while the system is decoupled from the heat bath along each one of adiabatic processes $2 \to 3$ and $4 \to 1$.

For each cycle, the work is produced only in the two adiabatic processes, without work done by the system in the isochoric processes. Initially, the time at instant 1 is assumed to be $t_1 = 0$. The Hamiltonian changes from $\omega(\tau_h)\hat{N}(\tau_h)$ to $\omega(\tau_h + \tau_{hc})\hat{N}(\tau_h + \tau_{hc})$ along the adiabatic expansion $2 \to 3$, and the Hamiltonian changes back to $\omega(0)\hat{N}(0)$ from $\omega(\tau_{cyc} - \tau_{ch})\hat{N}(\tau_{cyc} - \tau_{ch})$ during the adiabatic compression $4 \to 1$. Then the stochastic work done by the system in a single cycle, which is equivalent to the total work produced along the two (adiabatic) microscopic trajectories, can be given by

$$w[|n(\tau_h)\rangle^{sq};|n(\tau_{cyc}-\tau_{ch})\rangle] = (\omega_h-\omega_c)[{}^{sq}\langle n(\tau_h)|\hat{N}|n(\tau_h)\rangle^{sq} - \langle n(\tau_{cyc}-\tau_{ch})|\hat{N}|n(\tau_{cyc}-\tau_{ch})\rangle]. \tag{12}$$

In the adiabatic process the state of the system remains unchanged and thus the final state is identical to the initial one, namely, $\hat{\rho}(\tau_h+\tau_{hc}) = \hat{\rho}^{sq}(\tau_h)$ and $\hat{\rho}^{sq}(0) = \hat{\rho}(\tau_{cyc}-\tau_{ch})$, where $\hat{\rho}(t)$ and $\hat{\rho}^{sq}(t)$ were defined in Eqs. (3) and (4), respectively. The probability density of the work w is given by the formula

$$p(w) = \sum_n p_n(\tau_h)p_n(\tau_{cyc}-\tau_{ch})\delta\{w - w[|n(\tau_h)\rangle^{sq};|n(\tau_{cyc}-\tau_{ch})\rangle]\}, \tag{13}$$

where $\delta(\bullet)$ is the Dirac's δ function. Using $\rho^{sq}(t) = \sum_n p_n(t)\mathcal{S}(\gamma)|n(t)\rangle\langle n(t)|\mathcal{S}^\dagger(\gamma) = \sum_n p_n(t)|n(t)\rangle^{sq}\ {}^{sq}\langle n(t)|$, we obtain the average work output per cycle as

$$\mathcal{W} \equiv \langle w\rangle = \int wp(w)dw = (\omega_h-\omega_c)[\langle n(\tau_h)\rangle^{sq} - \langle n(\tau_{cyc}-\tau_{ch})\rangle], \tag{14}$$

where $\langle n(\tau_h)\rangle^{sq} = \mathrm{Tr}[\hat{\rho}^{sq}(\tau_h)\hat{N}]$ and $\langle n(\tau_{cyc}-\tau_{ch})\rangle = \mathrm{Tr}[\hat{\rho}^{sq}(\tau_{cyc}-\tau_{ch})\hat{N}]$ have been used, with $\langle n(\tau_h)\rangle^{sq} = \langle n(\tau_h)\rangle + [2\langle n(\tau_h)\rangle + 1]\sinh^2(\gamma)$. For each cycle, heat is transferred only in the isochore, while work is produced only along the adiabatic process. The heat absorbed from the hot squeezed bath is

$$\mathcal{Q}_h = \langle \hat{H}(\tau_h)\rangle^{sq} - \langle \hat{H}(0)\rangle^{sq} = \omega_h[\langle n(\tau_h)\rangle^{sq} - \langle n(0)\rangle^{sq}]. \tag{15}$$

Then the efficiency reads

$$\eta = \frac{\mathcal{W}}{\mathcal{Q}_h} = 1 - \frac{\omega_c}{\omega_h}. \tag{16}$$

We now turn to discussion on the dynamical evolution of the system during the thermalization processes $1\to 2$ and $3\to 4$, which allows for establishing the relation between the excitation number $\langle n(\tau_h)\rangle^{sq}$ ($\langle n(\tau_{cyc}-\tau_{ch})\rangle$) and its asymptotic value $\langle n_h\rangle^{sq,eq}$ ($\langle n_c\rangle^{eq}$), with $\langle n_h\rangle^{sq,eq} = \langle n(\tau_h\to\infty)\rangle^{sq}$ and $\langle n_c\rangle^{eq} = \langle n(\tau_c\to\infty)\rangle = 1/(e^{\beta_c\omega_c}-1)$.

For the isochore $1\to 2$, the energy transfer is realized merely via heat absorbed from the squeezed bath during the period τ_h. Although practically the system evolves form instant 1 to 2, it would relax to a nondisplaced squeezed thermal state after infinite long time, and its excitation number $\langle n_h(t\to\infty)\rangle^{sq} = \langle n_h\rangle^{sq,eq}$. From Eq. (8), we find that the excitation numbers of the initial and final states $\langle n(0)\rangle^{sq}$ and $\langle n(\tau_h)\rangle^{sq}$ satisfy the relation:

$$\langle n(\tau_h)\rangle^{sq} = \langle n_h\rangle^{sq,eq} + (\langle n(0)\rangle^{sq} - \langle n_h\rangle^{sq,eq})e^{-\Gamma_h\tau_h}. \tag{17}$$

For the cold isochore $3 \to 4$, the system is in contact with the cold reservoir at inverse temperature β_c in time of τ_c. Based on an analogy with hot isochore $1 \to 2$, the excitation number $\langle n(\tau_{cyc} - \tau_{ch})\rangle$ as a function $\langle n(\tau_h + \tau_{hc})\rangle$ is obtained,

$$\langle n(\tau_{cyc} - \tau_{ch})\rangle = \langle n_c\rangle^{eq} + (\langle n(\tau_h + \tau_{hc})\rangle - \langle n_c\rangle^{eq})e^{-\Gamma_c \tau_c}. \tag{18}$$

Here Γ_h (Γ_c) represents the heat conductivity between the working substance and the hot (cold) reservoir.

The adiabatic expansion $2 \to 3$ and compression $4 \to 1$ are realized by changing adiabatically the frequency between ω_c and ω_h, during which

$$\langle n(\tau_h)\rangle^{sq} = \langle n(\tau_h + \tau_{ch})\rangle, \langle n(0)\rangle^{sq} = \langle n(\tau_{cyc} - \tau_{hc})\rangle \tag{19}$$

due to constant Von Neumann entropy via unitary transformation. With consideration of Eqs. (17), (18) and (19), the heat transporting into the system (15) and the work output (14) turn out to be

$$\mathcal{Q}_h = (\langle n_h\rangle^{sq,eq} - \langle n_c\rangle^{eq})\omega_h g(\tau_c, \tau_h), \tag{20}$$

$$\mathcal{W} = (\langle n_h\rangle^{sq,eq} - \langle n_c\rangle^{eq})(\omega_h - \omega_c) g(\tau_c, \tau_h), \tag{21}$$

where $\langle n_h\rangle^{sq,eq} = \langle n_h\rangle^{eq} + (2\langle n_h\rangle^{eq} + 1)\sinh^2(\gamma)$ and $g(\tau_h, \tau_c) = \frac{(e^{\gamma_h \tau_h} - 1)(e^{\gamma_c \tau_c} - 1)}{e^{\gamma_c \tau_c + \gamma_h \tau_h} - 1}$. From Eqs. (14) and (21), one has

$$\langle n(\tau_h)\rangle^{sq} - \langle n(\tau_{cyc} - \tau_{ch})\rangle = (\langle n_h\rangle^{sq,eq} - \langle n_c\rangle^{eq}) g(\tau_h, \tau_c). \tag{22}$$

With consideration of Eqs. (17), (18) and (19), we have

$$\langle n(\tau_h)\rangle = \langle n_h\rangle^{eq} + \frac{\mathcal{A}}{2} \frac{e^{\gamma_c \tau_c} - 1}{e^{\gamma_h \tau_h + \gamma_c \tau_c} - 1} \tag{23}$$

and

$$\langle n(\tau_{cyc} - \tau_{ch})\rangle = \langle n_c\rangle^{eq} + \frac{\mathcal{B}}{2} \frac{e^{\gamma_h \tau_h} - 1}{e^{\gamma_h \tau_h + \gamma_c \tau_c} - 1}, \tag{24}$$

where $\mathcal{A} \equiv (2\langle n_c\rangle^{eq} + 1)\text{sech}(2\gamma) - 2\langle n_h\rangle^{eq} - 1$ and $\mathcal{B} \equiv (2\langle n_h\rangle^{eq} + 1)\cosh(2\gamma) - 2\langle n_c\rangle^{eq} - 1$. Using $\langle n^2\rangle = \text{Tr}(\hat{\rho}\hat{N}^2)$ and $\langle n^2\rangle^{sq} = \text{Tr}(\hat{\rho}^{sq}\hat{N}^2)$, we find that the second element of the work, $\langle w^2\rangle = \int w^2 p(w)dw$, can be given by $\langle w^2\rangle = (\omega_h - \omega_c)^2\{\langle n^2(\tau_h)\rangle^{sq} - [\langle n(\tau_h)\rangle^{sq}]^2 + \langle n^2(\tau_{cyc} - \tau_{ch})\rangle - \langle n(\tau_{cyc} - \tau_{ch})\rangle^2\}$. As a result, the work fluctuations, $\delta w^2 = \langle w^2\rangle - \langle w\rangle^2$, become

$$\begin{aligned}\delta w^2 &= (\omega_h - \omega_c)^2[\langle n(\tau_h)\rangle^2 \cosh(4\gamma) + \langle n(\tau_h)\rangle \cosh(4\gamma) + \frac{1}{2}\sinh(2\gamma) \\ &+ \langle n(\tau_{cyc} - \tau_{ch})\rangle^2 + \langle n(\tau_{cyc} - \tau_{ch})\rangle]. \end{aligned}\tag{25}$$

Since the heat capacity at given time $t = t_0$ reads $C[\beta(t_0)\omega(t_0)] = -\beta^2(t_0)\partial\langle \hat{H}(t_0)\rangle/\partial\beta(t_0)$, the work fluctuations [Eq. (25)] can be rewritten as

$$\delta w^2 = (\omega_h - \omega_c)^2 \left[\frac{\cosh(4\gamma)}{\beta_2^2\omega_h^2}C(\beta_2\omega_h) + \frac{1}{2}\sinh(2\gamma) + \frac{1}{\beta_4^2\omega_c^2}C(\beta_4\omega_c)\right], \tag{26}$$

where we have used $\beta_2 = \beta(\tau_h)$ and $\beta_4 = \beta(\tau_{cyc} - \tau_{ch})$. From Eq. (26) it shows that the work fluctuations δw^2 are dominated by squeezing when the temperature is very low due to small value of heat capacity. In contrast to the non-squeezing case, in the zero-temperature limit when the heat capacity $C(\beta\omega) \to 0$, the work fluctuations δw^2 for the heat engine under squeezing ($\gamma \neq 0$) are still positive as $\delta w^2 \to \sinh(\gamma)\cosh(\gamma)$.

Substituting Eqs. (23) and (24) into Eq. (25), the analytical time-dependent expression of the work fluctuations δw^2 is obtained as,

$$\begin{aligned}\delta w^2 \;=\;& (\omega_h - \omega_c)^2\big[(\langle n_h\rangle^{eq})^2\cosh(4\gamma) + \langle n_h\rangle^{eq}\cosh(4\gamma) + \frac{1}{2}\sinh(2\gamma) + (\langle n_c\rangle^{eq})^2 + \langle n_c\rangle^{eq}\\ &+\; \mathcal{A}(2\langle n_h\rangle^{eq} + 1)\cosh(4\gamma)\frac{g(\tau_h,\tau_c)}{(e^{\gamma_h\tau_h} - 1)} + \mathcal{A}^2\cosh(4\gamma)\frac{g^2(\tau_h,\tau_c)}{(e^{\gamma_h\tau_h} - 1)^2}\\ &+\; \mathcal{B}(2\langle n_c\rangle^{eq} + 1)\frac{g(\tau_h,\tau_c)}{(e^{\gamma_c\tau_c} - 1)} + \mathcal{B}^2\frac{g(\tau_h,\tau_c)}{(e^{\gamma_c\tau_c} - 1)}\big].\end{aligned} \tag{27}$$

In the quasi-static limit when τ_h and τ_c are very large, $\frac{e^{\gamma_\alpha\tau_\alpha}-1}{e^{\gamma_h\tau_h+\gamma_c\tau_c}-1} \to 0$ with $\alpha = h, c$, and thus the work fluctuations turn out to be

$$\delta w^2 = (\omega_h - \omega_c)^2[(\langle n_h\rangle^{eq})^2\cosh(4\gamma) + \langle n_h\rangle^{eq}\cosh(4\gamma) + \frac{1}{2}\sinh(2\gamma) + (\langle n_c\rangle^{eq})^2 + \langle n_c\rangle^{eq}], \tag{28}$$

which reduce to $\delta w^2 = (\omega_h - \omega_c)^2[(\langle n_h\rangle^{eq})^2 + \langle n_h\rangle^{eq} + (\langle n_c\rangle^{eq})^2 + \langle n_c\rangle^{eq}]$ under no squeezing ($\gamma = 0$). Accordingly, the fluctuations for the work as a function of heat capacity [Eq. (26)] become $\delta w^2 = (\omega_h - \omega_c)^2\left[(\beta_h\omega_h)^{-2}\cosh(4\gamma)C(\beta_h\omega_h) + \sinh(2\gamma)/2 + (\beta_c\omega_c)^{-2}C(\beta_c\omega_c)\right]$, which can reproduce the work fluctuations for the heat engine cycle without squeezing [26]: $\delta w^2 = (\omega_h - \omega_c)^2\left[(\beta_h\omega_h)^{-2}C(\beta_h\omega_h) + (\beta_c\omega_c)^{-2}C(\beta_c\omega_c)\right]$.

Since the stochastic power output reads $\dot{w}[|n(\tau_h)\rangle^{sq}; |n(\tau_{cyc} - \tau_{ch})\rangle] = w[|n(\tau_h)\rangle^{sq}; |n(\tau_{cyc} - \tau_{ch})\rangle]/\tau_{cyc}$, where $w[|n(\tau_h)\rangle^{sq}; |n(\tau_{cyc} - \tau_{ch})\rangle]$ is given by Eq. (12), the relative fluctuations of the power output are equivalent to those of work. From formulae (14) and (27), we obtain the relative power fluctuations as, $f_{\dot{w}} = f_w = \frac{1}{[\langle n_h\rangle^{eq}\cosh(2\gamma)+\sinh^2(\gamma)-\langle n_c\rangle^{eq}]^2 g^2(\tau_c,\tau_h)}\big[(\langle n_h\rangle^{eq})^2\cosh(4\gamma) + \langle n_h\rangle^{eq}\cosh(4\gamma) + \frac{1}{2}\sinh(2\gamma) + (\langle n_c\rangle^{eq})^2 + \langle n_c\rangle^{eq} + \mathcal{A}(2\langle n_h\rangle^{eq} + 1)\cosh(4\gamma)\frac{g(\tau_h,\tau_c)}{(e^{\gamma_h\tau_h}-1)} + \mathcal{A}^2\cosh(4\gamma)\frac{g^2(\tau_h,\tau_c)}{(e^{\gamma_h\tau_h}-1)^2} + \mathcal{B}(2\langle n_c\rangle^{eq} +$

$1)\frac{g(\tau_h,\tau_c)}{(e^{\gamma_c\tau_c}-1)}+\mathcal{B}^2\frac{g^2(\tau_h,\tau_c)}{(e^{\gamma_c\tau_c}-1)^2}\big]$. Consequently, the relative power fluctuations $f_{\dot{w}}$ decrease as the time allocations of τ_h and τ_c increase and they achieve their lower bound

$$
\begin{aligned}
f_{\dot{w}}^- &= \frac{1}{[\langle n_h\rangle^{eq}\cosh(2\gamma)+\sinh^2(\gamma)-\langle n_c\rangle^{eq}]^2} \\
&\times \left[(\langle n_h\rangle^{eq})^2\cosh(4\gamma)+\langle n_h\rangle^{eq}\cosh(4\gamma)+\frac{1}{2}\sinh(2\gamma)+(\langle n_c\rangle^{eq})^2+\langle n_c\rangle^{eq}\right] \qquad (29)
\end{aligned}
$$

in the quasi-static limit. Under no squeezing ($\gamma=0$), Eq. (29) shows that the relative root-mean-square power fluctuations can be obtained, $\sqrt{f_{\dot{w}}^-}=[\langle n_h\rangle^{eq}-\langle n_c\rangle^{eq}]^{-1}\left[(\langle n_h\rangle^{eq})^2+\langle n_h\rangle^{eq}+(\langle n_c\rangle^{eq})^2+\langle n_c\rangle^{eq}\right]^{1/2}$, or

$$
\sqrt{f_{\dot{w}}^-}=\frac{(\omega_h-\omega_c)}{[\langle n_h\rangle^{eq}-\langle n_c\rangle^{eq}](\omega_h-\omega_c)}\left[(\langle n_h\rangle^{eq})^2+\langle n_h\rangle^{eq}+(\langle n_c\rangle^{eq})^2+\langle n_c\rangle^{eq}\right]^{1/2}. \qquad (30)
$$

For the denominator, one can readily prove [37] that, in the quasi-static limit where the cycle duration is very long, $(\omega_h-\omega_c)(n_h^{eq}-n_c^{eq})\simeq(1/\beta_h-1/\beta_c)\Delta S$, with ΔS being the entropy change in the hot or cold isochoric thermalization process. In the classical limit where the temperature is high enough ($\beta_h\omega_h\ll 1$ and $\beta_c\omega_c\ll 1$), we have the approximation via making Taylor series expansion: $(\langle n_h\rangle^{eq})^2+\langle n_h\rangle^{eq}+(\langle n_c\rangle^{eq})^2+\langle n_c\rangle^{eq}\simeq 4/(\beta_h\omega_h)^2+4/(\beta_c\omega_c)^2$. For the engine efficiency (16), approaching the Carnot value η_C means the limit $\beta_h\omega_h\to\beta_c\omega_c$. The formula (30) for the quantum harmonic engine gives rise to

$$
\sqrt{f_{\dot{w}}^-}=\frac{2}{\Delta S}, \qquad (31)
$$

for the classical cyclic engine operating at the Carnot efficiency. It indicates from Eq. (31) that, unlike in the steady state heat engines [47, 48] in which trade-off between power and efficiency are overcome with the price of lager power fluctuations, the Otto engine can operate with efficiency η asymptotically close to η_C at positive power with finite and even vanishing fluctuations. Therefore, we recover the result obtained from the classical cyclic heat engines [7] in which a simplified system Hamiltonian as an illustrative example was adopted.

B. Efficiency at maximum power in the context of finite-time thermodynamics

Having obtained the time-dependent expressions of average heat and work, we can analyze the efficiency at maximum power to reveal the finite-time performance of our engine model.

Introducing $\mathcal{G}(\tau_c, \tau_h, \tau_{adi}) = g(\tau_h, \tau_c)/(\tau_h + \tau_c + \tau_{adi})$, where $\tau_{adi} \equiv \tau_{hc} + \tau_{ch}$ denotes the total time taken for the two adiabatic processes, we obtain the power output $\mathcal{P} = \mathcal{W}/\tau_{cyc}$ as

$$\mathcal{P} = (\omega_h - \omega_c)(\langle n_h \rangle^{sq,eq} - \langle n_c \rangle^{eq})\mathcal{G}(\tau_c, \tau_h, \tau_{adi}). \tag{32}$$

It is written as a product of two functions: a function $F(\beta_c, \omega_c, \beta_h, \omega_h) \equiv (\langle n_h \rangle^{sq,eq} - \langle n_c \rangle^{eq})(\omega_h - \omega_c)$, which explicitly depends on the external parameters β, ω, and γ, and the other one $\mathcal{G}$ determining the time allocations $(\tau_c, \tau_h, \tau_{adi})$ on the isochores and adiabats. When the external constraints are fixed, optimizing the power output $\mathcal{P}$ is realized via optimizing the time-dependent function $\mathcal{G}(\tau_c, \tau_h, \tau_{adi})$. In such a case, we set $\partial\mathcal{G}/\partial\tau_c = 0$ and $\partial\mathcal{G}/\partial\tau_h = 0$ to determine the optimal time allocations on the cold and hot isochores, leading to

$$\Gamma_c[\cosh(\Gamma_h \tau_h) - 1] = \Gamma_h[\cosh(\Gamma_c \tau_c) - 1], \tag{33}$$

which gives the optimal protocols for the engine cycle, and shows τ_c and τ_h depending on each other. Under maximum power $\Gamma_c = \Gamma_h$ leads to $\tau_c = \tau_h$. Either in the sudden adiabatic limit, where τ_{adi} can be negligible compared to τ_h and τ_c, or under the assumption when the time allocation on the adiabatic process τ_{adi} is proportional to that on the isochore, i.e., $\tau_{adi} \propto \tau_{h,c}$, the maximum power output $\mathcal{P}_{\max}$ increases with decreasing "effective time" $\gamma_h \tau_h (=\gamma_c \tau_c)$.

Now we consider the optimization on the external constrains of heat engine to get maximum power, assuming the time allocated on the adiabats τ_{adi} to be constant. From Eq. (32), optimizing the power output is equivalent to optimizing the frequencies ω_h and ω_c. By performing $\partial\mathcal{P}/\partial\omega_c = 0$ and $\partial\mathcal{P}/\partial\omega_h = 0$, we have

$$\langle n_h \rangle^{sq,eq} - \langle n_c \rangle^{eq} = \frac{1}{4}\left(\omega_h - \omega_c\right)\beta_c \mathrm{csch}^2\left(\frac{\omega_c \beta_c}{2}\right), \tag{34}$$

and

$$\langle n_h \rangle^{sq,eq} - \langle n_c \rangle^{eq} = \frac{1}{4}\left(\omega_h - \omega_c\right)\beta_h^{\mathrm{eff}} \mathrm{csch}^2\left(\frac{\omega_h \beta_h^{\mathrm{eff}}}{2}\right), \tag{35}$$

where $\langle n_h \rangle^{sq,eq} = 1/(e^{\beta_h^{\mathrm{eff}}\omega_h} - 1)$ and $\langle n_c \rangle^{eq} = 1/(e^{\beta_c \omega_c} - 1)$. For given bath temperatures (β_h and β_c) and the squeezing parameter (γ), this set of two nonlinear equations can be calculated for yielding the optimal values of ω_c and ω_h at maximum power. Combining Eqs. (34) and (35), and introducing $x_c = e^{-\beta_c \omega_c}$, $x_h = e^{-\beta_h^{\mathrm{eff}}\omega_h}$, and $\beta_c = r_s^2 \beta_h^{\mathrm{eff}}$, we arrive at

$$\frac{\beta_c x_c}{\beta_h^{\mathrm{eff}} x_h} = \frac{(x_c - 1)^2}{(x_h - 1)^2}, \tag{36}$$

which leads to $x_c = (2x_h)^{-1}\{2x_h + r_s(x_h-1)[r_s(x_h-1) + \sqrt{r_s^2(x_h-1)^2+4x_h}]\}$, where $r_s > 0$. For given x_h, x_c decreases very quickly and becomes much smaller than 1 (The numerical calculation for x_c as a function of r_s for given x_h with $0 < x_h < 1$ is not plotted here). By using Eq. (36), we can expand x_h (as a function x_c) up to the third term of x_c, $x_h = r_s^2 x_c + 2(1-r_s^2)x_c^2 + [3r_s^2(1-r_s^2) - 5r_s^4(1-r_s^2)]x_c^3 + \mathcal{O}(x_c^4)$. Substituting it into $n_h^{sq,eq} - n_c^{eq} = (x_h-1)^{-1} - (x_c-1)^{-1}$, yields the good approximation:

$$\langle n_h\rangle^{sq,eq} - \langle n_c\rangle^{eq} \simeq (x_h - x_c) = (r_s^2-1)x_c + \mathcal{O}(x_c^2). \tag{37}$$

In deriving Eq. (37), we have considered the two limits of $x_c \ll 1$ for $r_s \gg 1$, and $x_c \to 1$ with $r_s \to 0$. With consideration of these two limits, x_h can thus be approximated by

$$x_h = r_s^2 x_c. \tag{38}$$

Combination of Eqs. (34) and (35) yields $\left(\frac{\ln x_c}{\beta_c} - \frac{2\ln r_s + \ln x_c}{\beta_h^{\mathrm{eff}}}\right) = \frac{x_h - x_c}{\sqrt{x_h x_c \beta_c \beta_h^{\mathrm{eff}}}}$, which, together with Eq. (38), gives rise to

$$\ln x_c = \frac{(r_s^2-1)\sqrt{\beta_c\beta_h^{\mathrm{eff}}} + 2r_s\beta_c\ln(r_s)}{2r_s(\beta_h^{\mathrm{eff}} - \beta_c)}. \tag{39}$$

Substituting $r_s = 1/\sqrt{1-\eta_C^{\mathrm{gen}}}$ with $\eta_C^{\mathrm{gen}} = 1 - \beta_h^{\mathrm{eff}}/\beta_c$ into the expression: $\eta_{mp} = 1 - \omega_c/\omega_h = 1 - \frac{\beta_h^{\mathrm{eff}}\ln x_c}{\beta_c \ln x_h}$, we derive the expression of efficiency at maximum power as

$$\eta_{mp} = \frac{(\eta_C^{\mathrm{gen}})^2}{\eta_C^{\mathrm{gen}} - (1-\eta_C^{\mathrm{gen}})\ln(1-\eta_C^{\mathrm{gen}})}, \tag{40}$$

which reduces to that obtained from microscopic and mesoscopic heat engines [30, 32, 33] with vanishing squeezing $\gamma = 0$. We emphasize that the efficiency at maximum power obtained here holds well in the region of any finite temperatures. In the special case when the temperature is high enough, we find that $\beta_h^{\mathrm{eff}} = \beta_h/\cosh(2\gamma)$ is frequency-independent, and Eq. (40) becomes an analytical function of η_C,

$$\eta_{mp}^* = \frac{[\eta_C + \cosh(2\gamma) - 1]^2 \mathrm{sech}(2\gamma)}{\cosh(2\gamma) - (1-\eta_C)\{1 + \ln[(1-\eta_C)\mathrm{sech}(2\gamma)]\}}. \tag{41}$$

In the high-temperature limit when $\beta\omega \ll 1$ and thus $\langle n(\beta\omega)\rangle \simeq 1/(\beta\omega)$ as well as $\langle n(\beta\omega)\rangle^{sq} \simeq 1/(\beta^{\mathrm{eff}}\omega)$, the heat absorbed by the system during the hot (cold) isochore becomes the phenomenological heat-transfer law: $Q_h = \gamma_c(1/\beta_h^{\mathrm{eff}} - 1/\beta(t)]g(\tau_c,\tau_h)$ or $Q_c = \gamma_h[(1/\beta_c - 1/\beta(t)]g(\tau_c,\tau_h)$. We then find the efficiency at maximum power to be

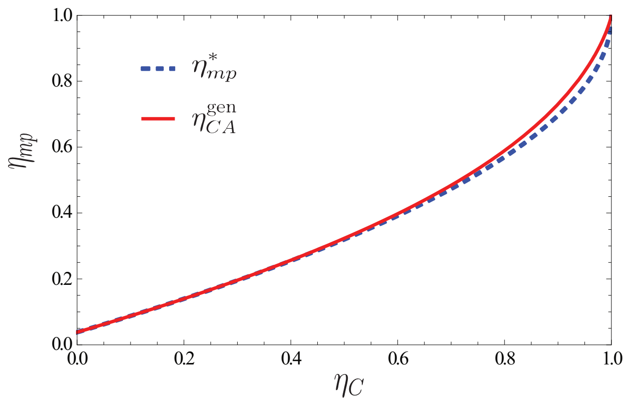

FIG. 2: (Color online) The efficiency at maximum power η_{mp} as a function of the Carnot efficiency for fixed squeezing parameter $\gamma = 0.2$. The analytical expression η^*_{mp} given by Eq. (41) and the generalized CA efficiency $\eta^{\rm gen}_{CA}$ are denoted by a red solid line and a blue dashed one, respectively.

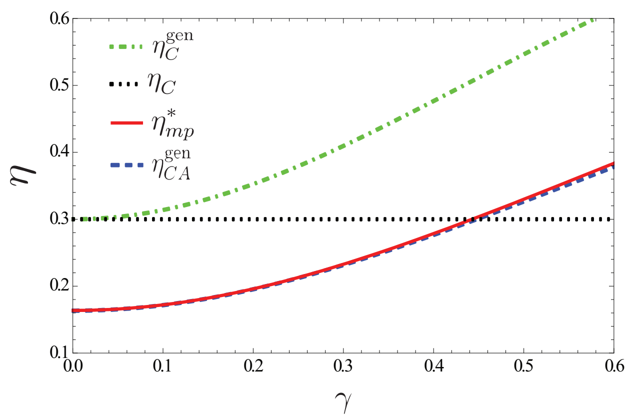

FIG. 3: (Color online) The efficiency η as a function of the squeezing parameter γ for given Carnot value (dotted black line). The solid red line and dashed blue one show η^*_{mp} and $\eta^{\rm gen}_{CA}$, both of which can surpasses the standard Carnot efficiency for finite squeezing parameters, but obey the generalized Carnot value (green dot-dashed line).

the generalized CA efficiency: $\eta_{mp} = \eta^{\rm gen}_{CA} = 1 - \sqrt{\mathrm{sech}(2\gamma)\beta_h/\beta_c}$, which, however, indicates less validity compared to Eq. (41), since Eq. (41) is obtained from quantum master equation in stochastic thermodynamics. The efficiency at maximum power (η^*_{mp}) as a function of the standard Carnot value for given squeezing parameter agrees well with the generalized CA efficiency, as demonstrated in Fig. 2.

The analytic expression for the efficiency at maximum power (η^*_{mp}) and the generalized CA efficiency ($\eta^{\rm gen}_{CA}$) for varying squeezing are plotted in Fig. 3, comparing the standard

Carnot efficiency (η_C) and generalized Carnot one (η_C^{gen}). Figure 3 shows that squeezing as a form of energy yields an increase in the work output [22] and thus results into the efficiency beyond the standard Carnot limit. As expected, the engine operation is limited by the generalized Carnot efficiency and it does obey the second law of thermodynamics.

C. Irreversible thermodynamic analysis on efficiency at maximum power

The expression of efficiency at maximum power (40) for the quantum Otto engine can be understood in terms of dissipations along the (isochoric) thermalization processes, within framework of irreversible thermodynamics. The time durations τ_h, τ_c, and τ_{adi} can be set to be constants, respectively, when they satisfy the optimal relation (33). Considering Eqs. (20) and (21) and using the relation $\mathcal{W} = \mathcal{Q}_h - \mathcal{Q}_c$, the average input heat current and output heat current can be expressed as

$$\dot{\mathcal{Q}}_h \equiv \mathcal{Q}_h \tau_{cyc}^{-1} = \tau_{cyc}^{-1}\omega_h(\langle n_h\rangle^{sq,eq} - \langle n_c\rangle^{eq})g, \tag{42}$$

$$\dot{\mathcal{Q}}_c \equiv \mathcal{Q}_c \tau_{cyc}^{-1} = \tau_{cyc}^{-1}\omega_c(\langle n_h\rangle^{sq,eq} - \langle n_c\rangle^{eq})g, \tag{43}$$

respectively. Since the change in system entropy is vanishing after a single cycle, the entropy production per cycle is merely coming from the two heat reservoirs. The average entropy production rate, $\dot{\sigma} = -\beta_h^{\text{eff}}\dot{\mathcal{Q}} + \beta_c\dot{\mathcal{Q}}_c$, can be expressed in terms of inverse temperature β,

$$\dot{\sigma} = -\Delta\omega\beta_h^{\text{eff}}\mathcal{I} + \beta_c\Delta(\beta^{-1})\omega_c\beta_h^{\text{eff}}\mathcal{I}, \tag{44}$$

where $\Delta(\beta^{-1}) = 1/\beta_h^{\text{eff}} - 1/\beta_c$ is the temperature difference and $\Delta\omega \equiv \omega_h - \omega_c$ is the difference between the maximum and minimum energy gaps (ω_h and ω_c), and

$$\mathcal{I} = \langle\dot{n}\rangle = g\tau_{cyc}^{-1}[\langle n_h\rangle^{sq,eq} - \langle n_c)\rangle^{eq}] \tag{45}$$

denotes the effective (average) particle current. With consideration of Eq. (44), we use $X = \Delta\omega$ and $X_0 = \omega_c\beta_c\Delta(\beta^{-1})$ to denote the thermodynamic forces, respectively. Introducing $S \equiv \beta_c\omega_c$ to denote the (average) entropy change due to a single particle transition in the cold quasistatic isochoric process, the average heat fluxes (42) and (43) can be rewritten as,

$$\dot{\mathcal{Q}}_h = (\beta_h^{\text{eff}})^{-1}S\mathcal{I} - (X_0 - X)\mathcal{I}, \tag{46}$$

$$\dot{\mathcal{Q}}_c = \beta_c^{-1}S\mathcal{I}. \tag{47}$$

These expressions satisfy the generalized forms derived in classical steady heat engines [39, 40]: $\dot{\mathcal{Q}}_h = \beta_h^{-1} S\mathcal{I} - \alpha R_{dis}\mathcal{I}^2$ and $\dot{\mathcal{Q}}_c = \beta_c^{-1} S\mathcal{I} + (1-\alpha) R_{dis}\mathcal{I}^2$, where α is the coefficient of partition of the dissipated heat between the two thermal baths and its values must be situated between $0 \leq \alpha \leq 1$. With consideration of Eqs. (46) and (47), the efficiency (16) can be rewritten as a function of thermodynamic forces X and X_0,

$$\eta = \frac{\eta_C^{\text{gen}} X}{X_0 - \eta_C^{\text{gen}}(X_0 - X)}. \tag{48}$$

Maximizing power $\mathcal{P}$ with respect to X and X_0 is equivalent to maximizing power through tuning of ω_c and ω_h, which thus allows one to obtain

$$X^{(m)} = \frac{\eta_C^{\text{gen}}}{(1-\eta_C^{\text{gen}})\beta_c}, X_0^{(m)} = \frac{\eta_C^{\text{gen}} - \ln(1-\eta_C^{\text{gen}})}{(1-\eta_C^{\text{gen}})\beta_c}. \tag{49}$$

As expected, the analytical expression for the efficiency at maximum power given by Eq. (40) is re-obtained by inserting Eq. (49) into Eq. (48). From Eq. (37), the particle current under maximal power can be approximated by

$$\mathcal{I} = g\tau_{cyc}^{-1} e^{-\beta_h^{\text{eff}}(\omega_c + X)} \left[1 - e^{-\beta_h(X_0 - X)}\right], \tag{50}$$

which has an exponential dependence on the thermodynamic force X. In order to consider the nonlinear case, we distinguish between the dynamical response of the system and the ability to dissipate energy [39, 40]. Let R_{dyn} be the dynamic resistance (also called differential resistance) and R_{dis} the dissipative resistance. While the dynamic resistance R_{dyn} is associated with small changes of X and $\mathcal{I}$ near a specific working point, the dissipative resistance R_{dis} refers to the dissipations that hinder the particle flow and thus result into dissipated power. In analogy with steady state heat engines [39, 40], we keep the following general definitions of these two resistances: $R_{\text{dyn}} = d(X_0 - X)/d\mathcal{I} = -dX/d\mathcal{I}$ and $R_{\text{dis}} = (X_0 - X)/\mathcal{I}$, which, together with Eq. (50), gives rise to

$$R_{\text{dyn}} = \frac{\tau_{cyc}}{g\beta_h^{\text{eff}}} e^{\beta_h^{\text{eff}}(\omega_c + X)}, R_{\text{dis}} = \frac{\beta_h^{\text{eff}}(X_0 - X) R_{\text{dyn}}}{1 - e^{-\beta_h^{\text{eff}}(X_0 - X)}}. \tag{51}$$

Substituting Eq. (49) into Eq. (51) , we find that R_{dyn} and R_{dis} as well as the ratio $R_{\text{dis}}/R_{\text{dyn}}$

$$\frac{R_{\text{dis}}}{R_{\text{dyn}}} = -\frac{\ln(1-\eta_C^{\text{gen}})}{\eta_C^{\text{gen}}} \tag{52}$$

are increasing significantly due to squeezing. From Eq. (52), we recover the general expression of the efficiency at maximum power ($\alpha = 1$)[39]:

$$\eta_{mp} = \frac{\eta_C^{\text{gen}}}{1 + (1 - \alpha\eta_C^{\text{gen}}) R_{\text{dis}}/R_{\text{dyn}}} \tag{53}$$

by extending nonlinear steady heat engines to nonlinear cyclic ones. Physically, squeezing yields an increase in the ratio $R_{\rm dis}/R_{\rm dyn}$ but a decrease in $(1-\eta_C^{\rm gen})R_{\rm dis}/R_{\rm dyn}$. Consequently, the efficiency at maximum power (53) is significantly enhanced by squeezing. The expression (53) is exactly the same as Eq. (40) and thus simplifies to Eq. (41) in the hight temperature limit. It is therefore demonstrated that efficiency at maximum power (53) can be used to describe the cyclic quantum heat engines beyond linear response regime.

IV. CONCLUSIONS

In summary, we have analyzed the finite-time performance of a quantum Otto engine driven by a squeezed thermal bath. Starting with quantum master equation, we have derived analytical expressions for efficiency, power, and power fluctuations, all of which are explicitly dependent on the time allocation on each step of the thermalization processes. The efficiency at maximum power was derived analytically by optimizing power output with respect to external control parameters. We find that at the high temperature limit the efficiency at maximum power derived here closely follows the generalized CA efficiency. From irreversible thermodynamics, this expression for the efficiency at maximum power was re-obtained by introducing the dynamical resistance and dissipative resistance as in classical steady heat engines. Our results show that the efficiency is significantly enhanced and can surpass the standard Carnot value due to the squeezing as an energy resource increasing the work output, but with larger power fluctuations compared to vanishing squeezing.

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