Acoustic self-oscillation in a spherical microwave plasma

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Acoustic Self-Oscillation in a Spherical Microwave Plasma

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We present a method of sound amplification and self-oscillation in high pressure partially ionized gas. Continuous microwaves incident on partially ionized gas may sustain and amplify an acoustic field if increased ionization during the sound field's adiabatic compression enhances RF power absorption. Amplifying sound in this way enables the generation of high amplitude sound in a cavity containing partially ionized gas without mechanical driving or precise knowledge of its resonance frequency. This method of amplification may open opportunities within thermoacoustics such as using 3D geometries and volumetric gain mechanisms.

I. INTRODUCTION

An acoustic cavity held out of thermal equilibrium by a heat source may spontaneously amplify its resonant modes to extraordinary amplitudes. Famous lecture demonstrations of the amplification of sound in the presence of a thermal gradient include the Rijke tube[1], Sondhauss tube[2], and Knipps tube[3]. Such amplification can both cause catastrophe by rattling apart jet engines[4] or benefit through the generation of electrical energy[5]. To explain these phenomena, Rayleigh wrote general criteria for amplification and what would become the guiding principle of thermoacoustics: "If heat be given to the air at the moment of greatest condensation, or be taken from it at the moment of greatest rarefaction, the vibration is encouraged" [6].

When a gas ionizes and becomes conducting, electromagnetic radiation may be used to effect heating throughout its volume. In the present study, we consider how a standing sound wave may modify a gas's conductivity and absorption of radiation so as to cause amplification. Similar amplification of sound in a plasma has been observed and explained in the context of rarefied, ionized gas such as glow discharges and fluorescent lamps[7, 8]. In such low density systems, the acoustic period is shorter than the electron-ion recombination time, and the temperature oscillations of the acoustic wave do not appreciably change the ionization fraction. In higher density systems, the ionization fraction oscillates in phase with pressure, which is expected to enhance amplification.

The current proposal is motivated by an observation made while studying acoustic plasma confinement [9, 10]. As reported in [10], the 180 dB re 20 μPa sound wave that confines a lightly ionized collisional plasma causes luminosity oscillations in phase with its acoustic pressure. It is well-known that there is a temperature oscillation associated with sound, and the amplitude quoted above corresponds to a temperature swing of more than 10 K. The opportunity here is that because the system sits at a temperature on the cusp of ionization, small changes in temperature lead to changes in electron density and microwave absorption that are in phase with acoustic compression. For these reasons, this high density, ionizing environment should be investigated for interesting nonlinear acoustic effects beyond those presented here and in those papers mentioned above.

In this letter, we present a type of acoustic self-oscillation that may occur in an acoustic cavity filled with partially ionized gas located within a microwave cavity as shown in Figure 1. For the present analysis, the configuration is assumed similar to that found in [9–11], where the acoustic cavity was a sealed quartz sphere with a radius around 2 cm, and the surrounding microwave cavity was a thin-walled metallic cavity with a resonance near 2.45 GHz. Future implementations may include a means to couple the sound out of the acoustic cavity, for example by attaching a horn shaped outlet.

The amplification conditions are determined via an analysis based on Saha's ionization equation[12], the Drude conductivity[13], Ohm's law, and the acoustic wave equation forced by a time-varying heat source[14]. We demonstrate that Rayleigh's criteria of acoustic amplification can be satisfied in achievable conditions when a gas's ionization fraction increases due to adiabatic acoustic compression.

II. WAVE EQUATION WITH HEAT SOURCE

Sound is typically generated by driving pressure or velocity oscillations with a moving object, but variable heating or cooling can also induce sound. In an ideal gas, this effect is manifest in the relation between internal energy, temperature, and pressure. The posibility of generating sound by variable heating was demonstrated more than a hundred years ago by heating thin filaments in the so-called Thermophone[15], but practical implementation of simultaneous high frequency and amplitude has been hindered by the difficulty of oscillating the filament's thermal mass at sonic frequencies[16]. High power RF sources such as magnetrons, however, can both directly heat a volume of partially ionized gas and undergo rapid modulation, which enables high amplitude

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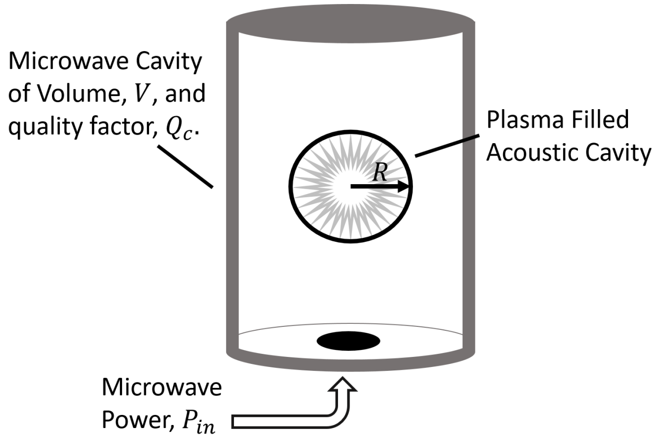

FIG. 1. A spherical acoustic resonator sits within a microwave cavity. Low levels of ionization in the hot gas within the resonator allow driving sound with amplitude modulated microwaves. Under the right conditions, sound will spontaneously develop in the presence of a continuous wave (i.e. no modulation) microwave field.

and high frequency sound generation.

For a general volumetric heating, a direct manipulation of the Euler, conservation of mass, and the heating equations produces a modified wave equation which highlights the ability of a time-varying, volumetric heating, H, to generate sound,

$$\frac{\partial^2}{\partial t^2}p_1 - c^2\nabla^2 p_1 = (\gamma - 1)\frac{\partial H}{\partial t}, \tag{1}$$

where p_1 is the acoustic pressure, c is the speed of sound, and γ is the ratio of specific heats[8, 14]. This form of the wave equation explains the sound generation of modulated flames[17] and the buzzing of overhead powerlines[18].

III. JOULE HEATING IN A PARTIALLY IONIZED GAS

At temperatures above a few thousand Kelvin, as gas begins to ionize, EM radiation can be used to directly add energy to the gas. At neutral density near that of air at STP, $N_0 \approx 2.5 \times 10^{19}$ /cc, the free electrons that absorb the EM radiation will collide with neutral particles within a collision time, τ, that is shorter than the microwave period, $2\pi/\omega = [2.45 \text{ GHz}]^{-1} \approx .4$ ns. Under such conditions, the EM power absorption can be accurately described as Joule heating of the gas with the familiar power absorption formula,

$$H = \sigma_p \mathrm{E}_p^2, \tag{2}$$

where σ_p is the electrical conductivity of the plasma and E_p is the electric field within the plasma[19, 20]. For the present case, we assume that the microwave penetration depth into the plasma is greater than its radius, $\delta_{rf} > R$, and E_p is constant throughout the plasma.

When the collision time, τ, is also less than the plasma frequency of the gas, the conductivity is well modeled by the Drude formula[13, 21],

$$\sigma_p = \frac{N_e e^2}{m_e}\tau, \tag{3}$$

where N_e is the free electron density, m_e is the electron mass, and e is the fundamental charge.

By calculating the collision time as the quotient of the mean free path and the mean thermal speed[13], the Drude conductivity can be directly rewritten in terms of the ionization fraction, temperature, a collision cross sectional area, a, and fundamental constants

$$\sigma_p = \frac{N_e}{N_0}\frac{e^2}{m_e a}\sqrt{\frac{\pi m_e}{8 k_B T}}. \tag{4}$$

By combining Eqs. (2) and (4), the volumetric heating of a partially ionized gas with a relatively high neutral density can be computed for a given E_p.

IV. SAHA'S IONIZATION EQUATION

The equilibrium ionization fraction of a gas, $x = N_e/N$, is determined by the temperature, number density, N, and ionization energy of the gas, χ, via the Saha ionization equation,

$$\frac{x^2}{1-x} = \frac{2g}{N\lambda^3}\exp\left(-\frac{\chi}{k_B T}\right), \tag{5}$$

where g is a statistical weight we've taken to be 1, and $\lambda = h/\sqrt{2\pi m_e k_B T}$ is the thermal de Broglie wavelength [12]. At low ionization rates $x \ll 1$, the Saha equation is often simplified to,

$$x = \sqrt{\frac{2}{N_0\lambda^3}}\exp\left(-\frac{\chi}{2k_B T}\right), \tag{6}$$

which is the form to be used in the following stability analysis.

V. ACOUSTIC PERTURBATION TO JOULE HEATING

By using the Saha equation to predict the ionization fraction, the conductivity is determined by the temperature and neutral density of a gas. Under the assumption that the ionization changes in phase with the temperature and pressure, one can expect that a sound wave passing through the gas will also cause oscillatory heating that is in phase with the acoustic pressure. This would satisfy the Rayleigh criterion.

To demonstrate that process, the temperature[22] and neutral density can be expanded to first order in terms of the acoustic pressure field p_1,

$$T = T_0 \left(1 + \frac{\gamma - 1}{\gamma} \frac{p_1}{p_0}\right), \quad (7)$$

$$N_0 = N_{00} \left(1 + \frac{1}{\gamma} \frac{p_1}{p_0}\right). \quad (8)$$

Here, T_0 and N_{00} are the temperature and neutral density in the absence of a sound field. By using the linearizations in Eqs. (7) and (8) in the Saha equation (6) and Drude conductivity equation (4), the Joule heating in the presence of a constant electric field can be written to first order in terms of the acoustic field as,

$$H = H_0 \left[1 + \left(\frac{1}{4}\frac{\gamma - 1}{\gamma} - \frac{1}{2}\frac{1}{\gamma} + \frac{\gamma - 1}{\gamma}\frac{\chi}{2k_B T_0}\right)\frac{p_1}{p_0}\right], \quad (9)$$

where

$$H_0 = \frac{e^2}{a}\sqrt{\frac{2}{N_{00}\lambda^3}}\sqrt{\frac{1}{2k_B T m_e}} \exp\left(\frac{-\chi}{2k_B T}\right) \mathrm{E}_p^2. \quad (10)$$

When $2k_B T < \chi$, the last term which is due to the varying ionization fraction dominates. Note that in general T and H_0 will vary as a function of position. In this initial analysis, however, we take them to be uniform.

VI. WAVE EQUATION WITH NEGATIVE DAMPING

Keeping only the largest, time dependent term in Eq. (9), the acoustic wave equation in the presence of a fixed power input can be written explicitly in terms of acoustic pressure as

$$\frac{\partial^2}{\partial t^2} p_1 - c^2 \nabla^2 p_1 = \frac{(\gamma - 1)^2}{c^2 \rho_0} H_0 \frac{\chi}{2k_B T_0} \frac{\partial}{\partial t} p_1. \quad (11)$$

Note that the sign of the term on the right hand side is opposite of a damping term. Negative damping is a characteristic of self-oscillation and often causes systems to depart from the linear regime. Examples of negative damping and other types of positive feedback can be found in [23, 24].

The possibility of amplification due to a sound wave traversing a partially ionized gas has been considered before in the literature and used to explain traveling striations in plasma tubes[7, 8, 14]. Those treatments, however, did not address the increased ionization fraction due to the temperature swing of the adiabatic compression of the gas, because the electron recombination time was assumed long compared to an acoustic period. In the previously reported cases, the neutral density was much lower than those considered here, so Saha equilibrium is not achieved within an acoustic oscillation. By considering how the adiabatic temperature swing causes increased ionization via Saha's equation, we show here that a collisional, partially ionized gas may demonstrate RF-fueled self-oscillation more readily than previously anticipated.

VII. SELF-OSCILLATION IN A SPHERICAL CAVITY

Self-oscillation will occur when amplification exceeds losses. In order to assess whether this amplification mechanism can generate acoustic energy sufficient to exceed the acoustic losses, we will solve Eq. (11) in the simplest representative case, a spherical cavity with rigid walls and a uniform temperature. Following the technique presented in [14], we will compare the growth time constant due to amplification to the decay constant due to acoustic damping. At the low acoustic amplitudes characteristic of the onset of amplification, these constants can be calculated independently. To determine conditions for the onset of amplification, a comparison of these time constants serves as a valid proxy for determining whether the energy added to the acoustic field exceeds that lost due to acoustic damping. The unique geometry of a spherical cavity simplifies the calculation of damping losses and offers other benefits as will be explained in the next section.

Amplification due to time-varying ionization can be studied by applying the wave equation in Eq. (11) in a spherical cavity of radius R containing a homogeneous gas of temperature T_0, speed of sound c, and density ρ_0. Here, we assume the walls are perfectly rigid and consider only the first breather mode, in which case the acoustic pressure field assumes the form,

$$p_1(t, r) = P_1 j_0\left(\frac{\pi\alpha_1 r}{R}\right) e^{i\omega_1 t}, \quad (12)$$

where j_0 is the spherical Bessel function and α_1 satisfies $j'(\pi\alpha_1) = 0$. Using this functional form, the wave equation in Eq. (11) generates the characteristic equation,

$$\omega_1^2 + i\frac{(\gamma - 1)^2}{c^2\rho_0} H_0 \frac{\chi}{2k_B T_0}\omega_1 - \frac{c^2\pi^2\alpha_1^2}{R^2} = 0. \quad (13)$$

The real term determines the resonance frequency and is dominated by the cavity's geometry. The imaginary term, which is the negative damping or amplification, will cause an exponential growth with time constant,

$$\tau_{amp} = \frac{2\rho_0 c^2 k_B T_0}{(\gamma - 1)^2 H_0 \chi}. \quad (14)$$

VIII. DAMPING DUE TO THERMAL DIFFUSION

Martin Greenspan et al [25] performed probably the most careful analysis of the resonant modes of a sphere in order to make an extraordinarily accurate thermometer which was also a device capable of determining the universal gas constant with an accuracy of 1.7 ppm. In so doing, they highlighted reasons why a spherical cavity is better for determination of thermodynamic properties and for acoustic resonance measurements than other shapes that also apply well for improving conditions for amplification: 1) In the breather mode the velocity is everywhere perpendicular to the surface, so there is no viscous damping at that surface. 2) Spheres have the smallest surface to volume ratio, so losses at the surface are minimized. 3) Acoustic energy density is peaked away from the walls. 4) The higher order resonance frequencies are not linear multiples of the breather and therefore less easily excited.

These issues under consideration, the main source of damping in a spherical cavity is thermal loss to the fixed temperature walls. The timescale of this damping is given in [25] as,

$$\tau_\kappa = \sqrt{\frac{2R^2}{(\gamma-1)^2\omega_1 D_T}}, \tag{15}$$

where D_T is the coefficient of thermal diffusivity. By comparing the growth time scale, Eq. (14), to the decay time scale, Eq. (15), it is possible to determine the feasibility of the amplification process. As microwave power is increased, $\tau_{amp}-\tau_\kappa$ decreases toward zero, and the acoustic quality factor diverges.When $\tau_{amp} < \tau_\kappa$, the cavity will amplify its resonant modes. Here we consider which configurations of temperature, incident power, and number density encourage amplification. This approach might be compared to the analysis of more typical thermoacoustic engines where the quality factor is found to diverge for a sufficient temperature gradient across the stack[26].

The growth and decay timescales are plotted in Figure 2 for Argon with a neutral density of 2.5×10^{19} /cc in a bulb with a radius of 2 cm, and subjected to 2.45 GHz microwaves with incident powers ranging from 500 W to 2 kW. The electron-neutral collision cross section, a was taken to be 2×10^{-17} cm^2, and the thermal diffusivity adapted from [27] was approximately 10^{-4} W/m K. With sufficient power applied, the amplification time is shorter than the damping time for a range of temperatures. The increased time constant at low temperatures occurs because the plasma doesn't absorb power at low conductivity as explained in the next section.

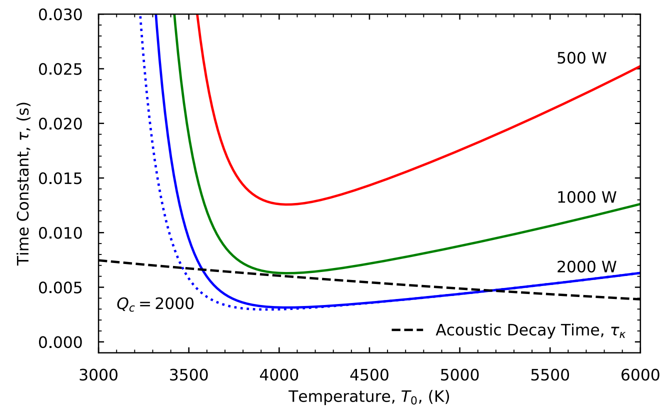

FIG. 2. A comparison of the acoustic damping time constant, τ_κ, and the acoustic amplification time constant, τ_{amp}, for three incident powers, neutral density $N_0 = 2.5 \times 10^{19}$ /cc, and microwave quality factor $Q_c = 1000$. When $\tau_{amp} < \tau_\kappa$, the energy added to the acoustic field by the absorbed microwave exceeds the acoustic losses. The increase in amplification time at low temperatures is due to the inability of low temperature plasma to absorb microwaves as described in the text. To illustrate how the quality factor impacts the amplification time, the 2 kW case is shown both with $Q_c = 1000$ (solid), and $Q_c = 2000$ (dotted).

IX. COUPLING SUFFICIENT MICROWAVE POWER

For microwave radiation to couple to the sound field, it must first be absorbed by the hot gas. If the conductivity of the hot gas is too low, the energy will ultimately dissipate elsewhere such as in the microwave cavity walls. In the appendix, we calculate the power absorbed per unit volume within the gas, H_0, as a function of temperature by comparing the microwave dissipation in the walls to that absorbed in the plasma. It is found to be,

$$H_0 = \frac{P_{in}}{V_p}\left[1 + A\frac{\epsilon_0\omega V}{\sigma_p Q_c V_p}\right]^{-1}, \tag{16}$$

where A is a geometrical factor that depends on the plasma's effect on the shape of the mode. Note that in the limit of low temperature where $\sigma \to 0, H_0 \to 0$, and at reasonably high temperatures, $H_0 \to P_{in}/V_P$.

The volumetric power absorption in Eq (16) is used to calculate the amplification time in Eq. (14). The effect this has can be seen in the low temperature side of Figure 2 where Q_c and A were set to 1000 and 1 respectively[11]. In general, both acoustic time scales depend on the gas, its density, and its temperature. An example of the range of parameter space in which amplification might be possible is shown in figure 3.

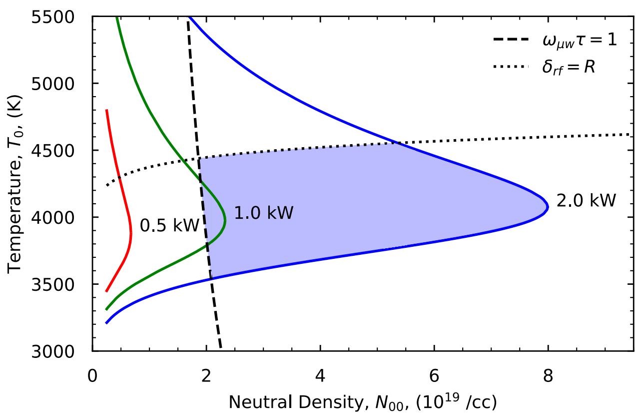

FIG. 3. For each power, acoustic amplification exceeds thermal acoustic damping, $\tau_{amp} < \tau_\kappa$, to the left of the solid curve. In the region to the right of the dashed line, the electron-neutral collision time is short enough to justify use of the Drude conductivity. Below the dotted line, the microwave penetration depth is larger than the plasma radius, which justifies use of a constant electric field throughout the plasma. The shaded area represents the region in parameter space in which this analysis predicts acoustic self-oscillation for the 2 kW case.

X. CONCLUSION

We have presented a theoretical outline of how acoustic self-oscillation may happen more readily than previously expected when heating due to acoustic compression causes enhanced ionization. We have also shown that this type of amplification is experimentally feasible. Further work will need to consider nonhomogeneous temperature profiles and electron recombination times. Self-oscillation provides a path toward generating extreme sound fields in a plasma. Further research should determine which nonlinear process limits the ultimate achievable amplitude, and what role plasma self-oscillation could then play in nonlinear acoustics and thermoacoustics.

XI. ACKNOWLEDGEMENT

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XII. APPENDIX

When the plasma conductivity is high enough, it is able to absorb all the incident microwave power. However, when the conductivity is low, some of the microwave invariably is absorbed by the microwave cavity walls. This crossover limits the acoustic gain at low temperatures. In this appendix we calculate the fraction of the input power that is absorbed by the plasma as a function of the plasma conductivity.

The power loss due to the finite conductivity of the metallic cavity walls is,

$$P_w = \frac{1}{2\sigma_w \delta} \int_S |\hat{n} \times \vec{\mathrm{H}}|^2 da, \tag{17}$$

where $\delta = \sqrt{2/\omega\mu\sigma_w}$ is the skin depth of the metallic cavity walls, σ_w is the conductivity of the walls, $\vec{\mathrm{H}}$ is the magnetic field vector (not to be confused with the volumetric heating, H), $\hat{n}$ is a unit vector perpendicular to the wall, and the integral is taken over the entire cavity surface S[28]. The power loss due to the finite conductivity of the plasma is,

$$P_p = \frac{\sigma_p}{2} \int_{V_p} |\vec{\mathrm{E}}|^2 dV, \tag{18}$$

where σ_p is the conductivity of the plasma assumed to be constant over its volume, $\vec{\mathrm{E}}$ is the electric field, and the integral is taken over the plasma volume V_p. Assuming a matched source, the source power, P_{in}, will equal the sum of P_w and P_p. We can then write the power per volume absorbed by the plasma as a function of the input power,

$$\begin{aligned} H_0 &= \frac{P_{in}}{V_p} \frac{P_p}{P_p + P_w} \\ &= \frac{P_{in}}{V_p} \left[1 + \frac{1}{\sigma_w \sigma_p \delta} \frac{\int_S |\hat{n} \times \vec{\mathrm{H}}|^2 da}{\int_{V_p} |\vec{\mathrm{E}}|^2 dV} \right]^{-1} . \end{aligned} \tag{19}$$

We can gain some insight into this equation by writing the wall losses in terms of the unloaded (no plasma) quality factor,

$$Q_c = \frac{\omega \epsilon_0 \int_{V_c} |\vec{\mathrm{E}}_0|^2 dV}{\frac{1}{\sigma_w \delta} \int_S |\hat{n} \times \vec{\mathrm{H}}_0|^2 da}, \tag{20}$$

where $\vec{\mathrm{E}}_0$ and $\vec{\mathrm{H}}_0$ indicate the fields in the absence of the plasma, and the energy integral is taken over the cavity volume V_c. H_0 can then be written,

$$H_0 = \frac{P_{in}}{V_p} \left[1 + A \frac{\epsilon_0 \omega V_c}{\sigma_p Q_c V_p} \right]^{-1}, \tag{21}$$

where the unitless constant A is,

$$A = \frac{V_p \int_{V_c} |\vec{\mathrm{E}}_0|^2 dV \ \int_S |\hat{n} \times \vec{\mathrm{H}}|^2 da}{V_c \int_{V_p} |\vec{\mathrm{E}}|^2 dV \ \int_S |\hat{n} \times \vec{\mathrm{H}}_0|^2 da}. \tag{22}$$

The constant A depends on the particular geometry of the microwave cavity, the location and size of the plasma bulb, as well as its conductivity. However, for low plasma conductivity, we may approximate A as being independent of σ and use Eq. (21) to determine the low temperature behavior of the acoustic gain.