

This is the accepted manuscript made available via CHORUS. The article has been published as:

Black hole remnants may exist if Starobinsky inflation occurred

Paul R. Anderson, Mathew J. Binkley, Jillian M. Bjerke, and P. Wilson Cauley

Phys. Rev. D **98**, 125011 — Published 18 December 2018

DOI: [10.1103/PhysRevD.98.125011](https://doi.org/10.1103/PhysRevD.98.125011)

Black hole remnants may exist if Starobinsky inflation occurred

Paul R. Anderson,* Mathew J. Binkley, Jillian M. Bjerke, and Paul W. Cauley

Department of Physics

Wake Forest University

Winston-Salem, NC 27109

Zero temperature black hole solutions to the semiclassical backreaction equations are investigated. Evidence is provided that certain components of the stress-energy tensors for free quantum fields at the horizon only depend on the local geometry near the horizon. This allows the semiclassical backreaction equations to be solved near the horizon. It is found that macroscopic uncharged zero temperature black hole solutions to the equations may exist if the coefficient of one of the higher derivative terms in the gravitational Lagrangian is large enough and of the right sign for Starobinsky inflation to have occurred in the early universe.

* anderson@wfu.edu

I. INTRODUCTION

With the discovery of black hole evaporation [1] came the fact that one can assign a temperature to a black hole which is equal to the temperature of the thermal radiation that it emits. This temperature is related to the surface gravity of the black hole. Two questions which have still not been resolved were raised by this discovery: What is the end point of the evaporation process and what happens to the information about how the black hole formed? It may be that a fully quantum theory of gravity is necessary to answer these questions. However, black hole solutions to the four dimensional semiclassical backreaction equations (SCE) have yet to be fully explored. Thus it remains a possibility that semiclassical gravity has something significant to say.

One possible answer to both the end point and information issues that has been suggested is that at late times the black hole evaporation process may shut off leaving a zero temperature black hole remnant [2]. It is usually expected that such remnants would have sizes which are Planck scale and thus would need to be described by a quantum theory of gravity. It has been argued [3] that generically one might expect there to be an infinite amount of pair production of such remnants to occur if the information is stored inside such remnants. However, it is also pointed out that there may be situations in which such infinite pair production does not occur.

Although it would be attractive to solve both the information and end point issues using black hole remnants, it is possible that they have separate solutions. In that case one can ask the question of whether such remnants could exist without being concerned about whether the information about how the black hole formed is inside of them. Here we take this approach and investigate solutions to the semiclassical backreaction equations that correspond to static spherically symmetric zero temperature black holes, SZTBHs. We focus on the solutions to the semiclassical backreaction equations near the event horizons of such black holes and consider black holes with and without electric charges. A macroscopic SZTBH must have an electric charge. However, we find that if Starobinsky inflation [4, 5] occurred then the coefficient of one of the terms in the SCE is large enough and of the right sign so that it is possible to have uncharged SZTBH solutions to the semiclassical backreaction equations which are significantly larger than the planck scale in size.

There is a long history of studying quantum effects in four dimensional zero temperature black hole spacetimes. The stress-energy for free massless quantized fields of spin 0 and $\frac{1}{2}$ has been numerically computed in extreme Reissner-Nordström (ERN) spacetimes in four dimensions and found to be regular on the event horizon [6, 7]. It has also been analytically computed in Bertotti-Robinson spacetime [8, 9] which becomes a good approximation to the ERN geometry near the

horizon.

SZTBH solutions to the linearized SCE in four dimensions for conformally invariant fields were investigated in [10, 11] and for massive fields were investigated in [12, 13]. In [11] and [13] it was shown that solutions to the equations exist with different relationships between the mass of the black hole, the electric charge, and the radius of the event horizon than occur for a classical ERN black hole.

The first solutions to the full nonlinear SCE in four dimensions that we are aware of which are relevant for SZTBHs, are for $AdS_2 \times S_2$ spacetimes in the case that a massless minimally coupled scalar field is present [14]. This is the asymptotic form of the geometry near the event horizon of a SZTBH with a metric on the horizon which is of the same general form as that for an ERN black hole near the horizon. Both exact and approximate solutions were found, with the approximate ones being exact in certain limits. It was found for a large range of values of a coefficient of the terms in the gravitational Lagrangian that are quadratic in the curvature that solutions exist with no electric charge.

Constraints on the behaviors of possible solutions to the full nonlinear SCE near the event horizons of SZTBHs were investigated in [15, 16]. Assuming the usual higher derivative terms in the gravitational Lagrangian necessary for the renormalization of free quantum fields in curved space along with conformally invariant fields and a possible electric charge for the black hole, the trace of the SCE was solved near the horizon. It was shown that there is a range of sizes for which no SZBHT solutions to the SCE are possible [16]. For metrics with power law behaviors for g_{tt} and g^{rr} near the horizon, constraints on the powers were obtained along with a relationship between the form of the metric near the horizon and the radius of the horizon [15].

Here we continue the exploration of SZTBH solutions to the full nonlinear SCE. We first add a constraint and then make the argument that the most likely form of the metric near the horizon is one with g_{tt} and g^{rr} quadratic in $r - r_0$, with r_0 the radius of the event horizon. Next we consider metrics which have these forms near the horizon but different forms away from it. We compute the stress-energy tensors for massless scalar fields with minimal and conformal coupling to the scalar curvature in these geometries. Our results provide strong evidence that the values of the $\langle T_t^t \rangle$, $\langle T_r^r \rangle$, and $\langle T_\theta^\theta \rangle$ components on the horizon only depend on the geometry near the horizon. This appears to be true for massless scalar fields with other couplings to the scalar curvature as well. We expect that this property will also hold for massless free fields of higher spin. Our results provide evidence that the solutions mentioned above in [14] can be used to describe the near horizon regions of SZTBH solutions to the SCE in the cases considered.

We have also computed the quantity

$$\frac{\langle T_r^r \rangle - \langle T_t^t \rangle}{g_{tt}} \quad (1.1)$$

at the horizon. This is related to the energy density a freely falling observer who passes through the horizon sees, and if it diverges at the horizon, then the observer sees an infinite energy density there. We find that its value and in general the values of $\langle T_t^t \rangle_{,r}$ and $\langle T_r^r \rangle_{,r}$ depend on the geometry away from the horizon as well as that near it. Thus the quantity in (1.1) does as well. We find that in some cases this quantity is finite on the horizon, but in many cases it is not.

We use our results to solve the SCE near the horizon when only conformally invariant fields are present along with the usual higher derivative terms which are necessary for the renormalization of these fields. Since the values of $\langle T_t^t \rangle$, $\langle T_r^r \rangle$, and $\langle T_\theta^\theta \rangle$ at the horizon depend only on the geometry near the horizon, we can solve the SCE for the values of these components at the horizon. If the stress-energy is finite on the horizon then $\langle T_t^t \rangle = \langle T_r^r \rangle$ there and it suffices to solve the trace equation and the rr component of the SCE. Since the radial derivatives of these components depend upon the geometry away from the horizon, we cannot say anything about SZTBH solutions to the SCE away from the horizon. Therefore the solutions we find tell us about the properties that physically acceptable SZTBH solutions to the SCE must have near the event horizon given the types of quantum fields that we consider.

We restrict our attention to conformally invariant fields because most fields in the Standard Model of particle physics are conformally invariant in the limit that their masses and interactions vanish. It was shown in [17] that the relevant quantity in determining the importance of the mass is mM in Planck units with m the mass of the scalar field and M the mass of the black hole. For $mM \gtrsim 2$ the DeWitt-Schwinger approximation, which is a large mass approximation, was found to be valid. Thus we expect the stress-energy tensor near the horizon to be approximately the same as that for a massless field if $mM \ll 1$.

For the form of the metrics that we use, the results of [15] for solutions to the trace equation indicate that there is a minimum size that a SZTBH can have which is independent of the coefficients of the higher derivative terms in the equations. Solving the rr component of the SCE, we find that in many cases there is a more severe lower bound on the size that a SZTBH can have. This lower bound corresponds to the case of zero electric charge and thus a solution satisfying this lower bound could serve as a black hole remnant. If the coupling constant for the higher derivative term that leads to Starobinsky inflation [4, 5] has the right sign and magnitude for Starobinsky inflation to occur in the early universe [18], and if it is significantly larger in magnitude than the

other coupling constant, then the lower bound results in a black hole whose size is large enough compared with the Planck scale that semiclassical gravity can be valid.

In Sec. II we review some results of [15, 16] and come up with a new constraint on SZTBH solutions to the semiclassical backreaction equations. In Sec. III we argue that the most likely form for a zero temperature black hole metric near the horizon is given by (3.1). We also show the specific form of the metric that we use for the numerical computations. In Sec. IV we present some of our numerical results for components of the stress-energy tensor in various candidate geometries. Our solutions to the semiclassical backreaction equations near the horizon are given in Sec. V. Sec. VI contains a summary and discussion of our results. Throughout we use units such that $\hbar = c = G = k_B = 1$ and our sign conventions are those of Misner, Thorne, and Wheeler [19].

II. CONSTRAINTS ON STATIC SPHERICALLY SYMMETRIC ZERO TEMPERATURE BLACK HOLES

In this section we first review constraints on the spacetime geometry near the event horizon of a SZTBH and then add a new constraint.

A. Previous constraints

Some constraints on the geometry of a SZTBH near the event horizon were obtained in [15, 16] by simply requiring that the components of the Riemann tensor in an orthonormal frame be finite at the horizon. Writing the metric in the form

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{k(r)} + r^2 d\Omega^2, \quad (2.1)$$

one finds the surface gravity is

$$\kappa = \frac{v}{2} \sqrt{fk}, \quad (2.2)$$

with

$$v \equiv \frac{f'}{f}. \quad (2.3)$$

Here primes denote derivatives with respect to r . To have an event horizon it is necessary that $f = 0$ there and therefore that $v = \infty$. To avoid a divergence of the Kretschmann scalar $R^{abcd}R_{abcd}$ at the horizon it is necessary that $k = 0$ there as well. To have a zero temperature black hole it is further necessary that $k' = 0$ at the horizon. It is also necessary for zero temperature black holes

that kv^2 be finite on the horizon and thus $kv = 0$ there. Finally, for all zero temperature black holes $\square R$ cannot approach a constant on the horizon. It thus either diverges or vanishes there.

In [15, 16] further constraints were obtained by considering conformally invariant quantum fields. The trace of the stress-energy tensor for such fields is the trace anomaly and is known in an arbitrary spacetime. It is given in terms of the scalar curvature R , the Ricci tensor R_{ab} and the Weyl tensor C_{abcd} by

$$\langle T^q \rangle = \alpha \square R + \beta \left(R_{ab} R^{ab} - \frac{1}{3} R^2 \right) + \gamma C_{abcd} C^{abcd}, \quad (2.4)$$

with

$$\alpha = \frac{1}{2880\pi^2} [N(0) + 6N(1/2) - 18N(1)], \quad (2.5a)$$

$$\beta = \frac{1}{2880\pi^2} [N(0) + 11N(1/2) + 62N(1)], \quad (2.5b)$$

$$\gamma = \frac{1}{2880\pi^2} [N(0) + \frac{7}{2}N(1/2) - 13N(1)]. \quad (2.5c)$$

Here $N(0)$, $N(1/2)$, and $N(1)$ are the numbers of conformally invariant scalar fields, four component spin 1/2 fields, and vector fields respectively. Thus for the trace of the stress-energy tensor for a given type of conformally invariant field to be finite at the horizon of a SZTBH, it is clear that $\square R$ cannot diverge there. Thus since there is also the constraint mentioned above that $\square R$ cannot be constant on the horizon, it is necessary that $\square R = 0$ there.

Solutions to the semiclassical backreaction equations were investigated when only conformally invariant quantized fields are present. The general form of these equations can be written as

$$G_{ab} = 8\pi [T_{ab}^c + \langle T_{ab}^q \rangle + h_1 {}^{(1)}H_{ab} + h_2 {}^{(C)}H_{ab}], \quad (2.6)$$

where the superscripts c and q correspond to classical matter and quantum fields respectively and

$${}^{(1)}H_{ab} = -\frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{ab}} \int d^4x \sqrt{-g} R^2 = -2g_{ab} \square R + 2\nabla_a \nabla_b R - 2RR_{ab} + \frac{1}{2}g_{ab} R^2, \quad (2.7a)$$

$${}^{(C)}H_{ab} = -\frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{ab}} \int d^4x \sqrt{-g} C_{abcd} C^{abcd} = -4\nabla^c \nabla^d C_{abcd} + 2R^{cd} C_{abcd}. \quad (2.7b)$$

The coefficients h_1 and h_2 are constants which must in principle be determined experimentally.

An important constraint was obtained from the trace of the SCE. The only classical matter we consider here is the classical electric field that occurs if the black hole has an electric charge Q . Since the electromagnetic field is conformally invariant, the trace of T_{ab}^c is zero. From (2.7b) one sees that the trace of ${}^{(C)}H_{ab}$ is also zero due to its dependence on the Weyl tensor. From (2.7a) it is easily seen that ${}^{(1)}H_a^a = -6\square R$. Setting $\square R = 0$ on the horizon gives

$$-R = 8\pi [\langle T^q \rangle]. \quad (2.8)$$

To derive the constraint the following component of the Riemann tensor in an orthonormal frame was considered:

$$A(r) \equiv R_{\hat{t}\hat{r}\hat{t}\hat{r}} = \frac{v'k}{2} + \frac{vk'}{4} + \frac{v^2k}{4} . \quad (2.9)$$

Clearly this must be finite or there is a curvature singularity at the horizon. Integrating one obtains

$$k = \frac{B_0}{v^2 f} + \frac{4}{v^2 f} \int_{r_0}^r f'(r_2) A(r_2) dr_2 , \quad (2.10)$$

where r_0 is the radius of the event horizon. Multiplying by $v^2 f$ and comparing with (2.2) one finds that so long as $A_0 \equiv A(r_0)$ is finite on the horizon, $B_0 = 4\kappa^2$. Thus for the zero temperature black holes we are considering, $B_0 = 0$. In [16] these results were used to solve (2.8) on the horizon with the result that

$$A_0 = \frac{1}{16\pi(\beta + 2\gamma)r_0^2} \left[3r_0^2 - 32\pi(\beta - \gamma) \pm (768\pi^2\beta^2 - 3072\pi^2\beta\gamma - 288\pi\beta r_0^2 + 9r_0^4)^{1/2} \right] \quad (2.11)$$

For physically acceptable solutions A_0 must be real which means there can be no solutions with r_0 in the range $r_- < r_0 < r_+$ with

$$r_{\pm} = 4(\pi\beta)^{1/2} \left[1 \pm \left(\frac{2}{3\beta} \right)^{1/2} (\beta + 2\gamma)^{1/2} \right]^{1/2} . \quad (2.12)$$

B. New constraints

A new constraint, that to our knowledge has not been presented elsewhere, can be obtained by first requiring that the curvature seen by a freely falling observer in an orthonormal frame be finite. In such a frame one component of the Einstein tensor near the horizon depends in part upon the combination¹

$$\frac{1}{f}(G_r^r - G_t^t) = \frac{kf'}{rf^2} - \frac{k'}{rf} \equiv -F(r) \quad (2.13)$$

For the curvature to be finite at the horizon, it is clear that $F(r_0)$ must be finite. This equation can be formally integrated with the result that

$$k = f \left[a_1 + \int_{r_0}^r r_1 F(r_1) dr_1 \right] , \quad (2.14)$$

¹ This combination of components for the stress-energy tensor is part of the energy density and pressure seen by a freely falling observer passing through the event horizon on a radial geodesic [20].

where a_1 is an integration constant. Equating (2.10) and (2.14) and using the definition (2.3) gives

$$(f')^2 = \frac{4 \int_{r_0}^r f'(r_2) A(r_2) dr_2}{a_1 + \int_{r_0}^r r_1 F(r_1) dr_1} . \quad (2.15)$$

In [16] it was shown that for SZTBH solutions to the SCE when only conformally invariant fields are present,

$$A_0 > 0 . \quad (2.16)$$

Then we find that to leading order near the horizon

$$\frac{(f')^2}{f} = \frac{4A_0}{a_1 + \int_{r_0}^r r_1 F(r_1) dr_1} . \quad (2.17)$$

Next we consider what this constraint implies for various values of a_1 and $F_0 \equiv F(r_0)$. First it is necessary that $a_1 \geq 0$ since if $a_1 \neq 0$ then it dominates the denominator near the horizon. If $a_1 > 0$ then near the horizon

$$\frac{(f')^2}{f} = \frac{4A_0}{a_1} . \quad (2.18)$$

Integrating and using (2.10) gives

$$\begin{aligned} f &= \frac{A_0}{a_1} (r - r_0)^2 , \\ k &= A_0 (r - r_0)^2 . \end{aligned} \quad (2.19)$$

If $a_1 = 0$ and $F(r_0) = F_0 > 0$ then one can integrate (2.17) and use (2.10) to show that

$$f = \frac{4A_0}{r_0 F_0} (r - r_0) \quad (2.20a)$$

$$k = 4A_0 (r - r_0)^2 \quad (2.20b)$$

Finally if $a_1 = F_0 = 0$ then near the horizon

$$\frac{(f')^2}{f} = \frac{4A_0}{\int_{r_0}^r r_2 F(r_2) dr_2} . \quad (2.21)$$

Computing the square root and integrating gives

$$2f^{1/2} = \int_{r_0}^r \left[\frac{4A_0}{\int_{r_0}^{r_3} r_2 F(r_2) dr_2} \right]^{1/2} dr_3 . \quad (2.22)$$

The minimum value of the right hand side would occur if $F(r_0) > 0$ and one would obtain the result (2.20a) for which f' is constant at the horizon. Thus f' must have an infinite value on

the horizon. Further the function $F(r)$ cannot vanish too rapidly as the horizon is approached or f would not be equal to zero at the horizon. As an example, suppose $f = a_4(r - r_0)^p$ near the horizon with $0 < p < 1$. Then it is not hard to show that near the horizon

$$\begin{aligned} k &= \frac{4A_0}{p^2}(r - r_0)^2, \\ F(r) &= \frac{4A_0}{r_0 p^2 a_4}(r - r_0)^{1-p}. \end{aligned} \quad (2.23)$$

III. METRICS CONSIDERED HERE

In the previous section constraints were found on the form of the metric for static spherically symmetric zero temperature black holes near the event horizon. It was found that if only conformally invariant fields are present, then for SZTBH solutions to the semiclassical backreaction equations metrics of the form (2.19) and (2.20) are allowed. It was shown that for all other solutions $f' \rightarrow \infty$ at the horizon, which means there is no smooth way to continue f across the horizon, and the coordinate system breaks down in a more significant way than it does for Schwarzschild or Reissner-Nordström spacetimes. If f is linear and k is quadratic at the horizon then the obvious way of continuing f and k across the horizon leads to Euclidean space. On the other hand if both f and k are quadratic near the horizon, then the metric is of the same form as that of an extreme Reissner-Nordström spacetime. From the point of view of the semiclassical backreaction equations, this is clearly the form of most interest and the one which will be pursued here.

In general the stress-energy tensor for a quantum field is a nonlocal quantity. Therefore it is necessary to know the geometry everywhere in the causal past of a given spacetime point in order to compute the stress-energy tensor at that point. For a SZTBH solution to the SCE outside the event horizon, this means knowing the geometry everywhere outside of the event horizon. One can of course guess the geometry, but it is extremely unlikely that any guess would correspond to a solution to the SCE. However, we have numerical evidence that for a SZTBH most components of the stress-energy tensor on the horizon depend only on the geometry near the horizon. This allows us to solve the semiclassical backreaction equations near the horizon to determine that geometry.

Our conjecture concerns metrics for SZTBHs which near the event horizon have the leading order behaviors

$$f \rightarrow \left(\frac{r - r_0}{r_0} \right)^2, \quad (3.1a)$$

$$k \rightarrow b_2 \left(\frac{r - r_0}{r_0} \right)^2. \quad (3.1b)$$

Note that the coefficient for f has been set to 1 here because it is always possible to do this by rescaling the coordinate time t in (2.1). The conjecture states that for a massless scalar field with arbitrary coupling to the scalar curvature, in SZTBH spacetimes for which f and k have the above form near the horizon, the values of the components $\langle T_t^t \rangle$, $\langle T_r^r \rangle$, $\langle T_\theta^\theta \rangle$, and $\langle T_\phi^\phi \rangle$ on the event horizon depend on the coefficient b_2 , but not on the behaviors of f and k away from the horizon.

Previous work provides some evidence for this conjecture. In [6] it was shown numerically that on the event horizon of an extreme Reissner-Nordström black hole ($b_2 = 1$) one finds that for a massless scalar field with arbitrary coupling to the scalar curvature

$$\langle T_t^t \rangle = \langle T_r^r \rangle = \langle T_\theta^\theta \rangle = \langle T_\phi^\phi \rangle = \frac{1}{2880\pi^2 M^4} . \quad (3.2)$$

It was also shown in [6] that these are the same values as those for the stress-energy tensor for the conformally coupled ($\xi = 1/6$) massless scalar field in the Bertotti-Robinson spacetime which is obtained by expanding the extreme Reissner-Nordström metric in a series about $r = r_0$ and keeping only the lowest order terms. In Section IV we give a more technical explanation of why the conjecture works along with numerical results for other values of b_2 that support it.

If the conjecture is correct then the following procedure will work to solve the semiclassical backreaction equations near the horizon. Choose metric functions which approach (3.1) near the horizon for various values of b_2 and which have any convenient form away from it. Then compute the stress-energy tensors for the quantum fields and evaluate their components at the horizon. Next evaluate the left hand sides of the trace and rr components of the SCE. They depend only on r_0 and b_2 at the horizon. Finally, since the ERN black hole has an electric charge, include on the right-hand side of the SCE the classical electromagnetic stress-energy tensor which occurs if the black hole has an electric charge Q . Then the trace of the SCE will be independent of Q and should yield a relationship between r_0 and b_2 . The rr component should yield a relationship between r_0 , b_2 , and Q^2 . Thus, for any desired size for the black hole, one could find the magnitude of the resulting electric charge and the leading order behavior of the metric near the horizon.

In this paper we use the dimensionless radial coordinate

$$s \equiv \frac{r - r_0}{r_0} , \quad (3.3)$$

and consider metrics of the following power series form near the event horizon

$$f = a_2 s^2 + a_3 s^3 + \dots , \quad (3.4a)$$

$$k = b_2 s^2 + b_3 s^3 + \dots . \quad (3.4b)$$

Note that without loss of generality we can absorb the value of the coefficient a_2 into the definition of the time coordinate t . We do this for the computations discussed here.

To compute the components of the stress-energy tensor it is necessary to specify the metrics everywhere outside of the event horizon. So the actual metrics we consider are of the general form

$$f = \frac{s^2}{(s+1)^2} + \frac{s^3}{(s+1)^3} A_{33} + \dots, \quad (3.5a)$$

$$k = \frac{s^2}{(s+1)^2} b_2 + \frac{s^3}{(s+1)^3} B_{33} + \dots, \quad (3.5b)$$

Note that for an asymptotically flat spacetime $f \rightarrow c$ for some constant $c > 0$ and $k \rightarrow 1$ as $s \rightarrow \infty$. The first condition is automatically satisfied by these metrics. For the second

$$b_2 + B_{33} + \dots = 1. \quad (3.6)$$

Since $b_2 > 1$ it is necessary that at least one of the other terms in the sum be negative.

It is tempting to make the conjecture that the first radial derivatives of the components of the stress tensor at the horizon depend only on the values of r_0, b_2, a_3 , and b_3 . However we have found that this is not the case. Thus it appears that this approach only allows one to find the behaviors of solutions to the SCE when its trace, rr , and tt components are evaluated at the horizon.

IV. NUMERICAL RESULTS

We begin with a constraint on two components of the stress-energy tensor at the horizon. The radial component of the conservation equation $\langle T_a^b \rangle_{;b} = 0$ is

$$\langle T_r^r \rangle_{,r} + \frac{1}{2f} \frac{df}{dr} (\langle T_r^r \rangle - \langle T_t^t \rangle) + \frac{2}{r} (\langle T_r^r \rangle - \langle T_\theta^\theta \rangle) = 0. \quad (4.1)$$

Note that since we consider only states which respect spherical symmetry, $\langle T_\phi^\phi \rangle = \langle T_\theta^\theta \rangle$. For the metrics we consider $f^{-1} \frac{df}{dr} \sim (r-r_0)^{-1}$ near the horizon. Thus for $\langle T_r^r \rangle_{,r}$ to be finite at the horizon it is necessary that $\langle T_t^t \rangle = \langle T_r^r \rangle$ there. This result is well known and our numerical results confirm that for the vacuum state this condition is always satisfied.

In the previous section a conjecture was presented which states that for a massless scalar field the components $\langle T_t^t \rangle$, $\langle T_r^r \rangle$, $\langle T_\theta^\theta \rangle$, and $\langle T_\phi^\phi \rangle$ depend only on r_0 and the metric parameter b_2 when the metric is of the form (3.1) near the horizon. It is possible to show using the general static spherically symmetric form of the expressions for $\langle T_a^b \rangle$ [17], the definition

$$r = r_0(1+s), \quad (4.2)$$

and the scaling $\omega \rightarrow \omega/r_0$, that the entire r_0 dependence for each of these components is r_0^{-4} .

In this section we first discuss the computation of these components on the horizon. In the process we provide a technical explanation for why the conjecture should be correct. Then we present the results of some of our numerical computations.

The method we use to compute the stress-energy tensor for a massless scalar field in a SZTBH spacetime is given in detail in [17]. In this approach the mode equation in the Euclidean space associated with the exterior region of the black hole is solved. For each value of the frequency ω and the angular momentum parameter ℓ there are two linearly independent solutions. One of them we call $p_{\omega\ell}$ and it is regular at the horizon² but diverges at infinity. The other we call $q_{\omega\ell}$. It is well behaved at infinity but diverges at the horizon. The two-point function $\langle\{\phi(x), \phi(x')\}\rangle$ is a sum and integral over the product of these two mode functions. The unrenormalized stress-energy tensor involves spacetime derivatives of the two-point function.

The fact that the stress-energy tensor depends only on the geometry near the horizon for ERN spacetimes and our conjecture that this is the case in general for SZTBHs can be understood in two different ways. First, it is easy to show that the proper distance to the horizon along a radial spacelike geodesic from any point outside of it is infinite [16]. In Euclidean space the distance is infinite for any path from outside the horizon to the horizon. Thus it makes sense qualitatively that the stress-energy tensor might depend only on the geometry near the horizon.

From a more technical point of view it is found that to leading order near the horizon $p_{\omega\ell}$ and $q_{\omega\ell}$ have exponential factors of the form

$$\exp(\pm\omega/(r-r_0)) . \quad (4.3)$$

Since the boundary conditions for $q_{\omega\ell}$ are fixed away from the horizon, changing these conditions simply amounts to adding some part of the $p_{\omega\ell}$ mode to the original $q_{\omega\ell}$ mode. Then a product of the $p_{\omega\ell}$ and $q_{\omega\ell}$ modes simply results in the original product plus a term which is damped exponentially as the horizon is approached. Therefore it is plausible that in the limit that the horizon is approached this exponentially damped term does not contribute to leading order to the mode sum that makes up the stress-energy tensor.

The method in [17] allows us to compute the components of the stress-energy tensor anywhere outside the event horizon. The results can be extrapolated to the horizon. There is a well known ambiguity which occurs for the value of $\langle T_{ab} \rangle$ which comes from the renormalization counter terms.

² There can be spacetimes where there are exceptions to this for small values of ω . However, in these cases the divergence is still less strong than for $q_{\omega\ell}$ at the horizon.

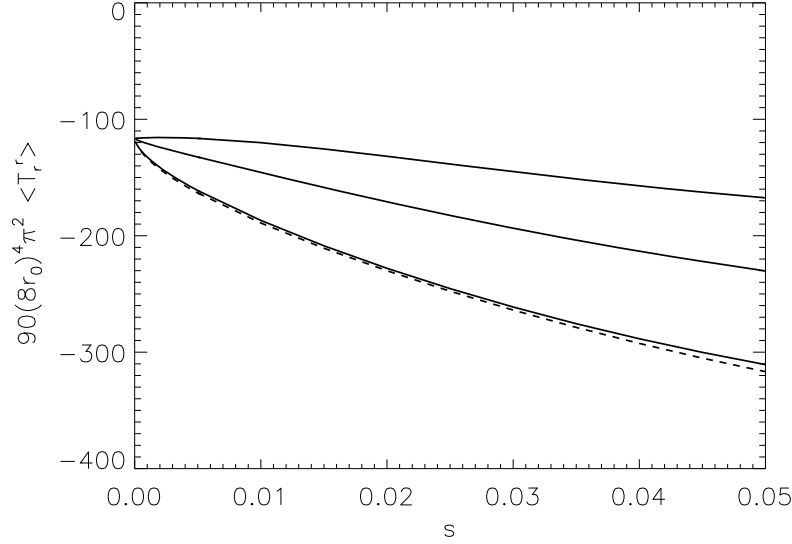


FIG. 1. The quantity $\langle T_r^r \rangle$ is plotted near the horizon for a massless scalar field with $\xi = \frac{1}{6}$ when $b_2 = 2$. All of the curves have the series (3.5) truncated at A_{33} and B_{33} . The solid curves have $A_{33} = 0$ and thus $a_3 = -2$. From top to bottom they have $B_{33} = 0$ ($b_3 = -4$), $B_{33} = 1$ ($b_3 = -3$), $B_{33} = 2$ ($b_3 = -2$). The dashed curve has $A_{33} = B_{33} = 2$ ($a_{33} = 0$, $b_3 = -2$).

For the conformally invariant scalar field this results in a finite renormalization of the parameter h_2 in the semiclassical backreaction equations (2.6). For the massless minimally coupled field it results in finite renormalizations of both h_1 and h_2 . For the method we use there is an arbitrary constant in one term of the stress-energy tensor which multiplies ${}^{(C)}H_{ab}$ in the case of the conformally invariant field and which multiplies a linear combination of ${}^{(1)}H_{ab}$ and ${}^{(C)}H_{ab}$ for the massless minimally coupled scalar field. More details are given in [17]. For the numerical results shown we chose the value of this constant to be zero.

The field is conformally invariant if it is massless and $\xi = \frac{1}{6}$. In this case the $\langle T_\theta^\theta \rangle$ component on the horizon is related through the trace anomaly with the $\langle T_r^r \rangle$ component, see the next section for details. Some of our results for $\langle T_r^r \rangle$ are shown in Fig. 1.

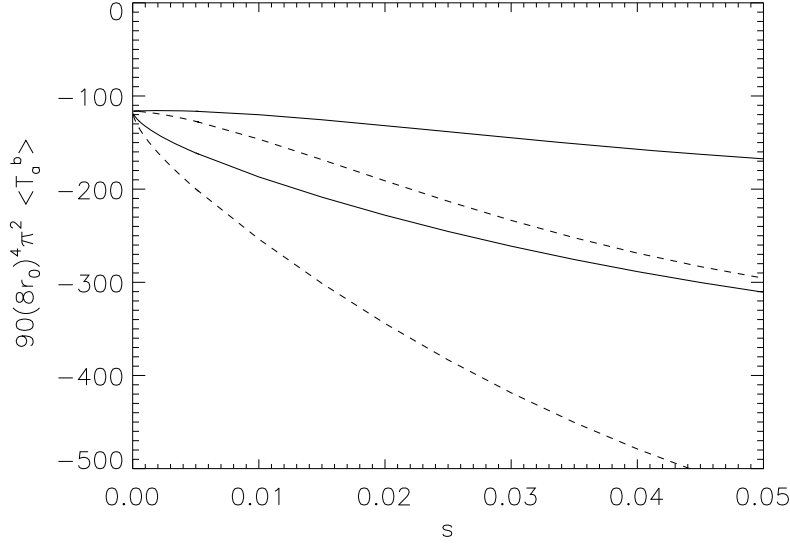


FIG. 2. Components of the stress-energy tensor are plotted near the horizon for a massless scalar field with $\xi = \frac{1}{6}$ when $b_2 = 2$. All of the curves have the series (3.5) truncated at A_{33} and B_{33} . The upper solid and dashed curves show $\langle T_r^r \rangle$ and $\langle T_t^t \rangle$ respectively for $A_{33} = B_{33} = 0$ ($a_3 = -2$, $b_3 = -4$). The lower solid curve and lower dashed curve show $\langle T_r^r \rangle$ and $\langle T_t^t \rangle$ respectively for $A_{33} = 0$ and $B_{33} = 2$ ($a_3 = b_3 = -2$). Note that in both cases the slope of the curve for $\langle T_r^r \rangle$ near the horizon is different from that of the curve for $\langle T_t^t \rangle$.

A. Results for another component

Computation of the stress-energy tensor in an orthonormal frame attached to a freely falling observer moving in the radial direction shows that as the observer falls through the horizon, the observer will observe an infinite stress-energy unless both $\langle T_t^t \rangle$, $\langle T_r^r \rangle$, and $g_{tt}^{-1}(\langle T_r^r \rangle - \langle T_t^t \rangle)$ are finite on the horizon [20]. Since $g_{tt} \sim (r - r_0)^2$, this component is divergent unless $\langle T_t^t \rangle_{,r} = \langle T_r^r \rangle_{,r}$ on the horizon. From Fig. 2 it is clear that this is not the case for all geometries of the form (3.5). In fact we have not found an example where this condition is satisfied for conformal coupling $\xi = \frac{1}{6}$. However, as shown in Fig. 3 we have found examples where it appears to be satisfied for minimal coupling $\xi = 0$.

The values of $\langle T_t^t \rangle$ and $\langle T_r^r \rangle$ on the horizon depend only on the geometry near the horizon and in particular on the values of b_2 and r_0 . However, some of our numerical results indicate that the

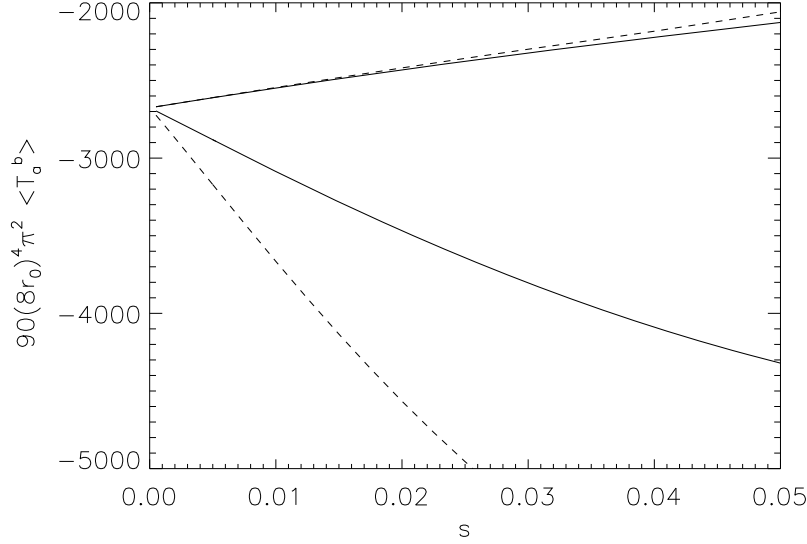


FIG. 3. Components of the stress-energy tensor are plotted near the horizon for a massless scalar field with $\xi = 0$ when $b_2 = 2$. All of the curves have the series (3.5) truncated at A_{33} and B_{33} . The upper solid and dashed curves show $\langle T_r^r \rangle$ and $\langle T_t^t \rangle$ respectively for $A_{33} = B_{33} = 0$ ($a_3 = -2$, $b_3 = -4$). The lower solid curve and lower dashed curve show $\langle T_r^r \rangle$ and $\langle T_t^t \rangle$ respectively for $A_{33} = 4$ and $B_{33} = 6$ ($a_3 = b_3 = 2$). Note that the slopes of the upper curves approach each other near the horizon but that for the lower curves the slope of the curve for $\langle T_r^r \rangle$ near the horizon is different from that of the curve for $\langle T_t^t \rangle$.

values of $\langle T_t^t \rangle_{,r}$ and $\langle T_r^r \rangle_{,r}$ at the horizon appear to depend on the geometry away from the horizon as well. Thus it is quite possible that there are spacetime geometries for which $\langle T_t^t \rangle_{,r} = \langle T_r^r \rangle_{,r}$ on the horizon for $\xi = \frac{1}{6}$.

V. SOLUTIONS TO THE SEMICLASSICAL BACKREACTION EQUATIONS NEAR THE HORIZON

In this section we solve the semiclassical backreaction equations near the horizon using our results in Sec. IV which assume a metric of the form (3.5). We begin by reviewing the solution to the trace equation. If only conformally invariant quantum fields are present then the trace equation is given by substituting (2.4) into (2.8). Evaluating at the horizon and recalling that $\square R = 0$ there,

one finds that the resulting equation can be solved for r_0 as a function of b_2 with the result that

$$r_0^2 = \frac{\pi}{3(b_2 - 1)} [8(\beta + 2\gamma)(b_2^2 + 1) + 32(\beta - \gamma)b_2] . \quad (5.1)$$

It is easy to show from (2.5) that $\beta + 2\gamma > 0$ and $\beta - \gamma > 0$. Thus physically acceptable solutions only exist if $b_2 > 1$. It is worth noting that for an ERN black hole, $b_2 = 1$. Thus the ERN solution to the classical Einstein equations is not a solution to the SCE if only conformally invariant fields are present.

There is a minimum radius which occurs for

$$(b_2)_{\min} = 1 + \sqrt{\frac{6\beta}{\beta + 2\gamma}} . \quad (5.2)$$

It is

$$(r_0^2)_{\min} = 16\pi \left(\sqrt{\frac{2}{3}} \sqrt{\beta(\beta + 2\gamma)} + \beta \right) . \quad (5.3)$$

For the Standard Model $N_0 = 4$, $N_{1/2} = 45$, and $N_1 = 12$ so

$$\beta = \frac{1243}{2880\pi^2} , \quad (5.4a)$$

$$\gamma = \frac{11}{5760\pi^2} , \quad (5.4b)$$

and

$$(b_2)_{\min} = 1 + \sqrt{\frac{113}{19}} \approx 3.4$$

$$(r_0)_{\min} = \frac{1}{\sqrt{\pi}} \sqrt{\frac{1243}{180} + \frac{11\sqrt{2147}}{90}} \approx 2.0 . \quad (5.5)$$

Thus for the Standard Model the minimum size is of order the Planck length. However there are many more particles in Grand Unified Theories, so the minimum size could be significantly larger than the Planck scale. Further this minimum is really only a constraint because it comes from just one of the backreaction equations. The actual minimum could be larger. Note that it does not depend on the coefficients h_1 and h_2 of the higher derivative terms in the semiclassical backreaction equations nor does it depend on the charge of the black hole.

Continuing the analysis of the solutions to the trace equation at the horizon, for small values of $b_2 - 1 > 0$ the radius is

$$r_0^2 \approx \frac{16\pi\beta}{b_2 - 1} . \quad (5.6)$$

and the scalar curvature at the horizon is

$$R = -\frac{2(b_2 - 1)}{r_0^2} \approx -\frac{(b_2 - 1)^2}{8\pi\beta} . \quad (5.7)$$

Thus for $1 < b_2 \leq (b_2)_{\min}$ the size of the event horizon ranges from infinity to its minimum value and the scalar curvature is small when the horizon size is large. Thus these values of b_2 result in physically acceptable solutions.

For very large values of b_2 ,

$$r_0^2 \approx \frac{8\pi}{3}(\beta + 2\gamma)b_2 , \quad (5.8)$$

and

$$R \approx -\frac{3}{4\pi(\beta + 2\gamma)} . \quad (5.9)$$

Since R does not get small as r_0 gets large, the solutions with $b_2 \gg (b_2)_{\min}$ are probably not physically acceptable.

To go further we examine the “rr” component of the semiclassical backreaction equations. At the horizon the equation is

$$-\frac{1}{r_0^2} = 8\pi \left[-\frac{Q^2}{8\pi r_0^4} + (T_r{}^r)_0 + \frac{2}{r_0^4}(b_2^2 - 1) \left(\frac{h_2}{3} - h_1 \right) \right] . \quad (5.10)$$

Here $(T_r{}^r)_0$ is the value of $\langle T_r{}^r \rangle$ evaluated at $r = r_0$. Thus the charge of the black hole which corresponds to a given value of r_0 and hence b_2 is

$$Q^2 = r_0^2 + 8\pi \left[r_0^4 (T_r{}^r)_0 + 2(b_2^2 - 1) \left(\frac{h_2}{3} - h_1 \right) \right] . \quad (5.11)$$

Note that $r_0^4 (T_r{}^r)_0$ depends on b_2 and not r_0 . Thus this equation gives a relationship between the charge Q , the radius r_0 , and the metric parameter b_2 for fixed values of h_1 , and h_2 .

It is of interest to see whether it is possible to have $Q = 0$. Since $b_2 > 1$ it is clearly not possible if $h_2 > 3h_1$ and $(T_r{}^r)_0 > 0$. Even for values of these quantities where it is possible to have $Q = 0$, the resulting radius of the black hole will be of the Planck scale or smaller unless there is a large number of fields and $(T_r{}^r)_0 < 0$, and/or $h_1 - h_2/3 \gg 1$. The latter condition can be satisfied if $h_2 \ll h_1$ and the universe underwent Starobinsky inflation which requires $h_1 \sim 10^9$ [18].

If $Q^2 = 0$ then (5.11) gives a second equation for r_0 . Combining (5.1) and (5.11) gives

$$(\beta + 2\gamma)(b_2^2 + 1) + 4(\beta - \gamma)b_2 + 3r_0^4 (T_r{}^r)_0 (b_2 - 1) + 6 \left(\frac{h_2}{3} - h_1 \right) (b_2 + 1)(b_2 - 1)^2 = 0 . \quad (5.12)$$

For a black hole much larger than the Planck scale, Eq. (5.1) implies that $b_2 \approx 1$ which in turn implies that the metric near the horizon is nearly the same as that of the extreme Reissner-Nordström metric. In that case one expects $(T_r{}^r)_0$ to be approximately equal to its value in an ERN spacetime which for conformally invariant fields is³

$$(T_r{}^r)_0 = \frac{\beta}{r_0^4}. \quad (5.13)$$

Using this as an approximation for $(T_r{}^r)_0$ in (5.12) along with $b_2 \approx 1$ gives

$$(b_2 - 1)^2 = \frac{\beta}{2h_1}. \quad (5.14)$$

Substituting this into (5.1) gives

$$r_0 \approx (512\pi^2\beta h_1)^{1/4}. \quad (5.15)$$

Using 10^9 for h_1 and the value of β for the Standard Model (5.4a) gives $r_0 \approx 700$ which is well above the Planck scale where $r_0 \sim 1$. For Grand Unified Theories β and hence r_0 are even larger. Thus if Starobinsky inflation occurred it is possible that black hole remnants could exist which are compatible with and predicted by semiclassical gravity.

VI. SUMMARY AND CONCLUSIONS

We have examined constraints on the form of the metric for static spherically symmetric zero temperature black hole solutions to the semiclassical backreaction equations and found that the most likely form the metric would take is that both g_{tt} and g^{rr} are quadratic in $r - r_0$ near the horizon. Restricting our attention to metrics of this form, we have numerically computed the stress-energy tensor for both the conformally invariant scalar field and the massless minimally coupled scalar field in spacetimes with metrics of the form (3.5). It has been found in all cases considered that the value of $\langle T_t{}^t \rangle = \langle T_r{}^r \rangle$ on the horizon depends only on the metric parameter b_2 and on the radius r_0 of the event horizon. This makes it possible to determine the leading order behaviors of solutions to the SCE near the horizon.

We have examined the solutions to the SCE near the horizon when only conformally invariant quantum fields are present. It was shown in [17] that for a massive scalar field the large mass condition is given by $mM \sim 2$ in Planck units. For the small mass limit ($mM \ll 1$), most massive

³ The ERN value for massless scalar fields was computed in [6] and for the spin $\frac{1}{2}$ was computed in [7]. In [10] it was argued from the conformal invariance of the spacetime near the horizon that for conformally invariant fields in general it is given by (5.13).

free fields are approximately conformally invariant. For small enough black holes this includes most of the fields in the Standard Model if their interactions can be neglected. We have found that near the horizon zero temperature solutions to the SCE can exist even if the black hole has no electric charge. Of course only knowing their behaviors near the horizon does not guarantee that these solutions have physically realistic geometries far from the horizon and that they could thus correspond to realistic zero temperature black holes. Even if the geometries are physically realistic, it does not guarantee that the black hole evaporation process really does shut off at late times when the black hole is small and therefore that black hole remnants exist. What one does expect however, is that backreaction effects due to quantum fields should be larger for black holes of smaller sizes. Thus it is possible that such effects could result in progressively smaller surface gravities and hence progressively lower temperatures for such black holes with the limit being the uncharged SZTBH solutions being discussed here.

The rr component of the semiclassical backreaction equations provides a relation between b_2 , r_0 , and the black hole charge Q along with the coefficients h_1 and h_2 of the R^2 and Weyl squared terms in the gravitational Lagrangian. If only conformally invariant fields are present, we have shown that this relationship allows for an electric charge of zero for the black hole if $\langle T_r{}^r \rangle$ on the horizon has a large enough negative value and/or $h_2 - 3h_1$ has a large enough negative value. For values of $|h_1|$ and $|h_2|$ less than or of order unity there would need to be an enormous number of quantum fields for the corresponding black hole to be larger in size than the Planck scale. However, if Starobinsky inflation occurred so that $h_1 \sim 10^9$, and if $h_2 \ll h_1$, then zero temperature black holes with sizes significantly above the Planck scale could exist even for the number of quantum fields in the Standard Model. Thus if Starobinsky inflation occurred then it is possible that black hole remnants could exist which are large enough that semiclassical gravity could be used to describe them. As such they could provide an answer to the question of what the end state of the black hole evaporation process is.

ACKNOWLEDGMENTS

P.R.A. would like to acknowledge helpful discussions with William Hiscock, Jacob Bekenstein, and Eric Carlson. He would like to thank Los Alamos National Laboratory, the Racah Institute of Physics, and the University of Valencia for hospitality. P.R.A. also acknowledges the Einstein Center at Hebrew University, the Forchheimer Foundation, and the Spanish Ministerio de Educacion y Ciencia for financial support. This work was supported in part by the National Sci-

ence Foundation under grant numbers PHY-9800971, PHY-0070981 PHY-0556292, PHY-0856050, PHY-1308325, and PHY-1506182.

-
- [1] S. W. Hawking, Commun. Math. Phys. **25**, 152 (1972).
 - [2] For a review see P. Chen, Y. C. Ong, and D. Yeom, Phys. Rep. **603**, 1 (2015).
 - [3] S. B. Giddings, Phys. Rev. D **49**, 947 (1994).
 - [4] A. A. Starobinsky, Phys. Lett. **91B**, 99 (1980).
 - [5] T. P. Netto, A. M. Pelinson, I. L. Shapiro, and A. A. Starobinsky, Eur. Phys. J. C **76**, 544 (2016)
 - [6] P. R. Anderson, W. A. Hiscock, and D. J. Loran, Phys. Rev. Lett. **74**, 4365 (1995).
 - [7] E. D. Carlson, W. H. Hirsch, B. Obermayer, P. R. Anderson, and P. B. Groves. Phys. Rev. Lett. **91**, 051301 (2003).
 - [8] L. A. Kofman and V. Sahni, Phys. Lett. **127B**, 197 (1983).
 - [9] A. C. Ottewill and P. Taylor, Phys. Rev. D **86**, 104067 (2012).
 - [10] P. R. Anderson, W. A. Hiscock, and B. E. Taylor, Phys. Rev. Lett. **85**, 2438 (2000).
 - [11] D. A. Lowe, Phys. Rev. Lett. **87**, 029001 (2001).
 - [12] P. R. Anderson, W. A. Hiscock, and B. E. Taylor, Phys. Rev. D **61**, 084021 (2000).
 - [13] J. Matyjasek, O. B. Zaslavskii, Phys. Rev. D **64**, 104018 (2001).
 - [14] S. N. Solodukhin, Phys. Lett. B **448**, 209 (1999).
 - [15] P. R. Anderson and C. D. Mull, Proceedings of the Eighth Marcell Grossmann Meeting on General Relativity, edited by T. Piran and R. Ruffini (World Scientific, 1999), p. 968.
 - [16] P. R. Anderson and C. D. Mull, Phys. Rev. D **59**, 044007 (1999).
 - [17] P. R. Anderson, W. A. Hiscock, and D. A. Samuel, Phys. Rev. D **51**, 4337 (1995).
 - [18] Q.-G. Huang, J. Cosmol. Astropart. Phys. **02** (2014) 035.
 - [19] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).
 - [20] D. J. Loran, W. A. Hiscock, and P. R. Anderson, Phys. Rev. D **52**, 4554 (1995).