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Signature change has been identified as a generic consequence of holonomy modifications in spherically symmetric models of loop quantum gravity with real connections, which includes modified Schwarzschild solutions. An extension of this result to 2-dimensional dilaton models, including in particular the Callan-Giddings-Harvey-Strominger (CGHS) solution, further elucidates properties of black holes in this setting. In addition, new obstructions are found to coupling matter and to including operator-ordering effects in an anomaly-free manner.

I. INTRODUCTION

Several bouncing black-hole scenarios based on loop-motivated models with real connection variables have been described recently, following [1]. The main mechanism that allows one to avoid the classical singularity is a modification of the dynamics, suggested by the use of holonomies instead of connection components in loop quantum gravity. However, constructing these models has so far been possible only by using classical gauge fixings or other assumptions about the structure of space-time, and therefore comes at the expense of not being able to discuss the anomaly problem of quantum gravity. It is then unclear whether there is any consistent space-time structure at all. In some cases, it has been possible to extend the modified dynamics to anomaly-free versions. These models have been shown to manifest signature change once the anomaly problem has been suitably addressed [2]. With a Euclidean phase in the core of black holes, deterministic evolution is impossible and the bounce picture no longer applies. In this paper, we further elaborate on these questions.

General covariance is deformed in most of the existing midisuperspace models of loop quantum gravity (LQG), in the sense that structure functions in the hypersurface deformation algebroid do not have the classical form [3]. In particular, non-singular signature change is possible in the presence of holonomy modifications in models based on real connection variables; for complex connections see [4, 5]. These results have been derived by effective [2, 6–8] and operator methods [9], and they appear in a related form in cosmological perturbation theory [10, 11].

Since the effective signature of space-time determines the form of well-posed initial or boundary value problems, modified space-time structures that allow for signature change have an important influence on the causal behavior of black-hole models in loop quantum gravity. Signature change results from the presence of holonomy modifications in the theory, which are the same effects often used to argue that singularities are removed in such models. However, even if curvature invariants remain bounded, signature change in the high-curvature region would imply that there is no deterministic evolution through high curvature. A black-hole model rather different from simple bounce models is then obtained, with problematic aspects owing to the presence of new Cauchy horizons [12]. While diverging curvature may be avoided, a serious problem (much like a naked singularity) persists in the sense of a space-time incompletely determined by initial data.

In order to extend space-time across the Euclidean region, one requires additional data on the part of its boundary that borders on the future Lorentzian space-time. This additional requirement is reminiscent of other proposals of black-hole models, for instance the final-state condition in [13]. To some degree, it is also related to 'stretched horizons', introduced in the context of black hole complementarity (see, for instance [14]), except that, with signature change, unexpected degrees of freedom are not located at a horizon. In any case, the detailed analysis of anomaly-free black hole models in loop quantum gravity points towards a much more subtle non-singular description of quantum space-time than usually postulated in simplified bounce models. Just as an outside observer finds the stretched horizon as a membrane storing and later releasing information in the form of microphysical degrees of freedom, additional information is encountered once an observer moves into the future of a Euclidean region embedded in space-time. However, in the case of black-hole models of loop quantum gravity, there is as yet no microscopic theory that would restrict or determine possible data around Euclidean regions. Geometrically, signature change demonstrates the existence of novel non-classical geometries in loop quantum gravity, from which classical space-time emerges in an effective picture [15].

In this paper, we extend the analysis of signature change in modified Schwarzschild space-times in several ways. First, we use the formalism of [16] in order to study signature change for all 2-dimensional dilaton models with holonomy modifications. (Loop quantum gravity also suggests inverse-triad corrections, which have a behavior rather different than holonomy modifications. In particular, inverse-triad corrections do not give rise to strong space-time effects such as signature change, and therefore they will be ignored here.) Secondly, in Section IV we incorporate a canonical transformation, suggested by [17], that may be used to modify the kinetic term of the Hamiltonian constraint (quadratic in extrinsic curvature). Although some of the formal aspects relevant for signature change seem to be absent for some choices of such canonical transformations, we will show that signature change is still realized. Finally, we obtain new partial obstructions to anomaly-free formulations with holonomy modifications when operator effects or matter terms are included (Sections IV C and V). This last result is another example of no-go theorems found in this context. As always, such statements are useful because they indicate how the usual assumptions may have to be modified in order to evade the conclusions of a no-go theorem. Our results therefore contribute to the ongoing construction of consistent models of loop quantum gravity, and to an analysis of some of their properties.

II. 2-DIMENSIONAL DILATON GRAVITY

We begin with a brief review of different choices of variables used to describe 2-dimensional models of dilaton gravity, referring to [16, 18] for further details. In two space-time dimensions, the general form of dilaton gravity models has the first-order action [19]

$$S = -\frac{1}{2G}\int_M \left(\phi \mathrm{d}\omega + \frac{1}{2}V(\phi)\epsilon + X_a \mathrm{D}e^a\right) \tag{1}$$

for a dyad e^a with volume form ϵ, the dilaton field ϕ and a connection 1-form ω which appears in the covariant derivative D. The fields X_a are Lagrange multipliers that ensure torsion-freedom. The potential $V(\phi)$ is an arbitrary function and characterizes different models.

A. First-order variables and Poisson Sigma model

After integrating by parts (ignoring boundary terms), we obtain

$$S = \frac{1}{2G}\int_M \left(e^a \wedge \mathrm{d}X_a + \omega \mathrm{d}\phi + X_a \epsilon^a{}_b \omega \wedge e^b + \frac{1}{2}V(\phi)\epsilon\right) . \tag{2}$$

As used in Poisson Sigma models [19–21], it is convenient to collect the fields in two triplets, $X^i = (X^-, X^+, \phi)$ and $A_i = (e_x^+, e_x^-, \omega_x)$ of canonical fields, and one triplet $\Lambda_i = (e_t^+, e_t^-, \omega_t)$ of multipliers. (Components with plus and minus superscripts refer to internal light-cone coordinates.) The action then takes the compact form

$$S = -\frac{1}{2G}\int_M \left(A_i \wedge \mathrm{d}X^i + \frac{1}{2}P^{ij}A_i \wedge A_j\right) \tag{3}$$

with a Poisson tensor

$$P = \begin{pmatrix} 0 & -\frac{1}{2}V(\phi) & -X^- \\ \frac{1}{2}V(\phi) & 0 & X^+ \\ X^- & -X^+ & 0 \end{pmatrix} . \tag{4}$$

The canonical formulation leads to Poisson brackets

$$\{X^i(x), A_j(y)\} = 2G\delta^i_j \delta(x-y) , \tag{5}$$

and three first-class constraints

$$\tilde{C}^i = \frac{1}{2G}\left((X^i)' + P^{ij}A_j\right) . \tag{6}$$

The component $\tilde{C}^3$ generates SO(1,1) rotations of the Lorentzian dyad and connection, while the linear combination $D = A_i\tilde{C}^i = A_i(X^i)'$ generates spatial diffeomorphisms. The remaining independent combination of $\tilde{C}^+$ and $\tilde{C}^-$ serves as a Hamiltonian constraint.

B. Variables invariant under SO(1,1)

A comparison with canonical variables commonly used in spherically symmetric models, based on a symmetry reduced Arnowitt–Deser–Misner (ADM) formulation [22] of canonical gravity, is facilitated by first introducing combinations of the fields invariant under the SO(1,1) transformations generated by $\tilde{C}^3$. Instead of $X^\pm$ and $e_x^\pm$, we follow [16] and work with

$$X := \sqrt{X^+X^-} \quad \text{and} \quad e := \sqrt{e_x^+ e_x^-} \tag{7}$$

and two boost parameters α and β such that

$$X^\pm = X\exp(\pm\beta) \quad \text{and} \quad e_x^\pm = e\exp(\pm\alpha) . \tag{8}$$

One can turn these into canonical variables by introducing

$$Q^e = 2X\cosh(\alpha-\beta) \quad \text{and} \quad Q^\alpha = 2eX\sinh(\alpha-\beta) \tag{9}$$

such that

$$\{Q^e(x), e(y)\} = \{Q^\alpha(x), \alpha(y)\} = \{\phi(x), \omega_x(y)\} = 2G\delta(x-y) . \tag{10}$$

The original variables can be obtained from the canonical ones as

$$X^\pm = \frac{eQ^e \mp Q^\alpha}{2e}\exp(\pm\alpha) . \tag{11}$$

We now have the constraints

$$\tilde{C}^{\pm} = \frac{1}{2G}\left(\left(\frac{eQ^e \mp Q^\alpha}{2e}\right)' \pm \frac{eQ^e \mp Q^\alpha}{2e}(\omega_x + \alpha') \pm \frac{1}{2}V(\phi)e\right)\exp(\pm\alpha) \tag{12}$$

and

$$\tilde{C}^3 = \frac{1}{2G}(\phi' + Q^\alpha)\,. \tag{13}$$

C. Transformation to standard variables of spherical symmetry

The usual connection or extrinsic-curvature variables of spherically symmetric gravity [23] are finally obtained by a canonical transformation from $(Q^e, e; Q^\alpha, \alpha; \phi, \omega_x)$ to $(K_\varphi, E^\varphi; K_x, E^x; \eta, P^\eta)$ with

$$Q^e = 2\sqrt{2}(E^x)^{1/4}K_\varphi \quad , \quad e = \frac{E^\varphi}{\sqrt{2}(E^x)^{1/4}} \tag{14}$$

$$Q^\alpha = P^\eta \quad , \quad \alpha = -\eta \tag{15}$$

$$\omega_x = -\left(K_x + \frac{E^\varphi}{2E^x}K_\varphi - \eta'\right) \quad , \quad \phi = E^x\,. \tag{16}$$

(See Eq. (42) in [16].) The spherically symmetric variables have Poisson brackets

$$\{K_x(x), E^x(y)\} = 2G\delta(x-y) \quad , \quad \{K_\varphi(x), E^\varphi(y)\} = G\delta(x-y) \tag{17}$$

as defined in [23]. A suitable combination of $\tilde{C}^+$ and $\tilde{C}^-$, written in these variables, takes the usual form of the Hamiltonian constraint

$$\begin{aligned} H[N] &= C^+[2^{-1/2}N\phi^{1/4}\exp(-\alpha)] - C^-[2^{-1/2}N\phi^{1/4}\exp(\alpha)] \\ &= -\frac{1}{2G}\int \mathrm{d}x N\left(\frac{K_\varphi^2 E^\varphi}{\sqrt{E^x}} + 2\sqrt{E^x}K_xK_\varphi - \frac{1}{2}E^\varphi V(E^x) - \frac{((E^x)')^2}{4E^\varphi\sqrt{E^x}} + \frac{\sqrt{E^x}(E^x)'(E^\varphi)'}{(E^\varphi)^2} - \frac{\sqrt{E^x}(E^x)''}{E^\varphi}\right) \end{aligned}$$

in spherically symmetric variables for the appropriate dilaton potential, $V(E^x) = -2/\sqrt{E^x}$; see Eq. (46) of [16]. For an arbitrary potential, the equations generalize the connection formulation of spherically symmetric canonical gravity to arbitrary dilaton models, including the CGHS model [24] for constant V.

At this point, we can compare the results with [17]; see also [18]. The expression for e in (14) is the same as (51) in [17], Q^e in (14) is (57), and ω_x in (16) is (60), just in slightly different notations. However, the formulation of the CGHS model presented [17] is different. In particular, the canonical transformation to connection variables used there is simpler than in the spherically symmetric model, and the Hamiltonian constraint has only a term of the form K_xK_φ in its kinetic part but no contribution of K_φ^2. It amounts to renaming the SO(1,1)-invariant variables by $\omega_x \equiv K_x$, $Q^e \equiv K_\varphi$ and $e \equiv E^\varphi$. These are the variables introduced in Eq. (20) of [16], corresponding to a standard first-order formulation in variables as used in Poisson Sigma models. Since the Hamiltonian constraint then has only one term $Q^e\omega_x = K_xK_\varphi$ of variables identified with extrinsic-curvature components, there is no term of the form K_φ^2 in the resulting Hamiltonian constraint, as is clear from (23) of [16] (our (12)) and rederived in [17].

D. Contribution to kinetic terms

As the examples discussed so far make clear, the form of the kinetic contribution to the Hamiltonian constraint is not invariant under canonical transformations. In general, if one starts with a generic combination $aK_\varphi^2 E^\varphi/\sqrt{E^x} + K_xK_\varphi\sqrt{E^x}$ with some real number a (equal to $a = 1/2$ in (18)), as it appears in the standard connection formulation of spherically symmetric canonical gravity, one can always transform to new canonical variables that amount to setting $a = 0$. To do so, we need a new

$$\tilde{K}_x = K_x + aK_\varphi E^\varphi/E^x \tag{18}$$

and $\tilde{K}_\varphi = K_\varphi(E^x)^b$ with a second parameter b. If we also choose $\tilde{E}^\varphi = (E^x)^{-b}E^\varphi$ and leave $E^x = \tilde{E}^x$ unchanged, the tilde variables are canonical provided that $b = \frac{1}{2}a$: The only non-trivial bracket is

$$\begin{aligned} 0 = \{\tilde{K}_x(x), \tilde{K}_\varphi(y)\} &= \left(K_\varphi(y)\{K_x(x), E^x(y)^b\} + aK_\varphi(x)\frac{E^x(y)^b}{E^x(x)}\{E^\varphi(x), K_\varphi(y)\}\right)\delta(x-y) \\ &= GK_\varphi(E^x)^{b-1}(2b-a)\delta(x-y) \end{aligned} \tag{19}$$

using (17). The new kinetic term is $(\tilde{E}^x)^{1/4}\tilde{K}_x\tilde{K}_\varphi$ without a contribution from $\tilde{K}_\varphi^2$. The choice made in [17] for the CGHS model amounts to applying such a canonical transformation.

Previous derivations of signature change in spherically symmetric models with holonomy modifications used both contributions to the kinetic term of the Hamiltonian constraint, K_φ^2 and $K_\varphi K_x$ based on (18). As we will review in the next section, there is an interesting interplay between allowed modifications of these two terms in an anomaly-free formulation. If one of them can be removed by a canonical transformation, one may wonder whether the same conclusions can still be drawn. In the present paper, we answer this question in the affirmative.

III. MODIFIED SCHWARZSCHILD MODELS

Spherically symmetric models have already been analyzed in detail [2]. At an effective level, one can work with different kinds of constraint systems, given by the usual hypersurface deformation brackets on one hand and a partially Abelianized system without structure functions on the other [25]. The latter is easier to deal with when one attempts a full quantization, but as shown in [2], it obscures the important question of covariance because it is not obvious that a consistent set of hypersurface-deformation generators may be recovered in the modified or quantized system. If one analyzes possible realizations of hypersurface-deformation brackets in the partially Abelianized system with quantum modifications, the structure functions turn out to be modified and signature change is obtained in the same way as in a direct treatment of the brackets. In [2], the analysis was done at an effective level; here we complement it with operator considerations, and further generalizations.

A. Deformed covariance

It is possible to modify (18) while keeping the system first-class. In spherically symmetric models, the possibility of anomaly-free holonomy corrections has been analyzed based on a Hamiltonian constraint

$$H[N]=-\frac{1}{2G}\int \mathrm{d}x N\left(\frac{f_1(K_\varphi)E^\varphi}{\sqrt{E^x}}+2\sqrt{E^x}K_x f_2(K_\varphi)+\frac{E^\varphi}{\sqrt{E^x}}-\frac{((E^x)')^2}{4E^\varphi\sqrt{E^x}}+\frac{\sqrt{E^x}(E^x)'(E^\varphi)'}{(E^\varphi)^2}-\frac{\sqrt{E^x}(E^x)''}{E^\varphi}\right)$$

with two free functions of K_φ, f_1 and f_2. The holonomy modification function $f_1(K_\varphi)$ is typically chosen to be $\sin(\rho K_\varphi)/\rho$, but may be kept more general. The system is anomaly-free if

$$f_2(K_\varphi)=\frac{1}{2}\frac{\mathrm{d}f_1}{\mathrm{d}K_\varphi} \tag{20}$$

in which case the classical structure functions are modified by an additional factor of [6]

$$\beta(K_\varphi)=\frac{\mathrm{d}f_2}{\mathrm{d}K_\varphi}=\frac{1}{2}\frac{\mathrm{d}^2 f_1}{\mathrm{d}K_\varphi^2}\,. \tag{21}$$

This function is negative around a local maximum of $f_1(K_\varphi)$, or in any regime where an upper bound on curvature is achieved by the modification.

B. Abelianization of the constraints and operator ordering

As shown in [2], it is possible to construct a partially Abelianized system with holonomy modifications, using the methods of [25]. If one replaces the local Hamiltonian constraint $\mathcal{H}$ by a linear combination

$$C:=\frac{(E^x)'}{E^\varphi}\mathcal{H}-2f_2(K_\varphi)\frac{\sqrt{E^x}}{E^\varphi}\mathcal{D} \tag{22}$$

with the local diffeomorphism constraint $\mathcal{D}=G^{-1}(-\frac{1}{2}(E^x)'K_x+K_\varphi'E^\varphi)$, the component K_x cancels out and greatly simplifies the bracket of two constraints C: We have

$$C=-\frac{1}{G}\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{E^x}\left(1-\left(\frac{(E^x)'}{2E^\varphi}\right)^2\right)\right)+\frac{1}{2}\frac{(E^x)'}{\sqrt{E^x}}f_1(K_\varphi)+2\sqrt{E^x}K_\varphi'f_2(K_\varphi)\,. \tag{23}$$

One can show that the Abelianization succeeds if C is a total derivative by x, which requires the same condition (20) that follows from anomaly freedom: with

$f_2 = \frac{1}{2}\mathrm{d}f_1/\mathrm{d}K_\varphi$, we have

$$C = -\frac{1}{G}\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{E^x}\left(1-\left(\frac{(E^x)'}{2E^\varphi}\right)^2\right)+\sqrt{E^x}f_1(K_\varphi)\right) . \tag{24}$$

If, on the other hand, one starts with the classical Abelianized system and modifies the dependence of C on K_φ, as proposed in [25], hypersurface-deformation brackets can be recovered only if there is a modification function β in the structure functions, implying signature change [2].

However, obstructions to recovering hypersurface-deformation brackets, and therefore obstructions to covariance, are obtained if one tries to couple a matter field to the modified system. While this can be done within the Abelianized system [26], it is no longer possible to recover hypersurface-deformation brackets from the modified system [2]. Another question not yet addressed is whether additional quantum effects, such as ordering choices, may affect the outcome.

It is possible to turn the modified, partially Abelianized constraints C and $\mathcal{D}$ into operators without anomalies. However, this quantization step could introduce further properties, in addition to holonomy modifications, that prevent one from recovering hypersurface-deformation brackets even in the vacuum case. This possibility turns out to be realized for the Schwarzschild model.

We demonstrate the sensitivity to ordering questions by working with quantized constraints at a formal level. We only indicate the non-commuting nature of our variables by paying attention to their position in products, but do not go into details of regularizations such as point splitting.

We begin with the partially Abelianized system of spherically symmetric constraints, as derived in [25] but written here as formal operators:

$$G\hat{\mathcal{C}}_{\mathrm{SS}} = -\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{\hat{E}^x}\left(1-\hat{\Gamma}_\varphi^2+\hat{f}_1(K_\varphi)\right)\right) \tag{25}$$

$$G\hat{\mathcal{D}}_{\mathrm{SS}} = -\frac{1}{2}(\hat{E}^x)'\hat{K}_x + \hat{K}_\varphi'\hat{E}^\varphi . \tag{26}$$

The brackets of these constraints are such that any factor ordering choice is allowed for $\hat{\mathcal{D}}_{\mathrm{SS}}$ while $\hat{\mathcal{C}}_{\mathrm{SS}}$ is free from operator ordering ambiguities. At the formal level we are working with here, it is easy to see that two smeared $\hat{\mathcal{C}}_{\mathrm{SS}}$ commute because any non-zero commutator introduces a delta function which gives zero once the antisymmetric bracket is imposed.

Let us start with a factor ordering choice for the diffeomorphism constraint as shown in Eq. (26). For simplicity, we will not write hats anymore, but all expressions in this subsection remain objects sensitive to ordering choices. We can recover the original Hamiltonian and diffeomorphism constraints as generators of the hypersurface deformation brackets, if we write them as linear combinations

$$\mathcal{H}_{\mathrm{SS}} := \frac{E^\varphi}{(E^x)'}\mathcal{C}_{\mathrm{SS}} + 2\frac{f_2(K_\varphi)\sqrt{E^x}}{(E^x)'}\mathcal{D}_{\mathrm{SS}} , \tag{27}$$

while $\mathcal{D}_{\mathrm{SS}}$ remains unmodified, and $f_2(K_\varphi) = \frac{1}{2}\mathrm{d}f_1(K_\varphi)/\mathrm{d}K_\varphi$ as before. (This procedure retraces the steps taken in [25].) We begin by rewriting $\mathcal{C}_{\mathrm{SS}}$ as

$$G\mathcal{C}_{\mathrm{SS}} = -\frac{(E^x)'}{2\sqrt{E^x}}\left(1-\Gamma_\varphi^2\right) + 2\sqrt{E^x}\,\Gamma_\varphi\Gamma_\varphi' - \frac{(E^x)'}{2\sqrt{E^x}}f_1(K_\varphi) - \sqrt{E^x}\,\frac{\mathrm{d}f_1(K_\varphi)}{\mathrm{d}K_\varphi}K_\varphi' . \tag{28}$$

(We use the spin-connection component $\Gamma_\varphi = -\frac{1}{2}(E^x)'/E^\varphi$ of spherically symmetric gravity as a shortcut.) Consequently, for our factor ordering choice for the diffeomorphism constraint, the Hamiltonian constraint (as defined in (27)) has the form

$$\begin{aligned} G\mathcal{H}_{\mathrm{SS}} = &-\frac{E^\varphi}{2\sqrt{E^x}}\left(1-\Gamma_\varphi^2\right) + 2\sqrt{E^x}\,\Gamma_\varphi' - \frac{E^\varphi}{2\sqrt{E^x}}f_1(K_\varphi) - \frac{E^\varphi\sqrt{E^x}}{(E^x)'}\,\frac{\mathrm{d}f_1(K_\varphi)}{\mathrm{d}K_\varphi}K_\varphi' \\ &-f_2(K_\varphi)\sqrt{E^x}K_x + \frac{2\sqrt{E^x}\,f_2(K_\varphi)}{(E^x)'}K_\varphi'E^\varphi . \end{aligned} \tag{29}$$

This expression does not reduce to the usual one of the Hamiltonian constraint because the last term in the first line of Eq. (29) does not cancel out with the last term in the second line even if $f_2 = \frac{1}{2}\mathrm{d}f_1/\mathrm{d}K_\varphi$. Instead, a non-zero commutator $[E^\varphi, f(K_\varphi)K_\varphi']$ remains. Thus, it is not possible to obtain the generators of the hypersurface deformation brackets from the newly defined system of constraints, once operator orderings are taken into account.

We could have started with the other factor ordering choice in the diffeomorphism constraint from the beginning. This is the only other choice left to be exploited because $\mathcal{C}_{\mathrm{SS}}$ does not have any factor ordering ambiguities. Had we started with the diffeomorphism constraint of the form

$$G\mathcal{D}_{\mathrm{SS}} = -\frac{1}{2}(\hat{E}^x)'\hat{K}_x + \hat{E}^\varphi\hat{K}_\varphi' \tag{30}$$

we would have, for the Hamiltonian constraint, the expression

$$G\mathcal{H}_{\rm SS} = -\frac{E^\varphi}{2\sqrt{E^x}}\left(1-\Gamma_\varphi^2\right) + 2\sqrt{E^x}\,\Gamma'_\varphi - \frac{E^\varphi}{2\sqrt{E^x}} f_1(K_\varphi) - \frac{E^\varphi\sqrt{E^x}}{(E^x)'}\frac{\mathrm{d}f_1(K_\varphi)}{\mathrm{d}K_\varphi} K'_\varphi - f_2(K_\varphi)\sqrt{E^x}K_x + \frac{2\sqrt{E^x}\,f_2(K_\varphi)}{(E^x)'} E^\varphi K'_\varphi\,. \quad (31)$$

Once again, the unwanted terms do not cancel out even if $f_2 = \frac{1}{2}\mathrm{d}f_1/\mathrm{d}K_\varphi$, but rather give a non-zero commutator $[E^\varphi, \hat{f}_2(K_\varphi)]$. We conclude that, with operator orderings, one cannot reproduce the hypersurface deformation brackets from the newly defined system of constraints with a (partially) Abelianized algebra.

IV. DILATON GRAVITY MODELS

For a general 2-dimensional dilaton gravity model with potential $V(\phi)$, the Hamiltonian constraint in the connection variables of [16] differs from the spherically symmetric one only in the term that does not depend on extrinsic curvature or spatial derivatives of the densitized triad; see Eq. (46) in [16], or (18) here. This term does not affect the constraint algebra, and therefore the same partial Abelianization found in [25] for spherically symmetric models can be applied to arbitrary 2-dimensional dilaton gravity models. The general Abelianized constraints are

$$G\mathcal{C} = -\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{E^x}\left(\frac{W(E^x)}{\sqrt{E^x}} - \Gamma_\varphi^2 + f_1(K_\varphi)\right)\right) \quad (32)$$

$$G\mathcal{D} = -\frac{1}{2}(E^x)'K_x + K'_\varphi E^\varphi \quad (33)$$

where $\mathrm{d}W(E^x)/\mathrm{d}E^x = -\frac{1}{4}V(E^x)$. This simple result can also be found in [27]. The same ordering obstructions as in the spherically symmetric model follow thanks to the closely related structure of the constraints, as do matter obstructions to be discussed in more detail in Sec. V.

A. Simplified kinetic term

Using a canonical transformation, the Hamiltonian and diffeomorphism constraints of 2-dimensional dilaton models can be brought to the form

$$H[N] = -\int \mathrm{d}x N\left(4K_\varphi K_x - \frac{1}{4}V(E^x)E^\varphi + \frac{1}{4}(E^x)'(E^\varphi)'(E^\varphi)^{-2} - \frac{1}{4}(E^x)''(E^\varphi)^{-1}\right)\,,$$
$$D[N^x] = \int \mathrm{d}x N^x\left(E^\varphi K'_\varphi - (E^x)'K_x\right)\,. \quad (34)$$

For simplicity, we have set the 2-dimensional Newton's constant, G_2, equal to one. Moreover, we have dropped tildes on the transformed variables for simpler notation even though we have applied the previous transformation from $(K_x, E^x, K_\varphi, E^\varphi)$ to $(\tilde{K}_x, \tilde{E}^x, \tilde{K}_\varphi, \tilde{E}^\varphi)$, described in Sec. II D. We have also applied a second transformation from $(\tilde{K}_x, \tilde{E}^x)$ to $(\tilde{\tilde{K}}_x, \tilde{\tilde{E}}^x)$ by defining $\tilde{K}_x = 4\tilde{\tilde{K}}_x$ and $\tilde{E}^x = \frac{1}{2}\tilde{\tilde{E}}^x$. The latter transformation turns the variables into strict canonical pairs without numerical factors. Finally, we have absorbed a factor of $\sqrt[4]{E^x}$ in the lapse function. Since E^x is a scalar without density weight, this last step does not change the behavior of the Hamiltonian density under spatial diffeomorphisms.

Classically, the hypersurface-deformation brackets do not change by a canonical transformation combined with absorbing a spatial scalar in the lapse function. They are still given by

$$\{D[N^x], D[M^x]\} = D[\mathcal{L}_{N^x}M^x] \quad (35)$$

$$\{H[N], D[N^x]\} = -H[\mathcal{L}_{N^x}N] \quad (36)$$

$$\{H[N_1], H[N_2]\} = D[q^{xx}(N_1N'_2 - N_2N'_1)]\,. \quad (37)$$

The only non-vanishing component of the (inverse of the) 1-dimensional spatial metric, $q^{xx} = 1/(E^\varphi)^2$, appears as a structure function. (After the transformation described above, the variables used here should be identified with $\tilde{E}^\varphi = E^\varphi/\sqrt[4]{E^x}$, using $a = 1/2$, $b = 1/4$ in the definitions of Sec. II D. Therefore, $1/(E^\varphi)^2$, dropping the tilde after the transformation, is the same as the usual structure function $E^x/(E^\varphi)^2$ in spherically symmetric models if one includes the factor of $\sqrt[4]{E^x}$ absorbed in the lapse function.)

If one follows the common steps to use loop quantum gravity as a motivation of a specific quantization of this model [23, 28], one has well-defined holonomy operators which are exponentiated versions of the extrinsic-curvature components but are not weakly continuous in those variables anymore. However, the holonomies corresponding to the K_x variables are extended along the edge of a (one-dimensional) spin network. As a consequence, the holonomy corrections arising from these are non-local in nature and difficult to implement [29]. (However, using partial Abelianization techniques as in [25], a suitable redefinition of the constraints makes it possible to eliminate the K_x variable.) On the other hand, the point-wise holonomy operators, corresponding to the K_φ component, act at the nodes of the spin networks. These are local modifications and can be included in our

Hamiltonian constraint

$$H[N]=-\int \mathrm{d}xN\left(4f(K_\varphi)K_x-\frac{1}{4}V(E^x)E^\varphi+\frac{1}{4}(E^x)'(E^\varphi)'(E^\varphi)^{-2}-\frac{1}{4}(E^x)''(E^\varphi)^{-1}\right)\,. \tag{38}$$

The diffeomorphism constraint remains unmodified as suggested by the simple geometrical action of finite diffeomorphisms on states.

The hypersurface deformation brackets that do not contain structure functions remain unaltered while the only modified bracket is

$$\begin{aligned}
\{H[N],H[M]\} &= \int \mathrm{d}x\mathrm{d}yN(x)M(y)\left[\left\{4f(K_\varphi(x))K_x(x),\frac{1}{4}(E^x(y))'(E^\varphi(y))'(E^\varphi(y))^{-2}\right\}\right.\\
&\qquad\left.+\left\{4f(K_\varphi(x))K_x(x),-\frac{1}{4}(E^x(y))''(E^\varphi(y))^{-1}\right\}+(x\leftrightarrow y)\right]\\
&=\int \mathrm{d}x(NM'-N'M)\left[-\dot{f}K_xE^x(E^\varphi)^{-2}-f(E^\varphi)'(E^\varphi)^{-2}\right.\\
&\qquad\left.+f(E^\varphi)'(E^\varphi)^{-2}+\dot{f}K_\varphi'(E^\varphi)^{-1}\right]\\
&=\int \mathrm{d}x(NM'-N'M)\frac{\dot{f}}{(E^\varphi)^2}\left(K_\varphi'E^\varphi-K_x(E^x)'\right)\\
&=D\left((NM'-N'M)\frac{\dot{f}}{(E^\varphi)^2}\right)\,.
\end{aligned} \tag{39}$$

As expected, the dilaton potential does not contribute to the bracket. The structure function q^{xx} is modified by the presence of $\dot{f}:=\mathrm{d}f/\mathrm{d}K_\varphi$, which is equal to one for the classical case but not if holonomy modifications are present. The underlying space-time symmetry is deformed for this system as in the Schwarzschild case. However, it is not as straightforward to conclude that this definition can give rise to signature change: The first derivative of the modification function appears here in the structure function, which unlike the second derivative in (21) does not necessarily change sign near a local maximum. Nevertheless, the same relationship can be established as we will do now.

B. Partial Abelianization of the constraint algebra

The unsmeared modified constraints are

$$\mathcal{H}=-4f(K_\varphi)K_x+\frac{1}{4}V(E^x)E^\varphi-\frac{1}{4}(E^x)'(E^\varphi)'(E^\varphi)^{-2}+\frac{1}{4}(E^x)''(E^\varphi)^{-1}\,, \tag{40}$$

$$\mathcal{D}=E^\varphi K_\varphi'-(E^x)'K_x\,. \tag{41}$$

We redefine the system of constraints by keeping the diffeomorphism constraint the same while defining a new constraint as a linear combination of the (old) Hamiltonian constraint and the diffeomorphism constraint as

$$\begin{aligned}
\mathcal{C}&=\frac{1}{4}\frac{(E^x)'}{E^\varphi}\mathcal{H}-\frac{f(K_\varphi)}{E^\varphi}\mathcal{D}\\
&=\frac{1}{16}(E^x)'V(E^x)-\frac{(E^x)'}{E^\varphi}\left(-\frac{1}{16}(E^x)'(E^\varphi)'(E^\varphi)^{-2}+\frac{1}{16}(E^x)''(E^\varphi)^{-1}\right)-f(K_\varphi)K_\varphi'\\
&=\frac{1}{16}(E^x)'V(E^x)+\frac{1}{16}\frac{(E^x)'}{E^\varphi}\frac{\mathrm{d}}{\mathrm{d}x}\left((E^x)'(E^\varphi)^{-1}\right)-f(K_\varphi)K_\varphi'\\
&=-\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}W(E^x)+g(K_\varphi)-\frac{1}{32}\left((E^x)'(E^\varphi)^{-1}\right)^2\right)
\end{aligned} \tag{42}$$

where $\mathrm{d}W(E^x)/\mathrm{d}E^x=-\frac{1}{8}V(E^x)$ (with an extra factor of 1/2 from our canonical transformation). We have introduced a new function $g(K_\varphi)$ via $f(K_\varphi)=\mathrm{d}g(K_\varphi)/\mathrm{d}K_\varphi$. The smeared version of the new constraint is obtained after integrating by parts and using L as the new smearing function:

$$C[L] = \int \mathrm{d}x L \left(\frac{1}{2} W(E^x) + g(K_\varphi) - \frac{1}{32} \left((E^x)'(E^\varphi)^{-1} \right)^2 + \text{const.} \right) . \tag{43}$$

The new constraint, so defined, is such that it commutes with itself

$$\{C[L_1], C[L_2]\} = 0 \,, \tag{44}$$

while its Poisson bracket with the diffeomorphism constraint is

$$\{C[L], D[N^x]\} = -C[\mathcal{L}_{N^x} L] \,. \tag{45}$$

The Poisson bracket between two diffeomorphism constraints remains unaltered.

The (partial) Abelianization of the constraint algebra helps us to demonstrate signature change in all dilaton models, including the CGHS black hole model. If one were to follow [25], one would first Abelianize the algebra classically and then insert the holonomy modification function in the new constraint. This procedure, sometimes called ‘polymerization’, replaces the extrinsic curvature component K_φ with a bounded function of K_φ, usually $\sin(\rho K_\varphi)/\rho$, in the new constraint. Claims about singularity resolution, as in [25, 30], are based on this boundedness property. However, as we have seen in Eq. (39), what appears in the deformation of the structure function is $\mathrm{d}f/\mathrm{d}K_\varphi = \mathrm{d}^2 g/\mathrm{d}K_\varphi^2$, the second derivative of the holonomy modification function in $C[L]$. At maximum curvature, the bounded function $g(K_\varphi)$ reaches its maximum and therefore its second derivative must be negative. We obtain hypersurface-deformation brackets with the same sign for normal deformations as one would have in the case of Euclidean gravity. Thus, singularity resolution in the vacuum CGHS black hole in the framework of loop quantum gravity, just like for the vacuum Schwarzschild black hole, cannot be divorced from signature-change.

C. Operator ordering in the Abelianization

For the spherically symmetric case, starting from the partially Abelianized constraints, we have shown that none of the possible factor ordering choices lead us to the required form of the hypersurface-deformation brackets. One can repeat the analysis for all 2-dimensional dilaton models, including the CGHS black hole model, following the same procedure and show that there is a similar obstruction. Instead, we shall arrive at the same conclusion following a different approach. We shall find a suitable operator ordering for the generators of the hypersurface-deformation brackets, the original Hamiltonian and diffeomorphism constraints, and then try to define the new, partially Abelianized constraint while keeping in mind that we are now dealing with objects sensitive to ordering choices. (Again, our treatment of operators is formal.)

Our first task is to find a factor ordering for the original system of constraints such that the hypersurface-deformation brackets are realized. Following [9], one factor ordering choice with closed brackets is

$$\hat{H}[N] = -\int \mathrm{d}x N(x) \left[4(\hat{E}^\varphi)^{-1} \hat{f}(K_\varphi) \hat{E}^\varphi \hat{K}_x - \frac{1}{4} \hat{V}(E^x) \hat{E}^\varphi + \frac{1}{4} (\hat{E}^x)'(\hat{E}^\varphi)'(\hat{E}^\varphi)^{-2} - \frac{1}{4} (\hat{E}^x)''(\hat{E}^\varphi)^{-1} \right] , \tag{46}$$

$$\hat{D}[N^x] = \int \mathrm{d}x N^x(x) \left[\hat{K}_\varphi' \hat{E}^\varphi - (\hat{E}^x)' \hat{K}_x \right] . \tag{47}$$

The operator ordering of the Hamiltonian constraint is subtle as can be seen from the first term in the Hamiltonian constraint, where there is no triad component E^φ classically, but it is important to introduce this variable into the quantum operator. The resulting operator version of the hypersurface-deformation brackets takes the form

$$\left[\hat{D}[N^x], \hat{D}[M^x] \right] = \hat{D}[\mathcal{L}_{N^x} M^x] \tag{48}$$

$$\left[\hat{H}[N], \hat{D}[N^x] \right] = -\hat{H}[\mathcal{L}_{N^x} N] \tag{49}$$

$$\left[\hat{H}[N_1], \hat{H}[N_2] \right] = \hat{D} \left[(\hat{E}^\varphi)^{-2} \left(\widehat{\frac{\mathrm{d}f}{\mathrm{d}K_\varphi}} \right) (N_1 N_2' - N_2 N_1') \right] \tag{50}$$

The details of the operator ordered constraints closing into an algebra are shown in the Appendix.

The next step is to start from these constraint operators and try to define a factor ordered new set of constraints for a (partially) Abelianized system. In fact what we shall find below is that there is no consistent way to carry out the Abelianization procedure anymore, once we start from these factor ordered constraint operators. We try to define the new constraint in such a way that we cancel out the $E^x K_x$ term coming from the Hamiltonian constraint operator with a similar term from the diffeomorphism constraint operator.

$$\hat{\mathcal{C}} = \frac{1}{4}\frac{(\hat{E}^x)'}{\hat{E}^\varphi}\hat{\mathcal{H}} - (\hat{E}^\varphi)^{-2}\hat{f}(K_\varphi)\hat{E}^\varphi\,\hat{\mathcal{D}} \tag{51}$$

As is evident from Eq. (51), operator ambiguities play a major role in choosing the pre-factor of the second term. This nontrivial operator ordering is chosen so as to make the cancellation mentioned above possible. However, the term which was a total derivative of the holonomy correction function earlier is more complicated due to the structure of the factor ordering chosen

$$\hat{\mathcal{C}} = \frac{1}{16}\hat{V}(E^x)(\hat{E}^x)' - (\hat{E}^x)'(\hat{E}^\varphi)^{-1}\left(-\frac{1}{16}(\hat{E}^x)'(\hat{E}^\varphi)'(\hat{E}^\varphi)^{-2} - \frac{1}{16}(\hat{E}^x)''(\hat{E}^\varphi)^{-1}\right) - (\hat{E}^\varphi)^{-2}\hat{f}(K_\varphi)\hat{E}^\varphi\hat{K}_\varphi'\hat{E}^\varphi \tag{52}$$

The non-zero commutator of K_φ and E^φ presents an obstruction to writing the last term as a total derivative, which is the reason why we do not have a partially Abelianized system of constraints anymore.

V. OBSTRUCTIONS TO ADDING MATTER TO THE HOLONOMY-MODIFIED CGHS MODEL

In this section, we refer specifically to the CGHS model because of its importance in discussions of Hawking radiation, which require a scalar field. However, the same conclusions can easily be achieved for general 2-dimensional dilaton models.

If we add the simplest type of matter, a minimally coupled scalar, to the CGHS model, the total constraints take the form

$$H_{\rm CGHS}[N] = \int {\rm d}x N\left[\mathcal{H}_{\rm grav} + \mathcal{H}_{\rm matter}\right] , \tag{53}$$

$$D_{\rm CGHS}[N^x] = \int {\rm d}x N^x\left[\mathcal{D}_{\rm grav} + \mathcal{D}_{\rm matter}\right] , \tag{54}$$

with the gravitational and matter parts of the constraints given by

$$\mathcal{H}_{\rm grav} = -4K_\varphi K_x - 4\lambda^2 E^\varphi - \frac{1}{4}(E^x)'(E^\varphi)'(E^\varphi)^{-2}$$

$$+\frac{1}{4}(E^x)''(E^\varphi)^{-1} , \tag{55}$$

$$\mathcal{H}_{\rm matter} = \frac{P_\varphi^2}{E^\varphi} + \frac{(\varphi')^2}{E^\varphi} , \tag{56}$$

$$\mathcal{D}_{\rm grav} = -K_x(E^x)' + E^\varphi K_\varphi' , \tag{57}$$

$$\mathcal{D}_{\rm matter} = P_\varphi \varphi' . \tag{58}$$

(The Hamiltonian of a spherically symmetric free and massless scalar field is given by $\frac{1}{2}\int {\rm d}x N q_{xx}^{-1/2}(P_\phi^2 + (\phi')^2)$, in which, after the canonical transformations, $q_{xx}^{-1/2}$ is turned into a factor of $(E^\varphi)^{-1}$ from the metric.) We have extended the gravitational phase space by a scalar field, with Poisson bracket $\{\varphi(x), P_\varphi(y)\} = \delta(x,y)$. The classical total constraints satisfy the usual hypersurface-deformation brackets, as expected.

However, once we incorporate holonomy effects in the constraints, $\mathcal{H}_{\rm grav}$ contains a modification function $(K_\varphi \to f(K_\varphi))$ whereas $\mathcal{H}_{\rm matter}$ does not since there are no K_φ components in the matter part of the Hamiltonian constraint. Thus, we have

$$\{H_{\rm grav}[N_1], H_{\rm grav}[N_2]\} = D_{\rm grav}\left[(NM' - N'M)\frac{\dot{f}(K_\varphi)}{(E^\varphi)^2}\right] , \tag{59}$$

$$\{H_{\rm matter}[N_1], H_{\rm matter}[N_2]\} = D_{\rm matter}\left[(NM' - N'M)\frac{1}{(E^\varphi)^2}\right] , \tag{60}$$

with the gravitational and matter parts of the Hamiltonian constraint now mismatched in their brackets. The Poisson bracket of the total Hamiltonian constraint with itself does not close into the full diffeomorphism constraint and one does not have a first-class system anymore. One natural recourse might be to introduce a holonomy correction function in the matter part of the Hamiltonian constraint, by hand, but then we have a non-vanishing cross term between the gravitational and matter parts,

$$\{H_{\rm grav}[N_1], H_{\rm matter}[N_2]\} - (N_1 \leftrightarrow N_2) \neq 0 , \tag{61}$$

which still leads to anomalies. For details, see [2].

Another way to address this problem would be to try and (partially) Abelianize the full constrained system (with both gravity and matter), as we did before. In this case, for the classical constraints, the Abelianization goes through due to some subtle cancellations as shown below. That the classical constraint algebra remains Abelianized has already been shown in [27]. We show crucial parts of the calculation here which help us to emphasize why the quantum algebra is not anomaly free. We write

$$\begin{aligned}\mathcal{C}_{\rm CGHS} &= \frac{1}{4}\frac{(E^x)'}{E^\varphi}\mathcal{H} - \frac{f(K_\varphi)}{E^\varphi}\mathcal{D} \\ &= \mathcal{C}_{\rm grav} + \mathcal{C}_{\rm matter}\,, \end{aligned} \tag{62}$$

where we have defined

$$\begin{aligned}\mathcal{C}_{\rm grav} &:= -\lambda(E^x)' - K_\varphi K_\varphi' \\ &\quad -\frac{1}{16}\frac{(E^x)'}{E^\varphi}\left((E^x)'' - \frac{(E^x)'(E^\varphi)'}{(E^\varphi)^2}\right)\,, \end{aligned} \tag{63}$$

$$\mathcal{C}_{\rm matter} := -\frac{1}{16}\frac{(E^x)'}{(E^\varphi)^2}P_\varphi^2 - \frac{1}{4}\frac{(E^x)'}{(E^\varphi)^2}(\varphi')^2 - \frac{K_\varphi}{E^\varphi}P_\varphi\varphi' \tag{64}$$

The Poisson bracket $\{C_{\rm CGHS}[N_1], C_{\rm CGHS}[N_2]\}$ can be decomposed as

$$\begin{aligned}\{C_{\rm CGHS}[N_1], C_{\rm CGHS}[N_2]\} &= \{C_{\rm grav}[N_1], C_{\rm grav}[N_2]\} \\ &\quad + \{C_{\rm matter}[N_1], C_{\rm matter}[N_2]\} \\ &= \{C_{\rm grav}[N_1], C_{\rm matter}[N_2]\} \\ &\quad -(N_1 \leftrightarrow N_2)\,. \end{aligned} \tag{65}$$

We already know from Eqs. (43) and (44) that $\{C_{\rm grav}[N_1], C_{\rm grav}[N_2]\} = 0$. (Although we write the gravitational part of the new constraint $C[N]$ in a different way, it is essentially the same as the total derivative term in (42) with the modification function equal to the classical one.) If we calculate the other brackets, we find that both $\{C_{\rm matter}[N_1], C_{\rm matter}[N_2]\}$ and $\{C_{\rm grav}[N_1], C_{\rm matter}[N_2]\} - (N_1 \leftrightarrow N_2)$ are non-zero but they cancel each other. The most crucial cancellation from our point of view is the following, where the Poisson bracket proportional to

$$\left\{-\frac{1}{16}\frac{(E^x)'}{(E^\varphi)^2}P_\varphi^2\,,\ -\frac{1}{4}\frac{(E^x)'}{(E^\varphi)^2}(\varphi')^2\right\} \tag{66}$$

from $\{C_{\rm matter}[N_1], C_{\rm matter}[N_2]\}$ is cancelled by terms proportional to

$$\left\{\frac{1}{16}\left(\frac{(E^x)'}{E^\varphi}\right)^2\frac{(E^\varphi)'}{(E^\varphi)}\,,\ -\frac{K_\varphi}{E^\varphi}P_\varphi\varphi'\right\} \tag{67}$$

coming from $\{C_{\rm grav}[N_1], C_{\rm matter}[N_2]\} - (N_1 \leftrightarrow N_2)$. This cancellation can take place precisely due to the linear factor of K_φ in the last term of the matter part of the new constraint. If this factor is replaced by the holonomy modification function as we should have to in case of holonomy modifications, these two terms do not cancel anymore and we, once again, end up with an anomaly. This is just another way of expressing the fact that we cannot add a massless scalar to the holonomy-modified CGHS model in a covariant quantization, at least in the standard regularization procedure of loop quantum gravity.

Our result extends a set of no-go theorems which had previously been proved for the Schwarzschild black hole [2] and Gowdy models [8], to the CGHS black hole and all other 2-dimensional dilaton models. An open question was left in [27, 30] whether one could add matter to the vacuum CGHS model and still have an anomaly free and covariant system. We have answered this question in the negative in the present article and, as a consequence, provided an obstruction to studying Hawking radiation in this context. It is possible to show that this obstruction can be generalized to other possible matter models, by a calculation which is essentially the same as what has already been shown for the spherically symmetry case in [2].

VI. CONCLUSIONS

We have discussed an extension of holonomy-modified anomaly-free models of loop quantum gravity from spherical symmetry to dilaton models. We have obtained new obstructions to anomaly-free midisuperspace models with holonomy modifications, given by ordering effects and matter terms. The second set of obstructions is particularly important because it spoils attempts to discuss Hawking radiation. Such a discussion is possible within a partially Abelianized system, only when one considers a background treatment, but such models cannot be covariant. Even for such partially Abelianized systems, there are obstructions to adding matter to the effective theory in a covariant manner [2]. In cases in which anomaly freedom can be achieved, holonomy modifications are accompanied by signature change. According to [15], new non-classical space-time structures are then obtained.

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Appendix A: Factor ordering for the CGHS constraints

We introduced an operator ordering for the gravitational CGHS constraints in (46) which gives a closed form of the constraint brackets (48). It is easy to observe that the subset of the full hypersurface-deformation brackets, involving at least one diffeomorphism constraint, works with this factor ordering. The most subtle calculation is for the bracket between two Hamiltonian constraint operators which has been shown below $[\hat{H}[N],\hat{H}[M]]+(N\leftrightarrow M)$. (We drop the hats from now on.)

The bracket between the first term and the third one of $[H[N],H[M]]$ is

$$\int \mathrm{d}x\,\mathrm{d}y\,N(x)M(y)\left[4(E^{\varphi}(x))^{-1}f(K_{\varphi}(x))E^{\varphi}(x)K_x(x)\;,\;\frac{1}{4}((E^{x\prime}(y))(E^{\varphi}(y))^{-2}(E^{\varphi\prime}(y))\right]\,. \tag{A1}$$

Combining this with the corresponding commutator between the last term of $H[N]$ and the first term of $H[M]$, we get

$$\int \mathrm{d}x\,(N'M-M'N)(E^{\varphi})^{-2}\left(\dot{f}(K_x)E^{x\prime}K_x+f(K_{\varphi})E^{\varphi\prime}\right)\,. \tag{A2}$$

The bracket between the first term and the last one in $[H[N],H[M]]+(N\leftrightarrow M)$ give the terms

$$\int \mathrm{d}x\,\left((N'M-M'N)\left(-2E^{\varphi\prime}(E^{\varphi})^{-3}f_2(K_{\varphi})E^{\varphi}\right)+(N''M-M''N)\left((E^{\varphi})^{-2}f(K_{\varphi})E^{\varphi}\right)\right)\,. \tag{A3}$$

Performing an intergration by parts on the last term, we have

$$-\int \mathrm{d}x\,(N'M-M'N)\left((E^{\varphi})^{-2}\dot{f}(K_{\varphi})K_{\varphi}'E^{\varphi}+(E^{\varphi})^{-2}f(K_{\varphi})E^{\varphi\prime}\right)\,. \tag{A4}$$

The required cancellation between the second term of (A4) and the second term of (A2), as it happens in the classical case, is obtained here with our choice of the factor ordering for the Hamiltonian constraint:

$$[H[N],H[M]]=\int \mathrm{d}x\,\left[N(x)M'(x)-N'(x)M(x)\right](E^{\varphi}(x))^{-2}\frac{\mathrm{d}f(K_{\varphi})}{\mathrm{d}K_{\varphi}}\left(K_{\varphi}'(x)E^{\varphi}(x)-E^{x\prime}(x)K_x(x)\right)\,. \tag{A5}$$

The term on the right-hand side is the required diffeomorphism constraint, with the proper operator ordering, as proposed in (46).

[1] A. Ashtekar and M. Bojowald, Quantum Geometry and the Schwarzschild Singularity, *Class. Quantum Grav.* 23 (2006) 391–411, [gr-qc/0509075]

[2] M. Bojowald, S. Brahma, and J. D. Reyes, Covariance in models of loop quantum gravity: Spherical symmetry, [arXiv:1507.00329]

[3] M. Bojowald and G. M. Paily, Deformed General Relativity, *Phys. Rev. D* 87 (2013) 044044, [arXiv:1212.4773]

[4] J. Ben Achour, S. Brahma, and A. Marciano, Spherically symmetric sector of self dual Ashtekar gravity coupled to matter: Anomaly-free algebra of constraints with holonomy corrections, [arXiv:1608.07314]

[5] J. Ben Achour, S. Brahma, J. Grain, and A. Marciano, A new look at scalar perturbations in loop quantum cosmology: (un)deformed algebra approach using self dual variables, [arXiv:1610.07467]

[6] J. D. Reyes, *Spherically Symmetric Loop Quantum Gravity: Connections to 2-Dimensional Models and Applications to Gravitational Collapse*, PhD thesis, The Pennsylvania State University, 2009

[7] M. Bojowald, J. D. Reyes, and R. Tibrewala, Nonmarginal LTB-like models with inverse triad corrections from loop quantum gravity, *Phys. Rev. D* 80 (2009) 084002, [arXiv:0906.4767]

[8] M. Bojowald and S. Brahma, Covariance in models of loop quantum gravity: Gowdy systems, [arXiv:1507.00679]

[9] S. Brahma, Spherically symmetric canonical quantum gravity, *Phys. Rev. D* 91 (2015) 124003, [arXiv:1411.3661]

[10] T. Cailleteau, J. Mielczarek, A. Barrau, and J. Grain, Anomaly-free scalar perturbations with holonomy corrections in loop quantum cosmology, *Class. Quant. Grav.* 29 (2012) 095010, [arXiv:1111.3535]

[11] A. Barrau, M. Bojowald, G. Calcagni, J. Grain, and M. Kagan, Anomaly-free cosmological perturbations in effective canonical quantum gravity, *JCAP* 05 (2015) 051, [arXiv:1404.1018]

[12] M. Bojowald, Information loss, made worse by quantum gravity, *Front. Phys.* 3 (2015) 33, [arXiv:1409.3157]

[13] G. Horowitz and J. Maldacena, The black hole final state *JHEP* 02 (2004) 008, [hep-th/0310281]

[14] L. Susskind, L. Thorlacius, and J. Uglum, The Stretched Horizon and Black Hole Complementarity, *Phys. Rev. D* 48 (1993) 3743–3761, [hep-th/9306069]

[15] M. Bojowald, S. Brahma, U. Büyükçam, and F. D'Ambrosio, Hypersurface-deformation algebroids and effective space-time models, *Phys. Rev. D* (2016) to ap-

pear

[16] M. Bojowald and J. D. Reyes, Dilaton Gravity, Poisson Sigma Models and Loop Quantum Gravity, *Class. Quantum Grav.* 26 (2009) 035018, [arXiv:0810.5119]

[17] R. Gambini, J. Pullin, and S. Rastgoo, New variables for 1+1 dimensional gravity, *Class. Quant. Grav.* 27 (2010) 025002, [arXiv:0909.0459]

[18] M. Bojowald, S. Brahma, and J. D. Reyes, Comment on "New variables for 1+1 dimensional gravity", *Class. Quant. Grav.* 34 (2017) 138001, [arXiv:1706.07379]

[19] T. Strobl, Gravity in Two Spacetime Dimensions, [hep-th/0011240]

[20] N. Ikeda, Two-Dimensional Gravity and Nonlinear Gauge Theory, *Ann. Phys.* 235 (1994) 435–464, [hep-th/9312059]

[21] P. Schaller and T. Strobl, Poisson Structure Induced (Topological) Field Theories, *Mod. Phys. Lett. A* 9 (1994) 3129–3136, [hep-th/9405110]

[22] R. Arnowitt, S. Deser and C. Misner, Republication of: The dynamics of general relativity, *Gen. Rel. Grav.* 40 (2008) 1997–2027

[23] M. Bojowald, Spherically Symmetric Quantum Geometry: States and Basic Operators, *Class. Quantum Grav.* 21 (2004) 3733–3753, [gr-qc/0407017]

[24] C. Callan, S. Giddings, J. Harvey, and A. Strominger, Evanescent Black Holes, *Phys. Rev. D* 45 (1992) 1005–1009, [hep-th/9111056]

[25] R. Gambini and J. Pullin, Loop quantization of the Schwarzschild black hole, *Phys. Rev. Lett.* 110 (2013) 211301, [arXiv:1302.5265]

[26] R. Gambini, J. Olmedo, and J. Pullin, Quantum black holes in Loop Quantum Gravity, [arXiv:1310.5996]

[27] A. Corichi, A. Karami, S. Rastgoo, and T. Vukašinac, Constraint Lie algebra and local physical Hamiltonian for a generic 2D dilatonic model, *Class. Quantum Grav.* 33 (2016) 035011, [arXiv:1508.03036]

[28] M. Bojowald and R. Swiderski, Spherically Symmetric Quantum Geometry: Hamiltonian Constraint, *Class. Quantum Grav.* 23 (2006) 2129–2154, [gr-qc/0511108]

[29] M. Bojowald, G. M. Paily, and J. D. Reyes, Discreteness corrections and higher spatial derivatives in effective canonical quantum gravity, *Phys. Rev. D* 90 (2014) 025025, [arXiv:1402.5130]

[30] A. Corichi, J. Olmedo, and S. Rastgoo, Vacuum CGHS in loop quantum gravity and singularity resolution, [arXiv:1608.06246]