

This is the accepted manuscript made available via CHORUS. The article has been published as:

Fermionic minimal dark matter in 5D gauge-Higgs unification

Nobuhito Maru, Nobuchika Okada, and Satomi Okada

Phys. Rev. D **96**, 115023 — Published 26 December 2017

DOI: [10.1103/PhysRevD.96.115023](https://doi.org/10.1103/PhysRevD.96.115023)

Fermionic Minimal Dark Matter in 5D Gauge-Higgs Unification

Nobuhito Maru^a, Nobuchika Okada^b, and Satomi Okada^c

^a*Department of Mathematics and Physics, Osaka City University,
Osaka 558-8585, Japan*

^b*Department of Physics and Astronomy, University of Alabama,
Tuscaloosa, Alabama 35487, USA*

^c*Graduate School of Science and Engineering, Yamagata University,
Yamagata 990-8560, Japan*

Abstract

We propose a Minimal Dark Matter (MDM) scenario in the context of a simple gauge-Higgs Unification (GHU) model based on the gauge group $SU(3) \times U(1)'$ in 5-dimensional Minkowski space with a compactification of the 5th dimension on S^1/Z_2 orbifold. A pair of vector-like $SU(3)$ multiplet fermions in a higher-dimensional representation is introduced in the bulk, and the DM particle is identified with the lightest mass eigenstate among the components in the multiplets. In the original model description, the DM particle communicates with the Standard Model (SM) particles only through the bulk gauge interaction, and hence our model is the GHU version of the MDM scenario. There are two typical realizations of the DM particle in 4-dimensional effective theory: (i) the DM particle is mostly composed of the SM $SU(2)_L$ multiplets, or (ii) the DM is mostly composed of the SM $SU(2)_L$ singlets. Since the case (i) is very similar to the original MDM scenario, we focus on the case (ii), which is a realization of the Higgs-portal DM scenario in the context of the GHU model. We identify an allowed parameter region to be consistent with the current experimental constraints, which will be fully covered by the direct dark matter detection experiments in the near future. In the presence of the bulk multiplet fermions in higher-dimensional $SU(3)$ representations, we reproduce the 125 GeV Higgs boson mass through the renormalization group evolution of Higgs quartic coupling with the compactification scale of 10 – 100 TeV.

1 Introduction

The existence of dark matter (DM) supported by the various cosmological observations is an evidence of physics beyond the Standard Model (SM). Clarifying the identities of the DM particle is one of the most important research topics in particle physics and cosmology. Among several possibilities, the so-called Weakly Interacting Massive Particle (WIMP) is a prime candidate for the DM. Since the WIMP DM is the thermal relic from the early universe, its relic abundance is determined independently of the detailed history of the universe before its decoupling from thermal plasma. Currently, many experiments aiming for directly/indirectly detecting DM particles are in operation and planned. The discovery of a DM particle may be around the corner.

The GHU scenario [1] is a unique candidate for new physics beyond the SM, in which the gauge hierarchy problem can be solved without invoking supersymmetry. An essential property of the GHU scenario is that the SM Higgs doublet is identified with an extra spatial component of a gauge field in higher dimensions. Thanks to the gauge symmetry in higher-dimensions, the GHU scenario, irrespective to the non-renormalizability of the scenario, predicts various finite observables, such as the effective Higgs potential [2, 3], the effective Higgs coupling with digluon/diphoton [4, 5, 6], the anomalous magnetic moment $g - 2$ [7], and the electric dipole moment [8].

Towards the completion of the GHU scenario as new physics beyond the SM, we need to supplement a DM candidate to the scenario. In order to keep the original motivation of the GHU scenario to solve the gauge hierarchy problem, the DM candidate to be introduced must be a fermion. Since the GHU scenario is defined in a higher dimensional space-time with a gauge group into which the SM gauge group is embedded, it is natural to introduce a DM candidate into the model as a bulk fermion multiplet. Hence, the DM particle communicates with the SM particles only through the original bulk gauge interactions. This picture is the same as the so-called minimal dark matter (MDM) scenario [9], where the SM $SU(2)_L \times U(1)_Y$ gauge interaction is a unique interaction through which the DM particle communicates with the SM particles.

In our previous work [10], we have proposed such a MDM scenario in the context of a simple 5-dimensional GHU model based on the gauge group $SU(3) \times U(1)'$ with an orbifold S^1/Z_2 compactification of the 5th dimension. We have introduced a pair of $SU(3)$ triplet fermions in

the bulk along with a bulk mass term and Majorana mass terms on the orbifold fixed point. The DM particle is identified as the lightest mass eigenstate, which is a linear combination of $SU(2)_L$ doublets and singlets in the decomposition of the bulk triplets into the SM gauge group of $SU(2)_L \times U(1)_Y$. There are two typical cases for a realistic DM scenario: (i) the DM particle is mostly composed of the SM $SU(2)_L$ doublet components, and (ii) the DM particle is mostly composed of the SM $SU(2)_L$ singlets. In the case (i), the DM particle is quite similar to the so-called Higgsino-like neutralino DM in the minimal supersymmetric SM, while the case (ii) is a realization of the so-called Higgs-portal DM scenario in the context of the GHU model. We have focused on the case (ii) and identified the allowed parameter region to reproduce the observed DM relic abundance and to satisfy the constraint from the LUX 2016 result [11] for the direct dark matter search. The entire allowed region will be covered by, for example, the LUX-ZEPLIN dark matter experiment [12] in the near future. We have also shown that in the presence of the bulk $SU(3)$ triplet fermions, the 125 GeV Higgs boson mass is reproduced through the renormalization group (RG) evolution of Higgs quartic coupling with the compactification scale of around 10^8 GeV. However, this compactification scale is too high in the naturalness point of view, and it is desirable to reduce the compactification scale to $\mathcal{O}(1$ TeV).

Similarly to the MDM scenario [9], we have a variety of choices for representations of the DM multiplets to be introduced. In this paper, we extend our previous model by introducing a pair of bulk $SU(3)$ multiplet fermions in higher-dimensional representations, such as **6**-plet, **10**-plet and **15**-plet. As has been pointed out in Refs. [6, 13], in the presence of such higher representations, the compactification scale can be as low as $\mathcal{O}(1$ TeV) while reproducing the 125 GeV Higgs boson mass. We identify an allowed parameter region to be consistent with the current experimental results. For related works on the DM physics in the context of the GHU scenario, see Ref. [14].

This paper is organized as follows. In the next section, we introduce the 5-dimensional GHU model based on the gauge group $SU(3) \times U(1)'$ with an orbifold S^1/Z_2 compactification. In this context, we propose the GHU version of the MDM scenario, where a DM particle is provided as the lightest mass eigenstate in a pair of $SU(3)$ multiplet fermions introduced in the bulk along with a bulk mass term, brane Majorana mass terms on an orbifold fixed point, and a periodic boundary condition. Here, we consider higher-dimensional $SU(3)$ representations, such as **6**-plet, **10**-plet, and **15**-plet. In Sec. 3, we focus on the case that the DM particle communicates

with the SM particles through the Higgs boson. Calculating the DM relic abundance, we identify an allowed parameter region of the model to reproduce the observed DM density. In Sec. 4, we further constrain the allowed parameter region by considering the upper limit of the elastic scattering cross section of the DM particle off nuclei from the current DM direct detection experiments. An effective field theoretical approach of the gauge-Higgs Unification scenario will be discussed in Sec. 5, and the 125 GeV Higgs boson mass is reproduced in the presence of the bulk $SU(3)$ triplet fermions with a certain boundary conditions. The compactification scale is determined in order to reproduce the Higgs boson mass of 125 GeV. The last section is devoted to conclusions.

2 Fermion MDM in GHU

We consider a GHU model based on the gauge group $SU(3) \times U(1)'$ [15] in a 5-dimensional flat space-time with orbifolding on S^1/Z_2 with radius R of S^1 . The boundary conditions should be suitably assigned so as to provide the SM fields as zero modes. While a periodic boundary condition corresponding to S^1 is applied to all of the bulk SM fields, the Z_2 parity for the bulk gauge fields is assigned as

$$A_\mu(-y) = P^\dagger A_\mu(y) P, \quad A_y(-y) = -P^\dagger A_y(y) P, \quad (2.1)$$

where $P = \text{diag}(-, -, +)$ is the parity matrix, and the subscripts μ (y) denotes the four (the fifth) dimensional component. With this choice of parities, the $SU(3)$ gauge symmetry is explicitly broken down to $SU(2) \times U(1)$. A hypercharge is a linear combination of $U(1)$ and $U(1)'$ in this setup. For more details of our setup, in particular, bulk fermions including the SM fermions, see Ref. [10].

Now we discuss the DM sector in our model. In addition to the bulk fermions corresponding to the SM quarks and leptons, we introduce a pair of extra bulk fermions, ψ and $\tilde{\psi}$, which are 6-dimensional, 10-dimensional and 15-dimensional representations under the bulk $SU(3)$ with $U(1)'$ charges of $2/3$, 1 and $4/3$, respectively. With this choice of the $U(1)'$ charge, the **6**-plet, **10**-plet and **15**-plet bulk fermions include electric-charge neutral components and a linear combination among the charge neutral components serves as the DM particle. While we impose the periodic boundary condition in the fifth dimension for all bulk fields, the Z_2 -parity

assignments for the bulk fermions are chosen to be

$$\begin{aligned}
\psi(-y) &= P\gamma^5\psi(y), \quad \tilde{\psi}(-y) = -P\gamma^5\tilde{\psi}(y) \quad (\text{triplet}), \\
\psi(-y) &= (P \otimes P)\gamma^5\psi(y), \quad \tilde{\psi}(-y) = -(P \otimes P)\gamma^5\tilde{\psi}(y) \quad (\mathbf{6}\text{-plet}), \\
\psi(-y) &= (P \otimes P \otimes P)\gamma^5\psi(y), \quad \tilde{\psi}(-y) = -(P \otimes P \otimes P)\gamma^5\tilde{\psi}(y) \quad (\mathbf{10}\text{-plet}), \\
\psi(-y) &= (P \otimes P \otimes P \otimes P)\gamma^5\psi(y), \quad \tilde{\psi}(-y) = -(P \otimes P \otimes P \otimes P)\gamma^5\tilde{\psi}(y) \quad (\mathbf{15}\text{-plet}).
\end{aligned} \tag{2.2}$$

After the electroweak symmetry breaking, the lightest mass eigenstate among the bulk multiplets is identified with the DM particle.

2.1 6-plet MDM

Let us first consider the case where the DM provided by a pair of $\mathbf{6}$ -plet bulk fermions. The Lagrangian of the $\mathbf{6}$ -plets relevant to our DM physics discussion is given by

$$\begin{aligned}
\mathcal{L}_{\text{DM6}} &= \text{Tr} \left[\bar{\psi}(\mathbf{6}) i \not{D} \psi(\mathbf{6}) + \bar{\tilde{\psi}}(\mathbf{6}) i \not{D} \tilde{\psi}(\mathbf{6}) - M(\bar{\psi}(\mathbf{6})\tilde{\psi}(\mathbf{6}) + \bar{\tilde{\psi}}(\mathbf{6})\psi(\mathbf{6})) \right] \\
&+ \delta(y) \left[\frac{m}{2} \overline{\nu_{sR}^{(0)c}} \nu_{sR}^{(0)} + \frac{\tilde{m}}{2} \overline{\tilde{\nu}_{sL}^{(0)c}} \tilde{\nu}_{sL}^{(0)} + \text{h.c.} \right],
\end{aligned} \tag{2.3}$$

where \not{D} is the covariant derivative for the $\mathbf{6}$ -plets. The bulk $SU(3)$ $\mathbf{6}$ -plet is expressed in a matrix form

$$\psi(\mathbf{6}) = \begin{pmatrix} \Sigma_{\uparrow} & \frac{1}{\sqrt{2}}\Sigma_0 & \frac{1}{\sqrt{2}}\nu \\ \frac{1}{\sqrt{2}}\Sigma_0 & \Sigma_{\downarrow} & \frac{1}{\sqrt{2}}e \\ \frac{1}{\sqrt{2}}\nu & \frac{1}{\sqrt{2}}e & \nu_s \end{pmatrix}, \tag{2.4}$$

where the quantum numbers for $SU(2)$ representation, the third component of the isospin I_3 , and the electric charge Q_{em} of various fields in the matrix are

$$\begin{aligned}
&\Sigma_{\uparrow}(\mathbf{3}, 1, 2), \quad \Sigma_0(\mathbf{3}, 0, 1), \quad \Sigma_{\downarrow}(\mathbf{3}, -1, 0), \\
&\nu(\mathbf{2}, 1/2, 1), \quad e(\mathbf{2}, -1/2, 0), \quad \nu_s(\mathbf{1}, 0, 0).
\end{aligned} \tag{2.5}$$

The corresponding mirror fermion $\mathbf{6}$ -plet $\tilde{\psi}(\mathbf{6})$ takes a similar form with tilde for all fields.

With the non-trivial orbifold boundary conditions, the $\mathbf{6}$ -plet fermions are decomposed into the SM $SU(2)$ triplet, doublet and singlet fermions. As we will see later, the DM particle is provided as a linear combination of the $SU(2)$ singlets and the electric charge neutral components in the $SU(2)$ doublets and triplets. In Eq. (2.3) we have introduced a bulk mass (M) to avoid exotic massless fermions. Here, we have also introduced Majorana mass terms on the brane at $y = 0$ for the zero-modes of the components of the $\mathbf{6}$ -plets ($(\nu_s^{(0)})_R$ and $(\tilde{\nu}_s^{(0)})_L$), which

are singlet under the SM gauge group. The superscript “ c ” denotes the charge conjugation. With the Majorana masses on the brane, the DM particle in 4-dimensional effective theory is a Majorana fermion, and hence its spin-independent cross section with nucleons through the Z -boson exchange vanishes in the non-relativistic limit.

Let us focus on the following terms in Eq. (2.3), which are relevant to the mass terms in 4-dimensional effective theory:

$$\begin{aligned}\mathcal{L}_{\text{mass6}} = & \text{Tr} \left[\bar{\psi} i \Gamma^5 (\partial_y - 2ig \langle A_y \rangle) \psi + \bar{\tilde{\psi}} i \Gamma^5 (\partial_y - 2ig \langle A_y \rangle) \tilde{\psi} - M (\bar{\psi} \tilde{\psi} + \bar{\tilde{\psi}} \psi) \right] \\ & + \delta(y) \left[\frac{m}{2} \overline{\nu_{sR}^{(0)c}} \nu_{sR}^{(0)} + \frac{\tilde{m}}{2} \overline{\tilde{\nu}_{sL}^{(0)c}} \tilde{\nu}_{sL}^{(0)} + \text{h.c.} \right],\end{aligned}\quad (2.6)$$

where $\Gamma^5 = i\gamma^5$. Expanding the bulk fermions in terms of KK modes as

$$\begin{aligned}\psi(x, y) = & \frac{1}{\sqrt{2\pi R}} \psi^{(0)}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \psi^{(n)}(x) \cos\left(\frac{n}{R}y\right) \\ & (\text{for } \Sigma_{\uparrow R, 0R, \downarrow R}, \nu_L, e_L, \nu_{sR}, \tilde{\Sigma}_{\uparrow L, 0L, \downarrow L}, \tilde{\nu}_R, \tilde{e}_R, \tilde{\nu}_{sL}),\end{aligned}\quad (2.7)$$

$$\begin{aligned}\psi(x, y) = & \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \psi^{(n)}(x) \sin\left(\frac{n}{R}y\right) \\ & (\text{for } \Sigma_{\uparrow L, 0L, \downarrow L}, \nu_R, e_R, \nu_{sL}, \tilde{\Sigma}_{\uparrow R, 0R, \downarrow R}, \nu_L, e_L, \nu_{sR}),\end{aligned}\quad (2.8)$$

and integrating out the fifth coordinate y , we obtain the expression in 4-dimensional effective theory. The zero-mode parts for the electric-charge neutral fermions are found to be

$$\begin{aligned}\mathcal{L}_{\text{mass}}^{\text{zero-mode}} = & i \frac{m_W}{\sqrt{2}} \left[\overline{e_L^{(0)}} (\Sigma_{\downarrow R}^{(0)} + \nu_{sR}^{(0)}) + (\tilde{\Sigma}_{\downarrow L}^{(0)} + \tilde{\nu}_{sL}^{(0)}) \tilde{e}_R^{(0)} + \text{h.c.} \right] \\ & - M \left(\overline{\Sigma_{\downarrow R}^{(0)}} \tilde{\Sigma}_{\downarrow L}^{(0)} + \overline{\nu_{sR}^{(0)}} \tilde{\nu}_{sL}^{(0)} + \overline{\tilde{e}_R^{(0)}} e_L^{(0)} + \text{h.c.} \right) \\ & + \left(\frac{m}{2} \overline{\nu_{sR}^{(0)c}} \nu_{sR}^{(0)} + \frac{\tilde{m}}{2} \overline{\tilde{\nu}_{sL}^{(0)c}} \tilde{\nu}_{sL}^{(0)} + \text{h.c.} \right) \\ \rightarrow & - \frac{m_W}{\sqrt{2}} \left[\overline{e_L^{(0)}} (\Sigma_{\downarrow R}^{(0)} + \nu_{sR}^{(0)}) + (\tilde{\Sigma}_{\downarrow L}^{(0)} + \tilde{\nu}_{sL}^{(0)}) \tilde{e}_R^{(0)} + \text{h.c.} \right] \\ & - M \left(\overline{\Sigma_{\downarrow R}^{(0)}} \tilde{\Sigma}_{\downarrow L}^{(0)} + \overline{\nu_{sR}^{(0)}} \tilde{\nu}_{sL}^{(0)} + \overline{\tilde{e}_R^{(0)}} e_L^{(0)} + \text{h.c.} \right) \\ & - \left(\frac{m}{2} \overline{\nu_{sR}^{(0)c}} \nu_{sR}^{(0)} + \frac{\tilde{m}}{2} \overline{\tilde{\nu}_{sL}^{(0)c}} \tilde{\nu}_{sL}^{(0)} + \text{h.c.} \right),\end{aligned}\quad (2.9)$$

where $m_W = gv/2$ is the W -boson mass, and the arrow means the phase rotations

$$\nu_s^{(0)} \rightarrow i\nu_s^{(0)}, \quad \tilde{\nu}_s^{(0)} \rightarrow i\tilde{\nu}_s^{(0)}, \quad \Sigma_{\downarrow}^{(0)} \rightarrow i\Sigma_{\downarrow}^{(0)}, \quad \tilde{\Sigma}_{\downarrow}^{(0)} \rightarrow i\tilde{\Sigma}_{\downarrow}^{(0)}.\quad (2.10)$$

It is useful to rewrite these mass terms in a Majorana basis defined as

$$\begin{aligned}
\chi &\equiv \nu_{sR}^{(0)} + \nu_{sR}^{(0)c}, & \tilde{\chi} &\equiv \tilde{\nu}_{sL}^{(0)} + \tilde{\nu}_{sL}^{(0)c}, \\
\omega &\equiv e_L^{(0)} + e_L^{(0)c}, & \tilde{\omega} &\equiv \tilde{e}_R^{(0)} + \tilde{e}_R^{(0)c}, \\
\eta &\equiv \Sigma_{\downarrow R}^{(0)} + \Sigma_{\downarrow R}^{(0)c}, & \tilde{\eta} &\equiv \tilde{\Sigma}_{\downarrow L}^{(0)} + \tilde{\Sigma}_{\downarrow L}^{(0)c},
\end{aligned} \tag{2.11}$$

and we then express the mass matrix (\mathcal{M}_{N6}) as

$$\begin{aligned}
\mathcal{L}_{6\text{mass}}^{\text{zero-mode}} &= -\frac{1}{2} \begin{pmatrix} \bar{\chi} & \bar{\tilde{\chi}} & \bar{\omega} & \bar{\tilde{\omega}} & \bar{\eta} & \bar{\tilde{\eta}} \end{pmatrix} \mathcal{M}_{N6} \begin{pmatrix} \chi \\ \tilde{\chi} \\ \omega \\ \tilde{\omega} \\ \eta \\ \tilde{\eta} \end{pmatrix} \\
&= -\frac{1}{2} \begin{pmatrix} \bar{\chi} & \bar{\tilde{\chi}} & \bar{\omega} & \bar{\tilde{\omega}} & \bar{\eta} & \bar{\tilde{\eta}} \end{pmatrix} \times \\
&\quad \begin{pmatrix} m & M & \sqrt{2}m_W & 0 & 0 & 0 \\ M & \tilde{m} & 0 & -\sqrt{2}m_W & 0 & 0 \\ \sqrt{2}m_W & 0 & 0 & M & \sqrt{2}m_W & 0 \\ 0 & -\sqrt{2}m_W & M & 0 & 0 & -\sqrt{2}m_W \\ 0 & 0 & \sqrt{2}m_W & 0 & 0 & M \\ 0 & 0 & 0 & -\sqrt{2}m_W & M & 0 \end{pmatrix} \begin{pmatrix} \chi \\ \tilde{\chi} \\ \omega \\ \tilde{\omega} \\ \eta \\ \tilde{\eta} \end{pmatrix}.
\end{aligned} \tag{2.12}$$

The zero modes of the charged fermions have a common Dirac mass of M .

To simplify our analysis, we set $m = \tilde{m}$, and in this case we find an approximate expression for the mass eigenvalues of \mathcal{M}_{N6} by treating m_W terms as perturbation ($m_W \ll |M|$):

$$\begin{aligned}
m_1 &\simeq M - m + \frac{2m_W^2}{2M - m}, \\
m_2 &= m_3 \simeq M + \frac{m_W^2}{M}, \\
m_4 &\simeq M + \frac{m_W^2}{M} + \frac{2M^2}{2M + m}, \\
m_5 &\simeq M + \frac{m_W^2}{M} + \frac{2m_W^2}{2M - m}, \\
m_6 &\simeq M + m + \frac{2m_W^2}{2M + m},
\end{aligned} \tag{2.13}$$

for the mass eigenstates defined as $U_{\mathcal{M}6} (\chi, \tilde{\chi}, \omega, \tilde{\omega}, \eta, \tilde{\eta})^T = (\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6)^T$ with a uni-

tary matrix

$$U_{\mathcal{M}_6} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 & a & 0 \\ 0 & 1 & 0 & 0 & -c & 0 \\ 0 & 0 & 1 & c & 0 & 0 \\ 0 & 0 & -c & 1 & 0 & -b \\ -a & c & 0 & 0 & 1 & 0 \\ 0 & 0 & b & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (2.14)$$

where

$$a = \frac{\sqrt{2}m_W}{m - 2M}, \quad b = \frac{\sqrt{2}m_W}{m + 2M}, \quad c = \frac{\sqrt{2}m_W}{2M}. \quad (2.15)$$

Note that without loss of generality, we can take $M, m \geq 0$. Considering the current experimental constraints from the search for an exotic charged fermion, we may take $M \gtrsim 1 \text{ TeV} \gg m_W$ [17]. In this case, the lowest mass eigenvalue (dark matter mass m_{DM}) is given by $|m_1|$. From the explicit form of the mass matrix \mathcal{M}_{N_6} in Eq. (2.12) and $M \gg m_W$, we notice two typical cases for the constituent of the DM particle: (i) the DM particle is mostly composed of SM singlets when $m = \tilde{m} \lesssim M$, or (ii) the DM particle is mostly composed of a linear combination of the components in the SM $SU(2)$ doublets and triplets when $m = \tilde{m} \gtrsim M$. In the case (i), the DM particle communicates with the SM particle essentially through the SM Higgs boson. On the other hand, the DM particle is quite similar to the so-called Higgsino/wino-like neutralino DM in the minimal supersymmetric SM for the case (ii). Since the Higgsino-like neutralino DM has been very well-studied in many literatures (see, for example, [18]), we focus on the case (i) in this paper. Note that the case (i) is a realization of the so-called Higgs-portal DM from the GHU scenario. We emphasize that in our scenario, the Yukawa couplings in the original Lagrangian are not free parameters, but are identical to the SM $SU(2)$ gauge coupling, thanks to the structure of the GHU scenario. This is in sharp contrast to this class of fermion DM scenarios [19], where the Yukawa couplings are all free parameters.

Now we describe the coupling between the DM particle and the Higgs boson. In the original

basis, the interaction can be read off from Eq. (2.12) by $v \rightarrow v + h$ as

$$\begin{aligned}
\mathcal{L}_{\text{Higgs-coupling}} &= -\frac{1}{2} \left(\frac{\sqrt{2}m_W}{v} \right) h \begin{pmatrix} \bar{\chi} & \bar{\tilde{\chi}} & \bar{\omega} & \bar{\tilde{\omega}} & \bar{\eta} & \bar{\tilde{\eta}} \end{pmatrix} \mathcal{M}_{h6} \begin{pmatrix} \chi \\ \tilde{\chi} \\ \omega \\ \tilde{\omega} \\ \eta \\ \tilde{\eta} \end{pmatrix} \\
&= -\frac{1}{2} \left(\frac{\sqrt{2}m_W}{v} \right) h \begin{pmatrix} \bar{\phi}_1 & \bar{\phi}_2 & \bar{\phi}_3 & \bar{\phi}_4 & \bar{\phi}_5 & \bar{\phi}_6 \end{pmatrix} \mathcal{C}_{h6} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \end{pmatrix}, \quad (2.16)
\end{aligned}$$

where h is the physical Higgs boson, and the explicit form of the matrix \mathcal{M}_{h6} and \mathcal{C}_{h6} are given by

$$\mathcal{M}_{h6} \equiv \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{pmatrix}, \quad (2.17)$$

$$\mathcal{C}_{h6} \equiv \begin{pmatrix} -2a & -a+c & 0 & 0 & -1-ac+a^2 & 0 \\ -a+c & 2c & 0 & 0 & -1-ac+c^2 & 0 \\ 0 & 0 & -2c & -1+b+c^2 & 0 & -1-bc \\ 0 & 0 & -1+b+c^2 & 2(1-b)c & 0 & -b+b^2+c \\ -1-ac+a^2 & -1-ac+c^2 & 0 & 0 & 2(a-c) & 0 \\ 0 & 0 & -1-bc & -b+b^2+c & 0 & -2b \end{pmatrix}. \quad (2.18)$$

The interaction Lagrangian relevant to the DM physics is given by

$$\begin{aligned}
\mathcal{L}_{\text{DM-H6}} &= \frac{\sqrt{2}m_W}{2M-m} \left(\frac{\sqrt{2}m_W}{v} \right) h \bar{\psi}_{\text{DM}} \psi_{\text{DM}} + \left[\left(\frac{\sqrt{2}m_W}{2M-m} + \frac{m_W}{\sqrt{2}M} \right) \left(\frac{\sqrt{2}m_W}{v} \right) h \bar{\phi}_2 \psi_{\text{DM}} \right. \\
&\quad \left. + \left(-1 + \frac{2m_W^2}{(2M-m)^2} + \frac{m_W^2}{(2M-m)M} \right) \left(\frac{\sqrt{2}m_W}{v} \right) h \bar{\phi}_5 \psi_{\text{DM}} + \text{h.c.} \right], \quad (2.19)
\end{aligned}$$

where we have identified the lightest mass eigenstate ϕ_1 as the DM particle (ψ_{DM}).

2.2 10-plet MDM

Next we consider that case that the DM particle is provided by a pair of **10**-plets. The Lagrangian relevant to our DM physics discussion is given by

$$\begin{aligned}\mathcal{L}_{\text{DM10}} = & \text{Tr} \left[\bar{\psi}(\mathbf{10}) i \not{D} \psi(\mathbf{10}) + \bar{\tilde{\psi}}(\mathbf{10}) i \not{D} \tilde{\psi}(\mathbf{10}) - M(\bar{\psi}(\mathbf{10}) \tilde{\psi}(\mathbf{10}) + \bar{\tilde{\psi}}(\mathbf{10}) \psi(\mathbf{10})) \right] \\ & + \delta(y) \left[\frac{m}{2} \overline{\nu_{sR}^{(0)c}} \nu_{sR}^{(0)} + \frac{\tilde{m}}{2} \overline{\tilde{\nu}_{sL}^{(0)c}} \tilde{\nu}_{sL}^{(0)} + \text{h.c.} \right],\end{aligned}\quad (2.20)$$

where \not{D} is the covariant derivative. It is useful to express the **10**-plet fermion in a following matrix form,

$$\psi_{ijk}(\mathbf{10}) = \psi_{1jk} + \psi_{2jk} + \psi_{3jk}, \quad (2.21)$$

where

$$\begin{aligned}\psi_{1jk} &\equiv \begin{pmatrix} \Delta^{++} & \frac{1}{\sqrt{3}}\Delta^+ & \frac{1}{\sqrt{3}}\Sigma_\uparrow \\ \frac{1}{\sqrt{3}}\Delta^+ & \frac{1}{\sqrt{3}}\Delta^0 & \frac{1}{\sqrt{6}}\Sigma_0 \\ \frac{1}{\sqrt{3}}\Sigma_\uparrow & \frac{1}{\sqrt{6}}\Sigma_0 & \frac{1}{\sqrt{3}}\nu \end{pmatrix}, & \psi_{2jk} &\equiv \begin{pmatrix} \frac{1}{\sqrt{3}}\Delta^+ & \frac{1}{\sqrt{3}}\Delta^0 & \frac{1}{\sqrt{6}}\Sigma_0 \\ \frac{1}{\sqrt{3}}\Delta^0 & \Delta^- & \frac{1}{\sqrt{3}}\Sigma_\downarrow \\ \frac{1}{\sqrt{6}}\Sigma_0 & \frac{1}{\sqrt{3}}\Sigma_\downarrow & \frac{1}{\sqrt{3}}e \end{pmatrix}, \\ \psi_{3jk} &\equiv \begin{pmatrix} \frac{1}{\sqrt{3}}\Sigma_\uparrow & \frac{1}{\sqrt{6}}\Sigma_0 & \frac{1}{\sqrt{3}}\nu \\ \frac{1}{\sqrt{6}}\Sigma_0 & \frac{1}{\sqrt{3}}\Sigma_\downarrow & \frac{1}{\sqrt{3}}e \\ \frac{1}{\sqrt{3}}\nu & \frac{1}{\sqrt{3}}e & \nu_s \end{pmatrix}.\end{aligned}\quad (2.22)$$

The quantum numbers for $SU(2)$ representation, the third component of the isospin I_3 , and the electric charge Q_{em} of newly introduced fermions Δ in the matrix are

$$\Delta^{++}(4, 3/2, 3), \quad \Delta^+(4, 1/2, 2), \quad \Delta^0(4, -1/2, 1), \quad \Delta^-(4, -3/2, 0). \quad (2.23)$$

The corresponding mirror fermion **10**-plet $\tilde{\psi}(\mathbf{10})$ takes a similar form with tilde for all fields.

With the non-trivial orbifold boundary conditions, the bulk $SU(3)$ **10**-plet fermions are decomposed into the SM $SU(2)$ quartet, triplet, doublet and singlet fermions. The DM particle is provided as a linear combination of the $SU(2)$ singlet, doublet, triplet and quartet components in the **10**-plet fermions. In Eq. (2.20), as in the case of **6**-plet fermions, we have also introduced a bulk mass (M) to avoid exotic massless fermions and Majorana mass terms on the brane at $y = 0$ for the zero modes of the SM singlet components of the **10**-plets ($(\nu_s^{(0)})_R$ and $(\tilde{\nu}_s^{(0)})_L$) to vanish its spin-independent cross section with nucleons through the Z -boson exchange in the non-relativistic limit.

Let us focus on the following terms in Eq. (2.20), which are relevant to the mass terms in 4-dimensional effective theory:

$$\begin{aligned}\mathcal{L}_{\text{mass10}} = & \text{Tr} \left[\bar{\psi} i \Gamma^5 (\partial_y - 3ig \langle A_y \rangle) \psi + \bar{\tilde{\psi}} i \Gamma^5 (\partial_y - 3ig \langle A_y \rangle) \tilde{\psi} - M(\bar{\psi} \tilde{\psi} + \bar{\tilde{\psi}} \psi) \right] \\ & + \delta(y) \left[\frac{m}{2} \overline{\nu_{sR}^{(0)c}} \nu_{sR}^{(0)} + \frac{\tilde{m}}{2} \overline{\tilde{\nu}_{sL}^{(0)c}} \tilde{\nu}_{sL}^{(0)} + \text{h.c.} \right].\end{aligned}\quad (2.24)$$

Expanding the bulk fermions in terms of KK modes by noticing Z_2 parity of Δ as

$$\Delta_L(-y) = +\Delta_L(y), \quad \tilde{\Delta}_L(-y) = -\tilde{\Delta}_L(y), \quad (2.25)$$

$$\Delta_R(-y) = -\Delta_R(y), \quad \tilde{\Delta}_R(-y) = +\tilde{\Delta}_R(y) \quad (2.26)$$

and integrating out the fifth coordinate y , we obtain the expression in 4-dimensional effective theory. The zero-mode parts for the electric-charge neutral fermions are found to be

$$\begin{aligned}\mathcal{L}_{\text{mass10}}^{\text{zero-mode}} = & 3im_W \left[\frac{1}{\sqrt{3}} \overline{\Delta_L^{-(0)}} \Sigma_{\downarrow R}^{(0)} + \overline{e_L} \left(\frac{2}{3} \Sigma_{\downarrow R} + \frac{1}{\sqrt{3}} \nu_{sR} \right) + \frac{1}{\sqrt{3}} \overline{\tilde{\Sigma}_{\downarrow L}} \tilde{\Delta}_R^- \right. \\ & \left. + \left(\frac{2}{3} \overline{\tilde{\Sigma}_{\downarrow L}} + \frac{1}{\sqrt{3}} \overline{\tilde{\nu}_{sL}^{(0)}} \right) \tilde{e}_R^{(0)} + \text{h.c.} \right] \\ & - M \left(\overline{\Delta_L^-} \tilde{\Delta}_R^- + \overline{\Sigma_{\downarrow R}^{(0)}} \tilde{\Sigma}_{\downarrow L}^{(0)} + \overline{\nu_{sR}^{(0)}} \tilde{\nu}_{sL}^{(0)} + \overline{\tilde{e}_R^{(0)}} e_L^{(0)} + \text{h.c.} \right) \\ & + \left(\frac{m}{2} \overline{\nu_{sR}^{(0)c}} \nu_{sR}^{(0)} + \frac{\tilde{m}}{2} \overline{\tilde{\nu}_{sL}^{(0)c}} \tilde{\nu}_{sL}^{(0)} + \text{h.c.} \right) \\ \rightarrow & -3m_W \left[\frac{1}{\sqrt{3}} \overline{\Delta_L^{-(0)}} \Sigma_{\downarrow R}^{(0)} + \overline{e_L} \left(\frac{2}{3} \Sigma_{\downarrow R} + \frac{1}{\sqrt{3}} \nu_{sR} \right) + \frac{1}{\sqrt{3}} \overline{\tilde{\Sigma}_{\downarrow L}} \tilde{\Delta}_R^- \right. \\ & \left. + \left(\frac{2}{3} \overline{\tilde{\Sigma}_{\downarrow L}} + \frac{1}{\sqrt{3}} \overline{\tilde{\nu}_{sL}^{(0)}} \right) \tilde{e}_R^{(0)} + \text{h.c.} \right] \\ & - M \left(\overline{\Delta_L^-} \tilde{\Delta}_R^- + \overline{\Sigma_{\downarrow R}^{(0)}} \tilde{\Sigma}_{\downarrow L}^{(0)} + \overline{\nu_{sR}^{(0)}} \tilde{\nu}_{sL}^{(0)} + \overline{\tilde{e}_R^{(0)}} e_L^{(0)} + \text{h.c.} \right) \\ & - \left(\frac{m}{2} \overline{\nu_{sR}^{(0)c}} \nu_{sR}^{(0)} + \frac{\tilde{m}}{2} \overline{\tilde{\nu}_{sL}^{(0)c}} \tilde{\nu}_{sL}^{(0)} + \text{h.c.} \right),\end{aligned}\quad (2.27)$$

where the arrow means the phase rotations defined in (2.10). Rewriting these mass terms in a Majorana basis defined in Rq. (2.11) and

$$\xi \equiv \Delta_L^{(0)-} + (\Delta_L^{(0)-})^c, \quad \tilde{\xi} \equiv \tilde{\Delta}_R^{(0)-} + (\tilde{\Delta}_R^{(0)-})^c, \quad (2.28)$$

we have

$$\mathcal{L}_{10\text{mass}}^{\text{zero-mode}} = -\frac{1}{2}(\bar{\chi} \quad \bar{\tilde{\chi}} \quad \bar{\omega} \quad \bar{\tilde{\omega}} \quad \bar{\eta} \quad \bar{\tilde{\eta}} \quad \bar{\xi} \quad \bar{\tilde{\xi}}) \mathcal{M}_{\text{N10}} \begin{pmatrix} \chi \\ \tilde{\chi} \\ \omega \\ \tilde{\omega} \\ \eta \\ \tilde{\eta} \\ \xi \\ \tilde{\xi} \end{pmatrix}, \quad (2.29)$$

where

$$\mathcal{M}_{\text{N10}} \equiv \begin{pmatrix} m & M & \sqrt{3}m_W & 0 & 0 & 0 & 0 & 0 \\ M & \tilde{m} & 0 & -\sqrt{3}m_W & 0 & 0 & 0 & 0 \\ \sqrt{3}m_W & 0 & 0 & M & 2m_W & 0 & 0 & 0 \\ 0 & -\sqrt{3}m_W & M & 0 & 0 & -2m_W & 0 & 0 \\ 0 & 0 & 2m_W & 0 & 0 & M & \sqrt{3}m_W & 0 \\ 0 & 0 & 0 & -2m_W & M & 0 & 0 & -\sqrt{3}m_W \\ 0 & 0 & 0 & 0 & \sqrt{3}m_W & 0 & 0 & M \\ 0 & 0 & 0 & 0 & 0 & -\sqrt{3}m_W & M & 0 \end{pmatrix}. \quad (2.30)$$

The zero modes of the charged fermions have a common Dirac mass of M .

As in the case of **6**-plet, we set $m = \tilde{m}$ to simplify our analysis, and we also find an approximate expression for the mass eigenvalues of \mathcal{M}_{N10} in the case of $m_W \ll |M|$:

$$\begin{aligned} m_1 &\simeq M - m + \frac{3m_W^2}{2M - m}, \\ m_2 &\simeq M + \frac{1}{2} \left[\frac{3m_W^2}{2M + m} + \frac{7m_W^2}{M} - \sqrt{\left(\frac{3m_W^2}{2M + m} + \frac{m_W^2}{2M} \right)^2 + \frac{12m_W^4}{M^2}} \right], \\ m_3 &\simeq M + \frac{1}{2} \left[\frac{3m_W^2}{2M - m} + \frac{7m_W^2}{M} - \sqrt{\left(\frac{3m_W^2}{2M - m} + \frac{m_W^2}{2M} \right)^2 + \frac{12m_W^4}{M^2}} \right], \\ m_4 = m_5 &\simeq M + \frac{7m_W^2}{2M}, \\ m_6 &\simeq M + \frac{1}{2} \left[\frac{3m_W^2}{2M + m} + \frac{7m_W^2}{M} + \sqrt{\left(\frac{3m_W^2}{2M + m} + \frac{m_W^2}{2M} \right)^2 + \frac{12m_W^4}{M^2}} \right], \\ m_7 &\simeq M + \frac{1}{2} \left[\frac{3m_W^2}{2M - m} + \frac{7m_W^2}{M} + \sqrt{\left(\frac{3m_W^2}{2M - m} + \frac{m_W^2}{2M} \right)^2 + \frac{12m_W^4}{M^2}} \right], \\ m_8 &\simeq M + m + \frac{3m_W^2}{2M + m}, \end{aligned} \quad (2.31)$$

for the mass eigenstates defined as $U_{\mathcal{M}10} (\chi, \tilde{\chi}, \omega, \tilde{\omega}, \eta, \tilde{\eta}, \xi, \tilde{\xi})^T = (\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6, \phi_7, \phi_8)^T$ with a unitary matrix

$$U_{\mathcal{M}10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & a' & 0 \\ 0 & 1 & 0 & 0 & 0 & -d' & 0 & 0 \\ 0 & 0 & 1 & 0 & -d' & 0 & -c' & 0 \\ 0 & 0 & 0 & 1 & 0 & -c' & 0 & -b' \\ 0 & 0 & d' & 0 & 1 & 0 & 0 & 0 \\ 0 & d' & 0 & c' & 0 & 1 & 0 & 0 \\ -a' & 0 & c' & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & b' & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (2.32)$$

where

$$a' = \frac{\sqrt{3}m_W}{m - 2M}, \quad b' = \frac{\sqrt{3}m_W}{m + 2M}, \quad c' = \frac{m_W}{M}, \quad d' = \frac{\sqrt{3}m_W}{2M}. \quad (2.33)$$

As in the case of **6**-plets, we take $M \gtrsim 1 \text{ TeV} \gg m_W$ [17], and the DM mass is given by $m_{\text{DM}} = |m_1|$. For $m = \tilde{m} \gtrsim M$, the DM particle is mostly composed of a linear combination of the components in the $SU(2)$ doublets, triplets and quartets. Since this case is also very similar to the Higgsino/wino-like neutralino DM, we focus on the Higgs-portal DM case with $m = \tilde{m} \lesssim M$ also for this **10**-plet case.

We describe the coupling between the DM particle and the Higgs boson in the **10**-plet case. In the original basis, the interaction can be read off from Eq. (2.30) by $v \rightarrow v + h$ as

$$\begin{aligned} \mathcal{L}_{\text{Higgs-coupling}} &= -\frac{1}{2} \left(\frac{m_W}{v} \right) h \begin{pmatrix} \bar{\chi} & \bar{\tilde{\chi}} & \bar{\omega} & \bar{\tilde{\omega}} & \bar{\eta} & \bar{\tilde{\eta}} & \bar{\xi} & \bar{\tilde{\xi}} \end{pmatrix} \mathcal{M}_{h10} \begin{pmatrix} \chi \\ \tilde{\chi} \\ \omega \\ \tilde{\omega} \\ \eta \\ \tilde{\eta} \\ \xi \\ \tilde{\xi} \end{pmatrix} \\ &= -\frac{1}{2} \left(\frac{m_W}{v} \right) h \begin{pmatrix} \bar{\phi}_1 & \bar{\phi}_2 & \bar{\phi}_3 & \bar{\phi}_4 & \bar{\phi}_5 & \bar{\phi}_6 & \bar{\phi}_7 & \bar{\phi}_8 \end{pmatrix} \mathcal{C}_{h10} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix}, \quad (2.34) \end{aligned}$$

where h is the physical Higgs boson, and the explicit form of the matrix \mathcal{M}_{h10} and \mathcal{C}_{h10} are given by

$$\mathcal{M}_{h10} \equiv \begin{pmatrix} 0 & 0 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{3} & 0 & 0 & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & -\sqrt{3} & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & \sqrt{3} & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 & 0 & -\sqrt{3} \\ 0 & 0 & 0 & 0 & \sqrt{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\sqrt{3} & 0 & 0 \end{pmatrix}, \quad (2.35)$$

and

$$\mathcal{C}_{h10} \equiv \begin{pmatrix} \mathcal{C}_{11} & 0 & \mathcal{C}_{13} & 0 & \mathcal{C}_{15} & 0 & \mathcal{C}_{17} & 0 \\ 0 & \mathcal{C}_{22} & 0 & \mathcal{C}_{24} & 0 & \mathcal{C}_{26} & 0 & \mathcal{C}_{28} \\ \mathcal{C}_{31} & 0 & \mathcal{C}_{33} & 0 & \mathcal{C}_{35} & 0 & \mathcal{C}_{37} & 0 \\ 0 & \mathcal{C}_{42} & 0 & \mathcal{C}_{44} & 0 & \mathcal{C}_{46} & 0 & \mathcal{C}_{48} \\ \mathcal{C}_{51} & 0 & \mathcal{C}_{53} & 0 & \mathcal{C}_{55} & 0 & \mathcal{C}_{57} & 0 \\ 0 & \mathcal{C}_{62} & 0 & \mathcal{C}_{64} & 0 & \mathcal{C}_{66} & 0 & \mathcal{C}_{68} \\ \mathcal{C}_{71} & 0 & \mathcal{C}_{73} & 0 & \mathcal{C}_{75} & 0 & \mathcal{C}_{77} & 0 \\ 0 & \mathcal{C}_{82} & 0 & \mathcal{C}_{84} & 0 & \mathcal{C}_{86} & 0 & \mathcal{C}_{88} \end{pmatrix}, \quad (2.36)$$

with

$$\begin{aligned} \mathcal{C}_{11} &= -2\sqrt{3}a', & \mathcal{C}_{22} &= 2\sqrt{3}d', & \mathcal{C}_{33} &= 2(2c' + \sqrt{3}d'), & \mathcal{C}_{44} &= 2(\sqrt{3}b' + 2c'), \\ \mathcal{C}_{55} &= -2\sqrt{3}d', & \mathcal{C}_{66} &= -2(2c' + \sqrt{3}d'), & \mathcal{C}_{77} &= 2(\sqrt{3}a' - 2c'), & \mathcal{C}_{88} &= -2\sqrt{3}b', \\ \mathcal{C}_{13} &= \mathcal{C}_{31} = -2a' + \sqrt{3}c', & \mathcal{C}_{15} &= \mathcal{C}_{51} = -2a'd', & \mathcal{C}_{17} &= \mathcal{C}_{71} = -2a'c' + \sqrt{3}(-1 + a'^2), \\ \mathcal{C}_{24} &= \mathcal{C}_{42} = \sqrt{3}c' + 2d', & \mathcal{C}_{26} &= \mathcal{C}_{62} = 2c'd' + \sqrt{3}(-1 + d'^2), & \mathcal{C}_{28} &= \mathcal{C}_{82} = 2b'd', \\ \mathcal{C}_{35} &= \mathcal{C}_{53} = 2c'd' + \sqrt{3}(-1 + d'^2), & \mathcal{C}_{37} &= \mathcal{C}_{73} = 2(1 - c'^2) + \sqrt{3}c'(-a' + d'), \\ \mathcal{C}_{46} &= \mathcal{C}_{64} = 2(-1 + c'^2) + \sqrt{3}c'(b' + d'), & \mathcal{C}_{48} &= \mathcal{C}_{84} = \sqrt{3}(-1 + b'^2) + 2b'c', \\ \mathcal{C}_{57} &= \mathcal{C}_{75} = -\sqrt{3}c' - 2d', & \mathcal{C}_{68} &= \mathcal{C}_{86} = -2b' - \sqrt{3}c'. \end{aligned} \quad (2.37)$$

The interaction Lagrangian relevant to the DM physics can be read as

$$\begin{aligned} \mathcal{L}_{\text{DM-H10}} &= \frac{2\sqrt{3}m_W}{2M - m} \left(\frac{\sqrt{3}m_W}{v} \right) h \overline{\psi_{\text{DM}}} \psi_{\text{DM}} \\ &+ \left[\left(\frac{2m_W}{2M - m} + \frac{m_W}{M} \right) \left(\frac{\sqrt{3}m_W}{v} \right) h \overline{\phi_3} \psi_{\text{DM}} + \frac{\sqrt{3}m_W^2}{(2M - m)M} \left(\frac{\sqrt{3}m_W}{v} \right) h \overline{\phi_5} \psi_{\text{DM}} \right. \\ &\left. + \left(-1 + \frac{3m_W^2}{(2M - m)^2} + \frac{2m_W^2}{(2M - m)M} \right) \left(\frac{\sqrt{3}m_W}{v} \right) h \overline{\phi_7} \psi_{\text{DM}} + \text{h.c.} \right], \end{aligned} \quad (2.38)$$

where we have identified the lightest mass eigenstate ϕ_1 as the DM particle (ψ_{DM}).

2.3 15-plet MDM

It is straightforward to extend our previous analysis further higher-representations. In this subsection, we finally consider a pair of **15**-plet bulk fermions, $\psi(\mathbf{15})$ and $\tilde{\psi}(\mathbf{15})$. The Lagrangian relevant to our DM physics discussion is given by

$$\begin{aligned}\mathcal{L}_{\text{DM15}} = & \text{Tr} \left[\bar{\psi}(\mathbf{15}) i \not{D} \psi(\mathbf{15}) + \bar{\tilde{\psi}}(\mathbf{15}) i \not{D} \tilde{\psi}(\mathbf{15}) - M(\bar{\psi}(\mathbf{15}) \tilde{\psi}(\mathbf{15}) + \bar{\tilde{\psi}}(\mathbf{15}) \psi(\mathbf{15})) \right] \\ & + \delta(y) \left[\frac{m}{2} \overline{\nu_{sR}^{(0)c}} \nu_{sR}^{(0)} + \frac{\tilde{m}}{2} \overline{\tilde{\nu}_{sL}^{(0)c}} \tilde{\nu}_{sL}^{(0)} + \text{h.c.} \right].\end{aligned}\quad (2.39)$$

It is useful to express the **15**-plet in a following matrix form,

$$\psi_{ijkl}(\mathbf{15}) = \psi_{11kl} + \psi_{12kl} + \psi_{13kl} + \psi_{22kl} + \psi_{23kl} + \psi_{33kl}, \quad (2.40)$$

where

$$\begin{aligned}\psi_{11kl} &\equiv \begin{pmatrix} \Theta_{\uparrow\uparrow} & \frac{1}{2}\Theta_{\uparrow} & \frac{1}{2\sqrt{3}}\Delta^{++} \\ \frac{1}{2}\Theta_{\uparrow} & \frac{1}{\sqrt{6}}\Theta_0 & \frac{1}{2\sqrt{3}}\Delta^+ \\ \frac{1}{2}\Delta^{++} & \frac{1}{2\sqrt{3}}\Delta^+ & \frac{1}{\sqrt{6}}\Sigma_{\uparrow} \end{pmatrix}, & \psi_{12kl} &\equiv \begin{pmatrix} \frac{1}{2}\Theta_{\uparrow} & \frac{1}{\sqrt{6}}\Theta_0 & \frac{1}{2\sqrt{3}}\Delta^+ \\ \frac{1}{\sqrt{6}}\Theta_0 & \frac{1}{2}\Theta_{\downarrow} & \frac{1}{2\sqrt{3}}\Delta^0 \\ \frac{1}{2\sqrt{3}}\Delta^+ & \frac{1}{2\sqrt{3}}\Delta^0 & \frac{1}{2\sqrt{3}}\Sigma_0 \end{pmatrix}, \\ \psi_{13kl} &\equiv \begin{pmatrix} \frac{1}{2}\Delta^{++} & \frac{1}{2\sqrt{3}}\Delta^+ & \frac{1}{\sqrt{6}}\Sigma_{\uparrow} \\ \frac{1}{2\sqrt{3}}\Delta^+ & \frac{1}{2\sqrt{3}}\Delta^0 & \frac{1}{2\sqrt{3}}\Sigma_0 \\ \frac{1}{\sqrt{6}}\Sigma_{\uparrow} & \frac{1}{2\sqrt{3}}\Sigma_0 & \frac{1}{2}\nu \end{pmatrix}, & \psi_{22kl} &\equiv \begin{pmatrix} \frac{1}{\sqrt{6}}\Theta_0 & \frac{1}{2}\Theta_{\downarrow} & \frac{1}{2\sqrt{3}}\Delta^0 \\ \frac{1}{2}\Theta_{\uparrow} & \Theta_{\downarrow\downarrow} & \frac{1}{2}\Delta^- \\ \frac{1}{2\sqrt{3}}\Delta^0 & \frac{1}{2}\Delta^- & \frac{1}{\sqrt{6}}\Sigma_{\downarrow} \end{pmatrix}, \\ \psi_{23kl} &\equiv \begin{pmatrix} \frac{1}{2\sqrt{3}}\Delta^+ & \frac{1}{2\sqrt{3}}\Delta^0 & \frac{1}{2\sqrt{3}}\Sigma_0 \\ \frac{1}{\sqrt{3}}\Delta^0 & \frac{1}{2}\Delta^- & \frac{1}{\sqrt{6}}\Sigma_{\downarrow} \\ \frac{1}{2\sqrt{3}}\Sigma_0 & \frac{1}{\sqrt{6}}\Sigma_{\downarrow} & \frac{1}{2}e \end{pmatrix}, & \psi_{33kl} &\equiv \begin{pmatrix} \frac{1}{\sqrt{6}}\Sigma_{\uparrow} & \frac{1}{2\sqrt{3}}\Sigma_0 & \frac{1}{2}\nu \\ \frac{1}{2\sqrt{3}}\Sigma_0 & \frac{1}{\sqrt{6}}\Sigma_{\downarrow} & \frac{1}{2}e \\ \frac{1}{2}\nu & \frac{1}{2}e & \nu_s \end{pmatrix}.\end{aligned}\quad (2.41)$$

The quantum numbers for $SU(2)$ representation, the third component of the isospin I_3 , and the electric charge Q_{em} of newly introduced fermions Θ in the matrix are

$$\Theta_{\uparrow\uparrow}(\mathbf{5}, 2, 4), \quad \Theta_{\uparrow}(\mathbf{5}, 1, 3), \quad \Theta_0(\mathbf{5}, 0, 1), \quad \Theta_{\downarrow}(\mathbf{5}, -1, 1), \quad \Theta_{\downarrow\downarrow}(\mathbf{5}, -2, 0). \quad (2.42)$$

The corresponding mirror fermion **15**-plet $\tilde{\psi}(\mathbf{15})$ takes a similar form with tilde for all fields.

With the non-trivial orbifold boundary conditions, the bulk $SU(3)$ **15**-plet fermions are decomposed into the SM $SU(2)$ quintet, quartet, triplet, doublet and singlet fermions. Let us focus on the mass terms in Eq. (2.39):

$$\begin{aligned}\mathcal{L}_{\text{mass15}} = & \text{Tr} \left[\bar{\psi} i \Gamma^5 (\partial_y - 4ig\langle A_y \rangle) \psi + \bar{\tilde{\psi}} i \Gamma^5 (\partial_y - 4ig\langle A_y \rangle) \tilde{\psi} - M(\bar{\psi} \tilde{\psi} + \bar{\tilde{\psi}} \psi) \right] \\ & + \delta(y) \left[\frac{m}{2} \overline{\nu_{sR}^{(0)c}} \nu_{sR}^{(0)} + \frac{\tilde{m}}{2} \overline{\tilde{\nu}_{sL}^{(0)c}} \tilde{\nu}_{sL}^{(0)} + \text{h.c.} \right].\end{aligned}\quad (2.43)$$

Expanding the bulk fermions in terms of KK modes by noticing Z_2 parity of Δ as

$$\Theta_L(-y) = -\Theta_L(y), \quad \tilde{\Theta}_L(-y) = +\tilde{\Theta}_L(y), \quad (2.44)$$

$$\Theta_R(-y) = +\Theta_R(y), \quad \tilde{\Theta}_R(-y) = -\tilde{\Theta}_R(y) \quad (2.45)$$

and integrating out the fifth coordinate y , we obtain the expression in 4-dimensional effective theory. The zero-mode parts for the electric-charge neutral fermions are found to be

$$\begin{aligned} \mathcal{L}_{\text{mass15}}^{\text{zero-mode}} &= -4im_W \left[\frac{1}{2} \overline{\Delta}_L^- \Theta_{\downarrow R} + \frac{3}{2\sqrt{6}} \overline{\Delta}_L^- \Sigma_{\downarrow R} + \frac{3}{2\sqrt{6}} \overline{e}_L \Sigma_{\downarrow R} + \frac{1}{2} \overline{e}_L \nu_{sR} + \frac{1}{2} \overline{\tilde{\Delta}}_R^- \tilde{\Theta}_{\downarrow L} \right. \\ &\quad \left. + \frac{3}{2\sqrt{6}} \overline{\tilde{\Delta}}_R^- \tilde{\Sigma}_{\downarrow L} + \frac{3}{2\sqrt{6}} \overline{\tilde{e}}_R \tilde{\Sigma}_{\downarrow L} + \frac{1}{2} \overline{\tilde{e}}_R \tilde{\nu}_{sL} + \text{h.c.} \right] \\ &\quad - M \left(\overline{\Theta}_{\downarrow R} \tilde{\Theta}_{\downarrow L} + \overline{\Delta}_L^- \tilde{\Delta}_R^- + \overline{\Sigma}_{\downarrow R} \tilde{\Sigma}_{\downarrow L} + \overline{e}_L \tilde{e}_R + \overline{\nu}_{sR} \tilde{\nu}_{sL} + \text{h.c.} \right) \\ &\quad + \left(\frac{m}{2} \overline{\nu}_{sR}^{(0)c} \nu_{sR}^{(0)} + \frac{\tilde{m}}{2} \overline{\tilde{\nu}}_{sL}^{(0)c} \tilde{\nu}_{sL}^{(0)} + \text{h.c.} \right) \\ &\rightarrow -4m_W \left[\frac{1}{2} \overline{\Delta}_L^- \Theta_{\downarrow R} + \frac{3}{2\sqrt{6}} \overline{\Delta}_L^- \Sigma_{\downarrow R} + \frac{3}{2\sqrt{6}} \overline{e}_L \Sigma_{\downarrow R} + \frac{1}{2} \overline{e}_L \nu_{sR} + \frac{1}{2} \overline{\tilde{\Delta}}_R^- \tilde{\Theta}_{\downarrow L} \right. \\ &\quad \left. + \frac{3}{2\sqrt{6}} \overline{\tilde{\Delta}}_R^- \tilde{\Sigma}_{\downarrow L} + \frac{3}{2\sqrt{6}} \overline{\tilde{e}}_R \tilde{\Sigma}_{\downarrow L} + \frac{1}{2} \overline{\tilde{e}}_R \tilde{\nu}_{sL} + \text{h.c.} \right] \\ &\quad - M \left(\overline{\Theta}_{\downarrow R} \tilde{\Theta}_{\downarrow L} + \overline{\Delta}_L^- \tilde{\Delta}_R^- + \overline{\Sigma}_{\downarrow R} \tilde{\Sigma}_{\downarrow L} + \overline{e}_L \tilde{e}_R + \overline{\nu}_{sR} \tilde{\nu}_{sL} + \text{h.c.} \right) \\ &\quad - \left(\frac{m}{2} \overline{\nu}_{sR}^{(0)c} \nu_{sR}^{(0)} + \frac{\tilde{m}}{2} \overline{\tilde{\nu}}_{sL}^{(0)c} \tilde{\nu}_{sL}^{(0)} + \text{h.c.} \right), \end{aligned} \quad (2.46)$$

where the arrow means the phase rotations defined in (2.10) and

$$\Theta_{\downarrow} \rightarrow i\Theta_{\downarrow}, \quad \tilde{\Theta}_{\downarrow} \rightarrow i\tilde{\Theta}_{\downarrow}. \quad (2.47)$$

Rewriting these mass terms in a Majorana basis with Eq. (2.11), Eq. (2.28) and

$$\rho \equiv (\Theta_{\downarrow}^{(0)})_R + (\Theta_{\downarrow}^{(0)c})_R, \quad \tilde{\rho} \equiv (\tilde{\Theta}_{\downarrow}^{(0)})_L + (\tilde{\Theta}_{\downarrow}^{(0)c})_L, \quad (2.48)$$

we have the following mass terms,

$$\mathcal{L}_{15\text{mass}}^{\text{zero-mode}} = -\frac{1}{2} \begin{pmatrix} \bar{\chi} & \bar{\tilde{\chi}} & \bar{\omega} & \bar{\tilde{\omega}} & \bar{\eta} & \bar{\tilde{\eta}} & \bar{\xi} & \bar{\tilde{\xi}} & \bar{\rho} & \bar{\tilde{\rho}} \end{pmatrix} \mathcal{M}_{\text{N15}} \begin{pmatrix} \chi \\ \tilde{\chi} \\ \omega \\ \tilde{\omega} \\ \eta \\ \tilde{\eta} \\ \xi \\ \tilde{\xi} \\ \rho \\ \tilde{\rho} \end{pmatrix}, \quad (2.49)$$

where

$$\mathcal{M}_{\text{N15}} \equiv \begin{pmatrix} m & M & 2m_W & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ M & \tilde{m} & 0 & -2m_W & 0 & 0 & 0 & 0 & 0 & 0 \\ 2m_W & 0 & 0 & M & \sqrt{6}m_W & 0 & 0 & 0 & 0 & 0 \\ 0 & -2m_W & M & 0 & 0 & -\sqrt{6}m_W & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{6}m_W & 0 & 0 & M & \sqrt{6}m_W & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{6}m_W & M & 0 & 0 & -\sqrt{6}m_W & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{6}m_W & 0 & 0 & M & 2m_W & 0 \\ 0 & 0 & 0 & 0 & 0 & -\sqrt{6}m_W & M & 0 & 0 & -2m_W \\ 0 & 0 & 0 & 0 & 0 & 0 & 2m_W & 0 & 0 & M \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2m_W & M & 0 \end{pmatrix}. \quad (2.50)$$

The zero modes of the charged fermions have a common Dirac mass of M .

3 Dark Matter Relic Abundance

In this section, we evaluate the DM relic abundance and identify an allowed parameter region to reproduce the observed DM relic density [20] (68 % confidence level):

$$\Omega_{\text{DM}} h^2 = 0.1198 \pm 0.0015. \quad (3.1)$$

In our model, the DM physics is controlled by only two free parameters, namely, m and M . As we discussed in the previous section, we focus on the Higgs-portal DM case with $0 \leq m \lesssim M$. In this section, we consider the **6**-plet and **10**-plet cases. Analysis for a more higher-dimensional representation is analogous to those presented in this section.

According to the interaction Lagrangian in Eqs. (2.19) and (2.38), we consider two main annihilation processes of a pair of DM particles as analyzed in [10]. One is through the s -channel Higgs boson exchange, and the other is the process $\psi_{\text{DM}} \psi_{\text{DM}} \rightarrow hh$ through the exchange of ϕ_5 (ϕ_7) in the t/u -channel for the **6**-plet (**10**-plet) case, when the DM particle is heavier than the SM Higgs boson. In evaluating this process, we may use an effective Lagrangian of the form,

$$\mathcal{L}_{\text{DM-H}}^{\text{eff}} = \begin{cases} \frac{1}{2} \left(\frac{\sqrt{2}m_W}{v} \right)^2 \frac{1}{m_5} h h \overline{\psi_{\text{DM}}} \psi_{\text{DM}} \text{ (6-plet)}, \\ \frac{1}{2} \left(\frac{\sqrt{3}m_W}{v} \right)^2 \frac{1}{m_7} h h \overline{\psi_{\text{DM}}} \psi_{\text{DM}} \text{ (10-plet)} \end{cases} \quad (3.2)$$

which is obtained by integrating ϕ_5 (ϕ_7) out, and calculate the DM pair annihilation cross section times relative velocity (v_{rel}) as

$$\sigma v_{\text{rel}} = \begin{cases} \frac{1}{64\pi} \left(\frac{\sqrt{2}m_W}{v} \right)^4 \left(\frac{1}{m_5} \right)^2 v_{\text{rel}}^2 \equiv \sigma_0 v_{\text{rel}}^2 \text{ (6-plet)}, \\ \frac{1}{64\pi} \left(\frac{\sqrt{3}m_W}{v} \right)^4 \left(\frac{1}{m_7} \right)^2 v_{\text{rel}}^2 \equiv \sigma_0 v_{\text{rel}}^2 \text{ (10-plet)}. \end{cases} \quad (3.3)$$

It is well-known that the observed DM relic density is reproduced by $\sigma_0 \sim 1$ pb. Since we find $\sigma_0 \sim 0.04$ (0.06) pb for m_5 (m_7) $\simeq M = 1$ TeV in the case of **6**-plet (**10**-plet), we conclude that the observed relic density is not reproduced by the process $\psi_{\text{DM}}\psi_{\text{DM}} \rightarrow hh$.

Next we consider the DM pair annihilation through the s -channel Higgs boson exchange when the DM particle is lighter than the Higgs boson. Since the coupling between the pair of DM particles and the Higgs boson is suppressed, an enhancement of the DM annihilation cross section through the Higgs boson resonance is necessary to reproduce the observed relic DM density. We evaluate the DM relic abundance by integrating the Boltzmann equation

$$\frac{dY}{dx} = -\frac{xs\langle\sigma v\rangle}{H(m_{\text{DM}})}(Y^2 - Y_{\text{EQ}}^2), \quad (3.4)$$

where the temperature of the universe is normalized by the DM mass as $x = m_{\text{DM}}/T$, $H(m_{\text{DM}})$ is the Hubble parameter as $T = m_{\text{DM}}$, Y is the yield (the ratio of the DM number density to the entropy density s) of the DM particle, Y_{EQ} is the yield of the DM in thermal equilibrium, and $\langle\sigma v_{\text{rel}}\rangle$ is the thermal average of the DM annihilation cross section times relative velocity for a pair of the DM particles. Various quantities in the Boltzmann equation are given as follows.

$$s = \frac{2\pi^2}{45} g_* \frac{m_{\text{DM}}^3}{x^3}, \quad H(m_{\text{DM}}) = \sqrt{\frac{\pi^2}{90}} g_* \frac{m_{\text{DM}}^2}{M_P}, \quad sY_{\text{EQ}} = \frac{g_{\text{DM}}}{2\pi^2} \frac{m_{\text{DM}}^3}{x} K_2(x), \quad (3.5)$$

where $M_P = 2.44 \times 10^{18}$ GeV is the reduced Planck mass, $g_{\text{DM}} = 2$ is the number of degrees of freedom for the DM particle, g_* is the effective total number of degrees of freedom for the particles in thermal equilibrium (in our analysis, we use $g_* = 86.25$ corresponding to $m_{\text{DM}} \simeq m_h/2$ with the Higgs boson mass of 125 GeV), and K_2 is the modified Bessel function of the second kind. For $m_{\text{DM}} \simeq m_h/2 = 62.5$ GeV, a DM pair annihilates into a pair of the SM fermions as $\psi_{\text{DM}}\psi_{\text{DM}} \rightarrow h \rightarrow f\bar{f}$, where f denotes the SM fermions. We calculate the cross section for the annihilation process as

$$\sigma(s) = \frac{y_{\text{DM}}^2}{16\pi} \left[3 \left(\frac{m_b}{v} \right)^2 + 3 \left(\frac{m_c}{v} \right)^2 + \left(\frac{m_\tau}{v} \right)^2 \right] \frac{\sqrt{s(s - 4m_{\text{DM}}^2)}}{(s - m_h^2)^2 + m_h^2 \Gamma_h^2}, \quad (3.6)$$

where y_{DM} can be lead from Eqs. (2.19) and (2.38) as

$$y_{\text{DM}} \simeq \begin{cases} \frac{2\sqrt{2}m_W}{2M-m} \frac{\sqrt{2}m_W}{v} & (\mathbf{6}\text{-plet}) \\ \frac{4\sqrt{3}m_W}{2M-m} \frac{\sqrt{3}m_W}{v} & (\mathbf{10}\text{-plet}), \end{cases} \quad (3.7)$$

and we have only considered pairs of bottom, charm and tau for the final states, neglecting the other lighter quarks, and used the following values for the fermion masses at the Z -boson mass scale [21]: $m_b = 2.82$ GeV, $m_c = 685$ MeV and $m_\tau = 1.75$ GeV. The total Higgs boson decay width Γ_h is given by $\Gamma_h = \Gamma_h^{\text{SM}} + \Gamma_h^{\text{new}}$, where $\Gamma_h^{\text{SM}} = 4.07$ MeV [22] is the total Higgs boson decay width in the SM and

$$\Gamma_h^{\text{new}} = \begin{cases} 0 & m_h < 2m_{\text{DM}} \\ \frac{m_h}{16\pi} \left(1 - \frac{4m_{\text{DM}}^2}{m_h^2}\right)^{3/2} y_{\text{DM}}^2 & m_h > 2m_{\text{DM}} \end{cases}, \quad (3.8)$$

is the partial decay width of the Higgs boson to a DM pair. The thermal average of the annihilation cross section is given by

$$\langle \sigma v \rangle = (sY_{\text{EQ}})^{-2} g_{\text{DM}}^2 \frac{m_{\text{DM}}}{64\pi^4 x} \int_{4m_{\text{DM}}}^{\infty} ds \hat{\sigma}(s) \sqrt{s} K_1 \left(\frac{x\sqrt{s}}{m_{\text{DM}}} \right), \quad (3.9)$$

where $\hat{\sigma}(s) = 2(s - 4m_{\text{DM}}^2)\sigma(s)$ is the reduced cross section with the total annihilation cross section $\sigma(s)$, and K_1 is the modified Bessel function of the first kind. We solve the Boltzmann equation numerically and find an asymptotic value of the yield $Y(\infty)$ to obtain the present DM relic density as

$$\Omega h^2 = \frac{m_{\text{DM}} s_0 Y(\infty)}{\rho_c / h^2}, \quad (3.10)$$

where $s_0 = 2890 \text{ cm}^{-3}$ is the entropy density of the present universe, and $\rho_c / h^2 = 1.05 \times 10^{-5} \text{ GeV/cm}^3$ is the critical density.

In Fig. 1, we show y_{DM} as a function of m_{DM} (solid line) along which the observed DM relic density $\Omega_{\text{DM}} h^2 = 0.1198$ is reproduced. Here, the current experimental upper bound from the XENON 1T result [23] and the prospective reach in the future LUX-ZEPLIN DM experiment [12] are also shown as the dashed and the dotted lines, respectively, which will be derived in Sec. 4. In order to satisfy the XENON 1T constraint, we find the parameter regions such as $58.3 \leq m_{\text{DM}} [\text{GeV}] \leq 62.4$ and $(0.00692 \leq) y_{\text{DM}} \leq 0.0150$. Using Eqs. (2.13) and (2.31), we can express M as a function of m_{DM} along the solid line in Fig. 1. Our results are shown in the Fig. 2. Corresponding to the parameter regions of $58.3 \leq m_{\text{DM}} [\text{GeV}] \leq 62.4$ and $(0.00692 \leq) y_{\text{DM}} \leq 0.0150$, we have found $6.97 \leq M [\text{TeV}] (\leq 15.1)$ and $10.5 \leq M [\text{TeV}] (\leq 22.6)$ for the **6**-plet and **10**-plet cases, respectively.

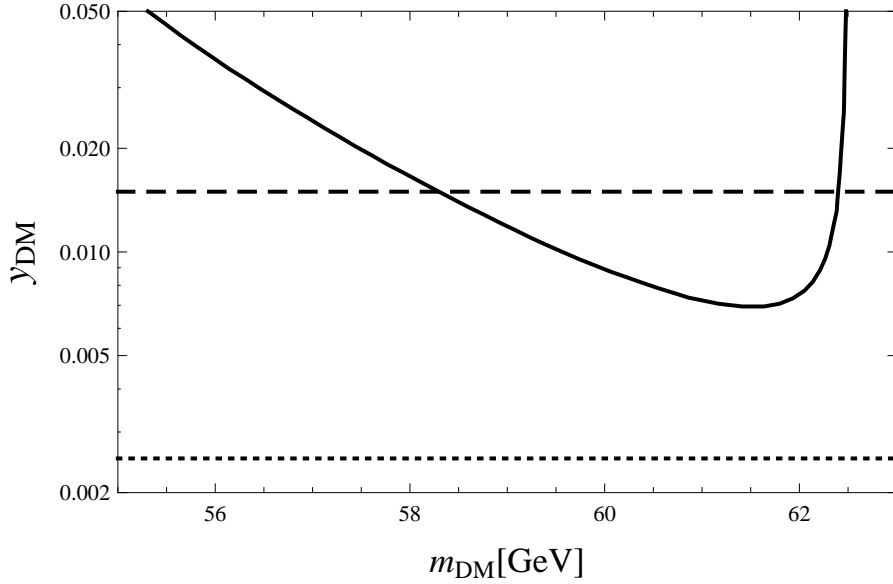


Figure 1: The Yukawa coupling y_{DM} as a function of m_{DM} (solid line) along which the observed DM relic density $\Omega_{\text{DM}}h^2 = 0.1198$ is reproduced. Here, the current experimental upper bound from the XENON 1T result [23] and the prospective reach in the future LUX-ZEPLIN DM experiment [12] are also shown as the dashed and the dotted lines, respectively.

4 Direct Dark Matter Detection Constraints

Currently, many experiments are in operation and also planned for directly detecting a dark matter particle through its elastic scattering off with nuclei. In this section, we calculate the spin-independent elastic scattering cross section of the DM particle via the Higgs boson exchange and derive the constraint on the model parameters from the current experimental results.

The spin-independent elastic scattering cross section with nucleon is given by

$$\sigma_{\text{SI}} = \frac{1}{\pi} \left(\frac{y_{\text{DM}}}{v} \right)^2 \left(\frac{\mu_{\psi_{\text{DM}}N}}{m_h^2} \right)^2 f_N^2, \quad (4.1)$$

where $\mu_{\psi_{\text{DM}}N} = m_N m_{\text{DM}} / (m_N + m_{\text{DM}})$ is the reduced mass of the DM-nucleon system with the nucleon mass $m_N = 0.939$ GeV, and

$$f_N = \left(\sum_{q=u,d,s} f_{T_q} + \frac{2}{9} f_{TG} \right) m_N \quad (4.2)$$

is the nuclear matrix element accounting for the quark and gluon contents of the nucleon. In evaluating f_{T_q} , we employ the results from the lattice QCD simulation [24] (see also [25]):

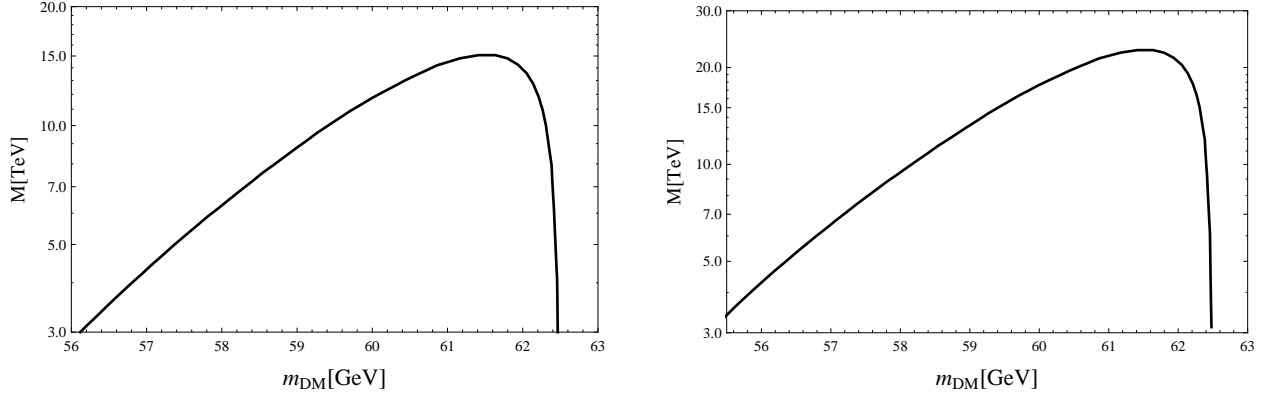


Figure 2: M as a function of m_{DM} for the **6**-plet case (left panel) and the **10**-plet case (right panel), along the solid line in Fig. 1.

$f_{T_u} + f_{T_d} \simeq 0.056$ and $|f_{T_s}| \leq 0.08$. To make our analysis conservative, we set $f_{T_s} = 0$. Using the trace anomaly formula, $\sum_{q=u,d,s} f_{T_q} + f_{TG} = 1$ [26], we obtain $f_N^2 \simeq 0.0706 m_N^2$, and hence the spin-independent elastic scattering cross section is approximately given by

$$\sigma_{\text{SI}} \simeq 4.47 \times 10^{-7} \text{ pb} \times y_{\text{DM}}^2 \quad (4.3)$$

for $m_{\text{DM}} = m_h/2 = 62.5 \text{ GeV}$.

For the upper bound on the spin-independent cross section, we refer the recent XENON 1T result [23]: $\sigma_{\text{SI}} \leq 1.0 \times 10^{-10} \text{ pb}$ for $m_{\text{DM}} \simeq 62.5 \text{ GeV}$. From Eq. (4.3), we find $y_{\text{DM}} \leq 0.0150$, which is depicted as the horizontal dashed line in Fig. 1. The next-generation successor of the LUX experiment, the LUX-ZEPLIN experiment [12], plans to improve the current upper bound on the spin-independent cross section by about two orders of magnitude. When we apply the search reach of the LUX-ZEPLIN experiment as $\sigma_{\text{SI}} \leq 2.8 \times 10^{-12} \text{ pb}$, we obtain $y_{\text{DM}} \leq 0.00251$. This prospective upper bound is shown as the dotted line in Fig. 1. The present allowed parameter region will be all covered by the future LUX-ZEPLIN experiment.

5 125 GeV Higgs boson mass

In this section, we reproduce the 125 GeV Higgs boson mass in our GHU model in the presence of the bulk fermion multiplets. For our calculations, we follow a 4-dimensional effective theory approach of the 5-dimensional GHU scenario developed in Ref. [27]. According to this approach, a low-energy effective Higgs quartic coupling in the GHU model can be easily obtained by solving

the renormalization group (RG) equation of the Higgs quartic coupling with a vanishing quartic coupling at the compactification scale. This so-called “gauge-Higgs condition” [27] corresponds to a restoration of the 5-dimensional gauge invariance at the compactification scale. For energies higher than the compactification scale, our theory behaves as the original 5-dimensional theory and no Higgs potential exists there.

We have two mass parameters important in our RG analysis, namely, the bulk mass $M \simeq m$ and the compactification scale $M_{\text{KK}} = 1/R$. Although the electroweak symmetry breaking causes the mass splittings among the bulk fermion zero modes, the mass splittings are all negligibly small due to $M \simeq m \gg m_W$. Thus, all mass eigenstates except for two mass eigenstates (ϕ_1 and ϕ_6 (ϕ_1 and ϕ_8) in the **6**-plet (**10**-plet) case) are degenerate. The non-degenerate mass eigenstates are mostly composed of the SM singlets, and we safely neglect their contributions to our RG analysis.

For the renormalization scale smaller than the bulk mass $\mu < M$, all bulk fermions are decoupled and we employ the SM RG equations at the two-loop level [28] (See [10] for explicit the SM RG equations.). In solving these RGEs, we use the boundary conditions at the top quark pole mass (M_t) given in [29]:

$$\begin{aligned}
g_1(M_t) &= \sqrt{\frac{5}{3}} \left(0.35761 + 0.00011(M_t - 173.10) - 0.00021 \left(\frac{M_W - 80.384}{0.014} \right) \right), \\
g_2(M_t) &= 0.64822 + 0.00004(M_t - 173.10) + 0.00011 \left(\frac{M_W - 80.384}{0.014} \right), \\
g_3(M_t) &= 1.1666 + 0.00314 \left(\frac{\alpha_s - 0.1184}{0.0007} \right), \\
y_t(M_t) &= 0.93558 + 0.0055(M_t - 173.10) - 0.00042 \left(\frac{\alpha_s - 0.1184}{0.0007} \right) \\
&\quad - 0.00042 \left(\frac{M_W - 80.384}{0.014} \right), \\
\lambda(M_t) &= 2(0.12711 + 0.00206(m_h - 125.66) - 0.00004(M_t - 173.10)).
\end{aligned} \tag{5.1}$$

We employ $M_W = 80.384$ GeV, $\alpha_s = 0.1184$, the central value of the combination of Tevatron and LHC measurements of top quark mass $M_t = 173.34$ GeV [30], and the central value of the updated Higgs boson mass measurement, $m_h = 125.09$ GeV from the combined analysis by the ATLAS and the CMS collaborations [31].

For the renormalization scale $\mu \geq M$, the SM RG equations are modified in the presence of the bulk fermions. In this paper, we take only one-loop corrections from the bulk fermions into

account. For the case with a pair of the bulk **6**-plet fermions, the beta functions of the $SU(2)$ and $U(1)_Y$ gauge couplings receive new contributions as

$$\Delta b_1 = 2 \left(\frac{2}{3} + \frac{24}{5} Q^2 \right), \quad \Delta b_2 = \frac{20}{3}, \quad (5.2)$$

where $Q = 2/3$ is the $U(1)'$ charge of the bulk **6**-plet fermions. The beta functions of the top Yukawa and Higgs quartic couplings are modified as

$$\begin{aligned} \beta_t^{(1)} &\rightarrow \beta_t^{(1)} + 2y_t (2|Y_S|^2 + 3|Y_D|^2), \\ \beta_\lambda^{(1)} &\rightarrow \beta_\lambda^{(1)} + 2 \left[\lambda (8|Y_S|^2 + 12|Y_D|^2) - (8|Y_S|^4 + 10|Y_D|^4 + 16|Y_S|^2|Y_D|^2) \right]. \end{aligned} \quad (5.3)$$

Here, the Yukawa couplings, Y_S and Y_D , are defined in the Yukawa interactions from Eq. (2.3) as

$$\mathcal{L}_{\text{DM6}} \supset -Y_S \bar{D} H S - Y_D \bar{D} T H^\dagger, \quad (5.4)$$

where S, D and T stand for the $SU(2)$ singlet, doublet and triplet fields in the decomposition of $\mathbf{6} = \mathbf{1} \oplus \mathbf{2} \oplus \mathbf{3}$ (see Eqs. (2.4) and (2.5)). The Yukawa couplings obey the following RG equations:

$$\begin{aligned} 16\pi^2 \frac{dY_S}{d \ln \mu} &= Y_S \left[3y_t^2 - \left(\frac{9}{20} g_1^2 + \frac{9}{4} g_2^2 \right) + 2 \left(\frac{7}{2} |Y_S|^2 + \frac{19}{4} |Y_D|^2 \right) \right. \\ &\quad \left. - \frac{18}{5} \left(\frac{2}{3} - Q \right) \left(\frac{1}{6} - Q \right) g_1^2 \right], \\ 16\pi^2 \frac{dY_D}{d \ln \mu} &= Y_D \left[3y_t^2 - \left(\frac{9}{20} g_1^2 + \frac{9}{4} g_2^2 \right) + 2 \left(\frac{9}{2} |Y_S|^2 + \frac{17}{4} |Y_D|^2 \right) \right. \\ &\quad \left. - 6g_2^2 - \frac{18}{5} \left(\frac{1}{6} - Q \right) \left(\frac{1}{3} - Q \right) g_1^2 \right]. \end{aligned} \quad (5.5)$$

In our RG analysis, we numerically solve the SM RG equations from M_t to M , at which the solutions connect with the solutions of the RG equations with the bulk **6**-plet fermions. For a fixed M values, we arrange input values of $|Y_S(M)|$ and $|Y_D(M)|$ so as to find numerical solutions which satisfy the the gauge-Higgs condition and the unification condition among the gauge and Yukawa couplings such that

$$\lambda(M_{\text{KK}}) = 0, \quad Y_S(M_{\text{KK}}) = Y_D(M_{\text{KK}}) = -ig_2(M_{\text{KK}}). \quad (5.6)$$

For the case with a pair of the bulk **10**-plet fermions, the beta functions of the $SU(2)$ and $U(1)_Y$ gauge couplings receive new contributions as

$$\Delta b_1 = 2 (3 + 12Q^2), \quad \Delta b_2 = 20, \quad (5.7)$$

where $Q = 1$ is the $U(1)'$ charge of the bulk **10**-plet fermions. The beta functions of the top Yukawa and Higgs quartic couplings are modified as

$$\begin{aligned}\beta_t^{(1)} &\rightarrow \beta_t^{(1)} + 2y_t (2|Y_S|^2 + 3|Y_D|^2), \\ \beta_\lambda^{(1)} &\rightarrow \beta_\lambda^{(1)} + 2 \left[4\lambda (2|Y_S|^2 + 3|Y_D|^2 + 4|Y_T|^2) \right. \\ &\quad \left. - \left(8|Y_S|^4 + 10|Y_D|^4 + \frac{112}{9}|Y_T|^4 + 16|Y_S|^2|Y_D|^2 + \frac{64}{3}|Y_D|^2|Y_T|^2 \right) \right].\end{aligned}\quad (5.8)$$

Here, the Yukawa couplings, Y_S , Y_D and Y_T , are defined in the Yukawa interactions from Eq. (2.20) as

$$\mathcal{L}_{\text{DM10}} \supset -Y_S \bar{D} H S - Y_D \bar{D} T H^\dagger - Y_T \bar{F} T H, \quad (5.9)$$

where S , D , T and F stand for the $SU(2)$ singlet, doublet, triplet and quartet fields in the decomposition of $\mathbf{10} = \mathbf{1} \oplus \mathbf{2} \oplus \mathbf{3} \oplus \mathbf{4}$, respectively (see Eqs. (2.4), (2.5), (2.22) and (2.23)). The Yukawa couplings obey the following RG equations:

$$\begin{aligned}16\pi^2 \frac{dY_S}{d\ln\mu} &= Y_S \left[3y_t^2 - \left(\frac{9}{20}g_1^2 + \frac{9}{4}g_2^2 \right) + 2 \left(\frac{7}{2}|Y_S|^2 + \frac{27}{4}|Y_D|^2 + 4|Y_T|^2 \right) \right. \\ &\quad \left. - \frac{18}{5}(1-Q) \left(\frac{1}{2} - Q \right) g_1^2 \right], \\ 16\pi^2 \frac{dY_D}{d\ln\mu} &= Y_D \left[3y_t^2 - \left(\frac{9}{20}g_1^2 + \frac{9}{4}g_2^2 \right) + 2 \left(\frac{9}{2}|Y_S|^2 + \frac{17}{4}|Y_D|^2 + \frac{22}{3}|Y_T|^2 \right) \right. \\ &\quad \left. - 6g_2^2 - \frac{18}{5} \left(Q - \frac{1}{2} \right) Q g_1^2 \right], \\ 16\pi^2 \frac{dY_T}{d\ln\mu} &= Y_T \left[3y_t^2 - \left(\frac{9}{20}g_1^2 + \frac{9}{4}g_2^2 \right) + 2 \left(2|Y_S|^2 + \frac{11}{2}|Y_D|^2 + \frac{31}{6}|Y_T|^2 \right) \right. \\ &\quad \left. - 15g_2^2 - \frac{18}{5}Q \left(Q + \frac{1}{2} \right) g_1^2 \right].\end{aligned}\quad (5.10)$$

As in the analysis for the bulk **6**-plet fermions, we numerically solve the SM RG equations from M_t to M , at which the solutions connect with the solutions of the RG equations with the bulk **10**-plet fermions. For a fixed M values, we arrange input values of $|Y_S(M)|$, $|Y_D(M)|$ and $Y_T(M)$ so as to find numerical solutions which satisfy the gauge-Higgs condition and the unification condition among the gauge and Yukawa couplings such that

$$\lambda(M_{\text{KK}}) = 0, \quad \sqrt{\frac{2}{3}}Y_S(M_{\text{KK}}) = \sqrt{\frac{1}{2}}Y_D(M_{\text{KK}}) = \sqrt{\frac{2}{3}}Y_T(M_{\text{KK}}) = -ig_2(M_{\text{KK}}). \quad (5.11)$$

The RG evolution of Higgs quartic coupling to reproduce the Higgs boson mass $m_H = 125.09$ GeV is shown in Fig. 3. The dotted line denotes the running quartic coupling in the SM, while

the dashed (solid) line corresponds to the result for the case with a pair of **6**-plet (**10**-plet) bulk fermions. For the dashed (solid) line, we find $M = 5$ TeV for $M_{\text{KK}} = 91$ (8.2) TeV, at which the gauge-Higgs condition is satisfied. When we trace the dashed and solid lines from M_t to higher energies we see that the running of the Higgs quartic coupling is drastically altered from the SM one (dotted line) due to the contributions from the bulk fermions with $M = 5$ TeV.

For a fixed M value, we numerically find a M_{KK} value. The relation between M and M_{KK} is depicted in Fig. 4. In the left (right) panel we show the relation in the **6**-plet (**10**-plet) case. Thanks to the contributions to the RG equations from the bulk fermions in higher-dimensional representations, no hierarchy between M and M_{KK} is needed to reproduce the 125 GeV Higgs boson mass. This is in sharp contrast to the case with the bulk $SU(3)$ triplet fermions considered in Ref. [10], where we have found $M_{\text{KK}} = \mathcal{O}(10^8)$ GeV for $M = \mathcal{O}(1)$ TeV. Such a compactification scale is too high for the GHU model to be natural. This unnaturalness of the model is significantly relaxed in the presence of the bulk fermions in higher-dimensional representations. This is a main point of this paper. In Sec. 3, we have found the parameter regions of $6.97 \leq M[\text{TeV}] \leq 15.1$ and $10.5 \leq M[\text{TeV}] \leq 22.6$ for the **6**-plet and **10**-plet cases, respectively, from the constraints on the DM relic abundance and the direct DM detection cross section. These M regions correspond to $137 \leq M_{\text{KK}}[\text{TeV}] \leq 238$ and $20.2 \leq M_{\text{KK}}[\text{TeV}] \leq 41.1$ for the **6**-plet and **10**-plet cases, respectively,

6 Conclusions

In this paper, we have proposed a MDM scenario in the context of a 5-dimensional GHU model based on the gauge group $SU(3) \times U(1)'$ with a compactification of the 5th dimension on S^1/Z_2 orbifold. We have introduced a pair of bulk $SU(3)$ multiplet fermions in higher-dimensional representations, along with a bulk mass term, Majorana mass terms at an orbifold fixed point, and a periodic boundary condition. Associated with the breaking of the $SU(3) \times U(1)'$ into the SM $SU(2) \times U(1)_Y$ gauge group by a non-trivial orbifold boundary condition, the DM particle is provided as a linear combinations of the electric charge neutral components in the multiplets. Since the DM particle communicates with the SM particles only through the bulk gauge interactions in the original framework, our model is the GHU version of the MDM scenario.

We have two typical cases for the constituent of the DM particles. One is that the DM

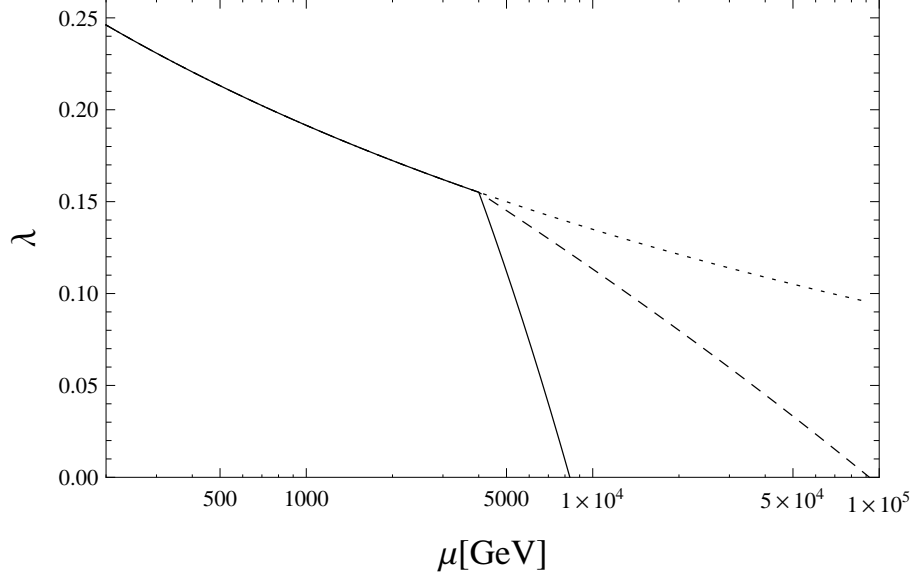


Figure 3: RG evolution of the Higgs quartic coupling with the bulk mass $M = 5$ TeV for the **6**-plet case (dashed line) and the **10**-plet case (solid line), along with the result in the SM (dotted line). The compactification scale is found to be $M_{\text{KK}} = 91$ TeV and 8.2 TeV for the **6**-plets and the **10**-plets, respectively, where the gauge-Higgs condition $\lambda(M_{\text{KK}}) = 0$, $|Y_S(M_{\text{KK}})| = g_2(M_{\text{KK}})/\sqrt{2}$ for the **6**-plets, and $\lambda(M_{\text{KK}}) = 0$, $\sqrt{\frac{2}{3}}Y_S(M_{\text{KK}}) = \sqrt{\frac{1}{2}}Y_D(M_{\text{KK}}) = \sqrt{\frac{2}{3}}Y_T(M_{\text{KK}}) = -ig_2(M_{\text{KK}})$ for the **10**-plets are satisfied.

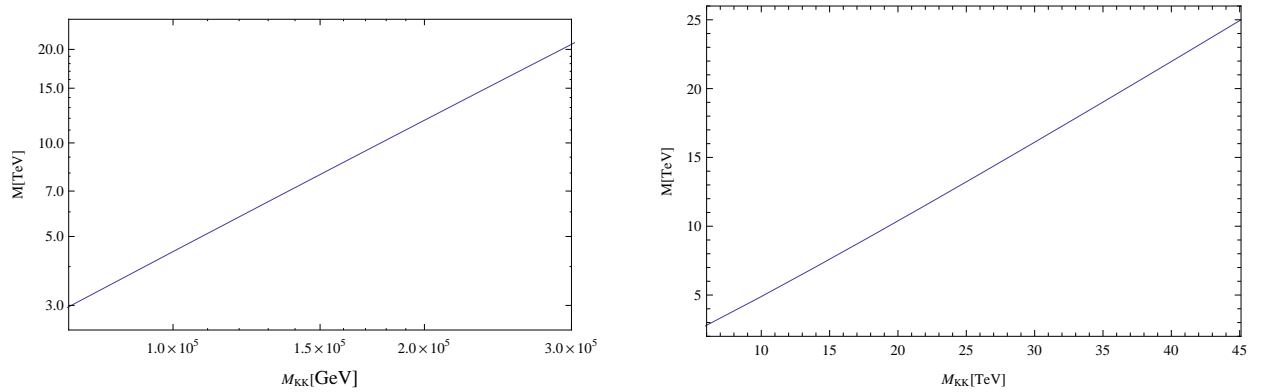


Figure 4: The relation between the bulk mass (M) and the compactification scale (M_{KK}) in the **6**-plet case (left) and **10**-plet case (right) so as to reproduce the 125 GeV Higgs boson mass.

particle is mostly a linear combinations of the electric charge neutral components in the SM $SU(2)$ multiplets. The other one is that the DM particle is mostly a singlet under the SM gauge group. The first one is very analogous to the Higgsino/wino-like neutralino DM in the minimal supersymmetric SM. Since the Higgsino/wino-like neutralino DM has been well studied, we have focused on the latter case, in which the DM particle communicates with the SM particles through the Higgs boson, namely, the Higgs-portal DM scenario. As an example, we have investigated the cases with the **6**-plet and the **10**-plet bulk fermions separately. We have identified an allowed parameter region to be consistent with the current experimental constraints, which will be fully covered by the direct dark matter detection experiments in the near future.

Employing the effective theoretical approach with the gauge-Higgs condition, we have also studied the RG evolution of Higgs quartic coupling and shown that the observed Higgs mass of 125 GeV is reproduced with the compactification scale of $M_{\text{KK}} = \mathcal{O}(100)$ TeV ($\mathcal{O}(10)$ TeV) for the case with the **6**-plets (**10**-plets), while satisfying the constraints from the DM relic abundance and the XENON 1T result. Comparing this result with $M_{\text{KK}} \simeq 10^8$ GeV that we have previously obtained in the case with a pair of bulk $SU(3)$ triplet fermions [10], the unnaturality of the model with $M_{\text{KK}} \gg 1$ TeV is drastically relaxed thanks to the bulk multiplets in higher-dimensional representations.

Finally, we comment on other possibilities for the realization of DM particle in our model. In this paper, we assign the $U(1)'$ charges of $2/3$ and -1 for the bulk **6**-plets and **10**-plets, respectively. In the SM gauge group decomposition, this charge assignment leads to fermions singlet under the SM gauge group, which are constituents of the DM particle. In general, there are other charge assignments to provide electric-charge neutral particle in the SM gauge group decomposition as shown in Table 1. Here, the DM particle is embedded in the SM $SU(2)$ multiplet(s), and this is a realization of the MDM scenario [9] as 4-dimensional effective theory of the GHU model.

Acknowledgments

S.O. would like to thank the Department of Physics and Astronomy at the University of Alabama for hospitality during her visit. This work is supported in part by JSPS KAKENHI Grant Number JP17K05420 (N.M.) and the DOE Grant Number DE-SC0012447 (N.O.).

$U(1)'$	$\mathbf{6} = \mathbf{1} \oplus \mathbf{2} \oplus \mathbf{3}$
$\frac{2}{3}$	$(0)_0 + \begin{pmatrix} 1 \\ 0 \end{pmatrix}_{1/2} + \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}_1$
$-\frac{1}{3}$	$(-1)_{-1} + \begin{pmatrix} 0 \\ -1 \end{pmatrix}_{-1/2} + \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}_0$
$-\frac{4}{3}$	$(-2)_{-2} + \begin{pmatrix} -1 \\ -2 \end{pmatrix}_{-\frac{3}{2}} + \begin{pmatrix} 0 \\ -1 \\ -2 \end{pmatrix}_{-1}$
	$\mathbf{10} = \mathbf{1} \oplus \mathbf{2} \oplus \mathbf{3} \oplus \mathbf{4}$
1	$(0)_0 + \begin{pmatrix} 1 \\ 0 \end{pmatrix}_{\frac{1}{2}} + \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}_1 + \begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \end{pmatrix}_{\frac{3}{2}}$
0	$(-1)_{-1} + \begin{pmatrix} 0 \\ -1 \end{pmatrix}_{-\frac{1}{2}} + \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}_0 + \begin{pmatrix} 2 \\ 1 \\ 0 \\ -1 \end{pmatrix}_{\frac{1}{2}}$
-2	$(-2)_{-2} + \begin{pmatrix} -1 \\ -2 \end{pmatrix}_{-\frac{3}{2}} + \begin{pmatrix} 0 \\ -1 \\ -2 \end{pmatrix}_{-1} + \begin{pmatrix} 1 \\ 0 \\ -1 \\ -2 \end{pmatrix}_{-\frac{1}{2}}$
-3	$(-3)_{-3} + \begin{pmatrix} -2 \\ -3 \end{pmatrix}_{-\frac{5}{2}} + \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}_{-2} + \begin{pmatrix} 0 \\ -1 \\ -2 \\ -3 \end{pmatrix}_{-\frac{3}{2}}$

(6.1)

Table 1: General $U(1)'$ charge assignments providing the DM candidates (electric charge neutral components) in the SM $SU(2) \times U(1)_Y$ decomposition of the $\mathbf{6}$ -plet and $\mathbf{10}$ -plets of the bulk $SU(3)$ gauge group. The numbers in the parenthesis denote the electric charges for the corresponding components of the multiplets. Subscripts of the multiplets are their hyper-charges.

References

- [1] N. S. Manton, “A New Six-Dimensional Approach To The Weinberg-Salam Model,” Nucl. Phys. B **158**, 141 (1979); D. B. Fairlie, “Higgs’ Fields And The Determination Of The Weinberg Angle,” Phys. Lett. B **82**, 97 (1979), “Two Consistent Calculations Of The Weinberg Angle,” J. Phys. G **5**, L55 (1979); Y. Hosotani, “Dynamical Mass Generation By Compact Extra Dimensions,” Phys. Lett. B **126**, 309 (1983); “Dynamical Gauge Symmetry Breaking As The Casimir Effect,” Phys. Lett. B **129**, 193 (1983); “DYNAMICS OF NONINTEGRABLE PHASES AND GAUGE SYMMETRY BREAKING,” Annals Phys. **190**, 233 (1989).
- [2] I. Antoniadis, K. Benakli and M. Quiros, “Finite Higgs mass without supersymmetry,” New J. Phys. **3**, 20 (2001) [arXiv:hep-th/0108005]; G. von Gersdorff, N. Irges and M. Quiros, “Bulk and brane radiative effects in gauge theories on orbifolds,” Nucl. Phys. B **635**, 127 (2002) [arXiv:hep-th/0204223]; R. Contino, Y. Nomura and A. Pomarol, “Higgs as a holographic pseudo-Goldstone boson,” Nucl. Phys. B **671**, 148 (2003) [arXiv:hep-ph/0306259]; C. S. Lim, N. Maru and K. Hasegawa, “Six Dimensional Gauge-Higgs Unification with an Extra Space S^{**2} and the Hierarchy Problem,” J. Phys. Soc. Jap. **77**, 074101 (2008) [hep-th/0605180].
- [3] N. Maru and T. Yamashita, “Two-loop calculation of Higgs mass in gauge-Higgs unification: 5D massless QED compactified on S^{**1} ,” Nucl. Phys. B **754**, 127 (2006) [arXiv:hep-ph/0603237]; Y. Hosotani, N. Maru, K. Takenaga and T. Yamashita, “Two loop finiteness of Higgs mass and potential in the gauge-Higgs unification,” Prog. Theor. Phys. **118**, 1053 (2007) [arXiv:0709.2844 [hep-ph]].
- [4] N. Maru and N. Okada, “Gauge-Higgs Unification at LHC,” Phys. Rev. D **77**, 055010 (2008) [arXiv:0711.2589 [hep-ph]].
- [5] N. Maru, “Finite Gluon Fusion Amplitude in the Gauge-Higgs Unification,” Mod. Phys. Lett. A **23**, 2737 (2008) [arXiv:0803.0380 [hep-ph]].
- [6] N. Maru and N. Okada, “Diphoton decay excess and 125 GeV Higgs boson in gauge-Higgs unification,” Phys. Rev. D **87**, no. 9, 095019 (2013) [arXiv:1303.5810 [hep-ph]].

- [7] Y. Adachi, C. S. Lim and N. Maru, “Finite Anomalous Magnetic Moment in the Gauge-Higgs Unification,” *Phys. Rev. D* **76**, 075009 (2007); [arXiv:0707.1735 [hep-ph]].
Y. Adachi, C. S. Lim and N. Maru, “More on the Finiteness of Anomalous Magnetic Moment in the Gauge-Higgs Unification,” *Phys. Rev. D* **79**, 075018 (2009) [arXiv:0901.2229 [hep-ph]].
- [8] Y. Adachi, C. S. Lim and N. Maru, “Neutron Electric Dipole Moment in the Gauge-Higgs Unification,” *Phys. Rev. D* **80**, 055025 (2009) [arXiv:0905.1022 [hep-ph]].
- [9] M. Cirelli, N. Fornengo and A. Strumia, “Minimal dark matter,” *Nucl. Phys. B* **753**, 178 (2006) [hep-ph/0512090].
- [10] N. Maru, T. Miyaji, N. Okada and S. Okada, “Fermion Dark Matter in Gauge-Higgs Unification,” *JHEP* **1707**, 048 (2017) [arXiv:1704.04621 [hep-ph]].
- [11] D. S. Akerib *et al.* [LUX Collaboration], “Results from a search for dark matter in the complete LUX exposure,” *Phys. Rev. Lett.* **118**, no. 2, 021303 (2017) [arXiv:1608.07648 [astro-ph.CO]].
- [12] M. Szydagis [LUX and LZ Collaborations], “The Present and Future of Searching for Dark Matter with LUX and LZ,” *PoS ICHEP* **2016**, 220 (2016) [arXiv:1611.05525 [astro-ph.CO]].
- [13] N. Maru and N. Okada, “125 GeV Higgs Boson and TeV Scale Colored Fermions in Gauge-Higgs Unification,” arXiv:1310.3348 [hep-ph]; J. Carson and N. Okada, “125 GeV Higgs boson mass from 5D gauge-Higgs unification,” arXiv:1510.03092 [hep-ph].
- [14] M. Regis, M. Serone and P. Ullio, “A Dark Matter Candidate from an Extra (Non-Universal) Dimension,” *JHEP* **0703**, 084 (2007) [hep-ph/0612286]; G. Panico, E. Ponton, J. Santiago and M. Serone, “Dark Matter and Electroweak Symmetry Breaking in Models with Warped Extra Dimensions,” *Phys. Rev. D* **77**, 115012 (2008) [arXiv:0801.1645 [hep-ph]]; M. Carena, A. D. Medina, N. R. Shah and C. E. M. Wagner, “Gauge-Higgs Unification, Neutrino Masses and Dark Matter in Warped Extra Dimensions,” *Phys. Rev. D* **79**, 096010 (2009) [arXiv:0901.0609 [hep-ph]]; Y. Hosotani, P. Ko and M. Tanaka, “Stable Higgs Bosons as Cold Dark Matter,” *Phys. Lett. B* **680**, 179 (2009) [arXiv:0908.0212 [hep-ph]].

- [hep-ph]]; N. Haba, S. Matsumoto, N. Okada and T. Yamashita, “Gauge-Higgs Dark Matter,” JHEP **1003**, 064 (2010) [arXiv:0910.3741 [hep-ph]].
- [15] C. A. Scrucca, M. Serone and L. Silvestrini, “Electroweak symmetry breaking and fermion masses from extra dimensions,” Nucl. Phys. B **669**, 128 (2003) [hep-ph/0304220].
- [16] G. Cacciapaglia, C. Csaki and S. C. Park, “Fully radiative electroweak symmetry breaking,” JHEP **0603**, 099 (2006) [hep-ph/0510366].
- [17] C. Patrignani *et al.* [Particle Data Group], “Review of Particle Physics,” Chin. Phys. C **40**, no. 10, 100001 (2016).
- [18] N. Arkani-Hamed, A. Delgado and G. F. Giudice, “The Well-tempered neutralino,” Nucl. Phys. B **741**, 108 (2006) [hep-ph/0601041].
- [19] T. Cohen, J. Kearney, A. Pierce and D. Tucker-Smith, “Singlet-Doublet Dark Matter,” Phys. Rev. D **85**, 075003 (2012) [arXiv:1109.2604 [hep-ph]]; C. Cheung and D. Sanford, “Simplified Models of Mixed Dark Matter,” JCAP **1402**, 011 (2014) [arXiv:1311.5896 [hep-ph]]; A. Dedes and D. Karamitros, “Doublet-Triplet Fermionic Dark Matter,” Phys. Rev. D **89**, no. 11, 115002 (2014) [arXiv:1403.7744 [hep-ph]]; A. Freitas, S. Westhoff and J. Zupan, “Integrating in the Higgs Portal to Fermion Dark Matter,” JHEP **1509**, 015 (2015) [arXiv:1506.04149 [hep-ph]]; C. E. Yaguna, “Singlet-Doublet Dirac Dark Matter,” Phys. Rev. D **92**, no. 11, 115002 (2015) [arXiv:1510.06151 [hep-ph]]; T. M. P. Tait and Z. H. Yu, JHEP **1603**, 204 (2016) [arXiv:1601.01354 [hep-ph]].
- [20] N. Aghanim *et al.* [Planck Collaboration], “Planck 2015 results. XI. CMB power spectra, likelihoods, and robustness of parameters,” Astron. Astrophys. **594**, A11 (2016) [arXiv:1507.02704 [astro-ph.CO]].
- [21] K. Bora, “Updated values of running quark and lepton masses at GUT scale in SM, 2HDM and MSSM,” Horizon **2** (2013) [arXiv:1206.5909 [hep-ph]].
- [22] A. Denner, S. Heinemeyer, I. Puljak, D. Rebuszi and M. Spira, “Standard Model Higgs-Boson Branching Ratios with Uncertainties,” Eur. Phys. J. C **71**, 1753 (2011) [arXiv:1107.5909 [hep-ph]].

- [23] E. Aprile *et al.* [XENON Collaboration], “First Dark Matter Search Results from the XENON1T Experiment,” arXiv:1705.06655 [astro-ph.CO].
- [24] H. Ohki *et al.*, “Nucleon sigma term and strange quark content from lattice QCD with exact chiral symmetry,” Phys. Rev. D **78**, 054502 (2008) [arXiv:0806.4744 [hep-lat]].
- [25] J. M. Alarcon, J. Martin Camalich and J. A. Oller, “The chiral representation of the πN scattering amplitude and the pion-nucleon sigma term,” Phys. Rev. D **85**, 051503 (2012) [arXiv:1110.3797 [hep-ph]]; J. M. Alarcon, L. S. Geng, J. Martin Camalich and J. A. Oller, “The strangeness content of the nucleon from effective field theory and phenomenology,” Phys. Lett. B **730**, 342 (2014) [arXiv:1209.2870 [hep-ph]].
- [26] R. J. Crewther, “Nonperturbative evaluation of the anomalies in low-energy theorems,” Phys. Rev. Lett. **28**, 1421 (1972); M. S. Chanowitz and J. R. Ellis, “Canonical Anomalies and Broken Scale Invariance,” Phys. Lett. **40B**, 397 (1972); “Canonical Trace Anomalies,” Phys. Rev. D **7**, 2490 (1973); J. C. Collins, A. Duncan and S. D. Joglekar, “Trace and Dilatation Anomalies in Gauge Theories,” Phys. Rev. D **16**, 438 (1977); M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, “Remarks on Higgs Boson Interactions with Nucleons,” Phys. Lett. **78B**, 443 (1978).
- [27] N. Haba, S. Matsumoto, N. Okada and T. Yamashita, “Effective theoretical approach of Gauge-Higgs unification model and its phenomenological applications,” JHEP **0602**, 073 (2006) [hep-ph/0511046]; “Effective Potential of Higgs Field in Warped Gauge-Higgs Unification,” Prog. Theor. Phys. **120**, 77 (2008) [arXiv:0802.3431 [hep-ph]].
- [28] M. E. Machacek and M. T. Vaughn, “Two Loop Renormalization Group Equations In A General Quantum Field Theory. 1. Wave Function Renormalization,” Nucl. Phys. B **222**, 83 (1983); “Two Loop Renormalization Group Equations In A General Quantum Field Theory. 2. Yukawa Couplings,” Nucl. Phys. B **236**, 221 (1984); “Two Loop Renormalization Group Equations In A General Quantum Field Theory. 3. Scalar Quartic Couplings,” Nucl. Phys. B **249**, 70 (1985); C. Ford, I. Jack and D. R. T. Jones, “The Standard Model Effective Potential at Two Loops,” Nucl. Phys. **B387** (1992) 373, [Erratum-ibid. **B504** (1997) 551]; H. Arason, D. J. Castano, B. Keszthelyi, S. Mikaelian, E. J. Piard, P. Ramond and B. D. Wright, Phys. Rev. D **46**, 3945 (1992); V. D. Barger, M. S. Berger and

- P. Ohmann, “Supersymmetric grand unified theories: Two loop evolution of gauge and Yukawa couplings,” *Phys. Rev. D* **47**, 1093 (1993); M. X. Luo and Y. Xiao, “Two-loop renormalization group equations in the standard model,” *Phys. Rev. Lett.* **90**, 011601 (2003).
- [29] D. Buttazzo, G. Degrandi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio and A. Strumia, “Investigating the near-criticality of the Higgs boson,” *JHEP* **1312**, 089 (2013) [arXiv:1307.3536 [hep-ph]].
- [30] [ATLAS and CDF and CMS and D0 Collaborations], “First combination of Tevatron and LHC measurements of the top-quark mass,” arXiv:1403.4427 [hep-ex].
- [31] G. Aad *et al.* [ATLAS and CMS Collaborations], “Combined Measurement of the Higgs Boson Mass in pp Collisions at $\sqrt{s} = 7$ and 8 TeV with the ATLAS and CMS Experiments,” *Phys. Rev. Lett.* **114**, 191803 (2015) [arXiv:1503.07589 [hep-ex]].