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Localization in the Rindler Wedge

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One of the striking features of QED is that charged particles create a coherent cloud of photons. The resultant coherent state vectors of photons generate a non-trivial representation of the localized algebra of observables that do not support a representation of the Lorentz group: Lorentz symmetry is spontaneously broken. We show in particular that Lorentz boost generators diverge in this representation, a result shown also in [1] (See also [2]). Localization of observables, for example in the Rindler wedge, uses Poincaré invariance in an essential way [3]. Hence in the presence of charged fields, the photon observables cannot be localized in the Rindler wedge.

These observations may have a bearing on the black hole information loss paradox, as the physics in the exterior of the black hole has points of resemblance to that in the Rindler wedge.

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I. INTRODUCTION

In Mikowski space, the vacuum state is known to become thermal or KMS for massive neutral fields restricted to a Rindler wedge. These fields are associated with uniformly accelerated particles. If the acceleration is in the 1-direction, the thermal or modular Hamiltonian is the boost K_1 in the 1-direction. We argue that if the fields are charged, for the photons, K_1 diverges and in fact all components K_i of $\mathbf{K}$ diverge. The reason is that the photon vacuum becomes dressed with an infrared cloud and breaks Lorentz invariance. Photon observables cannot thus be localized in the wedge in the presence of charged fields.

The work of [1] also shows a similar divergence of boosts (See also [2]). But the emphasis in that paper is on the breakdown of Lorentz invariance and not on localization problems as in this paper. Also the in state vector considered here is different from the state vector considered there for showing this divergence.

A consequence of this result is that the standard Tomita-Takesaki theory for the "symplectic" localization of observables [3] in a Lorentz covariant manner breaks down for charged fields.

These results may have a bearing on the information loss paradox for black holes.

Elsewhere [4] we have argued that equations of motion of electromagnetic fields generated by charged particles cannot be localized in the Rindler wedge because the charged particle itself is not localized.

II. THE RINDLER WEDGE FOR NEUTRAL FIELDS

The standard Rindler wedge W_1 in Minkowski space M_4 is the submanifold

$$W_1 = \{x = (x^0, x^1, x^2, x^3) \in M_4 : \; x^1 \geq |x^0|\} \tag{1}$$

Its causal complement is the opposite wedge W_1' (prime denoting causal complement),

$$W_1' = \{x \in M_4 : \; -x^1 \geq |x^0|\}. \tag{2}$$

For neutral free fields, there is a rigorous theory of localization in such wedges (and their intersections. See [3] and references therein.). It associates algebras of local observables A_W and $A_{W'}$ of W and W', respectively, compatibly with Poincaré covariance and causality. Thus this theory incorporates covariance and causality.

This theory of localization, called “modular localization”, is based in particular on the representation of the Poincaré group on the quantum fields. The construction of A_{W_1} for example uses the boost generator K_1.

If there are charged fields and their photons, then because of infrared effects, Lorentz group is spontaneously broken [5]. In particular, we shall see that K_1 diverges. The implications is that localizations in W and W' break down.

From another point of view [4], we have argued that equations of motion of charged field cannot be localized in W. We suspect that these results have implications for the black hole information paradox.

III. ON MODULAR LOCALIZATION

In non-relativistic quantum physics, given the spatial regions O_1 and O_2 at a fixed time with $O_1 \cap O_2 = \varnothing$, we have projection operators P_1 and P_2 such that $P_1 P_2 = 0$. Hence it is enough to set $\psi_1 = P_1\chi,\ \psi_2 = P_2\chi'$ for generic wave functions χ, χ' to see that there are wave functions ψ_1 and ψ_2 localized in O_1 and O_2 which are orthogonal, $\langle\psi_2|\psi_1\rangle = 0$. Such a localization is known as “Born localization”.

Let us next turn to relativistic quantum field theory and assume for the rest of this section that there are no infrared effects. Let us also denote by W the standard Rindler wedge (1), and by W' its causal complement (2).

As discussed by many authors [3], in relativistic physics, we cannot localize states. We can only localize algebras of observables in the “symplectic” or “modular” sense. That means the following in the present context: we can associate algebras of observables A_W and $A_{W'}$ to W and W' which are compatible with causality, that is, if ψ_W and $\psi_{W'}$ are elements of A_W and $A_{W'}$, then $[\psi_W, \psi_{W'}] = 0$. This association is also compatible with covariance as we presently discuss.

Thus in modular localization theory, we have a family of spacetime regions O_i to which one assigns the algebras of observables A_{O_i}. The regions O_i are obtained from W and W' by transforming them by the elements of the Poincaré group $\mathcal{P}_+ = \{g\}$ consisting of the connected Poincaré group and CPT and then by taking all their intersections. The algebras of observables A_{O_i} are such that we have

1. *covariance*: we have a representation $g \to U(g)$ of the Poincaré group $\mathcal{P}_+$ such that if

$g \cdot O$ is the Poincaré transform of O, then $A_{g \cdot O} = U(g) A_O U(g)^{-1}$;

2. *causality*: the algebra $A_{O'}$ is the commutant A'_O of A_O;

3. *isotony*: if $O_1 \subset O_2$, then $A_{O_1} \subseteq A_{O_2}$ (We will not discuss isotony further)

For our purposes in this paper, it is enough to consider A_W and $A_{W'}$. Let us first consider A_W and a free massive real scalar field φ. Let $\{f_W\}$ be a collection of smooth real test functions supported on W. Then the transformation

$$\mathcal{J}_W : (x^0, x^1, x^2, x^3) \;\rightarrow\; (-x^0, -x^1, x^2, x^3) \tag{3}$$

transforms $\{f_W\}$ to the test functions $\{f_{W'}\} = \{\mathcal{J}_W f_W\}$ supported in W'. In quantum theory $\mathcal{J}_W$ becomes

$$U(\mathcal{J}_W) \equiv J_W = \text{ CPT} \times \pi\text{-rotation around 1-axis.} \tag{4}$$

The algebra A_W is generated by

$$\varphi(f_W) \equiv \int d^4x \; f_W(x) \varphi(x), \tag{5}$$

or rather the unitaries $e^{i\varphi(f_W)}$, while $A_{W'}$ is generated by

$$J_W e^{i\varphi(f_W)} J_W^{-1} = e^{-i\varphi(\mathcal{J}_W f_W)}, \tag{6}$$

so that covariance is satisfied.

Since $[\varphi(x), \varphi(y)] = 0$ if x and y are spacelike separated, causality is also fulfilled.

There is thus a consistent assignment of A_W and $A_{W'}$: it is covariant and causal.

Let us ignore the transverse coordinates x^2 and x^3 in test functions and study this localization further. Since, with $K_1 \equiv K_W$,

$$e^{itK_W} : \; (x^0, x^1) \;\rightarrow\; (x^0 \cosh t - x^1 \sinh t, -x^0 \sinh t + x^1 \cosh t), \tag{7}$$

we have as $t \uparrow i\pi$,

$$e^{-\pi K_W} : (x^0, x^1) \rightarrow (-x^0, -x^1). \tag{8}$$

In quantum theory, $\mathcal{J}_W$ is represented by an anti-unitary operator J_W and

$$e^{-\pi K_W} \longrightarrow U(e^{-\pi K_W}) = \Delta_W^{1/2}. \tag{9}$$

Set

$$S_W \equiv J_W \Delta_W^{1/2}. \tag{10}$$

We remark that the continuation of t to $i\pi$ requires a positive energy representation U. See [3].

The effect of $\mathcal{J}_W$ is compensated by $e^{-\pi K_W}$, so that $\mathcal{J}_W e^{-\pi K_W}$ acts as identity on (x^0, x^1). Hence since $\varphi_W^* = \varphi_W$ (φ_W being a real field) and $\overline{f}_W = f_W$,

$$S_W \varphi(f_W) S_W^{-1} = \varphi(f_W). \tag{11}$$

We consider only free fields. Then since $\varphi(x)$ is linear in creation and annihilation operators, so is $\varphi(f_W)$ and

$$\varphi(f_W)|0\rangle \tag{12}$$

is a one-particle subspace.

Now, by (3) and (7),

$$\mathcal{J}_W e^{itK_W} = e^{itK_W} \mathcal{J}_W, \tag{13}$$

so that since J_W is anti-unitary,

$$J_W \Delta_W^{1/2} = \Delta_W^{-1/2} J_W \tag{14}$$

and so

$$S_W^2 = \mathbb{1}. \tag{15}$$

Further, by the Lorentz invariance of the vacuum,

$$J_W|0\rangle, \quad \Delta_W^{1/2}|0\rangle, \quad S_W|0\rangle \text{ are all } = |0\rangle. \tag{16}$$

Thus if $\mathcal{H}$ is the one-particle Hilbert space of Fock space, $\varphi(f_W)|0\rangle$ is a "real" subspace $\mathrm{Re}\mathcal{H}_W$ of $\mathcal{H}$:

$$S_W \varphi(f_W)|0\rangle = \varphi(f_W)|0\rangle. \tag{17}$$

It is real since S_W being anti-linear, $i\varphi(f_W)|0\rangle$ does not belong to this subspace $\mathrm{Re}\mathcal{H}_W$.

We can informally write

$$\mathrm{Re}\mathcal{H}_W = \frac{\mathbb{1} + S_W}{2} \mathcal{H}. \tag{18}$$

From $\mathrm{Re}\mathcal{H}_W$ we can construct A_W as Brunetti *et al.* (cf. [3]). discuss.

Summary

In the above we started by assuming that we have a free scalar field and arrived at S_W and therefrom at $\mathrm{Re}\mathcal{H}_W$. Since $\mathrm{Re}\mathcal{H}_W$ also determines A_W, we now have an approach to localization where we start from the one-particle representation ρ of the Poincaré group $\mathcal{P}_+$ on a complex Hilbert space $\mathcal{H}$. That supplies us with S_W and hence $\mathrm{Re}\mathcal{H}_W$ (18). From this we recover A_W, the algebra of local observables in the wedge W.

This approach is more intrinsic as it starts just from Wigner's representation theory of the Poincaré group. It can also be applied to the case where the covariance group is the conformal group [6]. It makes it clear that for localization in W compatibly with Poincaré covariance and causality, we need the existence of J_W and $\Delta_W^{1/2} = U\left(e^{-\pi K_W}\right)$.

IV. ON THE INFRARED EFFECT

We next consider a charged free massive scalar field φ of charge q. In this case, the Fock space states get dressed by an infrared factor which breaks Lorentz invariance.

Let

$$|0\rangle_\gamma |p\rangle \tag{19}$$

denote the state vector when photon is in the ground state and the free charged particle has momentum p. When the interaction is switched on, (19) leads to an in state, namely

$$|\mathrm{in}\rangle \equiv \Omega \; |0\rangle_\gamma |p\rangle, \tag{20}$$

where the calculation of the dressing factor Ω is indicated below.

Since we are interested in very soft photons, we can ignore back reactions and treat the charged particle as moving with momentum p. Then the current of the charged particle is

$$J^\mu(x) = q \int d\tau \; \delta^{(4)}(x - z(\tau)) \, \frac{dz^\mu}{d\tau}, \tag{21}$$

$$z^\mu(\tau) = \frac{p^\mu}{m} \, \tau. \tag{22}$$

The interaction term is thus

$$\int d^3x \; A_\mu(x) J^\mu(x), \tag{23}$$

where A_μ is the electromagnetic potential. This leads to

$$\Omega = \exp\left(-iq \int_{-\infty}^{0} dx^0 \, \frac{p^\mu}{m} \, A_\mu\left(\frac{p}{m} x^0\right)\right), \tag{24}$$

upto factors unimportant for us. This Ω was worked out in [7].

We will work in the radiation gauge $A_0 = \partial_i A^i = 0$ and in the interaction representation. Using the mode expansion of A_i,

$$A_i(x) = \int d\mu(\mathbf{k}) \ \left[a_i(\mathbf{k})e^{-ik\cdot x} + a_i^\dagger(\mathbf{k})e^{ik\cdot x}\right], \tag{25}$$

$$d\mu(\mathbf{k}) = \frac{d^3\mathbf{k}}{(2\pi)^{3/2}2k_0}, \tag{26}$$

$$\left[a_i(\mathbf{k}), a_j^\dagger(\mathbf{k}')\right] = (2\pi)^{3/2} \ 2k_0 \left(\delta_{ij} - \hat{k}_i\hat{k}_j\right) \delta^3(\mathbf{k} - \mathbf{k}'), \tag{27}$$

(with the rest of the commutators vanishing), we find

$$\Omega = \exp\left(q \int d\mu(\mathbf{k}) \left(a_i(\mathbf{k}) \ \hat{\omega}^i(k)^+ - a_i^\dagger(\mathbf{k}) \ \hat{\omega}^i(k)^-\right)\right), \tag{28}$$

$$\hat{\omega}^i(k)^\pm = \lim_{\varepsilon\downarrow 0} \frac{p_i - \mathbf{p}\cdot\hat{\mathbf{k}} \ \hat{k}_i}{k\cdot p + i\varepsilon}. \tag{29}$$

But since $k \cdot p > 0$ (k is light-like with $k^0 > 0$ and p is time-like with $p^0 > 0$), the $i\varepsilon$ can be dropped and we find dropping $\pm$ on $\hat{\omega}^i(k)^\pm$, that

$$\Omega = \exp\left(q \int d\mu(\mathbf{k}) \left(a_i(\mathbf{k}) - a_i^\dagger(\mathbf{k})\right) \hat{\omega}^i(k)\right). \tag{30}$$

Now,

$$\partial_0 A_i(x) = -i \int d\mu(\mathbf{k}) \ k_0 \ \left[a_i(\mathbf{k})e^{-ik\cdot x} - a_i^\dagger(\mathbf{k})e^{ik\cdot x}\right] = \text{Electric field } E_i. \tag{31}$$

We will return to this equation a little later.

Interpretation of (24)

Equation (24) is the exponential of the Dirac-Wilson line integral, but in the time-like direction. Thus,

$$\Omega = \exp\left(-iq \int_{-\infty}^{0} dz^\mu A_\mu(z)\right), \tag{32}$$

where z^μ is given in (22) with $\tau = x^0$, the time coordinate.

Under the gauge transformation

$$A_\mu \ \mapsto \ A_\mu + \partial_\mu\Lambda, \tag{33}$$

$$\Omega \ \mapsto \ \Omega \ e^{-iq\Lambda(0)} \ e^{+iq\Lambda(-\infty)}. \tag{34}$$

This shows that Ω is created by a charge q starting at time $-\infty$ and propagating to the origin at time 0.

V. THE BOOST IN THE INFRARED SECTOR

Let K_i be the Lorentz boosts in the Fock space. For the electromagnetic field, they are

$$K_i = \frac{1}{2}\int d^3x\; x_i \left[\mathbf{E}(x)^2 + \mathbf{B}(x)^2\right], \tag{35}$$

$$B_i = \varepsilon_{ijk} F_{jk}, \tag{36}$$

where $F_{jk} = \partial_j A_k - \partial_k A_j$ and E_i is the electric field conjugate to A_i:

$$[A_i(\mathbf{x},t), E_j(\mathbf{y},t)] = i\;\delta^T_{ij}(\mathbf{x}-\mathbf{y}), \tag{37}$$

where δ^T is the transverse δ-function,

$$\delta^T_{ij}(\mathbf{x}-\mathbf{y}) = \left(\delta_{ij} - \frac{\partial_{x_i}\partial_{x_j}}{\partial^2_{\mathbf{x}}}\right)\delta^3(\mathbf{x}-\mathbf{y}). \tag{38}$$

Then,

$$\Omega K_i \Omega^\dagger \tag{39}$$

acts on the in state vector.

The electric and magnetic fields E_i and B_i are shifted by the transformation (39). The shift of E_i is

$$\begin{aligned}\delta E_i &= \left[q\int d\mu(\mathbf{k}')\left(a_j(\mathbf{k}') - a_j(\mathbf{k}')^\dagger\right)\hat{\omega}^j(k'),\; i\int d\mu(\mathbf{k})k\left(a_i(\mathbf{k})^\dagger e^{ik\cdot x} - a_i(\mathbf{k}')e^{-ik\cdot x}\right)\right]\\ &= iq\int d\mu(\mathbf{k})\;k\;\hat{\omega}_i(k)\left(e^{ik\cdot x} - e^{-ik\cdot x}\right)\\ &\equiv \omega_i(x) - \omega_i(-x).\end{aligned} \tag{40}$$

A simple scaling argument shows that

$$\omega_i(\lambda x) \underset{\lambda\to\infty}{\sim} \mathcal{O}\left(\frac{1}{\lambda^2}\right), \tag{41}$$

or

$$\omega_i(x) \underset{x\to\infty}{\sim} \mathcal{O}\left(\frac{1}{x^2}\right). \tag{42}$$

Since

$$\frac{1}{p\cdot k} = \frac{1}{p_0 k_0 - \mathbf{p}\cdot\mathbf{k}} \tag{43}$$

is not even in $\mathbf{k}$, we do not expect the $\mathcal{O}(1/x^2)$ term to cancel (40). With that assumption, we find the following term in K_i to diverge logarithmically:

$$\int d^3x \; x_i \; \delta\vec{E}(x)^2. \tag{44}$$

After a cut-off, this term is positive.

If

$$\Omega B_i \Omega^{-1} = B_i + \delta B_i, \tag{45}$$

there is a similar contribution

$$\frac{1}{2}\int d^3x x_i \delta\vec{B}(x)^2 \tag{46}$$

from the $\vec{B}^2$-term. As it is also non-negative, it cannot cancel (44).

In [1], the divergence of K_i is shown for vectors obtained by replacing omega by another ("vertex") operator . Also that paper focuses on the breakdown of Lorentz invariance and not localization.

There is a physical interpretation of the above result. A Lorentz boost Λ transforms the photonic cloud of momentum p into the photonic cloud with momentum Λp. A consequence of this transformation law is that states of the coherent photon cloud do not belong to the domain of the infinitesimal generators of Lorentz boosts $\mathbf{K}$. The divergence found in the above calculation is also a proof of that behavior. An alternative argument can be obtained as follows. The expectation value of $\mathbf{K}$ in the photon cloud in particle mechanics is given by the sum of the contributions of each individual photon of the cloud. But that sum has the same degree of infrared divergence as

$$\langle N \rangle = \int d\mu(\mathbf{k}) \; n_{\mathbf{k}}, \tag{47}$$

$n_{\mathbf{k}}$ being the number of photons in the cloud with momentum $\mathbf{k}$. This is in agreement with the previous result. Notice that on the contrary the same coherent quantum state of the photon cloud belongs to the domain of the QED Hamiltonian. Indeed, once we renormalize the vacuum energy, the remaining energy is just the sum of the individual energies of each photon of the cloud

$$\mathcal{E} = \int d\mu(\mathbf{k}) k_0 n_{\mathbf{k}} < \infty, \tag{48}$$

which is finite.

This concludes our argument that modular localization fails for charged fields.

There is of course a general argument [5] that Lorentz invariance breaks down for charged sectors of QED. That is enough to affirm the failure of standard localization arguments for charged particles. The merit of this paper is perhaps the fact that it is explicit.

VI. REMARKS

It has been argued elsewhere [7] that non-abelian gauge theories, including QCD, breaks Lorentz invariance in sectors transforming non-trivially by the gauge group. Accordingly, standard localization arguments also fail in these sectors.

There is a striking resemblance between the Unruh effect and the physics of black holes. So we expect that our comments in this paper, which argue for the failure of localization arguments under generic conditions, have a bearing on the black hole information paradox.

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