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Deducing Cosmological Observables from the S-matrix

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ABSTRACT

We study one loop quantum gravitational corrections to the long range force induced by the exchange of a massless scalar between two massive scalars. The various diagrams contributing to the flat space S-matrix are evaluated in a general covariant gauge and we show that dependence on the gauge parameters cancels at a point considerably *before* forming the full S-matrix, which is unobservable in cosmology. It is possible to interpret our computation as a solution to the effective field equations — which could be done even in cosmology — but taking account of quantum gravitational corrections from the source and from the observer.

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1 Introduction

Primordial inflation produces a vast ensemble of long wave length gravitons, which is what causes the tensor power spectrum [1]. It is inconceivable that these gravitons simply exist, without interacting, at some level, with themselves and other particles. If such an ensemble were present today no one doubts that it would change the way particles propagate, or that it might affect the long range forces carried by virtual particles. Indeed, the effect of gravitational radiation on the propagation of photons is the basis for using pulsar timing to detect gravitational radiation [2, 3].

Although the actual geometry of inflation must show evolution, for many purposes one can employ the simpler, de Sitter geometry as a reasonable approximation. The effects of inflationary gravitons on a particle's kinematics, and on the force it carries, are studied in the same way. One first computes the one graviton loop correction to the particle's 1PI (one-particle-irreducible) 2-point function. Then one uses this result to quantum-correct the linearized effective field equation for the particle. Many effects have been studied in this way over the course of the past decade:

- Inflationary gravitons induce a progressive excitation of massless and light fermions which eventually becomes nonperturbatively strong [4, 5, 6, 7] due to the spin-spin coupling [8];
- Inflationary gravitons have little effect on massless, minimally coupled scalars [9, 10] owing to the absence of such a coupling;
- Inflationary gravitons secularly excite photons as they do fermions [11, 12, 13, 14], and they also induce secular modifications of electrodynamic forces [15];
- Inflationary gravitons secularly excite other gravitons [16, 17]; and
- Inflationary gravitons secularly excite conformally coupled scalars owing to the conformal coupling [18, 19, 20].

The physics behind these results seems plausible enough: inflationary gravitons scatter particles and force carriers, with the net deviation growing as the particle or force carrier propagates further. However, the reality of these effects is thrown into question by the notorious *gauge issue*. The graviton propagator depends upon an arbitrary gauge choice, which certainly affects full 1PI functions on flat space background. On the other hand, certain

parts of the flat 1PI N-point functions are gauge independent because sums of products of them combine to form the gauge independent S-matrix. So it seemed possible that the leading secular dependence on de Sitter background might be independent of the gauge [21].

Computations on de Sitter background are so terribly difficult that almost all work has been done in a single, particularly simple gauge [22, 23]. However, a determined effort at length produced a result for the graviton correction to the vacuum polarization [13] in a one-parameter family of de Sitter invariant gauges [24]. When this was used to quantum-correct Maxwell's equation the result was that dynamical photons suffer a progressive excitation which is independent of the gauge parameter [14]. The excitation has the same sign and time dependence as for the simple gauge [12], but the numerical coefficient is not quite the same. Hence it seems that even the leading secular effects of inflationary gravitons are somewhat gauge dependent.

On flat space background this sort of issue would be resolved by reference to the S-matrix, which is gauge independent. Unfortunately, that option is not available for inflationary cosmology. A synthetic S-matrix has been shown to exist for massive fields on de Sitter [25], but causality precludes an inflationary observer from making the global measurements it requires.

Another alternative would be to devise gauge invariant operators to quantify changes in particle kinematics and force laws, then compute the expectation values of these operators. One major disadvantage of this technique is that there are no local invariants in gravity, so any observables would be nonlocal composite operators. That vastly complicates renormalization. It is just possible to persevere through such a computation [26], but it seems worth looking for a simpler technique. It would be particularly nice to devise some way of modifying the effective field equations.

A promising approach is the one developed by Donoghue [27, 28], who noted that the leading quantum gravitational corrections to long range forces derive from a very special sort of nonanalytic correction to loop amplitudes. This is typically implemented by computing scattering amplitudes in Fourier momentum space, extracting the important contribution to the S-matrix, and then inferring corrections to the potential by inverse scattering. For example, this is how the first complete one loop computations were made of quantum gravitational corrections to the Newtonian potential [29, 30]. However, we suspect that the essential part of the technique can be separated from the full S-matrix, and phrased instead as a way of correcting the effective field equations — in position space — to include quantum gravitational correla-

tions with the source which disturbs the effective field and the observer who measures it.

To examine this possibility we have chosen to work on flat space background, with the object of computing the one graviton loop correction to the long range potential induced by a massless scalar. In section 2 we make the computation in the manner described above: calculating the scalar self-mass, then using it to quantum correct the effective field equation. By making the computation in the 2-parameter family of Poincaré invariant gauges we demonstrate that the result is highly gauge dependent. In section 3 we review an intriguing comment of the subject made by one of the great men of quantum gravity. In section 4 we abstract Donoghue's technique to position space, and show how it can be viewed as correcting the effective field equation. We explicitly demonstrate that the gauge dependence cancels when all the corrections are included. Section 5 summarizes what we have shown and discusses the implications for cosmology.

In addition to the obvious debt we owe to Donoghue, it should be noted that section 4 closely follows Bjerrum-Bohr's computation of the quantum gravitational correction to the Coulomb potential in SQED (scalar quantum electrodynamics) [31]. The relevant diagram topologies are the same once one replaces his photon lines with our massless scalar lines. Also, we merely translated to position space the three Fourier momentum integrals he used to extract the leading infrared contributions.

2 What We Wanted

In this section we discuss a simple, flat space analog of the sort of computations we have been doing of how inflationary gravitons change particle kinematics and force laws. The quantity we have chosen to correct is the long range force exerted by a massless scalar. We begin by reviewing the Feynman rules for the most general Poncaré invariant gauge. Then the one graviton loop contribution to the scalar self-mass is computed in dimensional regularization and fully renormalized. Finally, we use this result to quantum-correct the scalar field equation, and we solve for the response to a static point source.

2.1 Feynman Rules

The Lagrangian of gravity plus a massless, minimally coupled scalar is,

$$\mathcal{L}_1 = \frac{R\sqrt{-g}}{16\pi G} - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} \; . \tag{1}$$

We are perturbing around flat space with the usual definitions of the graviton field $h_{\mu\nu}$ and the loop counting parameter κ^2,

$$g_{\mu\nu}(x) \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu}(x) \qquad , \qquad \kappa^2 \equiv 16\pi G \; . \tag{2}$$

By convention graviton indices are raised and lowered using the Lorentz metric, $h^{\mu\nu} \equiv \eta^{\mu\rho}\eta^{\nu\sigma}h_{\rho\sigma}$, $h \equiv \eta^{\mu\nu}h_{\mu\nu}$. The expansions we require are,

$$\sqrt{-g} = 1 + \frac{1}{2}\kappa h + \ldots \qquad , \qquad g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \ldots \tag{3}$$

When using dimensional regularization on flat space background the order κ^2 interactions are not necessary for the sorts of diagrams we require.

To facilitate dimensional regularization we work in D-dimensional spacetime. Because temporal Fourier transforms are problematic in cosmology, we make this calculation in position space. The massless scalar propagator is,

$$i\Delta(x;x') = \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}}\Delta x^{D-2}} \quad , \quad \Delta x^\mu \equiv (x-x')^\mu \quad , \quad \Delta x^2 \equiv \eta_{\mu\nu}\Delta x^\mu \Delta x^\nu \; . \tag{4}$$

The scalar propagator obeys an equation of great significance for us,

$$\partial^2 i\Delta(x;x') = i\delta^D(x-x') \; . \tag{5}$$

We fix the gauge by adding to the Lagrangian the most general Poincaré invariant gauge fixing term,

$$\mathcal{L}_{GF} = -\frac{1}{2a}\eta^{\mu\nu}\mathcal{F}_\mu\mathcal{F}_\nu \qquad , \qquad \mathcal{F}_\mu \equiv \eta^{\rho\sigma}\Big(h_{\mu\rho,\sigma} - \frac{b}{2}h_{\rho\sigma,\mu}\Big) \; . \tag{6}$$

The resulting graviton propagator can be expressed in terms of the massless scalar propagator using the transverse projection operator $\Pi_{\mu\nu} \equiv \eta_{\mu\nu} - \partial_\mu\partial_\nu/\partial^2$ as [32],

$$\begin{aligned} i\Big[{}_{\mu\nu}\Delta_{\rho\sigma}\Big](x;x') = \Big\{ & 2\Pi_{\mu(\rho}\Pi_{\sigma)\nu} - \frac{2}{D-1}\Pi_{\mu\nu}\Pi_{\rho\sigma} \\ & - \frac{2}{(D-2)(D-1)}\Big[\eta_{\mu\nu} - \Big(\frac{Db-2}{b-2}\Big)\frac{\partial_\mu\partial_\nu}{\partial^2}\Big]\Big[\eta_{\rho\sigma} - \Big(\frac{Db-2}{b-2}\Big)\frac{\partial_\rho\partial_\sigma}{\partial^2}\Big] \\ & + 4a \times \frac{\partial_{(\mu}\Pi_{\nu)(\rho}\partial_{\sigma)}}{\partial^2} + \frac{4a}{(b-2)^2} \times \frac{\partial_\mu\partial_\nu\partial_\rho\partial_\sigma}{\partial^4}\Big\} i\Delta(x;x') \; . \end{aligned} \tag{7}$$

Here and henceforth parenthesized indices are symmetrized. The factors of $\partial_\mu \partial_\nu / \partial^2$ acting on the massless scalar propagator $i\Delta(x;x')$ can be written as [33],

$$\frac{\partial_\mu \partial_\nu}{\partial^2} i\Delta(x;x') = \frac{1}{2} \times \Bigl\{ \eta_{\mu\nu} - \frac{(D-2) \Delta x_\mu \Delta x_\nu}{\Delta x^2} \Bigr\} i\Delta(x;x') \; , \tag{8}$$

$$\frac{\partial_\mu \partial_\nu \partial_\rho \partial_\sigma}{\partial^4} i\Delta(x;x') = \frac{1}{8} \times \Bigl\{ 3 \eta_{(\mu\nu} \eta_{\rho\sigma)} - \frac{6(D-2) \eta_{(\mu\nu} \Delta x_\rho \Delta x_{\sigma)}}{\Delta x^2} + \frac{D(D-2) \Delta x_\mu \Delta x_\nu \Delta x_\rho \Delta x_\sigma}{\Delta x^4} \Bigr\} i\Delta(x;x') \; . \tag{9}$$

2.2 One Loop Self-Mass

The scalar self-mass $-iM^2(x;x')$ is the 1PI (1-particle irreducible) scalar 2-point function. The primitive one graviton loop correction to it is,

$$-iM^2(x;x') = \partial_\alpha \partial'_\beta \Bigl\{ i \Bigl[\mu\nu \Delta_{\rho\sigma} \Bigr](x;x') \times (-i\kappa) \Bigl[-\eta^{\alpha\mu} \partial^\nu + \frac{1}{2} \eta^{\mu\nu} \partial^\alpha \Bigr] \times (-i\kappa) \Bigl[-\eta^{\beta\rho} \partial'^\sigma + \frac{1}{2} \eta^{\rho\sigma} \partial'^\beta \Bigr] i\Delta(x;x') \Bigr\} . \tag{10}$$

After performing the tensor contractions and making use of the identities,

$$\frac{1}{\Delta x^{2D-2}} = \frac{\partial^2}{2(D-2)^2} \frac{1}{\Delta x^{2D-4}} \; , \tag{11}$$

$$\frac{\Delta x^\mu \Delta x^\nu}{\Delta x^{2D}} = \Bigl[\frac{\eta^{\mu\nu} \partial^2}{4(D-2)^2 (D-1)} + \frac{\partial^\mu \partial^\nu}{4(D-2)(D-1)} \Bigr] \frac{1}{\Delta x^{2D-4}} \; . \tag{12}$$

we reach an expression in terms of the square of scalar propagator (4),

$$-iM^2(x;x') = -\kappa^2 \mathcal{C}_0(a,b,D) \times \partial^4 \Bigl[i\Delta(x;x') \Bigr]^2 \; . \tag{13}$$

The gauge dependence resides in the multiplicative factor,

$$\mathcal{C}_0(a,b,D) = \frac{(D-2)(D+2)}{16} + \frac{1}{4} \Bigl(\frac{Db-2}{b-2} \Bigr) - \frac{1}{16} \Bigl(\frac{Db-2}{b-2} \Bigr)^2 - \frac{(D-2)(D-1)a}{8} + \frac{(D-2)(D-1)a}{8(b-2)^2} \; . \tag{14}$$

Expression (13) can be renormalized by extracting another d'Alembertian and then adding zero in the form of the equation (5) for $i\Delta(x;x')$ [34, 35],

$$\Big[i\Delta(x;x')\Big]^2 = \frac{\Gamma^2(\frac{D}{2}-1)}{16\pi^D} \frac{\partial^2}{2(D-3)(D-4)} \Big[\frac{1}{\Delta x^{2D-6}}\Big] \; , \tag{15}$$

$$= \frac{\Gamma^2(\frac{D}{2}-1)}{32(D-3)(D-4)\pi^D} \Bigg\{ \partial^2 \Big[\frac{1}{\Delta x^{2D-6}} - \frac{\mu^{D-4}}{\Delta x^{D-2}}\Big] + \frac{\mu^{D-4} 4\pi^{\frac{D}{2}} i\delta^D(x-x')}{\Gamma(\frac{D}{2}-1)} \Bigg\}, \tag{16}$$

$$= -\frac{\partial^2}{64\pi^4} \Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big] + \frac{\Gamma(\frac{D}{2}-1)\mu^{D-4} i\delta^D(x-x')}{8(D-3)(D-4)\pi^{\frac{D}{2}}} + O(D-4) \; . \tag{17}$$

The second term of (17) can be absorbed with a local (higher derivative) counterterm, which gives the renormalized result,

$$-iM^2_{\rm ren}(x;x') = \kappa^2 C_0(a,b) \times \frac{\partial^6}{64\pi^4} \Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big] \; . \tag{18}$$

The gauge dependent constant $C_0(a,b)$ is obtained by setting $D = 4$ in (14),

$$C_0(a,b) \equiv \mathcal{C}_0(a,b,4) = \frac{3}{4} \frac{(b-5)(b-1)}{(b-2)^2} - \frac{3}{4} \frac{(b-3)(b-1)a}{(b-2)^2} \; . \tag{19}$$

2.3 Effective Field Equation

The scalar self-mass is used to quantum correct its kinetic operator,

$$\partial^2\phi(x) \longrightarrow \partial^2\phi(x) - \int d^4x' \, M^2_{\rm ren}(x;x')\phi(x') \; . \tag{20}$$

We employ the Schwinger-Keldysh formalism [36, 37, 38, 39, 40] to obtain real and causal effective field equations. There are many good reviews on this subject[41, 42, 43] so we merely apply the well-known rules for converting an in-out result such as (18) into its Schwinger-Keldysh analog [44],

$$M^2_{\rm SK}(x;x') = -\kappa^2 C_0(a,b) \times \frac{\partial^8}{128\pi^3} \Bigg\{ \theta(\Delta t - \Delta r) \Bigg[\ln\Big[\mu^2(\Delta t^2 - \Delta r^2)\Big] - 1 \Bigg] \Bigg\}. \tag{21}$$

Here $\Delta t \equiv t - t'$ and $\Delta r \equiv \|\vec{x} - \vec{x}'\|$.

The equation which gives the effective scalar response to a static point source of unit strength is,

$$\partial^2\phi(x) - \int d^4x' \, M^2_{\rm SK}(x;x')\phi(x') = \delta^3(\vec{x}) \; . \tag{22}$$

Because the four factors of the d'Alermbertian in expression (21) could be considered as acting on either the primed or the un-primed coordinate, we can partially integrate one of them and extract the other three from the integration,

$$\partial^2\phi(x) + \frac{\kappa^2 C_0(a,b)\partial^6}{128\pi^3}\int d^4x' \Big\{ \qquad \Big\} \partial'^2\phi(x') = \delta^3(\vec{x}) = \partial^2\Big(-\frac{1}{4\pi r}\Big) \; , \quad (23)$$

where the curly bracketed terms of equations (21) and (23) are the same. Relation (23) is easy to recast as a perturbative solution for $\phi(x)$,

$$\begin{aligned} \phi(x) &= -\frac{1}{4\pi r} - \frac{\kappa^2 C_0(a,b)\partial^4}{128\pi^3}\int_{-\infty}^{t-r} dt' \Big[\ln\Big[\mu^2(\Delta t^2 - r^2)\Big] - 1\Big] + O(\kappa^4) \; , \quad (24) \\ &= -\frac{1}{4\pi r}\Big\{1 + \frac{\kappa^2 C_0(a,b)}{8\pi^2 r^2} + O(\kappa^4)\Big\} \; . \quad (25) \end{aligned}$$

Relation (25) purports to be the one loop quantum gravitational correction to the long range massless scalar potential induced by a static point source. Much of the result makes good sense. There should be a quantum gravitational correction to this potential because the tree order result distorts virtual gravitons in the vicinity of the source. The factional correction of κ^2/r^2 is dictated by dimensional analysis and the single loop counting parameter. However, the overall factor of $C_0(a,b)$ is completely unacceptable. By varying the parameters a and b in expression (19) we see that $C_0(a,b)$ can be made to range from $-\infty$ to $+\infty$!

3 DeWitt's Lost Theorem

This is not a new problem. As discussed in the Introduction, it is usually resolved by appealing to the S-matrix. However, in 1981 Bryce DeWitt made this intriguing statement about choosing different gauge fixing terms for the quantum correction Σ to the action in the background field formalism [45]:

> The functional form of Σ is not independent of the choice of these terms. However, the solutions of the *effective field equation*
>
> $$0 = \frac{\delta\Gamma}{\delta g_{\mu\nu}} = \frac{\delta S}{\delta g_{\mu\nu}} + \frac{\delta\Sigma}{\delta g_{\mu\nu}}$$
>
> can be shown to be the same for all choices.[15,16,20]

None of the references DeWitt cited[1] provides an explicit proof of this statement but we believe he was referring to how one uses asymptotic scattering data to parameterize solutions to the effective field equations.

The background field effective action is gauge invariant, but dependent upon the gauge which was used to compute quantum corrections to the classical action. When solving the resulting effective field equations for the metric, and other gauge fields, one must of course fix the gauge to get a definite solution, and the solution will depend in the usual way on that gauge choice. However, DeWitt was discussing the functional dependence upon the quantum gauge fixing term.

To simplify the argument we work in the context of a scalar field $\varphi(x)$ whose renormalized effective action is $\Gamma[\varphi]$. Just being a solution of the effective field equation does not eliminate gauge dependence. The key to getting a gauge independent result is to correctly normalize the linearized solution and then use perturbation theory to expand this into a full solution of the effective field equation. If the plane wave mode function for wave vector $\vec{k}$ is $u(t,k)$, and the full (gauge dependent) field strength renormalization is Z, the correct linearized solution is,

$$\varphi_1[\alpha,\alpha^*](x) = \int \frac{d^3k}{(2\pi)^3} \Biggl\{ \frac{u(t,k)}{\sqrt{Z}} e^{i\vec{k}\cdot\vec{x}} \alpha(\vec{k}) + \frac{u^*(t,k)}{\sqrt{Z}} e^{-i\vec{k}\cdot\vec{x}} \alpha^*(\vec{k}) \Biggr\} . \tag{26}$$

Here the complex parameters $\alpha(\vec{k})$ and $\alpha^*(\vec{k})$ characterize which of the infinitely many possible linearized solutions is desired.

The effective field equation can be written in terms of a "scattering current" which is only nonzero at some early time $t_{\rm in}$ and some late time $t_{\rm out}$ [48],

$$J_\infty[\alpha,\alpha^*](x) \equiv \varphi_1(x)(\overleftarrow{\partial}_t - \overrightarrow{\partial}_t)\delta(t-t_{\rm out}) - \varphi_1(x)(\overleftarrow{\partial}_t - \overrightarrow{\partial}_t)\delta(t-t_{\rm in}) . \tag{27}$$

The role of J_∞ is to inject linearized solutions in the asymptotic past and remove them in the asymptotic future. Applying perturbation theory to the full effective field equation allows one to develop (26) into a full solution,

$$\frac{\delta\Gamma[\varphi]}{\delta\varphi(x)} = -J_\infty[\alpha,\alpha^*](x) \quad \Longrightarrow \quad \varphi[\alpha,\alpha^*](x) = \varphi_1[\alpha,\alpha^*](x) + O\Bigl(\varphi_1^2\Bigr) . \tag{28}$$

[1] In order, his references are [15] a proceedings article by 't Hooft [46], [16] another proceedings article by DeWitt [47], and [20] the comment, *The S-matrix (built out of the tree amplitudes of* Γ*) is also choice independent.* The comment indicates that DeWitt distinguished between solutions and the effective action evaluated at a general solution, which is a generating functional for the S-matrix.

It is certainly true that evaluating the effective action at this solution gives a generating functional for the S-matrix, which is independent of the gauge used to compute quantum corrections to Γ. DeWitt seems to be claiming that the solutions themselves are also independent of this gauge choice.

We are not sure this claim is true, but the physics of how it would work seems clear enough. The point is that simply solving the effective field equation with a classical source — as we did in section 2.3 — is not enough. Some physical source must cause any disturbance in the effective field, and some physical observer must measure this disturbance. The source and observer both interact with quantum gravity and these interactions must be included to produce a gauge independent result. Once this dependence is included one can solve a modified effective field equation in a way that makes sense even in cosmology.

4 Including the Observer & the Source

In this section we add a source and observer in the form of a massive scalar ψ which couples minimally to gravity and to ϕ,

$$\mathcal{L}_2 = \frac{R\sqrt{-g}}{16\pi G} - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} - \frac{1}{2}\partial_\mu\psi\partial_\nu\psi g^{\mu\nu}\sqrt{-g} - \frac{1}{2}\Bigl(m^2+\lambda\phi\Bigr)\psi^2\sqrt{-g} \; . \tag{29}$$

From the scattering of two ψ's one can extract a gauge independent measure of the one loop quantum gravitational correction to the ϕ potential. To do this we find the order $\kappa^2\lambda^2$ contribution to the amputated 4-ψ vertex function. Although many diagrams contribute, the analysis can be simplified by exploiting Donoghue's crucial insight that only very special, nonanalytic terms modify the long range potential (25) [27, 28]. This essentially eliminates the massive scalar propagator. Our analysis closely follows Bjerrum-Bohr's computation of quantum gravitational corrections to the Coulomb potential [31]. We begin by working out how the self-mass (13) contributes. Next the graviton correlation between the two vertices is computed, and we see that it can be regarded as simply changing the gauge-dependent constant $C_0(a,b)$ in expression (25). Then each of the remaining diagrams is subjected to a similar analysis, and the gauge independence of the final result is manifest by the cancellation of all dependence on the parameters a and b.

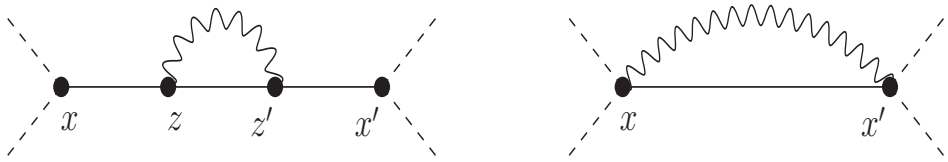

Figure 1: The left diagram shows how the self-mass contributes to the amputated 4-ψ vertex function. The diagram on the right shows how graviton correlations between the two vertices contribute. Solid lines represent the massless scalar, wavy lines represent the graviton, and dashed lines stand for the massive scalar. These graphs have the same topology as Bjerrum-Bohr's Diagrams 8 and 4, respectively [31].

The massive scalar propagator obeys,

$$(\partial^2 - m^2) i\Delta_m(x;x') = i\delta^D(x-x') \quad \Longrightarrow \quad i\Delta_m(x;x') = \frac{m^{D-2}}{(2\pi)^{\frac{D}{2}}} \frac{K_{\frac{D}{2}-1}(m\Delta x)}{(m\Delta x)^{\frac{D}{2}-1}} \; . \tag{30}$$

We do not need it to compute the contribution to the amputated 4-point function from the primitive scalar self-mass (13). This is the leftmost diagram of Figure 1, which has the same topology as Bjerrum-Bohr's Diagram 8 [31]. For our model (29) we can partially integrate the factors of ∂^2 in expression (13) to eliminate the outer propagators,

$$-iV_0(x;x') = (-i\lambda) \int d^D z \, i\Delta(x;z) \times (-i\lambda) \int d^D z' \, i\Delta(x';z') \times -iM^2(z;z') \; , \tag{31}$$

$$= \kappa^2 \lambda^2 \mathcal{C}_0(a,b,D) \int d^D z \, \partial_z^2 i\Delta(x;z) \int d^D z' \, \partial_{z'}^2 i\Delta(x';z') \times \Big[i\Delta(z;z') \Big]^2 \; , \tag{32}$$

$$= -\kappa^2 \lambda^2 \mathcal{C}_0(a,b,D) \times \Big[i\Delta(x;x') \Big]^2 \; , \tag{33}$$

where $\mathcal{C}_0(a,b,D)$ is given in (14) and the contribution to the renormalised self-mass is characterised by $C_0(a,b) = \mathcal{C}_0(a,b,4)$ in (19).

4.1 Correlations between the Vertices

The simplest extra contribution to the amputated 4-point function is the rightmost of the diagrams on Figure 1, which has the same topology as Bjerrum-Bohr's Diagram 4 [31]. This diagram represents quantum gravitational correlations between the source and observer vertices,

$$-iV_1(x;x') = -\frac{i\kappa\lambda}{2} \eta^{\mu\nu} \times i\Big[{}_{\mu\nu}\Delta_{\rho\sigma} \Big](x;x') \times -\frac{i\kappa\lambda}{2} \eta^{\rho\sigma} \times i\Delta(x;x') \; , \tag{34}$$

$$= -\kappa^2 \lambda^2 \Big[-\frac{2(D-1)}{(D-2)(b-2)^2} + \frac{a}{(b-2)^2} \Big] \times \Big[i\Delta(x;x') \Big]^2 \; . \tag{35}$$

Comparing expressions (33) and (35) reveals that we can think of $-iV_1(x;x')$ as a sort of contribution to the self-mass,

$$-iM_1^2(x;x) = -\kappa^2 \mathcal{C}_1(a,b,D) \times \partial^4 \Big[i\Delta(x;x') \Big]^2 \; , \tag{36}$$

where the gauge-dependent factor is,

$$\mathcal{C}_1(a,b,D) = -\frac{2(D-1)}{(D-2)(b-2)^2} + \frac{a}{(b-2)^2} \quad , \quad C_1(a,b) = \frac{(a-3)}{(b-2)^2} \; . \tag{37}$$

Because (36) takes the same form as (13), with the replacement of $\mathcal{C}_0(a,b,D)$ by $\mathcal{C}_1(a,b,D)$, we just add $C_0(a,b)$ and $C_1(a,b)$ in expression (25).

4.2 Vertex-Source and Vertex-Observer Correlations

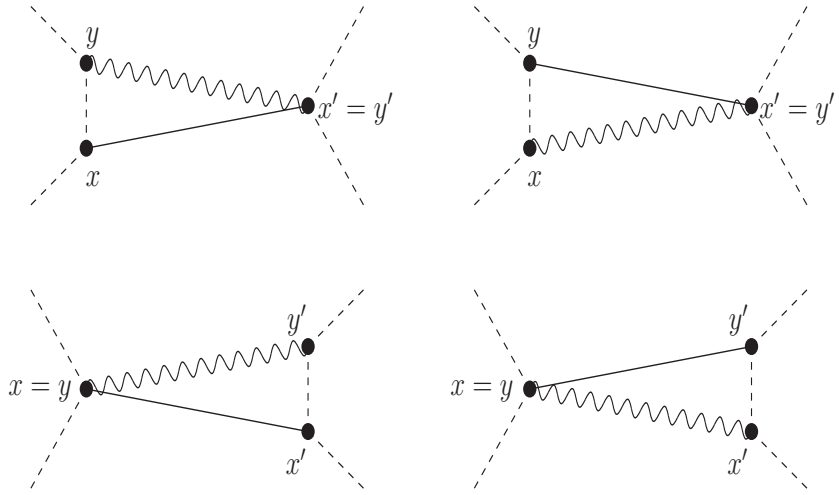

Figure 2: These diagrams show correlations between the source (primed) or observer (unprimed) and the opposite vertex. Solid lines represent the massless scalar, wavy lines represent the graviton, and dashed lines represent the massive scalar. These graphs have the same topology as Bjerrum-Bohr's Diagram 3 [31].

The remaining diagrams involve three or four distinct points. We shall reserve x^μ and y^μ for the in-coming and out-going observer, respectively, with x'^μ and y'^μ for the in-coming and out-going source. This section concerns the four diagrams of Figure 2, which have the same topology as Bjerrum-Bohr's

Diagram 3 [31]. For us these diagrams represent correlations between the source or observer and the more distant vertex. Correlations with the nearer vertex are cancelled by field strength renormalization and do not contribute to the long range potential (25).

The full contribution from these diagrams is,

$$-iV_2(x;y;x';y') = (-i\lambda)i\Delta(x;x')(-i\kappa)\Big[-\overline{\partial}^{\mu}_{y}\partial^{\nu}_{y}+\frac{\eta^{\mu\nu}}{2}(\overline{\partial}_y\cdot\partial_y+m^2)\Big]i\Delta_m(x;y)$$
$$\times i\Big[_{\mu\nu}\Delta_{\rho\sigma}\Big](y;x')\times\Big(-\frac{i}{2}\kappa\lambda\eta^{\rho\sigma}\Big)\times\delta^D(x'-y')+\Big(3\text{ permutations}\Big)\ , \quad (38)$$

where "(3 permutations)" indicates the other 3 diagrams of Figure 2. Here and henceforth an over-lined derivative indicates that it acts on the external state. For example, $\overline{\partial}^{\mu}_{y}$ means that the derivative acts on the out-going observer wave function.

Performing all the contractions and acting all the derivatives in expression (38) is quite tedious. It is also unnecessary if one only wants terms that can contribute to the long range potential (25), which are equivalent to the form,

$$-iV_2(x;y;x';y') \longrightarrow -\kappa^2\lambda^2\mathcal{C}_2(a,b,D)\times\Big[i\Delta(x;x')\Big]^2\times\delta^D(x\text{–}y)\delta^D(x'\text{–}y')\ . \quad (39)$$

For the purpose of identifying those terms which contribute to the long range potential one can make the simplification,

$$i\Delta_m(x;y)i\Delta(x;x')i\Delta(y;x') \longrightarrow \frac{i\delta^D(x-y)}{2m^2}\Big[i\Delta(x;x')\Big]^2\ . \quad (40)$$

Relation (40) is the position-space version of Bjerrum-Bohr's equation (B4), with a classical general relativistic contribution dropped [31]. This relation was originally derived by Donoghue [27, 28]. When it is used, along with the the propagator equations and the fact that $\overline{\partial}^2_y - m^2 = 0$, the result is surprising,

$$C_2(a,b) = \mathcal{C}_2(a,b,4) = 0\ . \quad (41)$$

Some details of the derivation of (41) are quite technical and are therefore given in the Appendix.

4.3 Vertex-Force Carrier Correlations

The massless scalar whose exchange carries the force between source and observer also interacts with gravity, so we must include quantum gravitational

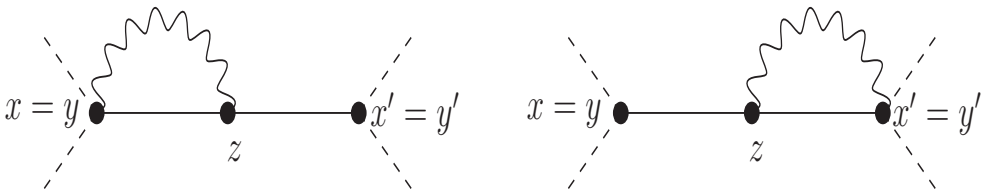

Figure 3: These diagrams show correlations between one of the vertices and the massless scalar force carrier. Solid lines represent the massless scalar, wavy lines represent the graviton, and dashed lines stand for the massive scalar. These graphs have the same topology as Bjerrum-Bohr's Diagram 7 [31].

correlations between it and the vertices. The relevant graphs are shown in Figure 3, and they have the same topology as Bjerrum-Bohr's Diagram 7 [31]. The contribution they make to the amputated 4-ψ vertex function is,

$$-iV_3(x;y;x';y') = \delta^D(x-y)\delta^D(x'-y')\int d^Dz\Big(-\frac{i}{2}\kappa\lambda\eta^{\mu\nu}\Big)i\Big[_{\mu\nu}\Delta_{\rho\sigma}\Big](x;z)$$
$$\times i\Delta(x;z)(-i\kappa)\Big[-\overleftarrow{\partial}^{\rho}_{z}\overrightarrow{\partial}^{\sigma}_{z}+\frac{\eta^{\rho\sigma}}{2}\overleftarrow{\partial}_{z}\cdot\overrightarrow{\partial}_{z}\Big]i\Delta(z;x')(-i\lambda)+\Big(\text{permutation}\Big). \quad (42)$$

The structure in expression (42) is simple enough that we can explain its reduction in detail. One key point is to evaluate the scalar derivatives on the second line,

$$i\Delta(x;z)\Big[-\overleftarrow{\partial}^{\rho}_{z}\overrightarrow{\partial}^{\sigma}_{z}+\frac{\eta^{\rho\sigma}}{2}\overleftarrow{\partial}_{z}\cdot\overrightarrow{\partial}_{z}\Big]i\Delta(z;x')$$
$$= \frac{(D-2)i\Delta(x;z)}{(x-z)^2}\Big[-(x-z)^{\rho}\eta^{\sigma\alpha}+\frac{\eta^{\rho\sigma}}{2}(x-z)^{\alpha}\Big]\frac{\partial i\Delta(z;x')}{\partial z^{\alpha}} \quad (43)$$

A second key point is the expressing the contracted graviton propagator in terms of the massless scalar propagator,

$$\eta^{\mu\nu}i\Big[_{\mu\nu}\Delta_{\rho\sigma}\Big](x;z) = \Bigg\{\frac{4\eta_{\rho\sigma}}{(D-2)(b-2)}$$
$$+\frac{2[(D-2)a-Db+2]}{(D-2)(b-2)^2}\Bigg[\eta_{\rho\sigma}-\frac{(D-2)(x-z)_{\rho}(x-z)_{\sigma}}{(x-z)^2}\Bigg]\Bigg\}i\Delta(x;z) \quad (44)$$

Contracting (44) into (43) can be expressed as a derivative of the square of $i\Delta(x;z)$,

$$\eta^{\mu\nu}i\Big[_{\mu\nu}\Delta_{\rho\sigma}\Big](x;z)\times i\Delta(x;z)\Big[-\overleftarrow{\partial}^{\rho}_{z}\overrightarrow{\partial}^{\sigma}_{z}+\frac{\eta^{\rho\sigma}}{2}\overleftarrow{\partial}_{z}\cdot\overrightarrow{\partial}_{z}\Big]i\Delta(z;x') = \Bigg[\frac{2(D-2)}{(b-2)}$$

$$+\frac{2(D-2)[(D-2)a-Db+2]}{(b-2)^2}\Bigr] \times \frac{(x-z)^\alpha}{(x-z)^2}\Bigl[i\Delta(x;z)\Bigr]^2 \times \frac{\partial i\Delta(z;x')}{\partial z^\alpha} \; , \quad (45)$$

$$= \Biggl[-\Bigl(\frac{D-1}{b-2}\Bigr)+\frac{(D-2)a-2(D-1)}{(b-2)^2}\Biggr] \times \eta^{\alpha\beta}\frac{\partial}{\partial z^\alpha}\Bigl[i\Delta(x;z)\Bigr]^2 \times \frac{\partial}{\partial z^\beta} i\Delta(z;x') \; . \quad (46)$$

Of course the next step is to partially integrate on z and use the propagator identity,

$$-\eta^{\alpha\beta}\frac{\partial}{\partial z^\alpha}\frac{\partial}{\partial z^\beta} i\Delta(z;x') = -i\delta^D(z-x') \; . \quad (47)$$

The same reductions apply to the other diagram and the final result takes the form,

$$-iV_3(x;y;x';y') = -\kappa^2\lambda^2 \mathcal{C}_3(a,b,D) \times \Bigl[i\Delta(x;x')\Bigr]^2 \times \delta^D(x-y)\delta^D(x'-y') \; , \quad (48)$$

where the gauge dependent multiplicative factor is,

$$\mathcal{C}_3(a,b,D) = \Bigl(\frac{D-1}{b-2}\Bigr) - \frac{[(D-2)a-2(D-1)]}{(b-2)^2} \implies C_3(a,b) = \frac{3}{b-2} - \frac{2(a-3)}{(b-2)^2} . \quad (49)$$

4.4 Source-Observer Correlations

Both the source and the observer interact with quantum gravity so we must include graviton correlations between them. The four graphs which contribute to the long range potential are shown in Figure 4. (Correlations from source to source or observer to observer do not affect long range potential.) The exact contribution for these diagrams is,

$$-iV_4(x;y;x';y') = (-i\lambda)^2 i\Delta(x;x')(-i\kappa)\Bigl[-\overline{\partial}^\mu_y \partial^\nu_y + \frac{\eta^{\mu\nu}}{2}(\overline{\partial}_y \cdot \partial_y + m^2)\Bigr] i\Delta_m(y;x)$$

$$\times i\Bigl[_{\mu\nu}\Delta_{\rho\sigma}\Bigr](y;y')(-i\kappa)\Bigl[-\overline{\partial}^\rho_{y'}\partial^\sigma_{y'} + \frac{\eta^{\rho\sigma}}{2}(\overline{\partial}_{y'} \cdot \partial_{y'} + m^2)\Bigr] i\Delta_m(y';x') + \Bigl(3 \text{ P's}\Bigr) . \quad (50)$$

Here "(3 P's)" stands for the other three diagrams of Figure 4, which are simple permutations of the expression shown. We also recall that an overlined derivative indicates it acting on the appropriate external state wave function.

The reduction of these diagrams proceeds according to the same methods as before. In addition to the propagator equations and the simplification (40)

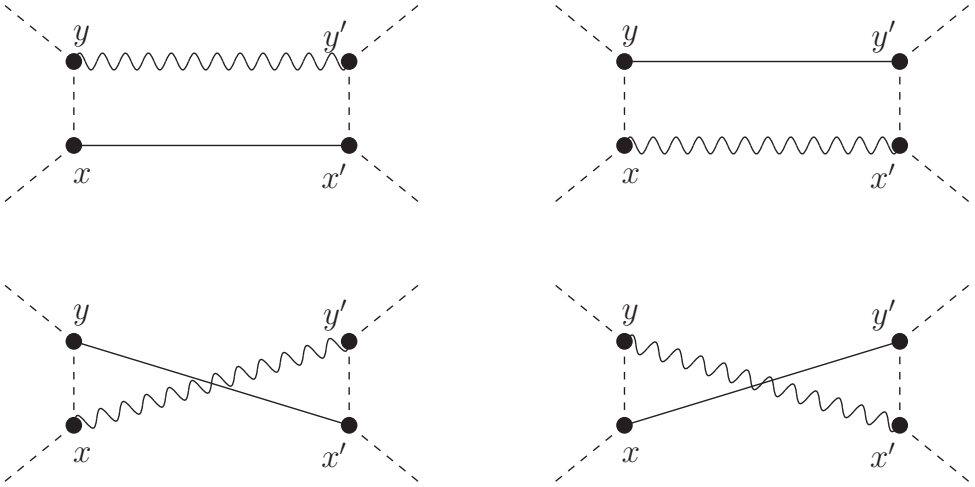

Figure 4: These diagrams show correlations between the source (primed) and the observer (unprimed). Solid lines represent the massless scalar, wavy lines represent the graviton, and dashed lines represent the massive scalar. These graphs have the same topology as Bjerrum-Bohr's Diagram 2 [31].

the long range potential (25) is not affected by the following simplifications,

$$m^2(\partial_x+\partial_y)^\alpha(\partial_x+\partial_y)_\alpha\Big[i\Delta(x;x')i\Delta(y;y')i\Delta_m(x;y)i\Delta_m(x';y')\Big] \longrightarrow -\Big[i\Delta(x;x')\Big]^2\delta^D(x-y)\delta^D(x'-y') \; , \quad (51)$$

$$m^2(\partial_x+\partial_y)^\alpha(\partial_x+\partial_y)_\alpha\Big[i\Delta(x;y')i\Delta(y;x')i\Delta_m(x;y)i\Delta_m(x';y')\Big] \longrightarrow +\Big[i\Delta(x;x')\Big]^2\delta^D(x-y)\delta^D(x'-y') \; , \quad (52)$$

Relations (51) and (52) are position-space versions of Bjerrum-Bohr's equations (B8) and (B9), respectively, with some classical general relativistic contributions neglected [31]. Both relations were originally derived by Donoghue and Torma [49].

Because $-iV_4(x;y;x';y')$ is ultraviolet finite we can take $D = 4$. The final result for the part relevant to the long range potential (25) takes the form,

$$-iV_4(x;y;x';y') \longrightarrow -\kappa^2\lambda^2 C_4(a,b)\times\Big[i\Delta(x;x')\Big]^2\times\delta^D(x-y)\delta^D(x'-y') \; . \quad (53)$$

The gauge-dependent multiplicative factor is,

$$C_4(a,b) = \frac{17}{4} - \frac{3}{4}a - \frac{1}{4}\frac{(a-3)}{(b-2)^2} \; . \quad (54)$$

4.5 Force Carrier Correlations with Source & Observer

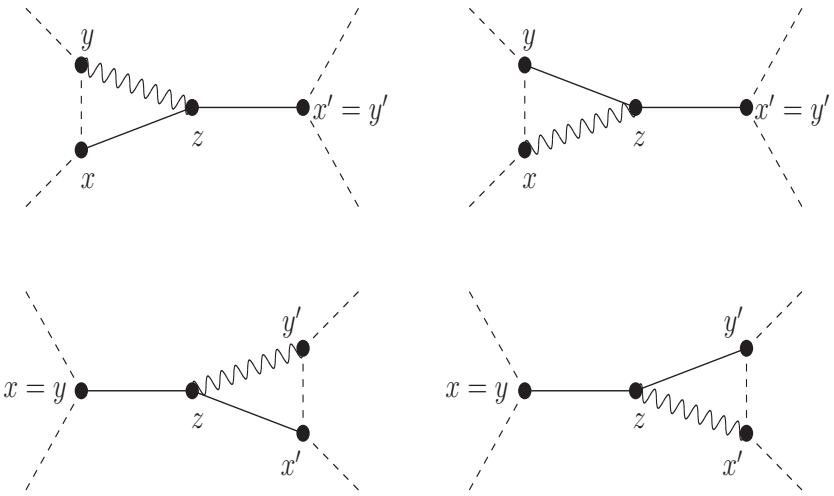

Figure 5: These diagrams show correlations between the source (primed) or observer (unprimed) and the massless scalar force carrier. Solid lines represent the massless scalar, wavy lines represent the graviton, and dashed lines represent the massive scalar. These graphs have the same topology as Bjerrum-Bohr's Diagram 6 [31].

Because the source, observer, and the massless scalar which carries the force between them, all interact with gravity we must include quantum gravitational correlations between them. The relevant Feynman diagrams are shown in Figure 5. Their full contribution to the amputated 4-ψ vertex function is,

$$-iV_5(x;y;x';y') = \kappa^2\lambda^2\delta^D(x'-y')\Big[-\overline{\partial}^\mu_y\partial^\nu_y + \frac{\eta^{\mu\nu}}{2}(\overline{\partial}_y\cdot\partial_y + m^2)\Big]i\Delta_m(y;x)\int d^Dz$$
$$\times i\Big[_{\mu\nu}\Delta_{\rho\sigma}\Big](y;z)\times i\Delta(x;z)\Big[-\overleftarrow{\overline{\partial}}^\rho_z\overrightarrow{\partial}^\sigma_z + \frac{\eta^{\rho\sigma}}{2}\overleftarrow{\overline{\partial}}_z\cdot\overrightarrow{\partial}_z\Big]i\Delta(z;x') + \Big(3\text{ P's}\Big). \quad (55)$$

As before, the symbol "(3 P's)" refers to the three other diagrams shown in Figure 5. Also as before, the over-lined derivative $\overline{\partial}_y$ acts on the out-going observer's external wave function.

The reduction of (55) follows previous reductions:

- The derivatives with respect to y on the first line of (55) are treated the same way as those of expression (38). They are first expressed as m^2

i	1	a	$\frac{1}{b-2}$	$\frac{(a-3)}{(b-2)^2}$
0	$+\frac{3}{4}$	$-\frac{3}{4}$	$-\frac{3}{2}$	$+\frac{3}{4}$
1	0	0	0	$+1$
2	0	0	0	0
3	0	0	$+3$	-2
4	$+\frac{17}{4}$	$-\frac{3}{4}$	0	$-\frac{1}{4}$
5	-2	$+\frac{3}{2}$	$-\frac{3}{2}$	$+\frac{1}{2}$
Total	$+3$	0	0	0

Table 1: The gauge dependent factors $C_i(a,b)$ for each contribution, where the index i ($i = 1, \ldots, 5$) refers to the diagrams shown in Figure i.

plus a sum of distinct kinetic operators, then acted on the propagators. Finally, the simplification (40) is invoked.

- The derivatives with respect to z on the second line of (55) are treated the same way as those of expression (42). We first act them on the two massless scalar propagators, then the derivative of $i\Delta(x;z)$ is combined with the factor of $i\Delta(x;z)$ in the graviton propagator to give a total derivative, which is partially integrated onto $i\Delta(z;x')$ to produce a delta function that eliminates the integration over z.

The final result takes the form,

$$-iV_5(x;y;x';y') = -\kappa^2\lambda^2 C_5(a,b) \times \Bigl[i\Delta(x;x')\Bigr]^2 \times \delta^D(x-y)\delta^D(x'-y') \; . \quad (56)$$

The gauge dependent multiplicative factor is,

$$C_5(a,b) = -2 + \frac{3}{2}a - \frac{\frac{3}{2}}{b-2} + \frac{1}{2}\frac{(a-3)}{(b-2)^2} \; . \quad (57)$$

4.6 Sum Total

No other diagrams contribute to the long range potential (25).[2] As we have seen, each diagram could be viewed as making a contribution to the self-mass of the form,

$$-iM_i^2(x;x') = -\kappa^2 C_i(a,b) \times \partial^4 \Big[i\Delta(x;x') \Big]^2 \; . \tag{58}$$

Table 1 gives our results for the gauge-dependent multiplicative factors $C_i(a,b)$. It is reassuring that all dependence on the gauge parameters a and b drops out in the sum, $\sum_{i=1}^{5} C_i(a,b) = +3$.

The simplest gauge is obtained by setting $a = b = 1$. This was the choice made by Donoghue [27, 28], and by Bjerrum-Bohr [31]. It is amusing to note that the actual self-mass, $-iM_0(x;x')$, we computed in (13) to motivate the problem, happens to vanish in that gauge. Of course our final result is independent of a and b, as would be those of Donoghue and Bjerrum-Bohr had they made their computations in a general gauge.

5 Discussion

The continual creation of horizon-scale gravitons during inflation tends to engender secular corrections to particle kinematics [4, 5, 6, 7, 11, 12, 13, 14, 16, 17, 18, 19, 20] and to force laws [15]. It has even been proposed that the self-gravitation between these gravitons induces a secular slowing of the expansion rate as more and more of them come into causal contact [50, 51]. However, behind all of these effects lurks the *gauge issue*: the simplest way to study what inflationary gravitons do is from solutions to the effective field equations and those solutions depend upon how the graviton's gauge freedom is fixed. Some researchers dismiss gauge-dependent Green's functions as completely unphysical [52]. Others reflect that even gauge-dependent Green's functions must contain physical information because the flat space S-matrix — which is gauge independent — is formed by taking sums of products of them [53]. The question is how to separate the physical information from the rest.

[2] In the SQED computation of Bjerrum-Bohr there was an additional contribution from what he termed Diagram 5 [31]. However, this diagram happens to vanish for the massless scalar process we consider.

Our goal has been to develop an analog of the S-matrix which does not involve the global integrations that preclude the S-matrix from being observable in cosmology. We believe the gauge dependence of solutions to the usual effective field equations derives from neglecting quantum gravitational correlations with the source which disturbs the effective field and the observer who measures the disturbance. Including these correlations leads to an improved effective field equation which can be solved quasi-locally. As a test of this idea we worked in the most general Poincaré invariant gauge (6) to compute the one graviton loop correction to the long range force exerted by a massless, minimally coupled scalar ϕ. The conventional result (25) is highly gauge dependent; by varying the two gauge parameters it can be made to go from $-\infty$ to $+\infty$! However, including a physical source and observer — in the form of a massive scalar ψ — led to the complete cancellation of gauge dependence which is evident in Table 1.

Our final result for the effective field equation takes the form,

$$\partial^2\phi(x) - \int d^4x' \, M^2_{\rm full}(x;x')\phi(x') = J(x) \; , \tag{59}$$

where the improved, gauge-independent scalar self-mass is,

$$M^2_{\rm full}(x;x') = -\frac{3\kappa^2\partial^8}{128\pi^3}\Biggl\{\theta(\Delta t-\Delta r)\Biggl[\ln\Bigl[\mu^2(\Delta t^2-\Delta r^2)\Bigr]-1\Biggr]\Biggr\} + O(\kappa^4) \; , \tag{60}$$

and we recall that $\Delta t \equiv t - t'$ and $\Delta r \equiv \|\vec{x} - \vec{x}'\|$. We actually only considered $J(t,\vec{x}) = \delta^3(\vec{x})$ but the equation is linear, so the passage to general $J(x)$ follows from superposition. Note that there is no dependence on the ψ mass m. It dropped out through using relations (40) and (51-52) to extract the special nonanalytic part of the general amplitude which contributes to the long range potential (25). Realizing that quantum gravitational corrections to low energy physics derive solely from these special sorts of terms was Donoghue's great contribution [27, 28]; we have merely translated his relations to position space. We should also comment that we were greatly aided in recognizing the handful of relevant diagrams by the computation Bjerrum-Bohr made of the quantum gravitational correction to the Coulomb potential in SQED [29].

The point of this exercise was to use the flat space S-matrix to abstract observables for cosmology. The essentials of how to study changes in kinematics and force laws seem clear enough now:

- We want to correct the linearized effective field equation in position space;
- We need to include quantum gravitational correlations with the source which disturbs the effective field, and with the observer who measures the disturbance; and
- Most details of the source and observer will drop out in the appropriate infrared limit.

What is not yet apparent is the correct generalization of the relations (40) and (51-52) which were used to extract the essential part of the full amplitude. In flat space background the appropriate infrared limit is large distances. For inflationary cosmology we suspect it is late times.

One crucial point which we have not addressed is what observables stand for the primordial power spectra when one includes loop corrections. The naive correlators cannot be right because they depend upon the infrared cutoffs which must be introduced to define the scalar and graviton propagators [54]. Nonlocal composite operator generalizations can be devised which avoid this dependence [55, 56] but these generalizations introduce new ultraviolet divergences and also disrupt the careful pattern by which loop corrections to the naive correlators are slow roll suppressed [21].

Finally, we should comment on the other alternative for extracting cosmological observables: taking expectation values of gauge invariant operators. A recent computation on de Sitter background shows how this can be done to invariantly quantify the back-reaction on inflation [26]. To invariantly study changes in particle kinematics, and the associated force laws, one might compute 2-point functions at geodesically fixed separations, as Fröb has recently done for a massless scalar on flat space background [57]. Fröb's calculation is interesting because the double logarithms he found would translate to corrections to the potential of the form $-\frac{1}{4\pi r} \times \frac{\kappa^2}{r^2} \times \ln(\mu r)$. (The same thing is bound to happen for the pure gravitational analog [58].) Of course the S-matrix technique we have pursued lacks the factor of $\ln(\mu r)$, so there is a clear difference between two, completely gauge independent results. It is not that one is right and the other wrong; they both represent correct answers to different questions. There is simply no alternative to thinking hard about how an effect is measured, and then correctly modelling that process.

Acknowledgments

We are grateful for D. Glavan for locating an old proceedings article [46]. We are also grateful for correspondence and conversation on this subject with S. Deser, J. F. Donoghue, M. B. Fröb and G. 't Hooft. This work was partially supported by Taiwan MOST grant 103-2112-M-006-001-MY3; by the D-ITP consortium, a program of the Netherlands Organization for Scientific Research (NWO) that is funded by the Dutch Ministry of Education, Culture and Science (OCW); by NSF grant PHY-1506513; and by the Institute for Fundamental Theory at the University of Florida.

Appendix: Reduction of the diagrams in Fig. 2

In this Appendix we present some details of the evaluation of the 4-ψ vertex (38). Upon partial integration of $\bar{\partial}_y$ derivatives, $-iV_2(x;y;x';y')$ becomes,

$$\begin{aligned} -iV_2(x;y;x';y') &= \frac{i\lambda^2\kappa^2}{2}\delta^D(x'-y')i\Delta(x;x') \\ &\times \Big\{\partial_y^\alpha\Big[\Big(\delta^\mu_\alpha\partial^\nu_y - \frac{1}{2}\eta^{\mu\nu}\partial^y_\alpha\Big)i\Delta_m(x;y)\eta^{\rho\sigma}i\Big[_{\mu\nu}\Delta_{\rho\sigma}\Big](y;x')\Big] \\ &+\frac{m^2}{2}i\Delta_m(x;y)\eta^{\mu\nu}\eta^{\rho\sigma}i\Big[_{\mu\nu}\Delta_{\rho\sigma}\Big](y;x')\Big\} + \Big(3 \text{ perm's}\Big) \,, \end{aligned} \tag{61}$$

where

$$\begin{aligned} \eta^{\rho\sigma}i\Big[_{\mu\nu}\Delta_{\rho\sigma}\Big](y;x') &= \frac{4\eta_{\mu\nu}}{(D-2)(b-2)}\,i\Delta(y;x') \\ &+4\Big[\frac{a}{(b-2)^2} - \frac{D}{(D-2)(b-2)} - \frac{2(D-1)}{(D-2)(b-2)^2}\Big]\frac{\partial^y_\mu\partial^y_\nu}{\partial^2_y}i\Delta(y;x') \end{aligned} \tag{62}$$

and

$$\eta^{\mu\nu}\eta^{\rho\sigma}i\Big[_{\mu\nu}\Delta_{\rho\sigma}\Big](y;x') = 4\Big[\frac{a}{(b-2)^2} - \frac{2(D-1)}{(D-2)(b-2)^2}\Big]i\Delta(y;x')\,. \tag{63}$$

When these are inserted in (61), and one acts the derivatives, one obtains,

$$\begin{aligned} -iV_2(x;y;x';y') &= \frac{i\lambda^2\kappa^2}{2}\delta^D(x'-y')i\Delta(x;x') \\ &\times\Big\{-\frac{1}{b-2}\Big[2\partial^2_y i\Delta_m(x;y)\times i\Delta(y;x') + 2\partial^\mu_y i\Delta_m(x;y)\partial^y_\mu i\Delta(y;x')\end{aligned} \tag{64}$$

$$
\begin{aligned}
&\qquad\qquad -\frac{2D}{D-2} m^2 i\Delta_m(x;y) i\Delta(y;x')\Big] \\
&+\Big[\frac{a}{(b-2)^2} - \frac{Db-2}{(D-2)(b-2)^2}\Big]\Big[4\partial_y^\mu \partial_y^\nu i\Delta_m(x;y) \times \frac{\partial_\mu^y \partial_\nu^y}{\partial_y^2} i\Delta(y;x') \\
&\qquad +2\partial_y^\mu i\Delta_m(x;y)\partial_\mu^y i\Delta(y;x') - 2(\partial_y^2 - m^2) i\Delta_m(x;y) \times i\Delta(y;x')\Big]\Big\} \\
&+\Big(3 \text{ perm's}\Big) \, .
\end{aligned}
$$

To reduce this expression further we shall need some identities. The first useful identity is the Bjerrum-Bohr's identity (40). The second one can be obtained by noting that,

$$
\begin{aligned}
\phi_m(y)\partial_y^2[i\Delta_m(x;y)i\Delta(y;x')] &= \phi_m(y) m^2 i\Delta_m(x;y) i\Delta(y;x') \\
&= \phi_m(y)\Big[m^2 i\Delta_m(x;y) i\Delta(y;x') + i\delta^D(x-y) i\Delta(y;x') \\
&\quad + i\Delta_m(x;y) i\delta^D(y-x') + 2\partial_y^\mu i\Delta_m(x;y)\partial_\mu^y i\Delta(y;x')\Big] \, , \quad (65)
\end{aligned}
$$

where $\phi_m(y)$ is the external leg field that satisfies, $(\partial_y^2 - m^2)\phi_m(y) = 0$. From (65) one immediately obtains,

$$
\begin{aligned}
2\partial_y^\mu i\Delta_m(x;y)\partial_\mu^y i\Delta(y;x') &= -i\Delta_m(x;y) i\delta^D(y-x') - i\delta^D(x-y) i\Delta(y;x') \\
&\longrightarrow \quad -i\delta^D(x-y) i\Delta(y;x') \, , \qquad (66)
\end{aligned}
$$

where the last implication selects only the term which contributes to the long range potential (25). Analogous (albeit more tedious) manipulations yield the following identity,

$$
4\partial_y^\mu \partial_y^\nu i\Delta_m(x;y) \frac{\partial_\mu^y \partial_\nu^y}{\partial_y^2} i\Delta(y;x') \quad \longrightarrow \quad 3i\delta^D(x-y) i\Delta(y;x') \, . \qquad (67)
$$

When these identities are employed in the vertex function (64), one sees that both the square bracket multiplying the factor $-1/(b-2)$, and the square bracket on the last two lines of (64), vanish in $D = 4$, implying that the whole set of diagrams in Figure 2 contributes zero to (25), thus proving (41).

References

[1] A. A. Starobinsky, JETP Lett. **30**, 682 (1979) [Pisma Zh. Eksp. Teor. Fiz. **30**, 719 (1979)].

[2] S. L. Detweiler, Astrophys. J. **234**, 1100 (1979). doi:10.1086/157593

[3] D. R. Lorimer, Living Rev. Rel. **11** (2008) 8 doi:10.12942/lrr-2008-8 [arXiv:0811.0762 [astro-ph]].

[4] S. P. Miao and R. P. Woodard, Class. Quant. Grav. **23**, 1721 (2006) doi:10.1088/0264-9381/23/5/016 [gr-qc/0511140].

[5] S. P. Miao and R. P. Woodard, Phys. Rev. D **74**, 024021 (2006) doi:10.1103/PhysRevD.74.024021 [gr-qc/0603135].

[6] S. P. Miao, arXiv:0705.0767 [hep-th].

[7] S. P. Miao, Phys. Rev. D **86**, 104051 (2012) doi:10.1103/PhysRevD.86.104051 [arXiv:1207.5241 [gr-qc]].

[8] S. P. Miao and R. P. Woodard, Class. Quant. Grav. **25**, 145009 (2008) doi:10.1088/0264-9381/25/14/145009 [arXiv:0803.2377 [gr-qc]].

[9] E. O. Kahya and R. P. Woodard, Phys. Rev. D **76**, 124005 (2007) doi:10.1103/PhysRevD.76.124005 [arXiv:0709.0536 [gr-qc]].

[10] E. O. Kahya and R. P. Woodard, Phys. Rev. D **77**, 084012 (2008) doi:10.1103/PhysRevD.77.084012 [arXiv:0710.5282 [gr-qc]].

[11] K. E. Leonard and R. P. Woodard, Class. Quant. Grav. **31**, 015010 (2014) doi:10.1088/0264-9381/31/1/015010 [arXiv:1304.7265 [gr-qc]].

[12] C. L. Wang and R. P. Woodard, Phys. Rev. D **91**, no. 12, 124054 (2015) doi:10.1103/PhysRevD.91.124054 [arXiv:1408.1448 [gr-qc]].

[13] D. Glavan, S. P. Miao, T. Prokopec and R. P. Woodard, Class. Quant. Grav. **32**, no. 19, 195014 (2015) doi:10.1088/0264-9381/32/19/195014 [arXiv:1504.00894 [gr-qc]].

[14] D. Glavan, S. P. Miao, T. Prokopec and R. P. Woodard, Class. Quant. Grav. **34** (2017) no.8, 085002 doi:10.1088/1361-6382/aa61da [arXiv:1609.00386 [gr-qc]].

[15] D. Glavan, S. P. Miao, T. Prokopec and R. P. Woodard, Class. Quant. Grav. **31**, 175002 (2014) doi:10.1088/0264-9381/31/17/175002 [arXiv:1308.3453 [gr-qc]].

[16] N. C. Tsamis and R. P. Woodard, Phys. Rev. D **54**, 2621 (1996) doi:10.1103/PhysRevD.54.2621 [hep-ph/9602317].

[17] P. J. Mora, N. C. Tsamis and R. P. Woodard, JCAP **1310**, 018 (2013) doi:10.1088/1475-7516/2013/10/018 [arXiv:1307.1422 [gr-qc]].

[18] S. Boran, E. O. Kahya and S. Park, Phys. Rev. D **90**, no. 12, 124054 (2014) doi:10.1103/PhysRevD.90.124054 [arXiv:1409.7753 [gr-qc]].

[19] S. Boran, E. O. Kahya and S. Park, Phys. Rev. D **96**, no. 2, 025001 (2017) doi:10.1103/PhysRevD.96.025001 [arXiv:1704.05880 [gr-qc]].

[20] S. Boran, E. O. Kahya and S. Park, arXiv:1708.01831 [gr-qc].

[21] S. P. Miao and R. P. Woodard, JCAP **1207**, 008 (2012) doi:10.1088/1475-7516/2012/07/008 [arXiv:1204.1784 [astro-ph.CO]].

[22] N. C. Tsamis and R. P. Woodard, Commun. Math. Phys. **162**, 217 (1994). doi:10.1007/BF02102015

[23] R. P. Woodard, gr-qc/0408002.

[24] P. J. Mora, N. C. Tsamis and R. P. Woodard, J. Math. Phys. **53**, 122502 (2012) doi:10.1063/1.4764882 [arXiv:1205.4468 [gr-qc]].

[25] D. Marolf, I. A. Morrison and M. Srednicki, Class. Quant. Grav. **30**, 155023 (2013) doi:10.1088/0264-9381/30/15/155023 [arXiv:1209.6039 [hep-th]].

[26] S. P. Miao, N. C. Tsamis and R. P. Woodard, Phys. Rev. D **95**, no. 12, 125008 (2017) doi:10.1103/PhysRevD.95.125008 [arXiv:1702.05694 [gr-qc]].

[27] J. F. Donoghue, Phys. Rev. Lett. **72**, 2996 (1994) doi:10.1103/PhysRevLett.72.2996 [gr-qc/9310024].

[28] J. F. Donoghue, Phys. Rev. D **50**, 3874 (1994) doi:10.1103/PhysRevD.50.3874 [gr-qc/9405057].

[29] N. E. J. Bjerrum-Bohr, J. F. Donoghue and B. R. Holstein, Phys. Rev. D **68**, 084005 (2003) Erratum: [Phys. Rev. D **71**, 069904 (2005)] doi:10.1103/PhysRevD.68.084005, 10.1103/PhysRevD.71.069904 [hep-th/0211071].

[30] N. E. J. Bjerrum-Bohr, J. F. Donoghue and B. R. Holstein, Phys. Rev. D **67**, 084033 (2003) Erratum: [Phys. Rev. D **71**, 069903 (2005)] doi:10.1103/PhysRevD.71.069903, 10.1103/PhysRevD.67.084033 [hep-th/0211072].

[31] N. E. J. Bjerrum-Bohr, Phys. Rev. D **66**, 084023 (2002) doi:10.1103/PhysRevD.66.084023 [hep-th/0206236].

[32] D. M. Capper, J. Phys. A **13**, 199 (1980). doi:10.1088/0305-4470/13/1/022

[33] K. E. Leonard and R. P. Woodard, Phys. Rev. D **85**, 104048 (2012) doi:10.1103/PhysRevD.85.104048 [arXiv:1202.5800 [gr-qc]].

[34] V. K. Onemli and R. P. Woodard, Class. Quant. Grav. **19**, 4607 (2002) doi:10.1088/0264-9381/19/17/311 [gr-qc/0204065].

[35] V. K. Onemli and R. P. Woodard, Phys. Rev. D **70**, 107301 (2004) doi:10.1103/PhysRevD.70.107301 [gr-qc/0406098].

[36] J. S. Schwinger, J. Math. Phys. **2**, 407 (1961). doi:10.1063/1.1703727

[37] K. T. Mahanthappa, Phys. Rev. **126**, 329 (1962). doi:10.1103/PhysRev.126.329

[38] P. M. Bakshi and K. T. Mahanthappa, J. Math. Phys. **4**, 1 (1963). doi:10.1063/1.1703883

[39] P. M. Bakshi and K. T. Mahanthappa, J. Math. Phys. **4**, 12 (1963). doi:10.1063/1.1703879

[40] L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1965)].

[41] K. C. Chou, Z. b. Su, B. l. Hao and L. Yu, Phys. Rept. **118**, 1 (1985). doi:10.1016/0370-1573(85)90136-X

[42] R. D. Jordan, Phys. Rev. D **33**, 444 (1986). doi:10.1103/PhysRevD.33.444

[43] E. Calzetta and B. L. Hu, Phys. Rev. D **35**, 495 (1987). doi:10.1103/PhysRevD.35.495

[44] L. H. Ford and R. P. Woodard, Class. Quant. Grav. **22**, 1637 (2005) doi:10.1088/0264-9381/22/9/011 [gr-qc/0411003].

[45] B. S. DeWitt, Phys. Rev. Lett. **47**, 1647 (1981). doi:10.1103/PhysRevLett.47.1647

[46] G. 't Hooft, "The Background Field Method in Gauge Field Theories," in *Functional and Probabilistic Methods in Quantum Field Theory*, ed. B. Jancewicz (Wydawnictwa Universytetu Wroclawskiego, Wroclaw, 1976).

[47] B. S. DeWitt, "A Gauge Invariant Effective Action," in *Quantum Gravity II*, ed. C. Isham, R. Penrose and D. Sciama (Oxford University Press, New York, 1981).

[48] B. S. DeWitt, "The spacetime approach to quantum field theory," in *Relativity groups and topology II*, ed. B. S. DeWitt and R. Stora (North Holland, Amsterdam, 1984).

[49] J. F. Donoghue and T. Torma, Phys. Rev. D **54**, 4963 (1996) doi:10.1103/PhysRevD.54.4963 [hep-th/9602121].

[50] N. C. Tsamis and R. P. Woodard, Nucl. Phys. B **474**, 235 (1996) doi:10.1016/0550-3213(96)00246-5 [hep-ph/9602315].

[51] N. C. Tsamis and R. P. Woodard, Annals Phys. **253**, 1 (1997) doi:10.1006/aphy.1997.5613 [hep-ph/9602316].

[52] J. Garriga and T. Tanaka, Phys. Rev. D **77**, 024021 (2008) doi:10.1103/PhysRevD.77.024021 [arXiv:0706.0295 [hep-th]].

[53] N. C. Tsamis and R. P. Woodard, Phys. Rev. D **78**, 028501 (2008) doi:10.1103/PhysRevD.78.028501 [arXiv:0708.2004 [hep-th]].

[54] S. B. Giddings and M. S. Sloth, JCAP **1101**, 023 (2011) doi:10.1088/1475-7516/2011/01/023 [arXiv:1005.1056 [hep-th]].

[55] Y. Urakawa and T. Tanaka, Phys. Rev. D **82**, 121301 (2010) doi:10.1103/PhysRevD.82.121301 [arXiv:1007.0468 [hep-th]].

[56] M. Gerstenlauer, A. Hebecker and G. Tasinato, JCAP **1106**, 021 (2011) doi:10.1088/1475-7516/2011/06/021 [arXiv:1102.0560 [astro-ph.CO]].

[57] M. B. Fröb, arXiv:1706.01891 [hep-th].

[58] N. C. Tsamis and R. P. Woodard, Annals Phys. **215**, 96 (1992). doi:10.1016/0003-4916(92)90301-2