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Scale-up of Λ_{3}: Massive gravity with a higher strong interaction scale

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A Scale-up of Λ_3

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Abstract

Pure massive gravity is strongly coupled at a certain low scale, known as Λ_3. I show that the theory can be embedded into another one, with new light degrees of freedom, to increase the strong scale to a significantly larger value. Certain universal aspects of the proposed mechanism are discussed, notably that the coupling of the longitudinal mode to a stress-tensor is suppressed, thus making the linear theory consistent with the fifth-force exclusion. An example of the embedding theory studied in detail is 5D AdS massive gravity, with a large cosmological constant. In this example the 4D strong scale can be increased by 19 orders of magnitude. Holographic duality then suggests that the strong scale of the 4D massive gravity can be increased by coupling it to a 4D non-local CFT, endowed with a UV cutoff; however, the 5D classical gravity picture appears to be more tractable.

1 An outline

This article addresses the strong coupling problem in the 4D nonlinear diff-invariant theory of massive gravity [1, 2]. It does so by a mechanism that raises the strong scale to a parametrically larger one, due to embedding of the theory into extra dimensions; the resulting theory does not seem to have obvious contradictions with the observations.

Among many classical solutions, Minkowski space is a solution of massive gravity, as it is of the massless theory. It is instructive to discuss properties of massive gravity, especially the ones distinguishing it from the massless theory, on the Minkowski background. These properties are determined by Poincaré invariance, and the fact that massive spin-2 representation of the Poincaré group has 5 degrees of freedom [3]. One of these 5, is the longitudinal mode, properties of which are vital to the viability of the theory [4], [5, 6, 7], [8] (see also, [9, 10, 11]); this mode will be the main focus of the present work.

Note that helicity is a good label for a massive state in the small mass, or equivalently, the high momentum approximation; this is an approximation adopted in the paper, and the longitudinal mode, referred as π, is identified with the helicity-0 state.

The first important property, dictated by Poincaré invariance, and the requirement of the absence of negative energy states [3], is that the π has no kinetic term in the linearized theory [8]. In the same field basis, π does not couple to the stress-tensor of matter. It has, however, a kinetic mixing with the helicity-2. The latter can be diagonalized, to get a kinetic term for π. The diagonalization induces unsuppressed coupling of π to the matter stress-tensor. Thus, in the linearized theory the matter particles interact via the field π, in addition to their interactions via the tensor field, leading to inconsistencies [4]. All the above properties are robust, guaranteed by Poincaré invariance, and the absence of any additional degrees of freedom beyond the 5.

The very same requirement of Poincaré invariance dictates the nonlinear properties of π: in the basis in which π has no kinetic term, it also has no self-interaction terms in the massless limit.[1] It only has interactions with the helicity-2 and helicity-1 states [1]. The diagonalization that induces the π kinetic term, also generates nonlinear self-interactions of the π mode [1]. Due to the fact that the kinetic term was absent before the diagonalization, the generated self-interactions end up being suppressed by a scale, [8, 9], that becomes small with smaller graviton mass, m,

$$\Lambda_3 = (M_{\rm P} m^2)^{1/3}. \tag{1}$$

The fact that this scale is low for a small graviton mass, is a blessing and a curse, at the same time: it's a virtue for the classical theory as it enables for an efficient Vainshtein mechanism [5, 7], that suppresses π near physical sources, thus rendering the nonlinear theory consistent with the observations [5]; such a low nonlinear scale is, however, an obvious impediment for an effective low energy quantum theory. Below Λ_3 the theory is weakly coupled, while at the strong scale an infinite number of new symmetry-preserving

[1] More precisely, we'll be using to the so-called decoupling limit: the mass goes to zero, simultaneously with Planck mass going to infinity, while their certain geometric mean stays constant [8].

non-linear terms could be induced, in the absence of any specific completion by new light degrees of freedom appearing below Λ_3.

Ideally, one would like to identify new degrees of freedom – *a la* Higgs – to soften massive gravity at the scale Λ_3 (see, [12] and references therein). Ideally, these new degrees of freedom would be part of a 4D local Poincaré invariant field theory. However, it does not appear to be easy to identify such a theory in a conventional setup. One could try to couple massive gravity to a local field theory in an unconventional way, via nonlinear terms involving Stückelberg fields. However, there are an infinite number of such couplings, and it is not straightforward to identify a good principle that would help to select a finite number of such terms, [13].

Under these circumstances, perhaps it then makes sense to adopt a provisional approach and seek just to raise parametrically the strong coupling scale, instead of striving to soften the interactions, *a la* Higgs. If successful, such an approach may also hint to a possible completion beyond the strong scale.

One could attempt to do this by introducing a full-fledged kinetic term for the π mode; such a term would rescale Λ_3 to a higher value. For instance, the Vainshtein mechanism operates due to the kinetic terms for π generated on various backgrounds [5, 7]. However, this cannot be done by any 4D local field theory on pure Minkowski background, where the lack of the π kinetic term is mandated by Poincaré invariance, and absence of ghosts.

Thus, one is prompted to think of an embedding of massive gravity into a theory that at low energies would not reduce to a local 4D theory; it would need though to preserve 4D Poincaré symmetry. In such a theory, the notion of a single 4D massive graviton could only be an approximation; fundamentally, the state that resembles a massive graviton has to have distinctive features. These very features, should also enable π to acquire a full-fledged kinetic term, and raise the strong scale. Needless to say, this theory should be consistent with the observations.

I'll show that such an embedding is possible. The larger theory is higher dimensional, but admits a 4D Minkowski background. The effective 4D theory is that of a massive graviton coupled to an infinite number of gapless 4D modes. This coupling induces a large non-local 4D "kinetic term" for π, even though the higher dimensional theory is local. In the leading approximation, the non-local 4D "kinetic term" reduces to just an ordinary large kinetic term for π; this leads to changing of Λ_3 to a new scale that can be made significantly larger. In the 5D example considered in Sections 3 and 4, the scale is increased by 19 orders of magnitude.

The lack of the π kinetic term was a consequence of Poincaré invariance. Yet, I claim the existence of a term for π that approximates its kinetic term. How is this possible? It is, since the theory is truly nonlocal from the 4D perspective. The 5D bulk theory is holographic dual to a non-local CFT that has no 4D stress-tensor [14]. Hence, the mechanism of scaling up Λ_3 can be attributed to interactions of 4D massive gravity with such a non-local stuff, which is better described by 5D classical local massive gravity.

Section 2 presents the above arguments in detail, and puts them in a general context. A reader who'd prefer to see a concrete nonlinear theory, could skip to Sections 3 and 4.

2 Scaling up

Let us begin with a telegraphic summary of known facts about the origin of (1), (for detailed discussions see [8], and [1]). Consider linearized massive gravity on the Minkowski background in the limit when the mass tends to zero, while $M_{\rm P}$ is very large, and only the leading relevant terms are retained. Keep track of the helicity-2 mode, denoted by h (all the indexes omitted), and helicity-0 mode, denoted by π; schematically, the quadratic Lagrangian for h and π looks as

$$\mathcal{L}_2 = (\partial h)^2 + m^2 h \partial\partial\pi + hT\,, \tag{2}$$

where T is the matter stress-tensor, and $M_{\rm P} = 1$, here and below, unless it's explicitly shown. The key is that π has no kinetic term; it only has a kinetic mixing with h [8]. The mixing term is proportional to m^2, however, m^2 can be absorbed into a definition of π, and should not appear in physical observables in the approximation considered. Indeed, one can diagonalize (2) by a field redefinition, $h = \hat{h} + m^2\pi$, to get

$$\mathcal{L}_2 = (\partial\hat{h})^2 - m^4(\partial\pi)^2 + \hat{h}T + m^2\pi T\,, \tag{3}$$

and rescale, $\pi \to \pi/m^2$. This makes the quadratic Lagrangian (3) independent of m^2, and renders the π coupling to the trace of the stress-tensor as strong (or as weak) as that of $\hat{h}$. Furthermore, the field redefinition and rescaling affect the nonlinear interaction terms of π – they end up being proportional to inverse powers of m:

$$m^2\pi(\partial\partial\pi)^2 \to \frac{\pi(\partial\partial\pi)^2}{M_{\rm P}m^2}\,. \tag{4}$$

In the last term $M_{\rm P}$ has been restored to show that the strong scale coincides with (1). The smaller the value of m, the lower the strong scale. Hence, the origin of the low strong scale is the lack of the π kinetic term in (2), that would not be proportional to m^4.

Could one generate a conventional kinetic term for π? Such a term is known to be present on curved backgrounds [15, 16]. For instance, on AdS_4, with the cosmological constant, $-\Lambda < 0$, one would obtain, $-\Lambda m^2(\partial\pi)^2$, in addition to the flat-space kinetic term, $-m^4(\partial\pi)^2$, generated after the diagonalization of the $h-\pi$ kinetic mixing (both kinetic terms are written here in terms of π that has not yet been rescaled to a canonically normalized field). This would raise the strong scale to a higher value, as long as the magnitude of the cosmological constant is large, $\Lambda >> m^2$.

The very same phenomenon of the enhancement of the kinetic term of π on various backgrounds is responsible for the Vainshtein mechanism [5, 7]; the mechanism is usually discussed in the context of spatially-localized sources, but it actually is universal, and applies to any background that has a characteristic physical scale.

Ideally, one would like the strong scale to be raised in an entire space-time, as in the example of AdS_4. However, gravity in the observed world cannot be approximated by AdS_4, or any other curved space-time. Hence, such a mechanism is not immediately

useful in a theory that aims to describe the world around us.

Nevertheless, the above considerations suggest a path forward: if instead we assume that 4D massive gravity is embedded into a D-dimensional ($D = 4+n > 4$) massive gravity with a large characteristic D-dimensional curvature scale, $\bar{\Lambda}$, then one would get a large D-dimensional kinetic term for the D-dimensional longitudinal mode, $\Pi(x^\mu, z^1, z^2, ..., z^n)$,

$$-M_D^{2+n}\bar{m}^2\bar{\Lambda}\left(\partial_D\Pi(x^\mu, z^1, z^2, ..., z^n)\right)^2 , \tag{5}$$

where M_D is the higher-dimensional Planck mass, $\bar{m}$ is the higher dimensional graviton mass, and $z^1, z^2, ..., z^n$, denote the extra space coordinates. For clarity, we consider the case, $m \sim \bar{m} << \sqrt{\bar{\Lambda}}$. If extra space is warped or compactified, one gets an effective 4D description below a certain energy scale. Thus, the large bulk kinetic term should imply, at least in some constructions, a large 4D kinetic term for the π

$$-M_D^{2+n}L^n\bar{m}^2\bar{\Lambda}(\partial\pi(x^\mu))^2 , \quad \pi(x^\mu) = \Pi(x^\mu, z^1 = 0, z^2 = 0, ..., z^n = 0) , \tag{6}$$

where $L \sim \bar{\Lambda}^{-1/2}$ is the radius-curvature of the extra space, and the above expression is valid at distance scales larger than L.

This is not enough though, one would still need to obtain a (nearly) flat 4D world, in spite of the D-dimensional space-time being curved. By no means this is automatic or trivial. Presumably, fine-tuning of free parameters of the theory will be needed to achieve this. In the 5D example considered in the next section the fine-tuning is explicit. Once this tuning is done, the goal is achieved: the π would have a large 4D kinetic term determined by the curvature of the embedding space, even though the 4D space is flat. In such a theory, the strong scale would be set by the parameters of the D-dimensional theory, and can be made much larger than Λ_3.

However, the above arguments appear puzzling: the lack of a conventional 4D kinetic term for π was a consequence of Poincaré invariance of a local 4D field theory, coupled to gravity in a conventional way. How could this be reconciled with the above proposal suggesting that there is a new kinetic term for π on a 4D Minkowski background? One option to resolve the puzzle is for the theory to be truly nonlocal from the 4D perspective, e.g., contain an infinite number of light 4D states which cannot be repackaged into a local 4D field theory. Such an arrangement would evade the apparent contradiction. Hence, the kinetic term (6) should only be an approximation, to a certain nonlocal 4D term reflecting an infinite number of the light states.

Where could such states come from? Due to the D-dimensional embedding there will be new degrees of freedom, that will appear as Kaluza-Klein (KK) modes from the point of view of the 4D effective theory. In general, KK modes might be discrete of continuum, with or without a gap. In the present case, however, an infinite number of the KK states should be light, with masses

$$m_{KK} < \Lambda_3 , \tag{7}$$

making the low energy theory to differ from a theory of a single massive graviton coupled

to a local 4D theory. Furthermore, there seems to be a more stringent requirement in this framework: from the 4D perspective, the bulk theory cannot have a mass gap greater than $m \sim \bar{m}$; if such a gap existed the 4D graviton could not be a state of mass m. In the explicit example considered in Section 3, this requirement is well-satisfied since there are an infinite number of light KK modes below the graviton mass scale,

$$m_{KK} \lesssim m \sim \bar{m} << \Lambda_3 \,. \tag{8}$$

These light modes, in general, might change the large distance behavior of massive gravity, even in the regime of validity of the effective field theory, and as noted earlier, should not be representable in terms of a local 4D theory. These aspects should be studied in concrete models, e.g., in the theory of Section 3.

Last but not least. The Λ_3 scale is a cornerstone of making massive gravity compatible with observations; the nonlinear terms, like the one in (5), lead to the Vainshtein mechanism through which the π mode is suppressed near any realistic astrophysical source. For this suppression to take place in an observable vicinity of any meaningful source, the strong scale should be low enough. Raising the strong scale would confine the Vainshtein mechanism to shorter distances, and would lead to contradictions with the observations.

Luckily, in the proposed theory, there is no problem to start with, since the large kinetic term for the longitudinal mode also leads to a suppression of its linear coupling to a stress-tensor. Hence, there is no need to invoke the nonlinear screening mechanism. This is straightforward to see from the following schematic quadratic Lagrangian, motivated by the higher-dimensional considerations of this section

$$\tilde{\mathcal{L}}_2 = M_{\rm P}^2(\partial\hat{h})^2 - M_{\rm P}^2 m^4(\partial\pi)^2 - M_D^{2+n} L^n \bar{m}^2 \bar{\Lambda}(\partial\pi)^2 + \hat{h}T \, + m^2\pi T, \tag{9}$$

where $\bar{m}$ and $\bar{\Lambda}$ are the bulk graviton mass, and bulk curvature scales respectively, while the new kinetic term is attributed to the existence of the extra dimensions (as before, it should just be approximating an essentially nonlocal term). As long as, $M_D^{2+n} L^n \bar{m}^2 \bar{\Lambda} >> M_{\rm P}^2 m^4$, the dimensionful coupling of the π mode to the stress-tensor, is proportional to

$$\frac{(m^2 L/\bar{m})}{\sqrt{M_D^{2+n} L^n}} \,, \tag{10}$$

and can be made much smaller than the gravitational coupling, $G_N \sim 1/M_{\rm P}$. We will illustrate the above general framework by a concrete 5D example in the next section.

3 Warped massive gravity

The action of a theory that realizes the mechanism outlined above is given in this Section. It contains both 4D and 5D parts; I'll start with the former, before specifying the latter.

The action for the 4D metric, $g_{\mu\nu}(x)$, $\mu, \nu = 0, 1, 2, 3$, contains the 4D Einstein-Hilbert term with the coefficient M_4^2, the cosmological constant, $\Lambda > 0$, and the 4D dRGT mass

term [2], with the mass parameter m:

$$S_4 = M_4^2 \int d^4x \sqrt{-g} \left(R(g) - 2\Lambda + 2m^2 \mathcal{U}\left(\mathcal{K}\right) \right), \tag{11}$$

where the diff-invariant potential $\mathcal{U}$ is a function of the inverse metric g^{-1}, and the fiducial Minkowski metric, $\gamma_{\mu\nu} = \partial_\mu \varphi^a \partial_\nu \varphi^a \eta_{ab}$ $(a, b = 0, 1, 2, 3;\ \eta_{ab} = diag(-1, 1, 1, 1))$, in an arbitrary coordinate system; this potential can be written in the following form [2, 17, 18]:

$$\mathcal{U}(\mathcal{K}) = \underset{2}{\det}(\mathcal{K}) + \alpha_3 \underset{3}{\det}(\mathcal{K}) + \alpha_4 \underset{4}{\det}(\mathcal{K}), \tag{12}$$

where the matrix $\mathcal{K} = 1 - A$, and the matrix A is defined as one of the roots of, $A^\mu_\alpha A^\alpha_\nu = g^{\mu\alpha}\gamma_{\alpha\nu}$, so that $\mathcal{K} = 1 - \sqrt{g^{-1}\gamma}$, [2]; φ^a denote four scalar fields (Stückelber fields).

The fiducial Minkowski metric, γ, is not dynamical. In a certain gauge, and in the high energy limit the four scalar fields, $\varphi^a(x)$, parametrize three degrees of freedom of a massive graviton, helicity ± 1 and 0. Geometrically, these four fields can be regarded as general coordinates of a certain fiducial 4D Minkowski space-time; they guarantee the full 4D diff invariance of the theory (see discussions in [19]).

The proposed extension of the theory is as follows: the bulk 5D gravity is massive, with the 5D mass $\bar{m}$, and is endowed with a negative 5D cosmological constant, $-\bar{\Lambda} < 0$. We assume that $\bar{\Lambda} >> \bar{m}^2$. There is a positive-tension brane in the 5D space; its tension is nothing but the 4D vacuum energy density, $M_4^2\, 2\Lambda > 0$, introduced in (11). Moreover, the tension will have to be tuned to $\bar{\Lambda}$ (in Planck units), to get the flat 4D world-volume solution in the absence of any additional brane stress-tensor, or brane gravity. The result is the Randall-Sundrum (RS) brane [20]. In spite of the mass term in the bulk, there is a solution identical to the RS solution, as will be shown below.

The nonlinear 5D massive gravity action for the 5D bulk metric $\bar{g}_{MN}$, $M, N = 0, 1, 2, 3, 5$, and 5D fiducial metric $\bar{f}_{MN}$, takes the form

$$S_5 = M_5^3 \int d^4x\, dz\, \sqrt{-\bar{g}} \left(\bar{R}(\bar{g}) + 2\bar{\Lambda} + 2\bar{m}^2 \mathcal{V}(\bar{\mathcal{K}}^M_N) \right), \tag{13}$$

where

$$\bar{\mathcal{K}}^A{}_B = \delta^A_B - \sqrt{\bar{g}^{AM} \bar{f}_{MB}}, \qquad \bar{f}_{MN} = \partial_M \Phi^I \partial_N \Phi^J \tilde{f}_{IJ}(\Phi), \tag{14}$$

and $\Phi^J(x^\mu, z)$, $(I, J = 0, 1, 2, 3, 5)$, denote the five scalar Stückelberg fields. The term $\mathcal{V}$ is the 5D dRGT potential, represented by a sum of all the determinants of the matrix $\bar{\mathcal{K}}$,

$$\mathcal{V}(\bar{\mathcal{K}}) = \underset{2}{\det}(\bar{\mathcal{K}}) + \beta_3 \underset{3}{\det}(\bar{\mathcal{K}}) + \beta_4 \underset{4}{\det}(\bar{\mathcal{K}}) + \beta_5 \underset{5}{\det}(\bar{\mathcal{K}}). \tag{15}$$

The replacement of the Minkowski fiducial metric by a more general one, $\bar{f}_{MN}(\Phi)$, was shown in [21] to retain the key property of the dRGT theory that enables it to eliminate the unwanted, ghostly, degree of freedom. Thus we proceed with (14).

Note that the 4D components of the 5D and 4D metrics, $\bar{g}$ and g, the 5D and 4D Stückelberg fields, and the fiducial metrics, are respectively related as follows:

$$\bar{g}_{\mu\nu}(x,z)|_{z=0} = g_{\mu\nu}(x)\,, \quad \delta^a_J \Phi^J(x,z)|_{z=0} = \varphi^a(x)\,, \quad \delta^I_a \delta^J_b \tilde{f}_{IJ}(\Phi)|_{\Phi^z=0} = \eta_{ab}\,. \tag{16}$$

The full theory is specified by the above boundary conditions,[2] applied to the total action of the theory that reads as follows

$$S_{total} = S_4 + S_5 + S_{GH}\,, \tag{17}$$

with S_4 and S_5 defined above; S_{GH} is the Gibbons-Hawking boundary term that guarantees that the bulk equations are those of Einstein, modified by the mass terms.[3]

In what follows I will regard the brane to be the boundary of the 5D space, at $z = 0^+$, and consider the equations in that setting, as was done in [22, 9] (Another space can be glued to it according to prescribed rules; for instance, by postulating Z_2 symmetry across the brane, as in RS, or using a more general setup, without imposing Z_2, [23, 24], however, this is not done here). Hence, one can separate the bulk and brane equations of motion using the general formalism of [25]. In the bulk, for $z > 0$,

$$M_5^3(\bar{G}_{AB} - \bar{\Lambda}\bar{g}_{AB}) = M_5^3 \bar{m}^2 \bar{\Theta}_{AB}\,, \tag{18}$$

while the equation at the brane takes the form:

$$M_4^2(G_{\mu\nu} + \Lambda g_{\mu\nu}) - M_5^3(k_{\mu\nu} - g_{\mu\nu}k) = M_4^2 m^2 \Theta_{\mu\nu} + M_5^3 \bar{m}^2 [\sqrt{-\bar{g}}\,\bar{\Theta}_{\mu\nu}/\sqrt{-g}]\,, \tag{19}$$

where $\bar{\Theta}_{AB}$ and $\Theta_{\mu\nu}$ are the stress-tensors derived from the 5D and 4D mass terms respectively, and $k_{\mu\nu}$ denotes the value of the extrinsic curvature at $z = 0^+$, while the square brackets, $[\cdots]$, denote the boundary term obtained via the z integration of the quantity in the brackets. Variation of the action (17) with respect to the Stückelberg fields, $\Phi^J(x^\mu, z)$, and supplied with the boundary conditions (16), gives the equations that are satisfied, as long as (18) and (19) are obeyed.

[2]Vanishing of the classical value of Φ^z at the boundary, as implied by (16), might rise a concern that it could lead to vanishing of a kinetic term for some fluctuations at the boundary. However, the kinetic term does not necessarily vanish when Φ^z does, since its strength is proportional to the first *derivative* of Φ^z at the boundary; hence, if Φ^z vanishes proportionally to z – as it will be the case for our solutions – no kinetic terms will vanish at the boundary, as will be seen in the next Section.

[3]Note that the 5D mass and potential terms in (15) do not contain second derivatives of Φ^J, hence no new boundary terms are introduced for the variational procedure. However, in a non-unitary gauges the fluctuations on the classical background, $\delta\Phi = \Phi - \Phi^{cl}$, are often decomposed in terms of the helicity-2,1, and 0 fields (see, the next Section). Such decomposition introduces in the action second derivatives acting of the helicity-0 field. In the bulk theory such terms can be converted into the first derivatives, plus total derivatives. The latter aren't important unless there is a boundary, as in the present case. Thus, one has to introduce more boundary terms in the action to guarantee that the variational principle for the helicity fields is well defined. Since the present Section is dealing with the classical equations of motion and not with the fluctuations (i.e., no variation is taken w.r.t. the helicity fields) I'll ignore these new boundary terms here (they'd vanish on classical solutions), but will discuss them in the next Section.

The fiducial metric is chosen to be the RS metric in the Φ space, with $\Phi^z \geq 0$:

$$ds^2_{Fid} = \tilde{f}_{IJ} d\Phi^I d\Phi^J = \frac{L^2}{(\Phi^z + L)^2} \left[\eta_{ab} d\Phi^a d\Phi^b + (d\Phi^z)^2\right] . \tag{20}$$

Thus, if we adopt a field configuration, $\Phi^J(x^\mu, z)\delta^\mu_J = x^\mu,\ \Phi^z(x^\mu, z) = z$, then the solution for the space-time metric with a flat 4D brane located at $z = 0$, does exist:

$$ds^2 = \bar{g}_{AB} dx^A dx^B = A^2(z) \left[\eta_{\mu\nu} dx^\mu dx^\nu + dz^2\right] , \quad A(z) \equiv \frac{L}{z + L} , \tag{21}$$

provided that the standard RS tuning between the brane and bulk cosmological constants is adopted, $M_4^2 \Lambda = M_5^3 \sqrt{6\bar{\Lambda}}$. This is so because on the above-chosen field configurations, $\bar{K}^A_B = 0 = \bar{\Theta}^A_B$, $\mathcal{K}^\mu_\nu = 0 = \Theta^\mu_\nu$, and the equations of motion (18) and (19) are satisfied due to the cancellations between the two terms on the left hand side of (18), and between the second, third, and fourth terms on the l.h.s. of (19). The terms that distinguish these equations from the RS equations are zero on the above solution.[4]

Note that instead of (20) one could have started with the fiducial metric

$$ds^2_{Fid} = \frac{L^2}{(\Phi^z)^2} \left[\eta_{ab} d\Phi^a d\Phi^b + (d\Phi^z)^2\right] , \tag{22}$$

in which case one would have still obtained the solution (21), given the following relations, $\Phi^J(x^\mu, z)\delta^\mu_J = x^\mu,\ \Phi^z(x^\mu, z) = z + L$. The latter, however, implies that $\Phi^z \geq L$, hence, the AdS boundary in (22) can't be reached. Therefore, the two theories, one with (20), and another one with (22), are equivalent to each another.

Last but not least, the fiducial metric, (20), was introduced by "hands" for the needs of the construction. It is however straightforward to obtain as a solution of dynamical equations. For this, one would amend the 5D action (13) with the 5D Einstein-Hilbert term for the metric, $\tilde{f}_{IJ}(\Phi)$, in a space-time parametrized by the coordinates Φ^J, i.e., would obtain bigravity [28]. One would not need to tune the Planck scale of the second gravity, $\tilde{M}_5$, to the existing one, M_5, however, would have to tune the bulk cosmological constant in the Φ space-time to $\bar{\Lambda}$

$$\tilde{M}_5^3 \int d^5\Phi \sqrt{f(\Phi)} \left(R(f(\Phi)) + 2\bar{\Lambda}\right) . \tag{23}$$

Furthermore, one would need to introduce a brane located at $\Phi^z = 0$, and tune its tension to the 5D quantity, $2\tilde{M}_5^3 \sqrt{6\bar{\Lambda}}$, as in RS. One can then see that the solutions presented above – (20) and (21) – satisfy the equations of motion of bigravity, for an arbitrary positive value of $\tilde{M}_5$. Thus, for simplicity, one can choose the value of $\tilde{M}_5$ to be fairly large as compared to M_5, to be able to neglect the dynamical fluctuations of the second metric, $\tilde{f}_{IJ}$, as it's done in the present work.

[4]The above considerations suggest that there should be other solutions for which neither $\bar{\mathcal{K}}^A_B$ or $\mathcal{K}^\mu_\nu$ are zero, and if so, then the 4D foliations might be either AdS or dS, as in [26, 27].

In the next Section we will consider quadratic fluctuations on the background solution obtained above, ignoring the fluctuations of $\tilde{f}_{IJ}$. We then estimate the value of the strong scale by looking at the nonlinear terms in the bulk, and on the brane. The fluctuations of $\tilde{f}_{IJ}$ do not affect these considerations since neglecting them neglects a massless graviton, which, unlike a massive graviton, has no strong interactions for weak sources.

4 Estimating the new strong scale

The quadratic massive gravity Lagrangian in the bulk of AdS_5 reads as follows

$$\mathcal{L}_{5D} = M_5^3 \sqrt{\bar{g}^{AdS}} \left(-\bar{h}_{AB} \bar{\mathcal{E}}^{ACBD} \bar{h}_{CD} - \frac{\bar{m}^2}{2} (\tilde{h}_{AB}^2 - \tilde{h}^2) \right) + \sqrt{\bar{g}^{AdS}}\ \bar{h}_{AB} \bar{T}^{AB} \,, \tag{24}$$

where $\bar{\mathcal{E}}^{ACBD}$ is the Einstein operator on AdS_5 (see, e.g., [14]); note that the Stückelberg fields, $\Phi^J \delta^A_J = x^A + \frac{1}{\bar{m}} V^A$, enter the Lagrangian via

$$\tilde{h}_{AB} \equiv \bar{h}_{AB} - \frac{1}{\bar{m}} \left(\nabla_A V_B \ + \nabla_B V_A \right) \,, \tag{25}$$

and the covariant derivative and all the index contractions are defined by the background metric, $(\bar{g}^{AdS})$. Furthermore, $\bar{T}^A{}_B$ is a 5D stress-tensor, which will for simplicity be set to zero below.[5]

The quadratic part of the 4D Lagrangian, on the other hand, reads as follows:

$$\mathcal{L}_{4D} = M_4^2 \left(-h_{\mu\nu} \mathcal{E}^{\mu\alpha\nu\beta} h_{\alpha\beta} - \frac{m^2}{2} (h'^2_{\mu\nu} - h'^2) \right) + h_{\mu\nu} T^{\mu\nu} \,, \tag{26}$$

where $\mathcal{E}^{\mu\alpha\nu\beta}$ is the Einstein operator for 4D Minkowski space, h' is defined as

$$h'_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{\bar{m}} \left(\partial_\mu v_\nu \ + \partial_\nu v_\mu \right) \,, \tag{27}$$

and all the indices are contracted by the inverse of the 4D Minkowski metric, $\eta^{\mu\nu}$. Furthermore, as specified in Section 3, the bulk and brane fields are related as follows:

$$h_{\mu\nu}(x) = \bar{h}_{\mu\nu}(x, z=0) \equiv \bar{h}_{\mu\nu}| \,, \quad v_\mu(x) = V_\mu(x, z=0) \equiv V_\mu| \,, \tag{28}$$

where the "$|$" sign denotes evaluation at $z = 0$. Also, note that $h'_{\mu\nu}(x) \neq \tilde{h}_{\mu\nu}|$, due to the nonzero connection terms in the covariant derivatives in the expression (25).[6]

Before proceeding further, a comment on the boundary terms: since there are second derivatives acting on $\bar{h}$ in (24), the Gibbons-Hawking boundary term on the brane is

[5] Due to the explicit $1/\bar{m}$ factor in (25), the Stückelberg fields, and in particular the Π mode, are normalized differently in this section as compared to the previous one, $\Pi(of\ Section\ 2) = \Pi(of\ Section\ 3)/\bar{m}$.

[6] This implies, in particular, that in the unitary gauge the brane will be bent. This gauge will not be used in the present work.

implied, to give the correct bulk equations for $\bar{h}$. Moreover, the vector field V_A will be further decomposed below, introducing more terms in the action with second derivatives applied to a field. Hence, I'll introduce below new boundary terms for V_A, to have the variational procedure well defined, at least in the limit specified below.

The precise limit that will be taken is

$$m \sim \bar{m} \to 0\,, \quad M_5 \to \infty\,, \quad M_4 \to \infty\,, \tag{29}$$

moreover, $\bar{\Lambda}$ will be held fixed, and it shall also become clear below as to why we will keep fixed the scales, $\Lambda_{5/2} \sim (M_5^{3/2}\bar{m})^{2/5}$, and $\Lambda_2 \sim (M_{\rm P}\bar{m})^{1/2}$.

A few words about the symmetries: $\tilde{h}$ is invariant under 5D linearized diffs, $\delta_d \bar{h}_{AB} = \nabla_A \Omega_B + \nabla_B \Omega_A$, and $\delta_d V_C = \bar{m}\Omega_C$, where Ω_C is a 5D vector. Hence, the bulk action is invariant under these transformations. The brane action, on the other hand, is invariant under the same transformations, only if $\omega_z = \Omega_z| = 0$.

For now, let us restrict the bulk diffs by imposing a 5D gauge,

$$\bar{h}_{\mu z} = 0 = \bar{h}_{zz}\,. \tag{30}$$

This leaves a residual diff invariance w.r.t. the following transformations: $\delta_d \bar{h}_{AB} = \nabla_A R_B + \nabla_B R_A$, and $\delta_d V_C = \bar{m} R_C$, where the components of the 5D vector, R_A, are defined as follows:

$$R_\mu = A^2(z)\omega_\mu(x) - \frac{L}{2}\partial_\mu \sigma(x)\,, \quad R_z = A(z)\sigma(x)\,, \tag{31}$$

with ω_μ, and σ, being an arbitrary 4D vector and scalar functions, respectively. Choosing $\sigma = 0$, renders the 4D theory to be invariant under the residual bulk transformations (31). Moreover, the brane is kept fixed at $z = 0$. Therefore, after the gauge fixing, both the 5D and 4D actions are invariant w.r.t. the following residual linearized diffs:

$$\begin{aligned} \delta_d \bar{h}_{\mu\nu} = A^2(z)\left(\partial_\mu \omega_\nu + \partial_\nu \omega_\mu\right), \quad \delta_d V_\mu = \bar{m} A^2(z)\omega_\mu, \\ \delta_d h_{\mu\nu} = \left(\partial_\mu \omega_\nu + \partial_\nu \omega_\mu\right), \quad \delta_d v_\mu = \bar{m}\omega_\mu, \end{aligned} \tag{32}$$

with the transformations of all the other components being zero.

The residual diff invariance, (32), is the key for counting the degrees of freedom. Generically, there are 9 degrees of freedom carried by a 5D massive graviton. In the high momentum limit, the 9 can be decomposed as, $5 + 3 + 1$, where 5 are carried by a 5D helicity-2 state, 3 by a 5D helicity-1, and 1 by the 5D helicity-0 mode. Given the 5D gauge choice (30), these degrees of freedom are distributed as follows: 5 are in $\bar{h}_{\mu\nu}(x, z)$, while 4 remain in $V_A(x, z)$. These translate into 4D massive spin-2, spin-1, and spin-0 towers of KK modes. In the limit (29), the spin-2 KK tower is identical to the RS tower [20]. For small nonzero masses, $m \sim \bar{m}$, the KK wave-functions would get distorted slightly, without yielding a gap, but turning the RS zero mode into a long-lived resonance, as in a scalar theory of [29]; in the case of gravity considered here, the zero mode needs to "eat up" three degrees of freedom to become a resonance. Let us see how this works:

The spin-1, and spin-0 towers, are in addition to the RS tower. Furthermore, the residual freedom, (32), is not general enough to eliminate 5D degrees of freedom, however, it could be used to remove 4D degrees of freedom from the 4D brane field $h_{\mu\nu}(x)$, rendering in it only 2 (at the expense of keeping 3 degrees of freedom in v_μ). On the other hand, the 4D brane field, $h_{\mu\nu}(x) = \bar{h}_{\mu\nu}|$, is nothing but a linear superposition of all the spin-2 KK states. To see this in the limit (29), recall the form of the KK expansion

$$\bar{h}_{\mu\nu}(x,z) = \int_0^\infty dk\, \bar{h}^k_{\mu\nu}(x) f_k(z)\,, \tag{33}$$

where $\bar{h}^k_{\mu\nu}(x)$ denotes a field for a KK graviton of mass k, satisfying the on-shell condition, $\partial^\mu \bar{h}^k_{\mu\nu} = \partial_\nu \bar{h}^{k\mu}_{\ \ \mu}$, while $f_k(z)$ can be expressed via the Bessel functions. Then, the residual symmetry transformations, (32), can be used to remove degrees of freedom from a linear superposition state of the spin-2 KK modes

$$h_{\mu\nu}(x) = \int_0^\infty dk\, \bar{h}^k_{\mu\nu}(x) f_k(0)\,, \tag{34}$$

by imposing on it a further 4D gauge fixing condition of one's choice.[7]

Based on the above symmetry and gauge fixing considerations, the KK spectrum, away from the limit (29), should host one special 4D collective massive state, described by the fields, $h_{\mu\nu}(x)$ (2 degrees of freedom) and $v_\mu(x)$ (3 degrees of freedom), in analogy with the scalar field of [29]. As noted, in the limit (29), this special state is massless. Its tensor part, described by $h_{\mu\nu}(x)$, is nothing but the RS zero mode, carrying 2 degrees of freedom; its vector part, v_μ, decouples from the tensor part in the limit (29), carrying 3 degrees of freedom. Away from the limit (29), the collective state can be thought as the RS zero mode that has "eaten up" 3 degrees of freedom of v_μ to become massive, and thus acquired a width to decay into the lighter KK modes. Hence, the tensor part of this metastable state would give leading interactions similar to those in the single-brane RS model, at distances greater than L and smaller than the inverse graviton mass. Furthermore, the strongly coupled sector is due to this resonance. The helicity-2 part of it is weakly interacting for weak sources, however, the helicity-1 and helicity-0, can get strong. Given the bulk and brane gauges discussed above, the helicity-1 and helicity-0 of the resonance reside in the vector V_A, and its pullback, v_μ. For these reasons, I will focus below on the vector fields, and their strong coupling.

It follows from (24) and (25), that the 5D vector field, V_A, acquires the Maxwell kinetic term, as well as a mass term, due to the background curvature. In the limit (29), the

[7] Once the KK solutions and the expansion (34) are adopted, the 4D gauge fixing should be consistent with the on-shell condition, $\partial^\mu h_{\mu\nu} = \partial_\nu h^\mu_{\ \ \mu}$. Using (32) in the latter, one finds the allowed residual transformations, with ω_μ satisfying, $\partial^\mu \omega_\mu = 0$, and $\Box_4 \omega_\mu = 0$; the solutions of the latter two equations enable one to remove 3 on-shell massless degrees of freedom from $h_{\mu\nu}(x)$. Away from the limit (29), the above KK relations get modified by small graviton mass corrections, however, the symmetry (32) remains, and still should enable one to remove 3 degrees of freedom from $h_{\mu\nu}(x)$.

vector is decoupled from the tensor perturbation, $\bar{h}$, and its Lagrangian is proportional to

$$M_5^3 \sqrt{\bar{g}^{AdS}} \left(-\frac{1}{4} F_{AB}^2 - \frac{1}{2} M_V^2 V_A^2 \right) , \tag{35}$$

where $M_V^2 = 4\bar{\Lambda}/3 = 8/L^2$. The helicity decomposition of the massive vector makes sense for the momenta higher than $M_V \sim \sqrt{\bar{\Lambda}}$. On the other hand, one is interested in the 4D theory, that emerges in the opposite limit, when the momenta are much smaller than $\sqrt{\bar{\Lambda}}$. In such a regime helicity appears to be an inappropriate label. However, due to AdS_5 warping, a nonzero M_V does not generate a mass gap in the KK spectrum [29], hence, from a 4D perspective we expect a state with mass of the order of, $m \sim \bar{m} << \bar{\Lambda}$. If so, then 4D helicity should be a good label for the momenta above the scale of, $m \sim \bar{m}$, and below that of $\sqrt{\bar{\Lambda}}$. Hence, the aim will be to use a 5D formalism that would lead to the 4D decomposition of the 5D vector in terms of the 4D helicity-0 and helicity-1 states.[8]

With the above goal in mind, one can decompose the vector into its transverse and longitudinal parts, as follows

$$V_A = V_A^T + \nabla_A \Pi \,, \quad \text{with} \quad \nabla^A V_A^T = 0 \,. \tag{36}$$

The actions for V^T and Π separate from one another, and the substitution of (36) into (35) generates a kinetic term for Π proportional to the background curvature

$$-\frac{M_5^3 M_V^2}{2} \sqrt{\bar{g}^{AdS}} \, (\nabla_A \Pi)^2 \,. \tag{37}$$

The following rescaling makes the above kinetic term canonically normalized

$$\Pi = \frac{\Pi^c}{\sqrt{M_5^3 M_V^2}} \,. \tag{38}$$

The term, (37), would appear in the theory away from the limit (29) in addition to the kinetic mixing between $\bar{h}$ and Π, which is proportional to $M_5^3 \bar{m}$. After the rescaling, (38), the mixing term is proportional to $M_5^{3/2} (\bar{m}/\sqrt{M_V})$. Since one already has the kinetic terms for both the helicity-2 (proportional to M_5^3) and helicity-0 (canonically normalized), then such a mixing can be neglected as long as one stays in the regime, $\bar{m} << M_V = \sqrt{4\bar{\Lambda}/3}$ (hence, the mixing vanishes in the limit (29)).

Note that in the basis of the fields used here there is no direct coupling of the helicity-0 to the 5D external stress-tensor. Such coupling would arise due to the diagonalization of the kinetic mixing term; however, the latter is negligible, as was just shown.

[8]Note that there is a total derivative term en route from (24) to (35). This total derivative induces a nonzero 4D surface term. I introduced a new boundary term in the action, proportional to, $\int d^4x \, (V^A \nabla_A V_z - V_z V^C \nabla_C)|$, to cancel the surface term generated by the total derivative. This very boundary term removes some of the induced surface terms for the field Π, for which the bulk action contains terms with second derivatives acting on Π; thus, due to the introduced boundary term, the quadratic action for Π, in the limit (29), will contain only its first derivatives, see below.

Let us now look at the consequence of the split (36) on the brane, and in particular, see what it implies for the respective 4D decomposition

$$v_\mu = v_\mu^T + \partial_\mu \pi \,, \tag{39}$$

where $v_\mu^T = V_\mu^T|$, $\pi = \Pi|$; note that v_μ^T is not a 4D transverse vector; in fact, from the 4D perspective, it's unconstrained. One would like to understand the meaning of (39) when substituted into the 4D mass term:

$$-\frac{M_4^2 m^2}{2\bar{m}^2}\left((\bar{m}h_{\mu\nu} - \partial_\mu v_\nu - \partial_\mu v_\nu)^2 - (\bar{m}h_\mu^\mu - 2\partial^\mu v_\mu)^2\right) \,. \tag{40}$$

To clarify the role of (39) in 4D, one should point out that the decomposition (36) is arbitrary up to the following "gauge" transformations, $\delta_g V_A^T = \nabla_A S \,, \delta_g \Pi = -S$, where the scalar S satisfies, $\nabla^2 S = 0$. The latter equation, in the presence of the brane, has a non-trivial decaying solution

$$S(x,z) = \left(\frac{z+L}{L}\right)^2 \frac{K_2((z+L)\sqrt{-\Box_4})}{K_2(L\sqrt{-\Box_4})}\, s(x) \,, \tag{41}$$

where $s(x)$ is an arbitrary 4D scalar field. The above "gauge" symmetry can be used to move the description of the 4D degrees of freedom between the 4D fields, $v_\mu^T = V_\mu^T|$ and $\pi = \Pi|$. Indeed, under the "gauge" transformation, $\delta_g v_\mu^T = \partial_\mu s \,, \delta_g \pi = -s$, where $s = S|$. Thus, one can use this freedom to remove 1 degree of freedom from v_μ^T, rendering it with only 2, while keeping the longitudinal mode in π. This appears to be a logical choice, since the progenitor of π, the Π field, propagates the bulk longitudinal mode in a very high momentum limit, above $\bar{H}$. Having this done, from now on I focus on Π, and $\pi = \Pi|$; since the separation of the degrees of freedom between v^T and π at low energies is not unique, then focusing on the Π-sector, while ignoring V^T, should be enough to estimate the strong scale both in the bulk and on the brane.

Hence, I turn to the nonlinear terms for Π in the bulk and estimate their strong scales. The representative terms that manifest strong interactions in 5D, and contain both helicity-2 and helicity-0, are proportional to

$$M_5^3 \bar{m}^2 \bar{h}\left(\left(\frac{\nabla\nabla\Pi}{\bar{m}}\right)^2 + \left(\frac{\nabla\nabla\Pi}{\bar{m}}\right)^3 + \left(\frac{\nabla\nabla\Pi}{\bar{m}}\right)^4\right) \,. \tag{42}$$

There are relative, order one parameters between the three terms in the parenthesis, however I will omit them here and below for simplicity of presentation. In addition, there are also nonlinear terms containing only Π's; these terms would have been total derivatives on a flat background, however, on AdS_5 they turn into full fledged terms in the action due to nonzero commutators of the covariant derivatives.[9] Schematically, they look as

[9]This still leaves total derivatives, which in the present case would induce nonzero surface terms. As was done in the quadratic action, I invoked nonlinear boundary terms to cancel the surface terms; this

follows:

$$M_5^3 \bar{m}^2 \bar{\Lambda} \left(\left(\frac{\nabla\Pi}{\bar{m}}\right)\left(\frac{\nabla\Pi}{\bar{m}}\right)\left(\frac{\nabla\nabla\Pi}{\bar{m}}\right) + ... + \left(\frac{\nabla\Pi}{\bar{m}}\right)\left(\frac{\nabla\Pi}{\bar{m}}\right)\left(\frac{\nabla\nabla\Pi}{\bar{m}}\right)^3 \right) . \tag{43}$$

After the rescaling to the canonically normalized fields, as in (38), and using, $\bar{h} = \bar{h}^c/M_5^{3/2}$, one gets for the nonlinear mixing terms

$$\Lambda_{7/2}^{7/2}\, \bar{h}^c \left(\frac{(\nabla\nabla\Pi^c)^2}{(M_5^{3/2}\bar{m}\sqrt{\bar{\Lambda}})^2} + \frac{(\nabla\nabla\Pi^c)^3}{(M_5^{3/2}\bar{m}\sqrt{\bar{\Lambda}})^3} + \frac{(\nabla\nabla\Pi^c)^4}{(M_5^{3/2}\bar{m}\sqrt{\bar{\Lambda}})^4} \right) , \tag{44}$$

and for the pure Π terms

$$\frac{(\nabla\Pi^c)(\nabla\Pi^c)(\nabla\nabla\Pi^c)}{(M_5^{3/2}\bar{m}\sqrt{\bar{\Lambda}})} + \frac{(\nabla\Pi^c)(\nabla\Pi^c)(\nabla\nabla\Pi^c)^2}{(M_5^{3/2}\bar{m}\sqrt{\bar{\Lambda}})^2} + \frac{(\nabla\Pi^c)(\nabla\Pi^c)(\nabla\nabla\Pi^c)^3}{(M_5^{3/2}\bar{m}\sqrt{\bar{\Lambda}})^3} , \tag{45}$$

where, $\Lambda_{7/2} = (M_5^{3/2}\bar{m}^2)^{2/7}$, is what would have been the strong scale of the 5D theory if it had no bulk cosmological constant. However, due to the large bulk CC, $\bar{\Lambda} \equiv \bar{H}^2 >> \bar{m}^2$, the strong scale is higher; its lowest value is obtained from the terms containing the Π self-interactions only, (45); hence, the mixing terms (44), and the respective boundary terms they'd call for, can then be ignored. The strong scale reads:

$$\Lambda_{5D} \simeq (M_5^{3/2}\bar{m}\bar{H})^{2/7} = \Lambda_{7/2} \left(\frac{\bar{H}}{\bar{m}}\right)^{2/7} >> \Lambda_{7/2} . \tag{46}$$

Having the 5D strong scale estimated, let us turn to the respective effective 4D theory, with the goal to estimate its strong scale. The 4D description should be valid at energies below the scale of curvature of 5th dimension, $E \lesssim \sqrt{\bar{\Lambda}} = \bar{H} \sim L^{-1}$.

As was already noted, the spectrum of the KK modes has no mass gap, and is continuous, in spite of the nonzero bulk and brane mass terms [29]. Nevertheless, for the 4D distance scale r, such that $L << r$, it is possible to argue that the 5th dimension can be integrated out approximately. To show this, one notes that the Π equation in the bulk that follows from (37), $\nabla^2\Pi = 0$, can be solved with the decaying boundary conditions at $z \to \infty$

$$\Pi(x,z) = \left(\frac{z+L}{L}\right)^2 \frac{K_2((z+L)\sqrt{-\Box_4})}{K_2(L\sqrt{-\Box_4})}\, \pi(x) . \tag{47}$$

Then, the 4D "kinetic term" for π can be obtained by substituting (47) into (37); as is well known, this leaves only a surface term proportional to, $(-\Pi\partial_z\Pi)|_{z=0}$, which in its

makes the variational problem for Π well-defined, at least in the limit (29), where all the mixing terms vanish. In general, for each of the four potential terms in (15), there is a nonlinear total derivative term for Π, that contains second derivatives of Π [1]; all of these terms will induce surface terms. The latter are cancelled by invoking the new boundary terms, written in terms of V, as discussed above.

turn gives rise to the following term in 4D

$$\frac{M_5^3 M_V^2}{2}\,\pi(x)\,\sqrt{-\Box_4}\,\frac{K_1(L\sqrt{-\Box_4})}{K_2(L\sqrt{-\Box_4})}\,\pi(x)\,. \tag{48}$$

This nonlocal term appears in the 4D effective Lagrangian as a "kinetic term" for π. It defines the π propagator, that has a gapless continuum of poles. These poles correspond to a gapless continuum of 4D particles.

In the leading approximation for the 4D effective description, when $L\sqrt{-\Box_4} << 1$, one expands the McDonald functions and finds that the π kinetic term is proportional to

$$L\,\frac{M_5^3 M_V^2}{2}\,\pi(x)\,\Box_4\,\pi(x)\,. \tag{49}$$

Thus, in the leading approximation, the scalar in the AdS_5 background "feels" its ambient space as if it were of a physical size L, [30].

Due to the large induced kinetic term (49), the 4D dynamics of π should be expected to differ significantly from that in pure 4D massive gravity. To understand those differences let us look at other 4D terms containing π. One of them is a kinetic mixing term between the tensor, h, and π

$$\frac{M_4^2 m^2}{\bar{m}} h_{\mu\nu}(\partial^\mu\partial^\nu\pi - \eta^{\mu\nu}\partial^2\pi)\,. \tag{50}$$

After rescaling (38), the brane mixing term ends up being proportional to the following ratio, $q = (M_4^2 m^2/\sqrt{M_5^3\bar{m}^2\bar{\Lambda}})$. Since, $\bar{m}\sim m << \sqrt{\bar{\Lambda}}$, and $M_4^2 \lesssim M_5^3/\sqrt{\bar{\Lambda}}$, we conclude that $q\sim\mathcal{O}(m)$ and, therefore, the brane mixing term can also be neglected as compared to the induced 4D kinetic term (49).

Last but not least, there are also genuine 4D non-linear terms involving h and π [1], and one would like to estimate their strong scale, in the presence of (49). To this end, one collects the following representative linear and nonlinear terms of tensor-scalar and scalar-scalar interactions

$$-LM_5^3 M_V^2(\partial\pi)^2 + (M_4^2 m^2 + LM_5^3\bar{m}^2)h\left(\Sigma_{n=1}^3\left(\frac{\partial\partial\pi}{\bar{m}}\right)^n\right) + \frac{LM_5^3 M_V^2}{\bar{m}}(\partial\pi)^2(\partial\partial\pi)^2\cdots \tag{51}$$

where the order one coefficients between the terms in the above schematic expression have been ignored.[10] Since, $m\sim\bar{m} << \bar{H} = \sqrt{\bar{\Lambda}}$, one can obtain the following expression for the canonically normalized π field

$$-(\partial\pi^c)^2 + (M_4^2 m^2 + M_5^3 L\bar{m}^2)h\left(\frac{(\partial\partial\pi^c)}{\Lambda_*^3} + \frac{(\partial\partial\pi^c)^2}{\Lambda_*^6} + \frac{(\partial\partial\pi^c)^3}{\Lambda_*^9}\right) + \frac{(\partial\pi)^2(\partial\partial\pi)^2}{\Lambda_*^6} + \ldots, \tag{52}$$

[10] The bulk cubic Galileon gives zero on the lowest order bulk equations of motion for Π, and hence was not included in (51).

where, $\pi^c \equiv \sqrt{L}\,\Pi^c|$, and

$$\Lambda_* \simeq (M_5^{3/2}\bar{m}\bar{H}^{1/2})^{1/3}\,, \tag{53}$$

is the lowest strong scale of the 4D theory due to the π self-interactions. Note that the 4D Planck mass is determined by two contributions, proportional to M_4^2, and LM_5^3, respectively; if for simplicity we assume the latter is greater than the former, then, it would follow that $M_5^{3/2} \sim M_{\rm P}\bar{H}^{1/2}$, and,

$$\Lambda_* \sim (M_{\rm P}\bar{m}\bar{H})^{1/3} = (\Lambda_2^2\bar{H})^{1/3}. \tag{54}$$

To estimate the numerical value of this scale, let us set, $\bar{H} \sim 10^{16}\,GeV$, $M_5 \sim 10^{18}\,GeV$, and $\bar{m} \sim m \sim 10^{-42}\,GeV$, then the 5D strong scale, $\Lambda_{5D} \sim GeV$, while the 4D strong scale is lower, $\Lambda_* \sim MeV$. The latter, however, is some 19 orders of magnitude greater than the strong scale of pure 4D massive gravity, $\Lambda_3 \sim 10^{-19}\,MeV$. [11]

A few important comments are in order. Only a simple setup was studied above with just one mass scale on the brane and in the bulk, $m \sim \bar{m}$; however, it is straightforward to see that all the results above apply to the case, $m << \bar{m}$, and in particular to, $(m/\bar{m}) \to 0$. This is so since the 4D massive theory of Section 3 is perturbatively continuous in the $m \to 0$ limit due to the bulk physics with $\bar{m} \neq 0$, and the scale of non-linear interactions, (54), is independent of m, in the leading approximation.

The estimate for the new strong scale, (53), was made above assuming generic values for the parameters, $\alpha_3, \alpha_4, \beta_3, \beta_4, \beta_5$, in the 4D and 5D potentials, (12) and (15). However, for certain specific relationships between some of these parameters there might be cancelations between at least some of the nonlinear terms, in analogy with the cancellations in a 4D flat space case [1]; if so, it is then conceivable that in those special cases the strong scale might perhaps be higher than (53).

Furthermore, none of the calculations above would have changed if one ignored the 4D Einstein-Hilbert (EH) term, but kept a fixed brane tension, $2M_4^2\Lambda$. However, the 4D EH term might be useful for more general solutions, and would in any case be induced by quantum loops in an effective theory, even if it was not introduced in the classical theory to begin with [6]. Hence, it was included for generality.

As a final comment, the 5D massive gravity on AdS_5 with the AdS_5 fiducial metric, was argued [14] to be holographic dual to a theory of unparticles [31] – a certain non-local CFT that has no conserved 4D stress-tensor. If so, then the mechanism of the present work could be thought in terms of the CFT with a UV cutoff: the π mode acquires a large kinetic term due to coupling of massive gravity to the 4D non-local CFT, while the existence of the RS brane translates into the existence of a UV cutoff of the 4D CFT. Since such a CFT appears to be pretty exotic, the 5D classical gravity description, used

[11] One could also wonder if the theory can transition to the 5D regime before reaching the 4D strong scale (53). That is possible if, $\Lambda_* \gtrsim \bar{H}$, or expressed differently, $M_5^3\bar{m}^2 \gtrsim \bar{H}^5$; the latter condition can be rewritten as, $\Lambda_2 \gtrsim \bar{H}$, if $LM_5^3 > M_4^2$. Since Λ_2 is of the order of $10^{-3}\ eV$, the effective size of such a dimension would be a millimeter or larger. In that case, the strong coupling of the theory would be given by the 5D scale, (46), which then can be estimated to be, $\Lambda_{5D} \sim 10^{-3}eV$; not much of a gain.

in this work, seems to be a simpler option. However, certain aspects might be clearer in the CFT; for instance, in the $m = 0, \bar{m} \neq 0$ theory, 4D massive graviton is a state that should perhaps better be viewed as a resonant spin-2 state emerging entirely from the CFT.

5 An outlook

Calculations in Section 3 suggest that the theory proposed in this paper does admit a self-accelerated solution, for some values of the parameters (for a review of self-acceleration in massive gravity and its extensions, see [32], and references therein). The main features of the helicity-0 mode – that its coupling to an external stress-tensor can be ignored, and that its strong scale is high – would remain valid on the self-accelerated background. However, the quadratic fluctuations on the background should be expected to receive additional terms as compared to pure massive gravity. It would be interesting to see if these solutions exhibit healthy fluctuations.

Furthermore, one can straightforwardly extend the theory in various directions, for instance, by introducing a scalar field that sets the mass scale, thus providing a dynamical mass generation; or introduce a more restricted scalar based on dilatation symmetry, as in the quasidilaton theory [33], and its generalizations [34]. One could study self-acceleration in the warped version of bigravity of [28], discussed in Section 3, when the metric $\tilde{f}(\Phi)$ becomes dynamical due to a 5D Einstein-Hilbert term integrated over the 5D invariant volume in the Φ space-time, together with the cosmological constant, $\int d^5\Phi \sqrt{f(\Phi)}(R(f(\Phi)) + 2\bar{\Lambda})$, and a "brane" in the Φ-space is also included.

It remains to be seen if the mechanism proposed in this work may or may not be understood as softening of the strong π amplitudes by the light modes of the non-local CFT, which by itself should have a strong coupling. Last but not least, would be interesting to study extensions of the theory beyond 5 dimensions, including unconventional ones along the lines of [35], with the goal to perhaps raise the strong scale even further, ideally toward the Planck scale.

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