μ-hybrid inflation with low reheat temperature and observable gravity waves

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μ-Hybrid Inflation with Low Reheat Temperature and Observable Gravity Waves

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Abstract

In μ-hybrid inflation a non-zero inflaton VEV induced by supersymmetry breaking is proportional to the gravitino mass $m_{3/2}$, which can be exploited to resolve the MSSM μ problem. We show how this scenario can be successfully implemented with $m_{3/2} \sim 1 - 100$ TeV and reheat temperature as low as 10^6 GeV by employing a minimal renormalizable superpotential coupled with a well defined non-minimal Kähler potential. The tensor to scalar ratio r, a canonical measure of primordial gravity waves in most cases is less than or of order of $10^{-6} - 10^{-3}$.

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I. INTRODUCTION

In its simplest formulation minimal supersymmetric hybrid inflation can be associated with a symmetry breaking $G \to H$, where G and H are usually assumed to be local gauge symmetries [1]. Successful inflation employs a unique renormalizable superpotential W and a canonical Kähler potential K_c [1, 2], and the temperature fluctuation $\delta T / T$ in this case is roughly proportional to $(M/m_P)^2$[1], where M and m_P denote the symmetry breaking scale of G and reduced Planck scale $(2.4 \times 10^{18}$ GeV) respectively. The model therefore predicts that the symmetry breaking scale M is on the order of $10^{15} - 10^{16}$ GeV, which means that Grand Unified Theories (GUTs) are naturally incorporated in supersymmetric hybrid inflation [3]. A scalar spectral index in agreement with the measurement $n_s = 0.9655 \pm 0.0062$ by Planck [4] is realized by including both the radiative corrections as well as the relevant soft supersymmetry breaking terms in the inflationary potential [5]. Without the soft supersymmetry breaking terms n_s lies close to 0.98, and an alternative way to bring this into agreement with the Planck measurement is to employ a non-minimal Kahler potential [6, 7].

An attractive extension of minimal supersymmetric hybrid inflation is the so-called 'μ-hybrid inflation' [8, 9], which is shorthand for supersymmetric hybrid inflation in the presence of the trilinear coupling SH_uH_d that yields the desired Minimal Supersymmetric Standard Model (MSSM) μ term. The scalar component of the G-singlet superfield S acquires, after supersymmetry breaking, a non-zero VEV (vacuum expectation value) proportional to $m_{3/2}$, where $m_{3/2}$ denotes the gravitino mass [8]. This can be used, as shown in [8], to resolve the μ problem encountered in the MSSM. This model has been explored in greater depth in [9], with the conclusion that successful μ-hybrid inflation based on the minimal superpotential W and a canonical Kähler potential K_c leads to split supersymmetry [10]. The gravitino mass $m_{3/2}$ and the soft supersymmetry breaking masses are predicted to be larger than 5×10^7 GeV, and the reheat temperature after inflation estimated to be $T_r \gtrsim 10^{12}$ GeV [9]. For a recent discussion on μ-hybrid inflation in no-scale supergravity see [11] and for a discussion including axions see [12].

In this paper we study an extension of minimal μ-hybrid inflation in which the canonical Kähler potential is replaced by a non-minimal K but the renormalizable superpotential W is retained. This will allow us to implement successful μ-hybrid inflation with $m_{3/2} \sim 1 - 100$ TeV and soft scalar masses in the TeV region, compatible with the resolution of the gauge hierarchy problem. The plan of the paper is as follows. In section II we review μ-hybrid inflation with minimal canonical

Kähler potential. In section III we study the consistency of μ-hybrid inflation with the gravitino problem and reheat temperature in the case of non-minimal Kähler potential. We discuss the prospect of observing primordial gravity waves in section IV. In addition, the impact of cosmic strings [13], if present, on the inflationary predictions is also briefly discussed in section IV. In section V we summarize our findings.

II. μ-HYBRID INFLATION WITH MINIMAL KÄHLER POTENTIAL

Minimal supersymmetric μ-hybrid inflation employs a canonical Kähler potential and a unique renormalizable superpotential W which respects a $U(1)$ R-symmetry [8],

$$W = \kappa S(\Phi\overline{\Phi} - M^2) + \lambda S H_u H_d, \tag{1}$$

where κ and λ are dimensionless real parameters. The scalar component of S, a gauge singlet chiral superfield plays the role of the inflaton. The dimension-full parameter M represents the non-zero VEV of the conjugate chiral superfields Φ and $\overline{\Phi}$ that belong to a non-trivial representation of a gauge group G. Since our main goal is to discuss the viability of μ-hybrid inflation with low reheat temperature and TeV scale SUSY breaking masses, we take G to be $U(1)_{B-L}$ [14], an attractive and presumably the simplest model for μ-hybrid inflation. The superpotential W and superfield S have unit R-charges, while the remaining superfields are assigned zero R-charges. This implies that in the supersymmetric limit, the VEV of the scalar component of superfield S is zero. The gravity mediated supersymmetry breaking yields a non-zero VEV proportional to $m_{3/2}$ for the scalar component of S. Then the last term in the superpotential, $\lambda S H_u H_d$, effectively describes the μ-term with $\mu \sim \frac{\lambda}{\kappa} m_{3/2}$, thereby solving the MSSM μ problem [8].

The minimal canonical Kähler potential is given by

$$K_c = |S|^2 + |\Phi|^2 + |\overline{\Phi}|^2 + |H_u|^2 + |H_d|^2. \tag{2}$$

Taking into account the well-known radiative corrections [1, 15], supergravity (SUGRA) corrections [3, 16] and the soft supersymmetry breaking terms [5, 17], the inflationary potential (along the D-flat direction with $|\Phi| = |\overline{\Phi}| = 0$ and $|H_u| = |H_d| = 0$) to a good approximation is given by:

$$V(x) \simeq \kappa^2 M^4 \left(1 + \frac{\kappa^2}{8\pi^2} F(x) + \frac{\lambda^2}{4\pi^2} F(y) + \frac{1}{2}\left(\frac{M}{m_P}\right)^4 x^4 + a\frac{m_{3/2}}{\kappa M} x\right). \tag{3}$$
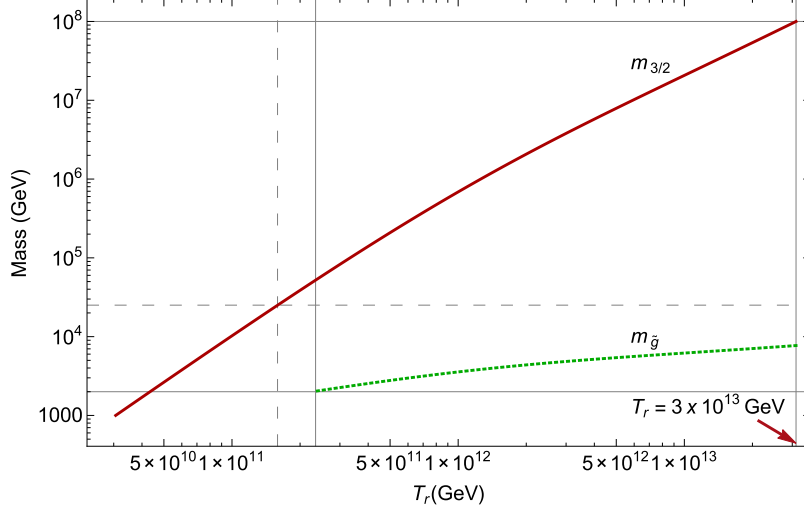
FIG. 1: Variation of gravitino mass $m_{3/2}$ with reheat temperature T_r is shown by solid-red curve for the minimal Kähler potential with the conditions, $n_s = 0.9655$, $\gamma = 2$ and $a = -1$. The gluino mass $m_{\tilde{g}}$, obtained via Eq. (8), is displayed with dotted-green curve. The LHC lower bound on gluino mass ($m_{\tilde{g}} \gtrsim 2$ TeV) is shown by a grey vertical line at $T_r \sim 2 \times 10^{11}$ GeV. Regarding the unstable gravitino scenario, we indicate $T_r \sim 10^{11}$ GeV corresponding to $m_{3/2} = 25$ TeV as shown by dashed-grey line.

The radiative corrections are described by the functions,

$$F(x) = \frac{1}{4}[(x^4 + 1) \ln\left(\frac{x^4 - 1}{x^4}\right) + 2x^2 \ln\left(\frac{x^2 + 1}{x^2 - 1}\right) + 2 \ln\left(\frac{\kappa^2 M^2 x^2}{Q^2}\right) - 3] \tag{4}$$

where $x = |S|/M$, $y = \sqrt{\gamma}\, x$ and following [9] we define $\gamma = \lambda/\kappa$ and take the coefficient of soft linear term $a = -1$ [5]. Note that the canonically normalized inflaton field is $\sigma = |S|/\sqrt{2}$ where we denote both the superfield and its scalar component by S. Note that in this paper we will ignore the imaginary component of S which has previously been analyzed in [7, 17, 18]. The μ-term coupling $\lambda S H_u H_d$ in Eq. (1) induces an inflaton decay into Higgsinos with a decay width given by [9],

$$\Gamma_S(S \to \tilde{H}_u \tilde{H}_d) = \frac{\lambda^2}{8\pi} m_S, \tag{5}$$

where $m_S = \sqrt{2}\kappa M$ is the inflaton mass. The reheat temperature T_r is estimated to be [19],

$$T_r \approx \sqrt[4]{\frac{90}{\pi^2 g_*}} \sqrt{\Gamma_S m_P}, \tag{6}$$

where g_* is 228.75 for MSSM.

Following [9] we briefly recapitulate μ-hybrid inflation with minimal K and W. Taking into account the inflationary constraints we obtain a lower bound on the reheat temperature $T_r \gtrsim 3 \times$

10^{10} GeV for a gravitino mass $m_{3/2} \gtrsim 1$ TeV (as shown by solid-red curve in Fig. 1). Assuming that the gravitino is the lightest supersymmetric particle (LSP), it's contribution to the relic density [20] is given by

$$\Omega_{3/2}h^2 = 0.23\left(\frac{T_r}{10^{10} \text{ GeV}}\right)\left(\frac{1 \text{ TeV}}{m_{3/2}}\right)\left(\frac{m_{\tilde{g}}}{2 \text{ TeV}}\right)^2, \tag{7}$$

where $m_{\tilde{g}}$ is the gluino mass. The relic density of the observed dark matter DM (that is $\Omega_{DM}h^2 = 0.11$) [21] is used to obtain the variation of the gluino mass $m_{\tilde{g}}$ with T_r and $m_{3/2}$,

$$\frac{m_{\tilde{g}}}{2 \text{ TeV}} = \left[0.46\left(\frac{10^{10} \text{ GeV}}{T_r}\right)\left(\frac{m_{3/2}}{1 \text{ TeV}}\right)\right]^{\frac{1}{2}}. \tag{8}$$

Using the LHC bound on the gluino mass, the plot of $m_{\tilde{g}}$ was cutoff at 2 TeV. It is clearly seen that the gravitino is heavier than gluino for all values above the LHC cutoff (Fig. 1), and hence the former cannot be the LSP.

In the light of the unstable gravitino problem [22] we review the two scenarios discussed in detail in [9]. For an unstable gravitino, the lifetime is (Fig. 1 of [23])

$$\tau_{3/2} \simeq 1.6 \times 10^4 \left(\frac{1 \text{ TeV}}{m_{3/2}}\right)^3. \tag{9}$$

A long-lived unstable gravitino with lifetime $\tau_{3/2} > 1$ sec and mass $m_{3/2} < 25$ TeV will decay after Big Bang Nucleosynthesis (BBN). Therefore, successful BBN limits the reheat temperature to be $T_r \lesssim 3 \times (10^5 - 10^6)$ GeV and $T_r \lesssim 2 \times 10^9$ GeV for gravitino masses of 1 TeV and 10 TeV respectively [23]. As shown in Fig. 1, the reheat temperature from inflationary constraints is 3×10^{10} GeV and 10^{11} GeV for gravitino masses of 1 TeV and 10 TeV respectively. Hence the possibility of long-lived unstable gravitino is inconsistent with the BBN bounds.

For a short lived unstable gravitino with $m_{3/2} > 25$ TeV, the BBN bound on reheat temperature is unimportant. As the gravitino decays into the LSP neutralino $\tilde{\chi}_1^0$, we find [23],

$$\Omega_{\tilde{\chi}_1^0}h^2 \simeq 2.8 \times 10^{11} \times Y_{3/2}\left(\frac{m_{\tilde{\chi}_1^0}}{1 \text{ TeV}}\right), \tag{10}$$

where $m_{\tilde{\chi}_1^0}$ is the mass of the lightest neutralino and $Y_{3/2}$ is gravitino yield given by

$$Y_{3/2} \simeq 2.3 \times 10^{-12}\left(\frac{T_r}{10^{10} \text{ GeV}}\right). \tag{11}$$

This relations fits well with the numerical result for a wide range of reheat temperature, that is $T_r \sim 10^5 \text{ GeV} - 10^{12} \text{ GeV}$ [23]. As the LSP neutralino density produced by gravitino decay should not exceed the observed DM relic density, we finally obtain an upper bound on the neutralino mass;

$$m_{\tilde{\chi}_1^0} \lesssim 18\left(\frac{10^{11} \text{ GeV}}{T_r}\right). \tag{12}$$

This is inconsistent with the lower limit set on the neutralino mass $m_{\tilde{\chi}_1^0} \gtrsim 18$ GeV [24] for reheat temperature $T_r \gtrsim 10^{11}$ GeV with $m_{3/2} > 25$ TeV (see Fig. 1).

To bypass this constraint the LSP neutralino is assumed to be in thermal equilibrium during gravitino decay in which case the neutralino abundance is independent of the gravitino yield. For a typical value of the freeze out temperature, $T_F \simeq 0.05\, m_{\tilde{\chi}_1^0}$, the gravitino lifetime is estimated to be:

$$\tau_{3/2} \lesssim 10^{-11} \text{ sec} \left(\frac{1 \text{ TeV}}{m_{\tilde{\chi}_1^0}}\right)^2. \tag{13}$$

Comparing Eqs.(9) and (13) we obtain a bound on $m_{3/2}$,

$$m_{3/2} \gtrsim 10^8 \text{ GeV} \left(\frac{m_{\tilde{\chi}_1^0}}{2 \text{ TeV}}\right)^{2/3}. \tag{14}$$

We thus arrive at the main conclusion of [9], namely that the successful realization of μ-hybrid inflation with the minimal Kähler potential requires split supersymmetry with gravitino mass $m_{3/2} \gtrsim 10^8$ GeV. The corresponding reheat temperature according to this scenario is $T_r \gtrsim 10^{13}$ GeV (Fig. 1). In summary, the gravitino mass is strongly dependent on reheat temperature due to inflationary constraints. In the next section we discuss how these problems are overcome by employing a non-minimal Kähler potential.

III. μ-HYBRID INFLATION WITH NON-MINIMAL KÄHLER POTENIAL

The Kähler potential including the relevant non-minimal terms is given by

$$K = K_c + \kappa_S \frac{|S|^4}{4m_P^2} + \kappa_{SS} \frac{|S|^6}{6m_P^4} + \dots \quad . \tag{15}$$

The corresponding scalar potential takes the following form,

$$V(x) \simeq \kappa^2 M^4 \left(1 + \frac{\kappa^2}{8\pi^2} F(x) + \frac{\lambda^2}{4\pi^2} F(y) + \frac{\gamma_S}{2}\left(\frac{M}{m_P}\right)^4 x^4 - \kappa_S \left(\frac{M}{m_P}\right)^2 x^2 + a \frac{m_{3/2}}{\kappa M} x\right), \tag{16}$$

where $\gamma_S = 1 + 2\kappa_S^2 - \frac{7\kappa_S}{2} - 3\kappa_{SS}$. Using this scalar potential we can obtain the usual slow roll parameters defined by,

$$\varepsilon = \frac{m_p^2}{4M^2}\left(\frac{V'}{V}\right)^2, \ \eta = \frac{m_p^2}{2M^2}\left(\frac{V''}{V}\right), \ \xi^2 = \frac{m_p^4}{4M^4}\left(\frac{V'V'''}{V^2}\right). \tag{17}$$

It is important to note that all derivatives (denoted by V', V'' and V''') in the above expressions are with respect to x and not with respect to the canonically normalized field $\sigma = \sqrt{2}|S|$. The scalar

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spectral index n_s, the tensor-to-scalar ratio r and the running of the scalar spectral index $dn_s/dlnk$, to leading order in slow roll approximation, are given by

$$n_s \simeq 1 - 6\varepsilon + 2\eta, \tag{18}$$

$$r \simeq 16\varepsilon, \tag{19}$$

$$\frac{dn_s}{dlnk} \simeq 16\varepsilon\eta - 24\varepsilon^2 - 2\xi^2. \tag{20}$$

The amplitude of power spectrum of scalar curvature perturbation A_s is given by,

$$A_s(k_0) = \frac{1}{6\pi^2}\left(\frac{M}{m_P}\right)^2\left(\frac{V^3/V'^2}{m_P^4}\right)_{x=x_0}, \tag{21}$$

where $A_s(k_0) = 2.196 \times 10^{-9}$ and x_0 is the value of the inflaton field at the pivot scale $k_0 = 0.05\,\text{Mpc}^{-1}$ [4]. The number of e-folds between the horizon exit (at pivot scale) and the end of inflation is given by

$$N_0 = 2\left(\frac{M}{m_P}\right)^2\int_1^{x_0}\left(\frac{V}{V'}\right)dx. \tag{22}$$

Assuming a standard thermal history, N_0 is related with T_r as,

$$N_0 = 53 + \frac{1}{3}\ln\left(\frac{T_r}{10^9\,\text{GeV}}\right) + \frac{2}{3}\ln\left(\frac{\sqrt{\kappa}M}{10^{15}\,\text{GeV}}\right), \tag{23}$$

where T_r is given by Eq. (6). The predictions for the various inflationary parameters is estimated numerically using these relations up to second order in the slow roll parameters. For most of the numerical work, we fix the scalar spectral index at the central value of the range given by Planck, $n_s = 0.9655 \pm 0.0062$ [4]. It is important to mention here that in the minimal Kähler potential case with TeV scale soft SUSY masses [5], we need to take $a = -1$ in order to obtain the red-tilted spectrum within the 1-σ Planck bounds. Alternatively, for $a = 1$ a similar range for n_s can be obtained by taking intermediate scale, negative soft mass-squared term for the inflaton [25]. However, with the non-minimal Kähler potential we can obtain the central value of n_s with TeV scale soft masses even for $a = 1$ [6, 7]. The appearance of a negative mass term for the inflaton with non-minimal coupling κ_s in the potential (Eq. (24)) actually plays a crucial role in realizing successful inflation. Most importantly it is shown in [7] that a low reheat temperature in the standard hybrid inflation can be obtained with appropriately small values of κ (see Fig. 10 of [7]). Therefore, with smaller values of κ and λ, we expect similar results in μ-hybrid inflation.

For suitably small κ and λ values, the last $50-60$ e-folds occur in the flat region of the potential with $|S|$ close to M (that is $x \to 1$). Therefore, we can ignore the radiative corrections and the
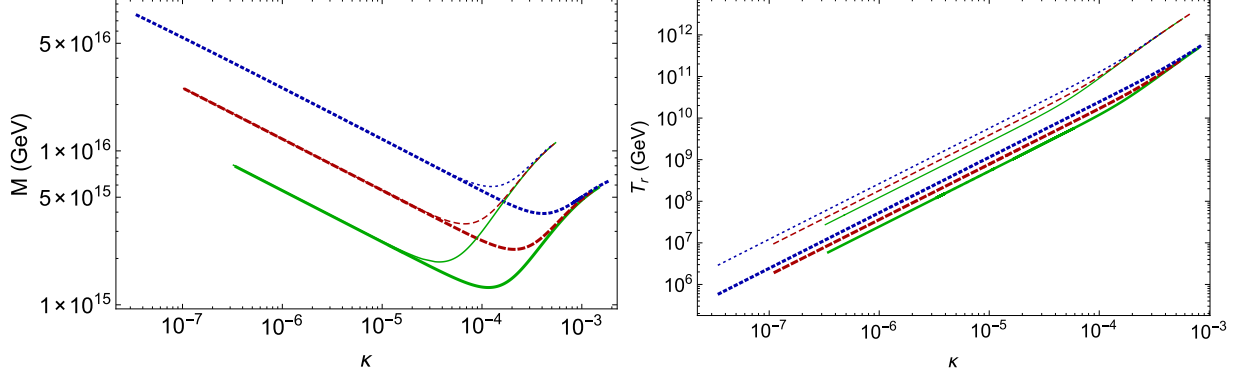
FIG. 2: Variation of the reheat temperature T_r and the symmetry breaking scale M with κ for gravitino mass of 1 TeV (solid-green), 10 TeV (dashed-red) and 100 TeV(dotted-blue). We fix the scalar spectral index $n_s = 0.9655$, $\kappa_S = 0.02$, $\kappa_{SS} = 0$ and represent $\gamma = 2(10)$ by thick (thin) curves.

potential becomes,

$$V(x) \simeq \kappa^2 M^4 \left(1 + \frac{\gamma_S}{2} \left(\frac{M}{m_P} \right)^4 x^4 - \kappa_S \left(\frac{M}{m_P} \right)^2 x^2 + a \frac{m_{3/2}}{\kappa M} x \right). \tag{24}$$

Apart from the linear term this potential resembles the model of Spontaneous Symmetry Breaking Inflation (SSBI) [26]. However, near $x \sim 1$, the linear term becomes important and develops a local maximum resulting in 'hilltop inflation' starting near this maximum [27]. These hilltop type solutions are a common feature of supersymmetric hybrid inflation models with non-minimal Kähler potential [28]. Near $x \sim 1$ Eq. (21) becomes

$$A_s \approx \frac{\kappa^2}{6\pi^2} \left(\frac{M}{m_P} \right)^6 \left(4\gamma_S \left(\frac{M}{m_P} \right)^4 - 2\kappa_S \left(\frac{M}{m_P} \right)^2 + a \frac{m_{3/2}}{\kappa M} \right)^{-2}. \tag{25}$$

Assuming $M \lesssim 10^{16}$ GeV, the quartic SUGRA correction term with co-efficient γ_S can be ignored and this further simplifies the above expression to,

$$A_s \approx \frac{\kappa^2}{6\pi^2} \left(\frac{M}{m_P} \right)^6 \left(-(1 - n_s) \left(\frac{M}{m_P} \right)^2 + a \frac{m_{3/2}}{\kappa M} \right)^{-2}, \tag{26}$$

where we have used Eq. (18) to eliminate κ_S in favour of n_s [7]. For the central value $n_s = 0.9655$ and other inflationary constraints from Eqs. (21) and (23), it is found numerically that there is one percent cancellation among the two terms in Eq. (26). As a result we arrive at a simple relation for M as function of κ,

$$M(\kappa) \approx \left(\frac{a m_{3/2}}{\kappa (1 - n_s) m_P} \right)^{1/3} m_P \approx 5.6 \times 10^{12} \left(\frac{m_{3/2}}{\kappa} \right)^{1/3} \propto \kappa^{-1/3}. \tag{27}$$

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The reheat temperature from Eq. (6) can be written as,

$$T_r(\kappa) \approx 1.7 \times 10^8 \gamma \sqrt{M\kappa^3} \approx 3.9 \times 10^{14} \gamma (m_{3/2})^{1/6} \kappa^{4/3} \propto \kappa^{4/3}. \tag{28}$$

This is an important relation which justifies our expectation of realizing a low reheat temperature with appropriately small values of κ in μ-hybrid inflation. The approximate results in Eqs. (27) and (28) are clearly in agreement with our numerical work as shown in Fig. 2. Here we have displayed the variation of T_r and M as a function of κ, for three different values of the gravitino mass $m_{3/2} = 1, 10$ and 100 TeV. As mentioned in [8], in order to have the 'good' point, that is $H_u = H_d = 0$ and $\Phi \overline{\Phi} = M^2$, as the unique local minimum of the superpotential (given by Eq. (1)), we should have $\gamma > 1$. Therefore, to see the effect of γ on various inflationary parameters in general and on reheat temperature in particular, we consider $\gamma = 2$ and 10 (as shown by the thick and thin curves respectively in Fig. 2). For the M versus κ curve of Fig. 2, the dependence of γ disappears in the low reheat temperature regime where the radiative corrections become unimportant (see Eq. (27)). For a given value of κ and M the reheat temperature varies linearly with γ as derived in Eq. (28) and shown in the right panel of Fig. 2. Therefore, to estimate the lower bounds on reheat temperature we implicitly assume $\gamma = 2$. As mentioned earlier the low reheat temperature values occur with x_0 close to one, so we impose a 0.01% fine tuning bound on the difference $x_0 - 1$. Consequently, we obtain the following lower bounds on the reheat temperature T_r:

$$T_r \gtrsim (6 \times 10^6, \ 2 \times 10^6, \ 6 \times 10^5) \text{ GeV } \text{ for } m_{3/2} \sim (1, 10, 100) \text{ TeV}, \tag{29}$$

respectively (Fig. 2).

We return to a discussion of the gravitino problem in the light of low reheat temperature. With reference to the previous discussion we consider the following three scenarios:

1. A stable LSP gravitino;

2. an unstable 'long-lived' gravitino with mass less than 25 TeV;

3. an unstable 'short-lived' gravitino with mass greater than 25 TeV.

Assuming the gravitino is LSP and using the dark matter relic density ($\Omega_{DM} h^2 = 0.11$) constraint, the gluino mass is calculated as a function of T_r via Eq. (7), as shown in Fig. 3, for three values of $m_{3/2}$. The upper bounds on the reheat temperature, shown by the vertical grey lines in Fig. 3 are

$$T_r \lesssim 2 \times (10^{10}, 10^9, 10^8) \text{ GeV for } m_{3/2} = (1, 10, 100) \text{ TeV}, \tag{30}$$
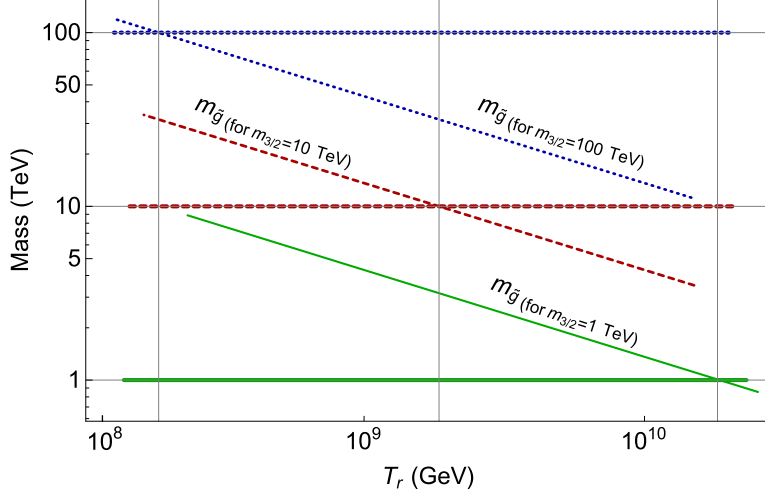
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FIG. 3: Possibility of LSP gravitino with masses $m_{3/2} = (1, 10, 100)$ TeV and corresponding gluino masses $m_{\tilde{g}}$, are shown by solid-green, dashed-red and dotted-blue curves, respectively, with $n_s = 0.9655$ and $\gamma = 2$. The lower bounds on reheat temperature are shown by vertical grey lines.

respectively. These upper bounds of T_r are consistent with the lower bound incorporated in our numerical work (i.e. Eq. (29)), and so the LSP gravitino scenario can be consistently realized in the non-minimal Kähler case.

For the second possibility, namely an unstable 'long-lived' gravitino (with $m_{3/2} \lesssim 25$ TeV), the gravitino problem arises as the gravitino decays right after BBN, which can adversely effect the light nuclei abundances and thereby ruin the success of BBN. To avoid this problem, as mentioned in section I, an upper bound on the reheat temperature is obtained [23]: $T_r \lesssim 3 \times (10^5 - 10^6)$ GeV for $m_{3/2} \sim 1$ TeV, and $T_r \lesssim 2 \times 10^9$ GeV for $m_{3/2} \sim 10$ TeV. But the inflationary constraints on the reheat temperature mentioned in Eq. (29) are $T_r \gtrsim 6 \times 10^6$ GeV for $m_{3/2} = 1$ TeV, and $T_r \gtrsim 2 \times 10^6$ GeV for $m_{3/2} = 10$ TeV, respectively. Thus the second scenario with a long-lived gravitino is marginally ruled out for 1 TeV gravitino mass but stays comfortably within the BBN bounds for a 10 TeV gravitino mass.

An unstable gravitino of mass $m_{3/2} = 100$ TeV falls in the third category of short-lived gravitino. Here, the BBN constraints are weakened because a heavy gravitino implies a shorter lifetime. But now the constraints from the abundance of the lightest LSP neutralino from the decay of gravitino will come into play. These constraints are less severe than those from BBN [23]. For a 100 TeV gravitino mass, the upper bound on the LSP neutralino as derived in Eq. (12) becomes

$$m_{\tilde{\chi}_1^0} \lesssim (18 - 10^5) \text{ GeV for } 10^{11} \text{ GeV} \gtrsim T_r \gtrsim 6 \times 10^5 \text{ TeV}. \tag{31}$$
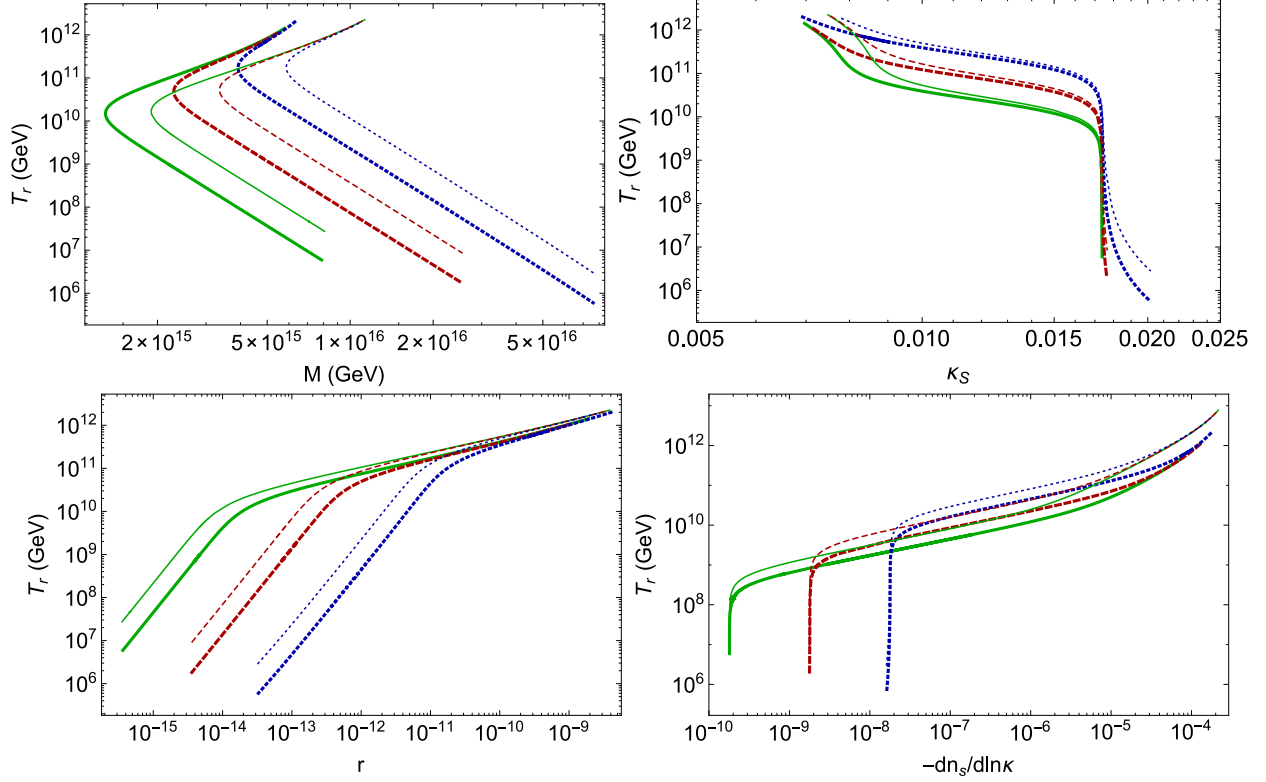
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FIG. 4: Variation of the reheat temperature T_r with the symmetry breaking scale M, coefficient κ_S, the tensor-to-scalar ratio r and the running of spectral index $dn_s/d\ln k$, for $m_{3/2} = 1$ TeV (solid-green), 10 TeV (dashed-red) and 100 TeV(dotted-blue). We have set the scalar spectral index $n_s = 0.9655$, $\kappa_S = 0.02$, $\kappa_{SS} = 0$ and represent $\gamma = 2(10)$ by thick (thin) curves.

The lower limit on the LSP neutralino mass from experiments is $m_{\tilde{\chi}_1^0} \gtrsim 18$ GeV [24]. Therefore, the non-LSP gravitino scenario with $m_{3/2} \sim 100$ TeV comfortably holds in a considerably larger domain: 10^6 GeV $\lesssim T_r \lesssim 10^{11}$ GeV. Hence, we have successfully realized μ-hybrid inflation with $m_{3/2} \sim 1 - 100$ TeV and low reheat temperature.

The predictions of other important inflationary parameters are displayed in Fig. 4. It is interesting to note that with low reheat temperature we can a achieve a gauge symmetry breaking scale M of the order of GUT scale, namely 2×10^{16} GeV. In the second panel of Fig. 4, the value of the parameter κ_S is seen to remain constant in the low reheat temperature range. As discussed earlier the radiative and the quartic-SUGRA corrections can be neglected in this region. Therefore, the scalar spectral index n_s in the low reheat region can be approximated as,

$$n_s \simeq 1 - 2\kappa_S \implies \kappa_S = \frac{1 - n_s}{2}. \tag{32}$$

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For the central value of scalar spectral index $n_s = 0.9655$, we obtain $\kappa_S = 0.0173$ in good agreement with the numerical estimate shown in Fig. 4. The tensor-to-scalar ratio r and the running of the scalar spectral index $dn_s/d\ln k$ in the low reheat limit are given by

$$r \lesssim \frac{2}{3\pi^2 \sqrt{\gamma/2} A_s(k_0)} \left(\frac{m_{3/2}}{(1-n_s)m_P}\right)^{5/4} \left(\frac{T_r}{m_P}\right)^{1/2}, \tag{33}$$

$$dn_s/d\ln k \lesssim - \left(\frac{6}{\pi^2 A_s(k_0)}\right)^{1/2} \frac{m_{3/2}}{(1-n_s)m_P}. \tag{34}$$

As shown in the lower two panels of Fig. 4, r and $dn_s/d\ln k$ in this case are very small which is a common feature of small field models.

IV. PRIMORDIAL GRAVITY WAVES AND COSMIC STRINGS

So far we have set the non-minimal coupling κ_{SS} to be zero in our calculations and only the non-minimal coupling κ_S has played a significant role in realizing μ-hybrid inflation. Next, with non-zero κ_{SS} we will mainly be interested in the large (observable) r solutions which might be detectable in future experiments; PRISM will be able to measure the tensor-to-scaler ratio $r \lesssim 5 \times 10^{-4}$ [29], whereas LiteBIRD will provide a precision of $\delta r < 0.001$ [30]. The large r solutions have previously been explored in standard hybrid inflation in [31] with intermediate scale soft masses, and in [32] with TeV scale soft masses. As explained in these references, large r solutions are possible with positive quadratic and negative quartic terms in the potential. This structure is not possible with $\kappa_{SS} = 0$ where, with $\kappa_S = 0.0173$, the quadratic term is negative and the quartic term is positive. This structure of the potential resembles the SSBI model of [26] if we ignore both the radiative corrections and the linear term in $V(x)$, Eq. (24). It turns out that although the linear term can be ignored for large r solutions, the radiative corrections provide a contribution that is comparable with the quadratic and quartic terms. Therefore, the predictions of our model are, in general, different from those of SSBI model but it lies within the same category of hilltop inflation [27]. The largest possible values of r consistent with the Lyth bound in our case are obtained with S_0 values comparable to m_P. Therefore, we take $S_0 = (0.1, 0.2, 0.5, 1)\, m_P$ and show in Fig. 5 the variation of reheat temperature T_r with r and $dn_s/d\ln k$ in the upper panel, and M versus κ and κ_{SS} versus κ_S in the lower panel. To avoid the gravitino problem the bound on reheat temperature $T_r \lesssim 10^{11}$ GeV is highlighted by darkening of curves, whereas the faded region of the curves show large r solutions that lie outside this bound.
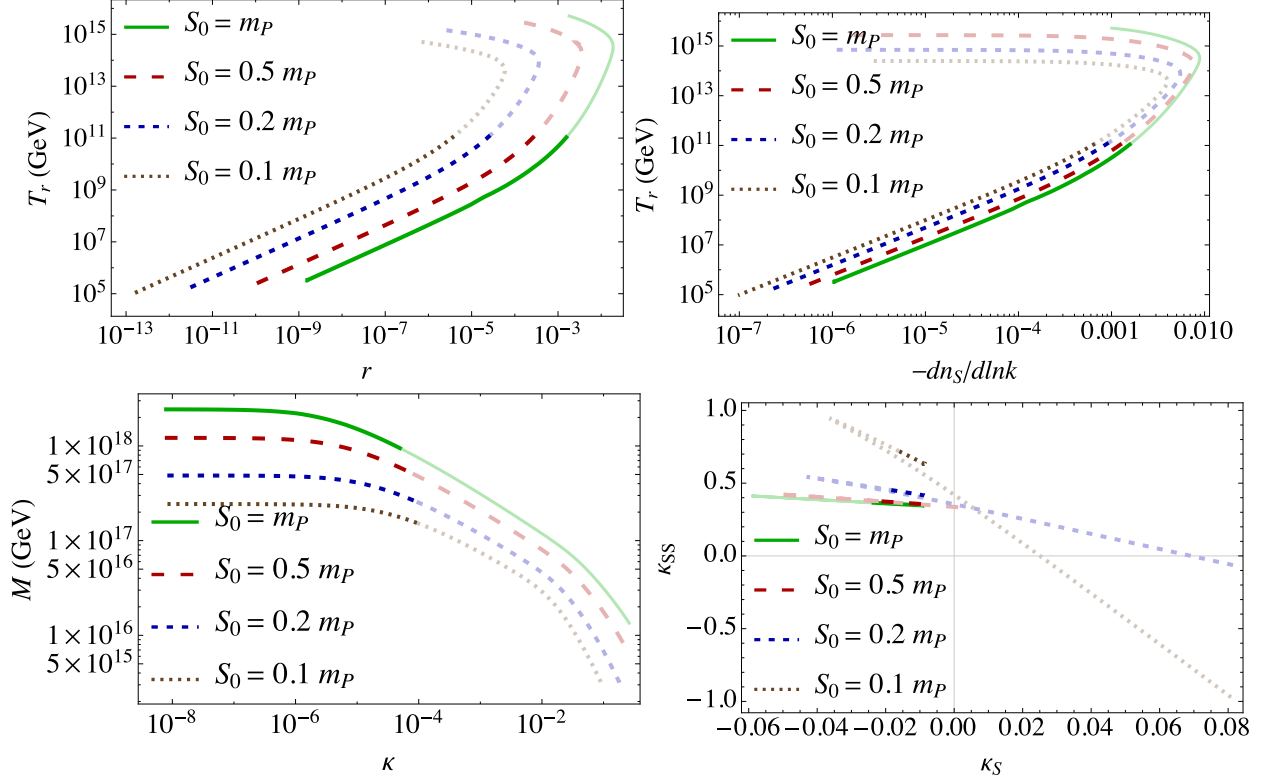
FIG. 5: The variation of the reheat temperature T_r with r and $dn_s/d\ln k$ (upper panel) and M versus κ and κ_{SS} versus κ_S (lower panel), for $S_0 = m_P$ (green), $m_P/2$ (red), $m_P/5$ (blue) and $m_P/10$ (brown), valid for gravitino mass range $m_{3/2} \sim 1 - 100$ TeV with $n_s = 0.9655$ and $\gamma = 2$. The faded region of the curves represents the reheat temperature $T_r \gtrsim 10^{11}$ GeV.

The choice $S_0 = m_P$ gives the upper bound on $r \lesssim 0.002$ mainly from the requirement of $T_r \lesssim 10^{11}$ GeV. However, this choice entails the fine tuning of higher order terms in the potential. Therefore, with $S_0 \sim 0.1\, m_P$, we obtain an upper bound $r \lesssim 4 \times 10^{-6}$ with reasonably natural suppression of higher order terms. This is also consistent with the value of r obtained by requiring the boundedness of the potential for large values of the field as discussed in [33].

Cosmic strings arise from the breaking of $U(1)_{B-L}$ at the end of inflation. The observational bounds on these strings are given in terms of the dimensionless quantity $G_N \mu_s$ which characterizes the strength of the the gravitational interaction of the strings. Here $G_N = \frac{1}{8\pi m_P^2}$ is Newton's constant and μ_s denotes the mass per unit length of the string. A recent Planck bound on $G_N \mu_s$, using field theory simulations of the Abelian-Higgs action, is given by [34, 35]
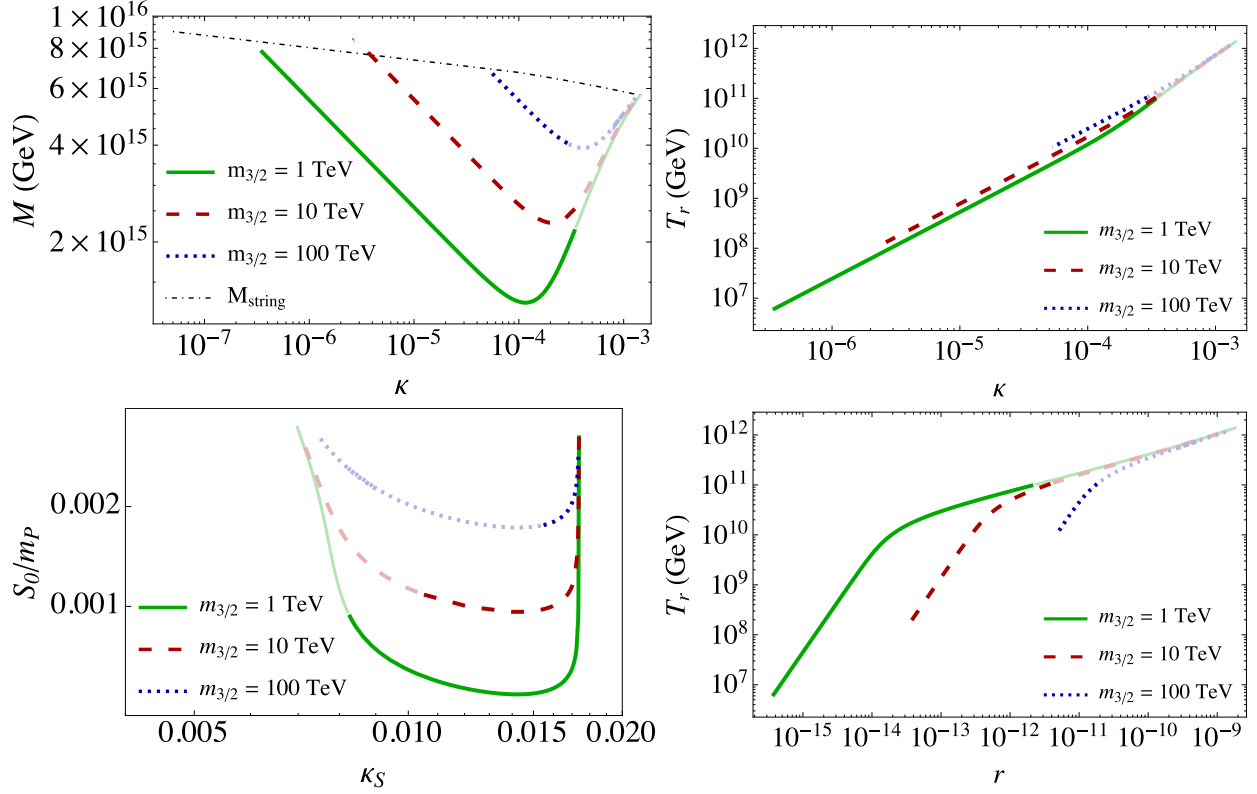
$$G_N \mu_s \lesssim 2.4 \times 10^{-7}. \tag{35}$$

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FIG. 6: The symmetry breaking scale M and T_r versus κ (upper panel) and S_0/m_P versus κ_S and T_r versus r (lower panel) are plotted for $m_{3/2} = 1$ (solid-green), 10 (dashed-red) and 100 (dotted-blue) TeV with $\gamma = 2$, $n_s = 0.9655$ and holds for $-1 \lesssim \kappa_{SS} \lesssim 1$. The cosmic string bounds M_{string} are shown by the grey dot-dashed curve. The faded region of the curves represents the reheat temperature $T_r \gtrsim 10^{11}$ GeV.

For local cosmic strings μ_s is given by,

$$\mu_s = 2\pi M^2 \varepsilon(\beta), \tag{36}$$

where,

$$\varepsilon(\beta) = \frac{2.4}{\log[2/\beta]} \text{ for } \beta = \frac{\kappa^2}{2g^2} < 10^{-2}. \tag{37}$$

For $g = 0.7$ and using above equations we calculate the Planck bound on the symmetry breaking scale M and denote it by M_{string}.

In Figure 6, we display the cosmic string bounds on the prediction of $U(1)_{B-L}$ μ-hybrid inflation for $m_{3/2} = 1, 10$ and 100 TeV. The cosmic strings bounds directly constrain M and κ as shown in the upper-left panel of Fig. 6. This, in turn, put bounds on the reheat temperature, especially for larger values of the gravitino mass. The cosmic string bounds also restricts the value of S_0 (with $0.008 \lesssim \kappa_S \lesssim 0.0173$) which, in turn, constrain the large r values, as shown in the lower panels

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TABLE I: The values of κ, r, M and T_r allowed by the Planck 2015 cosmic string bound $G_N\mu_s \lesssim 2.4 \times 10^{-7}$ for $m_{3/2} = 1$, 10, and 100 TeV for fixed values of $n_s = 0.9655$ and $\gamma = 2$.

$m_{3/2}$ (TeV)	κ	r	M (GeV)	T_r (GeV)
1	$3 \times 10^{-7} - 3 \times 10^{-4}$	$3 \times 10^{-16} - 2 \times 10^{-12}$	$8 \times 10^{15} - 2 \times 10^{15}$	$6 \times 10^{6} - 1 \times 10^{11}$
10	$4 \times 10^{-6} - 3 \times 10^{-4}$	$4 \times 10^{-14} - 4 \times 10^{-12}$	$7 \times 10^{15} - 3 \times 10^{15}$	$2 \times 10^{8} - 1 \times 10^{11}$
100	$5 \times 10^{-5} - 3 \times 10^{-4}$	$5 \times 10^{-12} - 2 \times 10^{-11}$	$6 \times 10^{15} - 4 \times 10^{15}$	$1 \times 10^{10} - 1 \times 10^{11}$

of Fig. 6. For further clarity the values obtained for κ, r, M, and T_r constrained by the cosmic string bounds are given in Table I. For 1 TeV gravitino mass a wide range of reheat temperature $10^6 - 10^{11}$ GeV will yield observationally acceptable values of the cosmic string tension μ_s. With the increase in gravitino mass the allowed phase space becomes more constrained. It is important to note that the allowed range of r permissible by cosmic string bounds is highly suppressed and unlikely to be observed in future experiments like PRISM and LiteBIRD. However, if we avoid the cosmic strings bound in some way (for instance, by employing the shifted hybrid inflation [36], or by using F_D-term hybrid inflation [37]), then the range of $r \lesssim 10^{-6} - 10^{-3}$ mentioned earlier is testable in the foreseeable future.

V. CONCLUSION

We have considered a $U(1)_{B-L}$ extension of MSSM in which successful supersymmetric hybrid inflation is realized in conjunction with the resolution of the well-known μ problem. This is achieved by invoking a non-minimal Kähler potential as a well-defined power series beyond the leading canonical terms. The reheat temperature lies below the conventional upper bound of around 10^9 GeV, and the gravitino and scalar sparticle masses can be of order $1 - 100$ TeV. Thus split supersymmetry, which appears if the Kähler potential is minimal, can be evaded. The upper bound on the reheat temperature obtained from the gravitino problem translates into the upper bound range on the tensor-to-scalar ratio $r \lesssim 10^{-6} - 10^{-3}$. This is expected to be observed in future experiments. However, it is important to note that this potentially observable interval alludes to the upper limits of a wider range of r predicted by the model. Generalization of μ-hybrid inflation to symmetry groups larger than $U(1)_{B-L}$ should be straightforward and will be discussed elsewhere [38].

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