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Heavy and Strange Holographic Baryons

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We extend the $D4$-$D8$ holographic construction to include three chiral and one heavy flavor, to describe heavy-light baryons with strangeness and their exotics. At strong coupling, the heavy meson always binds to the bulk instanton in the form of a flavor zero mode in the fundamental representation. We quantize the ensuing bound states using the collective quantization method, to obtain the spectra of heavy and strange baryons with both explicit and hidden charm and bottom. Our results confirm the existence of two low-lying charmed penta-quark states with $\frac{1}{2}^-, \frac{3}{2}^-$ assignments, and predict many new ones with both charm and bottom. They also suggest a quartet of low-lying neutral Ω_c^0 with assignments $\frac{1}{2}^\pm, \frac{3}{2}^\pm$ that are heavier than the quintuplet of neutral Ω_c^0 recently reported by LHCb.

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I. INTRODUCTION

Recently the Belle collaboration [1] and the BESIII collaboration [2] have reported many multiquark exotics uncommensurate with quarkonia, e.g. the neutral $X(3872)$ and the charged $Z_c(3900)^\pm$ and $Z_b(10610)^\pm$. These exotics have been also confirmed by the DO collaboration at Fermilab [3], and the LHCb collaboration at CERN [4]. Also recently, the same LHCb collaboration has reported new pentaquark states $P_c^+(4380)$ and $P_c^+(4450)$ through the decays $\Lambda_b^0 \rightarrow J\Psi pK^-, J\Psi p\pi^-$ [5], and five narrow and neutral excited Ω_c^0 baryon states that decay primarily to $\Xi_c^+K^-$ [6]. These flurry of experimental results support new physics involving heavy-light multiquark states, a priori outside the canonical classification of the quark model.

Some of the tetra-states exotics maybe understood as molecular bound states mediated by one-pion exchange much like deuterons or deusons [7–14]. Non-molecular heavy exotics were also discussed using constituent quark models [16], heavy solitonic baryons [17, 18], instantons [19] and QCD sum rules [20]. A flurry of quark-based descriptions of the reported neutrals Ω_c^0 states have also been proposed [21] following earlier descriptions [22], including sum rules calculations [23] and a recent lattice simulation [24].

The penta-states exotics reported in [5] have been foreseen in [25] and since addressed by many using both molecular and diquark constructions [26], as well as a bound anti-charm to a Skyrmion [27]. String based pictures using string junctions [28] have also been suggested for the description of exotics, including a recent proposal in the context of the holographic inspired string hadron model [29].

In QCD the light quark sector (u, d, s) is dominated by the spontaneous breaking of chiral symmetry, while the heavy quark sector (c, b, t) exhibits heavy-quark symmetry [30]. Both symmetries are at the origin of the chiral doubling in heavy-light mesons [31, 32], as measured by both the BaBar collaboration [33] and the CLEOII collaboration [34]. As most of the heavy hadrons and their exotics exhibit radiative decays through light or heavy-light mesons it is important to formulate a non-perturbative model of QCD that honors both chiral and heavy quark symmetry.

The initial holographic construction offers a framework for addressing chiral symmetry and confinement in the double limit of large N_c and large t′Hooft coupling $\lambda = g^2N_c$. A concrete model was proposed by Sakai and Sugimoto [35] using a $D4$-$D8$ brane construction. The induced gravity on the probe N_f $D8$ branes due to the large stack of N_c $D4$ branes, causes the probe branes to fuse in the holographic direction, providing a geometrical mechanism for the spontaneous breaking of chiral symmetry. The Dirac-Born-Infeld (DBI) action on the probe branes yields a low-energy effective action for the light pseudoscalars with full global chiral symmetry, where the vectors and axial-vector light mesons are dynamical gauge particles of a hidden chiral symmetry [36]. This construction was recently extended to accomodate heavy mesons with explicit heavy quark symmetry [37]. The construction makes use use of an additional heavy probe $D8$ brane in bulk [37].

In the $D4$-$D8$ brane construction, baryons are identified with small size instantons by wrapping $D4$ around S^4, and are dual to Skyrmions on the boundary [38, 39]. Remarkably, this identification provides a geometrical description of the baryonic core that is so elusive in most Skyrme models [40]. A first principle description of the baryonic core is paramount to the understanding of heavy hadrons and their exotics since the heavy quarks bind over their small Compton wavelength. In a recent analysis we have shown how heavy baryons and their exotics can be derived from the zero modes of bulk instantons using two light flavors [41]. This paper extends this analysis to the case of three light and one heavy flavors with both chiral and heavy quark symmetry. There are may

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new features and results following from this construction: 1/ the three flavor case involves a new contribution through the Chern-Simons term which is subtle in the present holographic set up [42, 43]; 2/ the Chern-Simons contribution fixes uniquely the baryonic hypercharge in the presence of a heavy flavor; 3/ a finite strange quark mass is introduced through a bulk instanton holonomy and treated perturbatively [44]; 4/ a large number of single and double heavy baryon states with explicit and hidden charm and bottom can be described by the present construction. The inclusion of the strange quark mass improves the $N_f = 2$ results in [41]. Our approach extends the bound state approach developed in the context of the Skyrme model with heavy mesons [27, 45] to holography. We note that alternative holographic models for the description of heavy hadrons have been developed in [46, 47] without the dual strictures of chiral and heavy quark symmetrty.

The organization of the paper is as follows: In section 2 and 3 we briefly recall the geometrical set up for the derivation of the heavy-light effective action for three flavors in terms of the bulk DBI and CS actions. We detail the heavy-meson interactions to the flavor instanton, and the ensuing heavy meson bound state to the instanton in bulk in the double limit of large coupling and heavy meson mass. In section 4 and 5, we use the collective quantization approach to derive the pertinent spectra for holographic heavy baryons and their exotics with strangeness. Our conclusions are in section 6. In the Appendix we briefly review the collective quantization of the light baryons for $N_f = 2, 3$.

II. HOLOGRAPHIC EFFECTIVE ACTION

A. DBI Action

The holographic brane set-up for heavy light hadrons with spontaneously broken chiral symmetry was recently discussed by us in [37] for the case of two- lavors with $N-f = 2$. Here, we extend to three flavors with $N_f = 3$. Since the two constructions are very similar modulo the Chern-Simons action, we will only recall the necessary steps and refer the reader to [37] for the complementary details. In brief, the construction consists of N_f light $D8$-$\bar{D}8$ (L) and one heavy (H) probe branes in the cigar-shaped geometry that spontaneously breaks chiral symmetry as illustrated in Fig. 1. The L-branes are embedded in $[0-3+5-9]$-dimensions and set at the antipodes of S^1. The warped $[5-9]$-space has a horizon at U_{KK}.

The effective action on the probe L-branes consists of the non-Abelian DBI and CS action. In leading $1/\lambda$ order it is given by

$$S_{\rm DBI} \approx -\kappa \int d^4x dz\, {\rm Tr}\left(f(z)\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} + g(z)\mathbf{F}_{\mu z}\mathbf{F}^{\nu z}\right) \quad (1)$$

Our conventions are $(-1, 1, 1, 1)$ with $A_M^\dagger = -A_M$. The

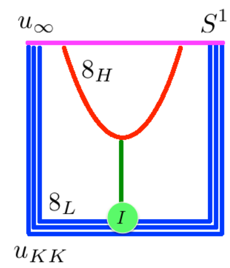

FIG. 1: $N_f = 3$ antipodal 8_L light branes, and one 8_H heavy brane shown in the τU plane, with a bulk $SU(3)$ instanton embedded in 8_L and a massive HL-string connecting them.

warping factors are

$$f(z) = \frac{R^3}{4U_z}\,, \qquad g(z) = \frac{9}{8}\frac{U_z^3}{U_{KK}} \quad (2)$$

with $U_z^3 = U_{KK}^3 + U_{KK}z^2$, and $\kappa \equiv a\lambda N_c$ and $a = 1/(216\pi^3)$ [35]. The effective fields in the field strengths are (M, N run over (μ, z))

$$\mathbf{F}_{MN} = \begin{pmatrix} F_{MN} - \Phi_{[M}\Phi^\dagger_{N]} & \partial_{[M}\Phi_{N]} + A_{[M}\Phi_{N]} \\ -\partial_{[M}\Phi^\dagger_{N]} - \Phi^\dagger_{[M}A_{N]} & -\Phi^\dagger_{[M}\Phi_{N]} \end{pmatrix} \quad (3)$$

The matrix valued 1-form gauge field is

$$\mathbf{A} = \begin{pmatrix} A & \Phi \\ -\Phi^\dagger & 0 \end{pmatrix} \quad (4)$$

For N_f coincidental branes, the Φ multiplet is massless. However, for the set up of Fig. 1 the Φ multiplet is massive with a contribution to (1) of the form

$$\frac{1}{2}m_H^2 {\rm Tr}\left(\Phi_M^\dagger \Phi_M\right) \quad (5)$$

The mass m_H is related to the separation between the light and heavy branes [48], which is about the length of the HL string. Below, m_H will be taken as the heavy meson mass for the heavy-light $(0^-, 1^-)$, i.e. (D, D^*) for charm and (B, B^*) for bottom. The introduction of a finite non-zero strange quark mass will be discussed separatly at the end.

B. Chern-Simons action

For $N_f > 2$, the naive Chern-Simons 5-form

$$S_{CS} = \frac{iN_c}{24\pi^2}\int_{M_5} \mathrm{Tr}\left(AF^2 - \frac{1}{2}A^3F + \frac{1}{10}A^5\right) \quad (6)$$

fails to reproduce the correct transformation law under the combined gauge and chiral transformations [42]. In particular, when addressing the $N_f = 3$ baryon spectra, (6) fails to reproduce the important hypercharge constraint [42]

$$J_8 = \frac{N_c}{2\sqrt{3}} \quad (7)$$

This issue was recently revisited in [43] where boundary contributions were added to (6) to address these shortcomings. Specifically, the new Chern-Simons (nCS) contribution is [43]

$$\begin{aligned} &S_{nCS} = S_{CS} \\ &+ \int_{N_5} \frac{1}{10}\,\mathrm{Tr}\left(h^{-1}dh\right)^5 + \int_{\partial M_5} \alpha_4\left(dhh^{-1}, A\right) \end{aligned} \quad (8)$$

Here N_5 is a 5-dimensional manidold whose boundaries are $\partial N_5 = \partial M_5 = M_{4+\infty} - M_{4-\infty}$, with the asymptotic flavor gauge field

$$A|_{z\to\pm\infty} = \hat{A}^{\pm h^{\pm}} = h^{\pm}(d + \hat{A}^{\pm})h^{\pm -1} \quad (9)$$

The gauged 4-form α_4 is given in [43]. $\hat{A}^{\pm}$ refer to the external gauge fields, and $h|_{\partial M_5} = (h^+, h^-)$. A is assumed to be well defined throughout M_5 and produces no-boundary contributions. In other words, in this gauge all topological information is moved to the holographic boundaries at $z = \pm\infty$. We can actually work in the $A_z = 0$ gauge, and for the instanton profile (as discussed below) we have

$$(h^-, h^+) \equiv \left(1, Pe^{-\int_{-\infty}^{\infty} A_z dz}\right) \quad (10)$$

Note that in our case $A \to \mathbf{A}$ as defined in (4). As a result, the contributions from (6) are similar to those in the $N_f = 2$ case discussed in [37]. The contributions from the new terms in (8) will be detailed in the quantization approach below.

III. HEAVY-LIGHT BARYONS

In the original Sakai and Sugimoto model [35] light baryons are identified with small size flavor instantons in bulk [38]. This construction carries to our current set up as we have recently shown for the $N_f = 2$ case in [37]. For the present $N_f = 3$ case shown in Fig. 1, a small size instanton translates to a flat space 4-dimensional instanton in the $[1-4]$ directions. Specifically, the SU(3) flavor instanton A_M and its time components are [42]

$$\begin{aligned} A_M =&\quad \mathrm{diag}\left(-\bar{\sigma}_{MN}\frac{x_N}{x^2+\rho^2}, 0\right) \quad (11)\\ A_0 =&\quad \frac{-1}{8\pi^2 a x^2}\sqrt{\frac{2}{3}}\left(1 - \frac{\rho^2}{(x^2+\rho^2)^2}\right)\mathrm{diag}(1,1,0) \\ &+ \frac{1}{16\pi^2 a x^2}\left(1 - \frac{\rho^2}{(x^2+\rho^2)^2}\right)\mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right) \end{aligned}$$

where the rescaling

$$\begin{aligned} &x_0 \to x_0, x_M \to x_M/\sqrt{\lambda}, \sqrt{\lambda}\rho \to \rho \\ &(A_0, \Phi_0) \to (A_0, \Phi_0), \\ &(A_M, \Phi_M) \to \sqrt{\lambda}(A_M, \Phi_M) \end{aligned} \quad (12)$$

was used. From here on M, N runs only over $1, 2, 3, z$ unless specified otherwise.

A. Heavy-light effective action

To order λ^0 the rescaled contributions describing the interactions between the light gauge fields A_M and the heavy fields Φ_M to quadratic order split to several contributions

$$\mathcal{L} = aN_c\lambda\mathcal{L}_0 + aN_c\mathcal{L}_1 + \mathcal{L}_{CS} \quad (13)$$

The contributions $\mathcal{L}_{0,1}$ are similar to those given [37] and will not be repeated here. The contribution $\mathcal{L}_{CS}$ is new and reads

$$\begin{aligned} \mathcal{L}_{CS} =&\quad -\frac{iN_c}{24\pi^2}(d\Phi^\dagger A d\Phi + d\Phi^\dagger dA\Phi + \Phi^\dagger dAd\Phi) \\ &- \frac{iN_c}{16\pi^2}(d\Phi^\dagger A^2\Phi + \Phi^\dagger A^2 d\Phi + \Phi^\dagger(AdA + dAA)\Phi) \\ &- \frac{5iN_c}{48\pi^2}\Phi^\dagger A^3\Phi + S_C(\Phi^4, A) \end{aligned} \quad (14)$$

The additional boundary contributions in (8) do not generate any new heavy meson contribution besides those generated by the standard Chern-Simons contributions quoted in (14).

B. Zero mode bound state

We now consider the bound state solution of the heavy meson field Φ_M in the (rescaled) instanton background 11). We note that the field equation for Φ_M is independent of Φ_0 and is similar to the one derived in [41], so it will not be repeated here. However, the (contraint) field equation for Φ_0 depends on Φ_M also through the Chern-Simons term

$$
\begin{aligned}
&D_M(D_0\Phi_M - D_M\Phi_0) \\
&-F^{0M}\Phi_M - \frac{\epsilon_{MNPQ}}{64\pi^2 a}K_{MNPQ} = 0
\end{aligned}
\tag{15}
$$

with K_{MNPQ} defined as

$$
\begin{aligned}
K_{MNPQ} = \quad &+\partial_M A_N \partial_P \Phi_Q + A_M A_N \partial_P \Phi_Q \\
&+\partial_M A_N A_P \Phi_Q + \frac{5}{6} A_M A_N A_P \Phi_Q
\end{aligned}
\tag{16}
$$

In the heavy meson mass limit it is best to redefine $\Phi_M = \phi_M e^{-im_H x_0}$ for particles ($m_H \to -m_H$ for anti-particles). In the double limit of $m_H, \lambda \to \infty$, the leading contributions are of order λm_H^0 from the ligh effective action, and of order $\lambda^0 m_H$ from the heavy-light interaction term $\mathcal{L}_1$,

$$
\frac{\mathcal{L}_{1,m}}{aN_c} = 4im_H \phi_m^\dagger D_0 \phi_m - 2im_H(\phi_0^\dagger D_M \phi_M - \text{c.c.}) \tag{17}
$$

and the Chern-Simons term

$$
\frac{m_H N_c}{16\pi^2}\epsilon_{MNPQ}\phi_M^\dagger F_{NP}\phi_Q = \frac{m_H N_c}{8\pi^2}\phi_M^\dagger F_{MN}\phi_N \tag{18}
$$

In this limit, (15) implies that ϕ_M is transverse with $D_M\phi_M = 0$. This observation, when combined with the classical field equations stemming from (13) as detailed in [41] are equivalent to a first order equation for the spinor combination $\psi = \bar{\sigma}_M \phi_M$, i.e. $\sigma_M D_M \psi = D\psi = 0$ with

$$
\psi^a_{\alpha\beta} = \epsilon_{\alpha a}\chi_\beta \frac{\rho}{(x^2+\rho^2)^{\frac{3}{2}}} \qquad \textit{with} \qquad a = 1, 2 \tag{19}
$$

Here χ_α is a constant two-component spinor, with only the first two components that are non-zero. In the presence of the instanton, the spin-1 vector field binds and transmutes to a spin $\frac{1}{2}$ spinor.

IV. QUANTIZATION

The classical bound instanton-zero-mode breaks iso-rotational, rotational and translational symmetries. To quantize it, we promote the solution to a slowly moving and rotating solution. The leading contribution for large λ is purely instantonic and its quantization is standard and can be found in [39], so we will assume it here. The quantization of the subleading $\lambda^0 m_H$ contribution involves the zero-mode and for $N_f = 2$ was recently addressed in [41]. Here, we will address the new elements of the quantization for $N_f = 3$.

The collective quantization method proceeds by first slowly rotating and translating the instanton configuration in bulk using

$$
\Phi \to V(a_I(t))\Phi(X_0(t), Z(t), \rho(t), \chi(t)) \tag{20}
$$

with $\Phi_0 = 0$. Here X_0 is the center in the 123 directions and Z is the center in the z directon. a_I is the SU(3) gauge rotation moduli. The moduli is composed of the collective coordinates $X_\alpha \equiv (X, Z, \rho)$ and by the collective SU(3) rotation a_I. The time-dependent configuration is then introduced in the heavy-light effective action described earlier and expanded in leading order in the time-derivatives as we now detail.

A. The new Chern-Simons contributions

The additional Chern-Simons contributions in (8) picks up from the collectively quantized instanton by defining

$$
\begin{aligned}
h^- &= \text{diag}\left(a_I(t)^{-1}, 1\right) \\
h^+ &= h_0\, \text{diag}\left(a_I(t)^{-1}, 1\right)
\end{aligned}
\tag{21}
$$

We now note that the field $\mathbf{A}$ composed of the instanton solution A plus the zero-mode solution $\mathbf{\Phi}$, carries the same topological number as the field with the instanton solution A but $\mathbf{\Phi} = 0$. Therefore, h_0 in (21) can be represented by only the latter. With this in mind, we insert (21) in the new contributions in (8) to obtain

$$
S_{nCS} = S_{CS} - \frac{iN_c}{48\pi^2}\int_{M^4} dt\, \text{Tr}\left(({a_I}^{-1}\partial_t a_I)(h_0^{-1}dh_0)^3\right) \tag{22}
$$

The heavy-light contributions from S_{CS} are those in (14), while the new second contribution is identical to the one obtained in the light sector [43]

$$
\frac{N_c}{2\sqrt{3}}a^8 \tag{23}
$$

When combined to terms emerging from the heavy sector it will give rise to the correct hypercharge constraint as we will show next.

B. Heavy contributions in leading order

There are four contributions to order $\lambda^0 m_H$ from the heavy meson sector, namely

$$\frac{\mathcal{L}}{aN_c} = \quad +16im_H\chi^\dagger\partial_t\chi f^2 - 16m_H\chi^\dagger\chi f^2\frac{2\sqrt{6}+1}{6}A_0 \\ -m_H f^2\chi^\dagger\sigma_\mu\Phi\bar{\sigma}_\mu\chi + m_H\chi^\dagger\chi f^2\frac{3}{a\pi^2}\frac{\rho^2}{(x^2+\rho^2)^2} \tag{24}$$

The second contribution is from the A_0 coupling, and the third contribution simplifies for the zero-mode

$$\chi^\dagger\sigma_\mu\Phi\bar{\sigma}_\mu\chi = a^8\frac{8\chi^\dagger\chi}{\sqrt{3}} \tag{25}$$

The last contribution originates from the heavy terms in naive CS term, and also simplifies using the instanton field strength and the zero-mode

$$\frac{im_H N_c}{8\pi^2}\phi_M^\dagger F_{MN}\phi_N = \frac{i3m_H N_c}{\pi^2}\frac{f^2\rho^2}{(x^2+1)^2}\chi^\dagger\chi \tag{26}$$

In addition to the terms retained in (24) the $\chi^\dagger\chi$ coupling to the U(1) flavor gauge field A_0 induces a Coulomb-like correction of the form $(\chi^\dagger\chi)^2$ as we have shown in [41]. With this in mind and after using the rescaling $\chi\to\chi/\sqrt{4aN_c m_H}$ in (24) we obtain

$$\mathcal{L} = \quad +\mathcal{L}_0[a_I, X_\alpha] \\ +i\chi^\dagger\partial_t\chi + \frac{\eta\,\chi^\dagger\chi}{32\pi^2 a\rho^2} - \frac{\mu\,(\chi^\dagger\chi)^2}{24\pi^2 aN_c\rho^2} \\ +a^8\frac{N_c}{2\sqrt{3}}\left(1-\frac{\chi^\dagger\chi}{N_c}\right) \tag{27}$$

where the parameters η,μ are given by

$$\eta \equiv 2x+1 \equiv \frac{2\sqrt{6}+1}{3}+1 \approx 2.966 \;\; and \;\; \mu = \frac{13}{12} \tag{28}$$

Here $\mathcal{L}_0[a_I, X_\alpha]$ refers to the effective action density on the moduli stemming from the contribution of the light degrees of freedom in the instanton background without the a^8 term [38] .

The term linear in a^8 in (27) couples to the hypercharge $J_8 = \frac{N_c}{2\sqrt{3}}(1-\frac{\chi^\dagger\chi}{N_c})$. So (27) can be seen as an action density of light and heavy degrees of freedom supplemented by a hypercharge constraint, namely

$$\mathcal{L}\to\mathcal{L}_0[a_I, X_\alpha] + \chi^\dagger i\partial_t\chi + \frac{\eta\,\chi^\dagger\chi}{32\pi^2 a\rho^2} - \frac{\mu\,(\chi^\dagger\chi)^2}{24\pi^2 aN_c\rho^2} \\ J^8 = \frac{N_c}{2\sqrt{3}}\left(1-\frac{\chi^\dagger\chi}{N_c}\right) \tag{29}$$

From (28) we note that $\eta\approx 3$ and $\mu\approx 1$ which are remarkably close to the same parameters derived in [41] for the $N_f = 2$ case. These terms are inertial and not sensitive to the value of N_f.

C. Heavy-light spectra

The quantization of (29) follows the same arguments as those presented in [38, 42] for $\mathcal{L}_0[a_I, X_\alpha]$ as we briefly recall in the Appendix. Let H_0 be the Hamiltonian associated to $\mathcal{L}_0[a_I, X_\alpha]$, then the full heavy-light Hamiltonian for (29) is

$$H = H_0[\pi_I, \pi_X, a_I, X_\alpha] - \frac{\eta\,\chi^\dagger\chi}{32\pi^2 a\rho^2} + \frac{\mu\,(\chi^\dagger\chi)^2}{24\pi^2 aN_c\rho^2} \tag{30}$$

with the new quantization rule for the spinor and the hypercharge constraint

$$\chi_i\chi_j^\dagger + \chi_j^\dagger\chi_i = \delta_{ij} \\ J^8 = \frac{N_c}{2\sqrt{3}}\left(1-\frac{\chi^\dagger\chi}{N_c}\right) \tag{31}$$

We recall that the statistics and parity of χ were fixed in [41]. Specifically, χ is a fermion in the spin $\frac{1}{2}$ representation with positive parity. With this in mind, the total spin $\mathbf{J}$ of the bound state is

$$\vec{\mathbf{J}} = -\vec{\mathbf{I}}_{SU(2)} + \vec{\mathbf{S}}_\chi \equiv -\vec{\mathbf{I}}_{SU(2)} + \chi^\dagger\frac{\vec{\tau}}{2}\chi \tag{32}$$

Here for a general SU(3) representation, $\vec{\mathbf{I}}_{SU(2)}$ means the induced representation for the first three generators, $J_{1,2,3}$ as noted in the Appendix.

The spectrum of (30) follows from the one discussed in [38, 42] and recalled in the Appendix, with two key modifications

$$Q \equiv \frac{N_c}{40a\pi^2}\to\frac{N_c}{40a\pi^2}\left(1-\frac{5\eta}{4N_c}\chi^\dagger\chi + \frac{5\mu(\chi^\dagger\chi)^2}{3N_c^2}\right) \tag{33}$$

and the change of the hypercharge as obtained in (31). The quantum states with a single bound state $N_Q = \chi^\dagger\chi = 1$ and the general (p,q) representation for SU(3) and spin j are labeled by

$$|N_Q, p, q, j, n_z, n_\rho\rangle \;\; with \;\; IJ^\pi = \frac{l}{2}\left(\frac{l}{2}\pm\frac{1}{2}\right)^\pi \tag{34}$$

with $n_z = 0, 1, 2, ..$ counting the number of quanta associated to the collective motion in the holographic direction, and $n_\rho = 0, 1, 2, ..$ counting the number of quanta associated to the radial breathing of the instanton core, a sort of Roper-like excitations. Following [38], we identify the parity of the heavy baryon bound state as $(-1)^{n_z}$. Using (33), the mass spectrum for the bound heavy-light states is

$$M_{NQ} = M_0 + N_Q\, m_H + \sqrt{\frac{49}{24} + \frac{\mathbf{K}}{3}} + \sqrt{\frac{2}{3}}(n_z + n_\rho + 1)M_{KK} \qquad (35)$$

with

$$\mathbf{K} = \quad +\frac{2N_c^2}{5}\left(1 - \frac{5\eta N_Q}{4N_c} + \frac{5\mu N_Q^2}{3N_c^2}\right) - \frac{N_c^2}{3}\left(1 - \frac{N_Q}{N_c}\right)^2 + \frac{4}{3}(p^2 + q^2 + pq + 3(p+q)) - 2j(j+1) \qquad (36)$$

with M_{KK} the Kaluza-Klein mass and $M_0/M_{KK} = 8\pi^2\kappa$ the bulk instanton mass. The Kaluza-Klein scale is usually set by the light meson spectrum and is fit to reproduce the rho mass with $M_{KK} \sim m_\rho/\sqrt{0.61} \sim 1$ GeV [35].

(35) is to be contrasted with the mass spectrum for baryons with no heavy quarks or $N_Q = 0$, where the nucleon state is idendified as $N_Q = 0, l = 1, n_z = n_\rho = 0$ and the Delta state as $N_Q = 0, l = 3, n_z = n_\rho = 0$ [38]. The radial excitation with $n_\rho = 1$ can be identified with the radial Roper excitation of the nucleon and Delta, while the holographic excitation with $n_z = 1$ can be interpreted as the odd parity excitation of the nucleon and Delta.

D. Single-heavy baryons

Since the bound zero-mode transmutes to a spin $\frac{1}{2}$, the lowest heavy baryons with one heavy quark are characterized by $n_z, n_\rho = 0, 1, N_Q = 1$, and $(p, q, j) = (0, 1, 0)$ for $\bar{\mathbf{3}}$ and $(p, q, j) = (2, 0, 1)$ for $\mathbf{6}$. The $\bar{\mathbf{3}}$-plet states have spin and parity $\frac{1}{2}^+$. We identify them with $\Lambda_Q, \Xi_Q(\bar{\mathbf{3}})$. The $\mathbf{6}$-plet states have $J = \frac{1}{2}, \frac{3}{2}$. We identify them with $\Sigma_Q, \Xi_Q(\mathbf{6}), \Omega_Q$ and $\Sigma_Q^\star, \Xi_Q(\mathbf{6})^\star, \Omega_Q^\star$, respectively. In the absence of symmetry breaking, the mass spectra are degenerate

$$M_{\bar{\mathbf{3}}} = \quad +M_0 + m_H + 1.75M_{KK} + \frac{2(n_\rho + n_z) + 2}{\sqrt{6}}M_{KK} \qquad (37)$$

$$M_{\mathbf{6}} = \quad +M_0 + m_H + 2.103M_{KK} + \frac{2(n_\rho + n_z) + 2}{\sqrt{6}}M_{KK} \qquad (38)$$

or equivalently

$$M_{\bar{\mathbf{3}}} - M_{p=q=1, N_Q=0, j=1/2} - m_H = -0.570\, M_{KK}$$
$$M_{\mathbf{6}} - M_{p=q=1, N_Q=0, j=1/2} - m_H = -0.236\, M_{KK} \qquad (39)$$

with the mass splitting $M_{\mathbf{6}} - M_{\bar{\mathbf{3}}} = 0.334\, M_{KK}$.

E. Double-heavy baryons: QQ

While the binding of a pair of heavy mesons with QQ or $Q\bar{Q}$ content is always BPS-like to leading order in $1/\lambda$, the Chern-Simons contribution is twice more attractive with the QQ content than with the $Q\bar{Q}$ content (see below), although the Coulomb induced contribution penalizes the former and not the latter. With this in mind, heavy baryons with two heavy quarks follow the same construct with $N_Q = 2$ or $\chi^\dagger\chi \to 2$ in (30-31) and $J^8 = 1/2\sqrt{3}$. As a result, the lowest heavy baryons with two bound heavy mesons are now characterized by $n_z, n_\rho = 0, 1$ and $(p, q, j) = (1, 0, 0)$ for the flavor $\mathbf{3}$-plet with assignment $\frac{1}{2}^+$, which we identify as Ξ_{QQ} with u, d light content, and Ω_{QQ} with s content. To this order, their degenerate masses are given by

$$M_{\mathbf{3}} - M_{p=q=1, N_Q=0, j=1/2} - 2m_H = -0.844\, M_{KK} \quad (40)$$

F. Double-heavy baryons: $Q\bar{Q}$

For heavy baryons containing also anti-heavy quarks we note that a rerun of the preceding arguments using instead the reduction $\Phi_M = \phi_M e^{+im_H x_0}$, amounts to binding an anti-heavy-light meson to the bulk instanton also in the form of a zero-mode in the fundamental representation of spin, much like the heavy-light meson binding. Most of the results are unchanged except for pertinent minus signs. For instance, when binding one heavy-light and one anti-heavy-light (29) now reads

$$\mathcal{L} = \quad \mathcal{L}_0[a_I, X_\alpha] + \chi_Q^\dagger i\partial_t\chi_Q + \frac{\eta}{32\pi^2 a\rho^2}\chi_Q^\dagger\chi_Q - \chi_{\bar{Q}}^\dagger i\partial_t\chi_{\bar{Q}} - \frac{\eta}{32\pi^2 a\rho^2}\chi_{\bar{Q}}^\dagger\chi_{\bar{Q}} - \frac{\mu(\chi_Q^\dagger\chi_Q - \chi_{\bar{Q}}^\dagger\chi_{\bar{Q}})^2}{24\pi^2 a N_c\rho^2} \qquad (41)$$

with the hypercharge constraint

$$J_8 = \frac{N_c}{2\sqrt{3}}\left(1 - \frac{\chi_Q^\dagger\chi_Q}{N_c} + \frac{\chi_{\bar{Q}}^\dagger\chi_{\bar{Q}}}{N_c}\right) \qquad (42)$$

The mass spectrum for baryons with N_Q heavy-quarks and $N_{\bar{Q}}$ anti-heavy quarks is the same as in (35) with the substitution $N_Q \to N_Q - N_{\bar{Q}}$ to the present order of the analysis or $\lambda^0 m_H$. For $N_Q = N_{\bar{Q}} = 1$ the hypercharge constraint is simply $J_8 = \sqrt{3}/2$. Therefore the lowest states carry $(p, q, j) = (1, 1, 1/2)$ and are identified with the baryonic states in the $\mathbf{8}$-plet representation with the J^π assignments $\frac{1}{2}^-$ and $\frac{3}{2}^-$, and $(p, q, j) = (3, 0, 3/2)$ in the **10**-plet representation with J^π assignments (one) $\frac{5}{2}^-$, (two) $\frac{3}{2}^-$ and (one) $\frac{1}{2}^-$. Their masses are given by

$$M^{\mathbf{8}}_{\bar{Q}Q} = M_N + 2m_H + \frac{2(n_z+n_\rho)}{\sqrt{6}} M_{KK}$$
$$M^{\mathbf{10}}_{\bar{Q}Q} = M_N + 2m_H + 0.386 M_{KK} + \frac{2(n_z+n_\rho)}{\sqrt{6}} M_{KK} \tag{43}$$

with the mass splitting $M^{\mathbf{10}}_{\bar{Q}Q} - M^{\mathbf{8}}_{\bar{Q}Q} = 0.386\, M_{KK}$.

V. STRANGE QUARK MASS CORRECTION

To compare the previous results for single-heavy and double-heavy baryons to some of the reported physical spectra, we need to address the role of a finite strange quark mass. In so far, the light flavor branes $D\bar{8}$-$D8$ only connect at U_{KK} because of the bulk gravity induced by $D4$, thereby spontaneously breaking chiral symmetry. To break explicitly chiral symmetry, say by introducing a finite strange quark mass, an additional bulk $D6$ brane can be introduced to connect $D\bar{8}$ to $D8$ [44, 53]. For the $N_f = 3$ case with $m_u = m_d = 0$ and finite m_s, the world-sheet instanton in $D6$ interpolating $D\bar{8}$ to $D8$, induces an explicit light mass breaking term for the light baryons, which takes the following form on the moduli [53]

$$H_{SB} = \tau\rho^3(1 - D_{88}(a_I)) \tag{44}$$

with $\tau \approx |m_s \langle \bar{s}s\rangle|$. Aside from the dependence on the moduli parameter through ρ^3, the explicit symmetry breaking term (44) is standard. An estimate of τ follows from holography, but here we will use τ as a free parameter to be adjusted below through the baryonic spectrum. (44) will be treated in perturbation theory by averaging ρ^3 using the radial baryonic wavefunctions $\phi_{n_\rho,\mathbf{K}}$ discussed in the Appendix. For $n_\rho = n_z = 0$, the averaged result is

$$\langle \rho^3 \rangle_{n_\rho=0,\mathbf{K}} = \frac{1}{f_\pi^3}\left(\frac{\sqrt{6}}{4\pi^3}\right)^{\frac{3}{2}} \frac{\Gamma\left(1+\sqrt{\frac{49}{4}+2\mathbf{K}}+\frac{3}{2}\right)}{\Gamma\left(1+\sqrt{\frac{49}{4}+2\mathbf{K}}\right)} \tag{45}$$

The emergence of the pion decay constant $f_\pi = 93$ MeV follows from the holographic ρ-wavefunction as discussed in the Appendix. For the $\bar{\mathbf{3}}$-plet and **6**-plet representations, we have specifically

$$\langle \rho^3 \rangle_{\bar{3}} = \frac{1}{f_\pi^3}\left(\frac{\sqrt{6}}{4\pi^3}\right)^{\frac{3}{2}} \times 13.65$$
$$\langle \rho^3 \rangle_{6} = \frac{1}{f_\pi^3}\left(\frac{\sqrt{6}}{4\pi^3}\right)^{\frac{3}{2}} \times 16.70 \tag{46}$$

The corresponding mass shifts induced by the explicit symmetry breaking term (44) on the heavy-light baryonic spectra is then

$$\Delta M_i = b_i(1-a_i)\frac{\tau}{f_\pi^3}\left(\frac{\sqrt{6}}{4\pi^3}\right)^{\frac{3}{2}} \equiv b_i(1-a_i)m_0 \tag{47}$$

with the representation dependent parameters

$$b_i = \frac{\Gamma(1+\sqrt{\frac{49}{4}+2\mathbf{K}_i}+\frac{3}{2})}{\Gamma(1+\sqrt{\frac{49}{4}+2\mathbf{K}_i})}$$
$$a_i = \langle pq, j|D_{88}|pq, j\rangle \tag{48}$$

For the specific representations of relevance to our analysis we have

$$a_N = \frac{3}{10}, \qquad b_N = 18.97$$
$$a_\Lambda = \frac{1}{4}, \qquad a_{\Xi^3} = -\frac{1}{8}$$
$$a_\Sigma = \frac{1}{10}, \qquad a_{\Xi^6} = -\frac{1}{20}, \qquad a_\Omega = -\frac{1}{5} \tag{49}$$

A. Single-heavy baryon spectrum

Combining all the previous results for the heavy-light masses, including the correction induced by the strange quark mass symmetry breaking term (44) yield the following mass spectrum for the single-heavy baryons

$$m_{\Lambda_Q} = m_N + m_H - 0.57 M_{KK} - 3.04 m_0$$
$$m_{\Xi(\bar{3})_Q} = m_N + m_H - 0.57 M_{KK} + 2.08 m_0$$
$$m_{\Sigma_Q} = m_N + m_H - 0.236 M_{KK} + 1.75 m_0$$
$$m_{\Xi(6)_Q} = m_N + m_H - 0.236 M_{KK} + 4.25 m_0$$
$$m_{\Omega_Q} = m_N + m_H - 0.236 M_{KK} + 6.76 m_0 \tag{50}$$

In the original Sakai and Sugimoto analysis, the Kaluza-Klein parameter is fixed by the light rho mass as indicated earlier with $M_{KK} \approx 1$ GeV. Although we will use this value for all the heavy-light baryon masses to follow, we note that this value of M_{KK} was noted to be large in [38, 42]. The nucleon mass $m_N = 938$ MeV is set to its empirical value. The symmetry breaking parameter m_0 will be fitted to reproduce the mass splitting between the nucleon in the octet and the $\Omega^- = sss$ in the decuplet as it is the baryon with the largest strangeness. Specifically, we set

$$m_{\Omega^-} - m_N = 0.386\, M_{KK} + 15.32\, m_0 = 732\,\text{MeV} \tag{51}$$

which fixes $m_0 = 22.6$ MeV.

So for $n_z = n_\rho = 0$, the lowest heavy-light mass spectra corrected in first order by the light strange quark symmetry breaking, with their J^π assignments are

$$
\begin{aligned}
&\Lambda_Q(\frac{1}{2})^+, M = m_N + m_H - 0.57M_{KK} - 3.04m_0\\
&\Xi^{\bar 3}_Q(\frac{1}{2})^+, M = m_N + m_H - 0.57M_{KK} + 2.08m_0\\
&\Sigma_Q(\frac{1}{2})^+, M = m_N + m_H - 0.236M_{KK} + 1.75m_0\\
&\Xi^{6}_Q(\frac{1}{2})^+, M = m_N + m_H - 0.236M_{KK} + 4.25m_0\\
&\Omega_Q(\frac{1}{2})^+, M = m_N + m_H - 0.236M_{KK} + 6.76m_0\\
&\Sigma^\star_Q(\frac{3}{2})^+, M = m_N + m_H - 0.236M_{KK} + 1.75m_0\\
&\Xi^{6\star}_Q(\frac{3}{2})^+, M = m_N + m_H - 0.236M_{KK} + 4.25m_0\\
&\Omega^\star_Q(\frac{3}{2})^+, M = m_N + m_H - 0.236M_{KK} + 6.76m_0
\end{aligned}
\tag{52}
$$

The lowest excited states of these heavy-light baryons carry finite n_ρ, n_z. For instance, for $n_\rho = 1$, $n_z = 0$ we have the even-parity or Roper-like excitation corresponding to $\Omega_{EQ}(\frac{1}{2})^+$, and for $n_\rho = 0$ and $n_z = 1$ we have the odd-parity excitation corresponding to $\Omega_Q(\frac{1}{2})^-$. Their masses are

$$
\begin{aligned}
\Omega_Q(\frac{1}{2})^-, M &= m_N + m_H + 0.580M_{KK} + 6.76m_0\\
\Omega_{EQ}(\frac{1}{2})^+, M &= m_N + m_H + 0.580M_{KK} + 10.74m_0
\end{aligned}
\tag{53}
$$

The masses of the single-heavy light baryons with charm follow by setting the charm heavy meson mass m_H to its empirical value $m_H = m_D = 1870$ MeV, and similarly for the bottom heavy meson mass $m_H = m_B = 5279$ MeV. The specifics mass values are quoted below in [MeV] with the measured masses from [54] indicated in bold numbers.

1. *Charm baryon masses* [MeV]

$$
\begin{aligned}
\Lambda_c(\frac{1}{2})^+, M &= 2117\,[\mathbf{2286}]\\
\Xi^{\bar 3}_c(\frac{1}{2})^+, M &= 2320\,[\mathbf{2468}]\\
\Sigma_c(\frac{1}{2})^+, \Sigma^\star_c(\frac{3}{2})^+, M &= 2641\,[\mathbf{2453}, \mathbf{2518}]\\
\Xi^{6}_c(\frac{1}{2})^+, \Xi^{6\star}_c(\frac{3}{2})^+, M &= 2740\,[\mathbf{2576}, \mathbf{2646}]\\
\Omega_c(\frac{1}{2})^+, \Omega^\star_c(\frac{3}{2})^+, M &= 2840\,[\mathbf{2695}, \mathbf{2766}]\\
\Omega_c(\frac{1}{2})^-, \Omega^\star_c(\frac{3}{2})^-, M &= 3656\,[\mathbf{3050}, \mathbf{3066}]\\
\Omega_{Ec}(\frac{1}{2})^+, \Omega^\star_{Ec}(\frac{3}{2})^+, M &= 3813\,[\mathbf{3090}, \mathbf{3119}]
\end{aligned}
\tag{54}
$$

2. *Bottom baryon masses* [MeV]

$$
\begin{aligned}
\Lambda_b(\frac{1}{2})^+, M &= 5580\,[\mathbf{5619}]\\
\Xi^{\bar 3}_b(\frac{1}{2})^+, M &= 5696\,[\mathbf{5799}]\\
\Sigma_b(\frac{1}{2})^+, \Sigma^\star_b(\frac{3}{2})^+, M &= 6022\,[\mathbf{5813}, \mathbf{5834}]\\
\Xi^{6}_b(\frac{1}{2})^+, \Xi^{6\star}_b(\frac{3}{2})^+, M &= 6079\,[****, \mathbf{5955}]\\
\Omega_b(\frac{1}{2})^+, \Omega^\star_b(\frac{3}{2})^+, M &= 6153\,[\mathbf{6048}, ****]\\
\Omega_b(\frac{1}{2})^-, \Omega^\star_b(\frac{3}{2})^-, M &= 6951\\
\Omega_{Eb}(\frac{1}{2})^+, \Omega^\star_{Eb}(\frac{3}{2})^+, M &= 7041
\end{aligned}
\tag{55}
$$

B. Double-heavy baryon spectrum

The double-heavy baryons with hidden charm or bottom are currently referred to as pentaquarks. Their masses in the **8**-plet of the flavor representation (43) corrected by the strange quark mass are

$$
\begin{aligned}
N_{\bar QQ}^{(\frac{1}{2},\frac{3}{2})^-}, M &= m_N + 2m_H\\
\Lambda_{\bar QQ}^{(\frac{1}{2},\frac{3}{2})^-}, M &= m_N + 2m_H + 3.80m_0\\
\Sigma_{\bar QQ}^{(\frac{1}{2},\frac{3}{2})^-}, M &= m_N + 2m_H + 7.59m_0\\
\Xi_{\bar QQ}^{(\frac{1}{2},\frac{3}{2})^-}, M &= m_N + 2m_H + 9.48m_0
\end{aligned}
\tag{56}
$$

The penta-quark masses in the **10**-plet representation corrected by the strange quark mass are

$$\begin{aligned}
\Delta_{\bar{Q}Q}^{(\frac{1}{2},\frac{3}{2},\frac{5}{2})^-}, M &= m_N + 2m_H + 0.386M_{KK} + 6.74m_0 \\
\Sigma_{\bar{Q}Q}^{\star(\frac{1}{2},\frac{3}{2},\frac{5}{2})^-}, M &= m_N + 2m_H + 0.386M_{KK} + 9.60m_0 \\
\Xi_{\bar{Q}Q}^{\star(\frac{1}{2},\frac{3}{2},\frac{5}{2})^-}, M &= m_N + 2m_H + 0.386M_{KK} + 12.46m_0 \\
\Omega_{\bar{Q}Q}^{(\frac{1}{2},\frac{3}{2},\frac{5}{2})^-}, M &= m_N + 2m_H + 0.386M_{KK} + 15.32m_0
\end{aligned} \tag{57}$$

The double heavy baryons consisting of two heavy bound mesons with explicit charm or bottom will be referred to by Ξ_{QQ} and Ω_{QQ} in the flavor **3**-plet representation as we noted earlier. Their strangeness corrected masses are

$$\begin{aligned}
\Xi_{QQ}^{(\frac{1}{2})^+}, M = m_N + 2m_H - 0.844M_{KK} - 2.67\, m_0 \\
\Omega_{QQ}^{(\frac{1}{2})^+}, M = m_N + 2m_H - 0.844M_{KK} - 0.54\, m_0
\end{aligned} \tag{58}$$

It is clear, that the holographic construct also describes their excited Roper-like with even parity as well as their odd parity partners, which can be retrieved from our formula.

1. Charm penta-quark masses [MeV]

$$\begin{aligned}
N_{\bar{c}c}(\frac{1}{2},\frac{3}{2})^-, M &= 4680\,[\mathbf{4380}, \mathbf{4450}] \\
\Lambda_{\bar{c}c}(\frac{1}{2},\frac{3}{2})^-, M &= 4766 \\
\Sigma_{\bar{c}c}(\frac{1}{2},\frac{3}{2})^-, M &= 4852 \\
\Xi_{\bar{c}c}(\frac{1}{2},\frac{3}{2})^-, M &= 4894 \\
\Delta_{\bar{c}c}(\frac{1}{2},\frac{3}{2},\frac{5}{2})^-, M &= 5218 \\
\Sigma^\star_{\bar{c}c}(\frac{1}{2},\frac{3}{2},\frac{5}{2})^-, M &= 5283 \\
\Xi^\star_{\bar{c}c}(\frac{1}{2},\frac{3}{2},\frac{5}{2})^-, M &= 5348 \\
\Omega_{\bar{c}c}(\frac{1}{2},\frac{3}{2},\frac{5}{2})^-, M &= 5412
\end{aligned} \tag{59}$$

2. Mixed penta-quark masses [MeV]

$$\begin{aligned}
N_{\bar{b}c}(\frac{1}{2},\frac{3}{2})^-, M &= 8089 \\
\Lambda_{\bar{b}c}(\frac{1}{2},\frac{3}{2})^-, M &= 8175 \\
\Sigma_{\bar{b}c}(\frac{1}{2},\frac{3}{2})^-, M &= 8261 \\
\Xi_{\bar{b}c}(\frac{1}{2},\frac{3}{2})^-, M &= 8303 \\
\Delta_{\bar{b}c}(\frac{1}{2},\frac{3}{2},\frac{5}{2})^-, M &= 8627 \\
\Sigma^\star_{\bar{b}c}(\frac{1}{2},\frac{3}{2},\frac{5}{2})^-, M &= 8692 \\
\Xi^\star_{\bar{b}c}(\frac{1}{2},\frac{3}{2},\frac{5}{2})^-, M &= 8757 \\
\Omega_{\bar{b}c}(\frac{1}{2},\frac{3}{2},\frac{5}{2})^-, M &= 8821
\end{aligned} \tag{60}$$

3. Bottom penta-quark masses [MeV]

$$\begin{aligned}
N_{\bar{b}b}(\frac{1}{2},\frac{3}{2})^-, M &= 11498 \\
\Lambda_{\bar{b}b}(\frac{1}{2},\frac{3}{2})^-, M &= 11583 \\
\Sigma_{\bar{b}b}(\frac{1}{2},\frac{3}{2})^-, M &= 11670 \\
\Xi_{\bar{b}b}(\frac{1}{2},\frac{3}{2})^-, M &= 11712 \\
\Delta_{\bar{b}b}(\frac{1}{2},\frac{3}{2},\frac{5}{2})^-, M &= 12036 \\
\Sigma^\star_{\bar{b}b}(\frac{1}{2},\frac{3}{2},\frac{5}{2})^-, M &= 12101 \\
\Xi^\star_{\bar{b}b}(\frac{1}{2},\frac{3}{2},\frac{5}{2})^-, M &= 12166 \\
\Omega_{\bar{b}b}(\frac{1}{2},\frac{3}{2},\frac{5}{2})^-, M &= 12230
\end{aligned} \tag{61}$$

4. Charm and bottom **3***-plet masses* [MeV]

$$\begin{aligned}
\Xi_{cc}(\frac{1}{2})^{+},\, M &= 3776\,[\mathbf{3519}] \\
\Omega_{cc}(\frac{1}{2})^{+},\, M &= 3848 \\
\Xi_{cb}(\frac{1}{2})^{+},\, M &= 7184 \\
\Omega_{cb}(\frac{1}{2})^{+},\, M &= 7257 \\
\Xi_{bb}(\frac{1}{2})^{+},\, M &= 10584 \\
\Omega_{bb}(\frac{1}{2})^{+},\, M &= 10657 \qquad (62)
\end{aligned}$$

VI. CONCLUSIONS

We have presented a top-down holographic approach to the single- and double-heavy baryons in the variant of $D4$-$D8$ we proposed recently [37] (first reference). To order λm_H^0, the heavy baryons emerge from the zero mode after binding a heavy meson in the multiplet $(0^-, 1^-)$ to the instanton. Remarkably, in the bulk instanton field the spin 1 and odd parity heavy meson transmutes equally to a spin $\frac{1}{2}$ and even parity massless fermion and anti-fermion. At subleading order, the Chern-Simons term is attractive for the bound meson with a heavy quark content and repulsive for the bound meson with heavy anti-quark content.

One of the key differences between the $N_f = 2$ and $N_f = 3$ case is the role played by the amended form of the Chern-Simons term which results in a good hypercharge quantization rule [42, 43]. We have shown that the rule gets modified by the presence of the bound zero mode states, leading to a rich heavy-light spectra for single-heavy and double-heavy baryons with hidden charm and bottom. In particular, the formers follow from the $\bar{\mathbf{3}}$ and **6** flavor representations, while the latters from the **8** and **10** representations for the lowest states. The holographic set up allows for a simple description of the low-lying odd-parity and Roper-like excitations of the heavy baryons. Our results for $N_f = 3$ with massive strangeness confirm and extend our previous findings for massless $N_f = 2$.

To compare our results with currently known heavy-light charm and meson spectra, it is necessary to account for the light strange quark mass. In holography this is induced by a worldsheet instanton that connects $D8$ and $D\bar{8}$ [44]. By accounting for this correction in leading order perturbation theory, we have found reasonable agreement for the lowest single-heavy baryons with a single adjustable parameter, namely the overall strength of the symmetry breaking term. The holographic model describes 2 neutral $\Omega_c^0, \Omega_c^{*0}$ states with $\frac{1}{2}^+, \frac{3}{2}^+$ assignments as the odd parity partners of the lowest $\Omega_c^0, \Omega_c^{*0}$ states, and 2 Roper-like neutral states with $\frac{1}{2}^+, \frac{3}{2}^+$ assignments as the even parity partners also of the lowest $\Omega_c^0, \Omega_c^{*0}$ states. The $\frac{1}{2}^- \frac{3}{2}^-$ are predicted to be lighter than the excited $\frac{1}{2}^+ \frac{3}{2}^+$ states, however both pairs are found to be heavier than the 5 neutral Ω_c^0 states reported recently by the LHCb collaboration.

The holographic set up for the heavy baryons is remarkable by the limited number of parameters it carries. Once the initial parameter κ is traded for the pion decay constant f_π, and the Kaluza-Klein scale M_{KK} is fixed by the rho meson mass, only the symmetry breaking parameter m_0 is left to be fixed in either the light or heavy sector. We choose the latter to fix it. Clearly, the model can and should be made more realitic through the use of improved holographic QCD [55].

The shortcomings of the heavy-light holographic approach stem from the triple limits of large N_c, strong 't Hooft coupling $\lambda = g^2 N_c$, and heavy meson mass. The corrections in $1/m_H$ are straighforward but laborious and should be studied as they shed important light on the hyperfine type splittings. Also, it should be useful to explore the sensitivity of our results by relaxing the value of M_{KK} as fixed in the light meson sector and addressing the strangeness mass correction beyond leading order perturbation theory. The one-meson radiative decays of the heavy baryons and their exotics can be addressed in this model for further comparison with the experimentally reported partial widths.

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VIII. APPENDIX

In this Appendix we briefly recall the key steps in the collective quantization of the holographic light baryons for both $N_f = 2, 3$ [38, 42, 53]. For $N_f = 2$ and no heavy-meson (1) describes the light meson sector. In the large λ limit and under the rescaling (12) the classical field equations yield a zero-size instanton. The latter is characterized by the moduli (a_I, X_α). Here a_I refers to the moduli of the global SU(2) gauge transformation. The quantum spectrum follows by promoting the moduli to be time-dependent $(a_I, X_\alpha) \rightarrow (a_I(t), X_\alpha(t))$. The ensuing Hamiltonian for the collective coordinates is [38]

$$
\begin{aligned}
H_0 &= M_0 + H_Z + H_\rho \\
H_Z &= -\frac{\partial_Z^2}{2m_z} + \frac{m_z\omega_z^2}{2}Z^2 \\
H_\rho &= -\frac{\nabla_y^2}{2m_y} + \frac{m_y\omega_\rho^2}{2}\rho^2 + \frac{Q}{\rho^2} \\
y &= \rho(a_1, a_2, a_3, a_4), a_I = a_4 + i\vec{a}\cdot\vec{\tau} \\
m_z &= \frac{m_y}{2} = 8\pi^2 a N_c, \omega_z^2 = \frac{2}{3}, \omega_\rho^2 = \frac{1}{6}
\end{aligned} \tag{63}
$$

So for $N_f = 2$, the eigenstates of H_ρ are given by $T^l(a)R_{l,n_\rho}(\rho)$, where T^l are the spherical harmonics on S^3. Under $SO(4) = SU(2) \times SU(2)/Z_2$ they are in the $(\frac{l}{2}, \frac{l}{2})$ representations, where the two SU(2) factors are defined by the isometry $a_I \rightarrow V_L a_I V_R$. The left factor is the isospin rotation, and the right factor is the space rotation. This quantization describes $I = J = \frac{l}{2}$ states. The nucleon is realized as the lowest state with $l = 1$ and $n_\rho = n_z = 0$.

For the SU(3) case most of the analysis remains the same except for two differences: 1/ the Chern-Simons term needs amendment as explained in main text; 2/ both A_0 and $\hat{A}_0$ need to be solved to a non-zero value at the static level as also explained in the main text. With this in mind, a general time-dependent SU(3) rotation a_I generates the new collective Hamiltonian H_ρ as [42]

$$
\begin{aligned}
H_\rho = \quad & -\frac{1}{2m_y}\frac{1}{\rho^\eta}\partial_\rho(\rho^\eta\partial_\rho) + \frac{1}{2}m_y\omega_\rho^2\rho^2 + \frac{Q}{\rho^2} \\
& + \frac{2\sum_{a=1}^3 J_a^2}{m_y\rho^2} + \frac{4\sum_{a=4}^7 J_a^2}{m_y\rho^2}
\end{aligned} \tag{64}
$$

We note that in holography, the inertia in the $1,2,3$ directions is twice larger than the inertia in the $4,5,6,7$ directions reflecting on the inherent SU(2) character of the flavor instanton in bulk. The J_a are the generators of the right representation on the group manifold associated to a_I. Given a representation (p, q) and right-spin j, we have

$$
\begin{aligned}
\sum_{a=1}^8 J_a^2 &= \frac{1}{3}(p^2 + q^2 + pq + 3(p+q)) \\
\sum_{a=1}^3 J_a^2 &= j(j+1)
\end{aligned} \tag{65}
$$

The radial wavefunctions and energies associated to the full Hamiltonian

$$
H_0 = -\frac{1}{2m_y}\frac{1}{\rho^\eta}\partial_\rho(\rho^\eta\partial_\rho) + \frac{1}{2}m_y\omega_\rho^2\rho^2 + \frac{\mathbf{K}}{m_y\rho^2} \tag{66}
$$

are found in the form

$$
\begin{aligned}
\phi_{n_\rho,\rho,\mathbf{K}} &= e^{-\frac{m_y\omega_\rho\rho^2}{2}}\rho^{\beta-\frac{\eta+1}{2}}F(-n_\rho, \beta, m_y\omega_\rho\rho^2) \\
\beta &= 1 + \left(\frac{(\eta-1)^2}{2} + 2\mathbf{K}\right)^{\frac{1}{2}} \\
E_{n_\rho} &= \omega_\rho\left(2n_\rho + 1 + \frac{\sqrt{(\eta-1)^2 + 8\mathbf{K}}}{2}\right)
\end{aligned} \tag{67}
$$

The combination $m_y\omega_\rho \equiv 16\pi^2\kappa/\sqrt{6}$ if we remember to unwind the rescaling $\sqrt{\lambda}\rho \rightarrow \rho$ from (12). The value of κ is fixed by the pion decay constant $f_\pi^2/M_{KK}^2 = \kappa/(54\pi^4)$ [35]. The explicit wavefunctions for the SU(3) representation with assignment $\mu = (p, q)$ are given by

$$
|\mu, YII_3, Y_RJ_sM_s >= (-1)^{J_s - M_s} D^\mu_{YII,Y_RJ_sM_s}(a_I) \tag{68}
$$

and the total state with one spinor attached (for a single-heavy baryon) follows by re-coupling

$$
\begin{aligned}
&\Phi_{\mu,YII_3,Y_RJJ_3} = \\
&\sum_{h=\pm, M_s+h=J_3} C^{\frac{1}{2},J_s,J}_{h,M_s,J_3}\chi_h|\mu, YII_3, Y_RJ_sM_s >
\end{aligned} \tag{69}
$$

A similar re-coupling holds for the double-heavy baryons. When evaluating the symmetry breaking contribution through $\langle D_{88}\rangle$, we note that the Clebsch-Gordon coefficients play no role since they depend only on μ, YI, Y_RJ_s and not on M_s.

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