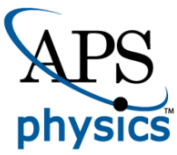

Quarkonium h states as arbiters of exoticity

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Quarkonium h States As Arbiters of Exoticity

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The mass splitting between the quarkonium spin-singlet state h ($J^{PC} = 1^{+-}$) and the spin average of the quarkonium spin-triplet states χ ($J^{PC} = 0^{++}, 1^{++}, 2^{++}$) is seen to be astonishingly small, not only in the charmonium and bottomonium cases where the relevant masses have been measured, but in positronium as well. We find, both in nonrelativistic quark models and in NRQCD, that this hyperfine splitting is so small that it can be used as a test of the pure $Q\bar{Q}$ content of the states. We discuss the $2P$ states of charmonium in the vicinity of 3.9 GeV, where the putative exotics $X(3872)$ and $X(3915)$ have been seen and a new $\chi_{c0}(2P)$ candidate has been observed at Belle.

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I. INTRODUCTION

The spectrum of charmonium-like states in the region near 3.9 GeV is exceptionally intricate and interesting. In addition to containing states that are believed to be the conventional $c\bar{c}$ 1^3D_1 [$\psi(3770)$], 1^3D_2 [$\psi(3823)$], and 2^3P_2 [$\chi_{c2}(2P)$] [1], this range has produced several unexpected states, including the most famous exotic candidate $X(3872)$ ($J^{PC} = 1^{++}$), as well as the 0^{++} (or even possibly 2^{++} [6]) $X(3915)$, the 1^{+-} $Z_c^0(3900)$ that is the neutral isospin partner of the $Z_c^+(3900)$, and the $X(3940)$, whose J^{PC} remains unknown. For a review of these states and more, see Ref. [2].

Missing from this list are several expected states in the $2P$ band, such as the conventional 0^{++} $\chi_{c0}(2P)$ and 1^{++} $\chi_{c1}(2P)$, and the 1^{+-} $h_c(2P)$. Indeed, the $X(3872)$ has long been argued to have at least a substantial $\chi_{c1}(2P)$ component, while the $X(3915)$ was briefly listed by the Particle Data Group as $\chi_{c0}(2P)$ until serious doubts were raised about this identification (especially its lack of $D\bar{D}$ final states) [3–5]; for example, $X(3915)$ might even be the lightest $c\bar{c}s\bar{s}$ state [7]. The crucial importance of sorting out the states in the $2P$ charmonium sector, in order to determine which states are (mostly) exotic and which are not, was emphasized as a central experimental goal in Ref. [8]. A very recent attempt in this direction appears in Ref. [9].

The latest chapter in this saga is the Belle observation [10] of a $\chi_{c0}(2P)$ candidate decaying to $D\bar{D}$, with mass 3862^{+26+40}_{-32-13} MeV and width $201^{+154+88}_{-67-82}$ MeV. While these uncertainties are quite large, the significance of the signal is substantial (6.5σ). With the $\chi_{c2}(2P)$ and hopefully the $\chi_{c0}(2P)$ now in hand, one can at last begin a serious study of mass splittings within this multiplet, with an eye toward testing the expectations for pure $c\bar{c}$ composition versus mixing with multiquark or hybrid components.

The mass difference of interest in this paper is the hyperfine splitting between the quarkonium spin-singlet state h ($^1P_1$, $J^{PC} = 1^{+-}$) and the spin average of the quarkonium spin-triplet states χ ($^3P_{0,1,2}$, $J^{PC} = 0^{++}, 1^{++}, 2^{++}$):

$$\Delta \equiv M_h - \frac{1}{9}\left[1\cdot M_{\chi_0} + 3\cdot M_{\chi_1} + 5\cdot M_{\chi_2}\right] . \qquad (1)$$

A complete set of experimental data for determining Δ is currently available in only 4 cases: $2P$ positronium (which, in the standard notation for positronium, is the lowest P wave) [11, 12], $1P$ charmonium, and $1P$ and $2P$ bottomonium [1]. The corresponding Δ values are presented in Table I. In every case, the value of Δ is zero to within experimental uncertainties, making the tight relationship among P-level states highly predictive for cases (such as $2P$ charmonium) in which some of the states have not yet been observed.

In this short paper we explore the physical reason for this remarkable relationship in quarkonium; indeed, Δ is so small that one may call it an *ultrafine* splitting. We then show how it may be applied to the confusing set of charmonium states around 3.9 GeV to uncover an unambiguous signal of exoticity, by which we mean a non-$c\bar{c}$ state component. Finally, we remind the reader of the ongoing and proposed experiments designed to uncover missing quarkonium states.

This paper is organized as follows. In Sec. II we identify and discuss the operators potentially contributing to the "ultrafine" mass difference Eq. (1) and related combinations. Section III identifies the origin of the relevant operator in quark potential models and explains the origin of its numerical suppression; Sec. IV does the same for the non-relativistic QCD effective theory. In Sec. V we discuss the nonperturbative heavy-quark limit and the effect of the appearance of partonic degrees of freedom

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TABLE I: Experimental values of Δ in MeV for quarkonium and in MHz for positronium. For quarkonium the state masses entering Eq. (1) are listed, while for positronium the differences $(2^3S_1 - 2^{2S+1}P_J)$ are presented.

System	$h(^1P_1)$	$\chi_0(^3P_0)$	$\chi_1(^3P_1)$	$\chi_2(^3P_2)$	Δ
$c\bar{c}(1P)$	3525.38(11)	3414.75(31)	3510.66(7)	3556.20(9)	+0.08(13)
$c\bar{c}(2P)$	–	3862^{+26+40}_{-32-13}	–	3927.2(2.6)	–
$b\bar{b}(1P)$	9899.3(8)	9859.44(42)(31)	9892.78(26)(31)	9912.21(26)(31)	−0.57(88)
$b\bar{b}(2P)$	10259.8(1.2)	10232.5(4)(5)	10255.46(22)(50)	10268.65(22)(50)	−0.44(1.31)
Ps	11180(5)(4)	18499.65(1.20)(4.00)	13012.42(67)(1.54)	8624.38(54)(1.40)	+4.31(6.50)

beyond the heavy quark-antiquark pair, and in Sec. VI describe the use of the "ultrafine" relation in identifying the presence of exotic (non-$Q\bar{Q}$) components in the candidate states and the prospects for observing the missing states. Section VII summarizes and concludes.

II. OPERATORS CONTRIBUTING TO Δ

The hyperfine interaction is defined, as usual, as a direct coupling between the intrinsic spins of the component fermions of the state. In the case of $f\bar{f}$ bound states, where f is a spin-$\frac{1}{2}$ fermion, one can produce only a finite number of linearly independent operators contributing to the mass from the basic ingredients of quark-spin $\mathbf{S}_f$, $\mathbf{S}_{\bar{f}}$ and orbital angular momentum $\mathbf{L}$ operators. For example, a quark-spin operator that transforms under an irreducible representation with spin greater than two cannot, by the Wigner-Eckart theorem, contribute to matrix elements of states containing only two spin-$\frac{1}{2}$ quarks. On the other hand, operators sensitive to arbitrarily high powers of squared quark momenta (but no spin dependence) might be generated by the fine details of quark distributions within the hadron, but their contributions to hadron masses are proportional to those arising from any spin-symmetric operator, such as the quark-mass operator.

To put the discussion on a firm footing, we define the usual operators in configuration space:

$$\mathbf{S}_f \cdot \mathbf{S}_{\bar{f}} \quad \text{(hyperfine)}\,, \tag{2}$$

$$\mathbf{S} \cdot \mathbf{L} \quad \text{(spin-orbit)}\,, \tag{3}$$

$$\overleftrightarrow{T} \equiv (\mathbf{S}_f \cdot \hat{\mathbf{r}})(\mathbf{S_f} \cdot \hat{\mathbf{r}}) - \frac{1}{3}\mathbf{S}_f \cdot \mathbf{S}_{\bar{f}} \quad \text{(tensor)}\,, \tag{4}$$

where $\mathbf{S} \equiv \mathbf{S}_f + \mathbf{S}_{\bar{f}}$. The operators can be expressed just as easily in momentum space by replacing f-$\bar{f}$ relative position operator $\mathbf{r}$ with the relative momentum operator $\mathbf{q}$ and replacing $\mathbf{L}$ with $\mathbf{q} \times \mathbf{p}$, where $\mathbf{p}$ is the total momentum operator. In any case, these are the only three independent spin-dependent operators that arise up to quadratic order in $\mathbf{S}_{f,\bar{f}}$; and since all linearly independent operators arising beyond quadratic order transform as spin greater than two, the list in Eqs. (2)–(4) is complete. For example, the operator $(\mathbf{S}\cdot\mathbf{L})^2$ can be shown[1] for any given multiplet of $f\bar{f}$ states to be linearly dependent on the ones above plus the operator $\mathbf{S}^2\mathbf{L}^2$.

Now consider any multiplet of $f\bar{f}$ states that, in the language of a quark potential model, carry the same principal quantum number n and orbital angular momentum L. For states with $S = 0$ [and hence $J = L$: quarkonium η and h], matrix elements of both the spin-orbit and tensor operator vanish by the Wigner-Eckart theorem since the operators transform as $S = 1$ and $S = 2$, respectively. The same matrix elements vanish for all $L = 0$ states, so one immediately sees that any n^3S_1-n^1S_0 hyperfine splitting—in which all spin-independent mass terms cancel—depends only on the hyperfine operator, as one might expect. For $L > 0$ and $S = 1$, $J = L - 1$, L, and $L + 1$ are allowed, and using the results

$$\langle \mathbf{S}\cdot\mathbf{L} \rangle = \frac{1}{2}\left[J(J+1) - L(L+1) - S(S+1)\right]\,, \tag{5}$$

and

$$\left\langle \overleftrightarrow{T} \right\rangle = \begin{cases} -\frac{L+1}{6(2L-1)}\,, & J = L-1\,, \\ +\frac{1}{6}\,, & J = L\,, \\ -\frac{L}{6(2L+3)}\,, & J = L+1\,, \end{cases} \tag{6}$$

one can quickly check that adding these matrix elements weighted by the $2J + 1$ degenerate spin states for each level gives a vanishing result. In other words, the spin-averaged matrix elements of any trio of spin-triplet states $n^3L_{J=L-1}$, $n^3L_{J=L}$, $n^3L_{J=L-1}$ with orbital angular momentum larger than zero vanish for the spin-orbit and tensor operators. The reason is not so mysterious: Although expressed in the $|J, J_z, L, S = 1\rangle$ basis, the states form a complete multiplet in the $|L, L_z, S = 1, S_z\rangle$ basis, while the spin-orbit and tensor operators, being irreducible operators of rank greater than zero, are traceless. On the other hand, all of these states have the same spin-independent mass terms, which is also the same as that of the corresponding spin-singlet $n^1S_{J=L}$. In total, the mass combination for orbital angular momentum larger

[1] This fact provides one convenient method [13] for calculating matrix elements of $\overleftrightarrow{T}$, such as those given in Eq. (6).

than zero defined by:

$$\begin{aligned}\Delta_{n,L} \;\equiv\; & M(n^1L_{J=L}) \\ & - \frac{2L-1}{3(2L+1)} M(n^3L_{J=L-1}) \\ & - \frac{2L+1}{3(2L+1)} M(n^3L_{J=L}) \\ & - \frac{2L+3}{3(2L+1)} M(n^3L_{J=L+1})\,, \end{aligned} \tag{7}$$

of which Eq. (1) is simply the case $L = 1$ for quarkonium, receives contributions only through the hyperfine operator.

All mass combinations $\Delta_{n,0} \equiv M(n^1S_0) - M(n^3S_1)$ and $\Delta_{n,L}$ of Eq. (7) for $L > 0$ are thus pure hyperfine splittings. In order to see why the latter deserve the label "ultrafine," we consider the origin of the hyperfine operator in three useful dynamical formalisms: quark-potential models, nonrelativistic QCD (pNRQCD), and the heavy-quark expansion of QCD, including nonperturbative effects.

III. QUARK POTENTIAL MODELS

The form of the operator multiplying $\mathbf{S}_f \cdot \mathbf{S}_{\bar{f}}$ is clearly crucial for determining the relative size of various hyperfine interactions. In the case of electromagnetic interactions, the direct spin-spin coupling is pointlike, being proportional to the wave function of the state at zero spatial separation $\mathbf{r}$ between the spins, a fact first noted by Fermi [14]. Such a term is proportional to $\delta^{(3)}(\mathbf{r})$, and arises naturally in the nonrelativistic reduction of terms in the Dirac equation (and more generally, QED) describing the interaction of two charged particles, known as the Breit Hamiltonian [15]. In that context, it appears through the Laplacian operator acting upon the Coulomb potential $1/r$. Of course, this $1/r$ simply occurs as the Fourier transform of the momentum-space propagator $1/q^2$ of the massless photon.

The equivalent Breit Hamiltonian for the case of potential interactions in quark systems, as applied to hadron masses, was expressed in De Rujula, Georgi, and Glashow [16]. Since QCD also has massless gauge bosons in the form of gluons, one finds that the corresponding spin-spin term is proportional to $\delta^{(3)}(\mathbf{r})$ in the short-distance limit in which the interaction is dominated by one-gluon exchange. *Any* quark potential model in which the potential $V(r)$ contains a piece representing one-gluon exchange will exhibit this feature. This term is represented for example, in the most thorough recent analysis of charmonium masses [17] as a Gaussian of width $1/\sigma$:

$$\delta^{(3)}(\mathbf{r}) \to \tilde{\delta}_\sigma(r) \equiv \left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r^2}\,. \tag{8}$$

"Smearing" of this sort is necessary to regularize the delta function (thereby making it a well-defined three-dimensional quantum-mechanical operator), and because nonzero hyperfine splitting is evident in the spectrum for radially excited S-wave states.

The most interesting feature of the $\delta^{(3)}(\mathbf{r})$ dependence is that it is only supported by wave functions that are nonvanishing at the origin. Of course, one well-known feature of quantum mechanics is that wave functions with orbital angular momentum quantum number L scale as r^L near the origin. Therefore, one naturally expects the S-wave hyperfine splittings to be numerically much larger than those with $L > 0$, hence the term "ultrafine." The evidence from Table I strongly supports this conclusion; for example, the S-wave hyperfine splitting in the $n = 1$ level of charmonium is $m_{J/\psi} - m_{\eta_c} = 113.5(5)$ MeV. One anticipates that the D-wave splittings would be even smaller than those in the P-wave.

Using the smearing function of Eq. (8) and the parameter value [17] $\sigma = 1.0946$ GeV, we find values of $\Delta_{n,1}$ of order 3–10 MeV, which are much larger than those observed. This result is a reflection of the relatively small value of σ used in the model, which is driven by the much larger observed S-wave hyperfine splittings, $\Delta_{n,0}$.

IV. NRQCD

In the heavy-quark limit of QCD, all operators dependent upon the heavy-quark flavor or spin are suppressed by powers of the heavy-quark mass m_Q, and the relevant finite dynamical parameter becomes the heavy-quark four-velocity v. Effects at energy scales higher than m_Q are integrated out in the usual Wilsonian fashion, leading to the *heavy-quark effective theory* [18]. Spin-spin and tensor operators, containing two heavy-quark spin operators, are therefore suppressed by $1/m_Q^2$.

When more than one heavy quark is present, as in quarkonium, new scales arise, and their modes must be integrated out successively: in decreasing magnitude, these are the "hard" scale m_Q (leading to the effective theory of *non-relativistic QCD* (NRQCD) [19, 20], the "soft" scale $m_Q v$, and the "potential" scale (energies $\sim m_Q v^2$, momenta $\sim m_Q v$), leading to the effective theory called *potential non-relativistic QCD* (pNRQCD) [21, 22]. The remaining modes are "ultrasoft" (energies and momenta $\sim m_Q v^2$).

The state-of-the-art calculations in this program are now performed at next-to-next-to-next-to leading order (NNNLO) in NRQCD; the corresponding Hamiltonian was obtained in Ref. [23] and the heavy-quarkonium spectrum for states of arbitrary quantum numbers, including terms of $O(m_Q \alpha_s^5 \ln \alpha_s)$, was presented in Ref. [24]. The corresponding expressions including $O(m_Q \alpha_s^5)$ terms, which we use here, appear in Ref. [25].

Using the NNNLO results from Ref. [25], one computes

the $1S$ hyperfine splitting:

$$\Delta_{1,0} = \frac{1}{3} m_Q \alpha_s^4 C_F^4 - \frac{m_Q \alpha_s^5 C_F^4}{108\pi} \Bigg[6\pi^2 \beta_0 - 72\beta_0 + 20 C_A + 18 C_F + 63 C_A \log\left(\frac{1}{\alpha_s C_F}\right) - 72\beta_0 \log\left(\frac{\mu}{\alpha_s C_F m_Q}\right) + 8 T_F n_\ell - 54 T_F + 54 T_F \log 2 \Bigg] , \quad (9)$$

where $\beta_0 = (11C_A - 2n_\ell)/3$, with n_ℓ being the number of light fermion species appearing in loop corrections (*i.e.*, as short-distance degrees of freedom only) and μ being the renormalization point. The usual color and trace factors $C_A = N_c \to 3$, $C_F = (N_c^2 - 1)/2N_c \to \frac{4}{3}$, and $T_F = \frac{1}{2}$ also appear. It is worth noting that the leading $[O(\alpha_s^4)]$ term in this expression gives only 13 MeV for $\alpha_s = 0.3$ and $m_c = 1.5$ GeV – too low by a factor of nearly 9 compared to the experimental value given above; however, the $O(\alpha_s^5)$ term not only exhibits a strong dependence upon μ but turns out to be of the same numerical order as the leading term. One concludes that the splitting $\Delta_{1,0}$ is not yet under control in the NRQCD result Eq. (9).

In comparison, the ultrafine combination at this order in NRQCD computed using Ref. [25] is much simpler:

$$\Delta_{n,1} = \frac{m_Q C_F^4 \alpha_s^5}{432\pi (n+1)^3} \left(8 T_F n_\ell - C_A \right) . \quad (10)$$

This expression is smaller both parametrically [by a power of $\alpha_s(m_Q)$] and numerically (by the large denominator factor) than the usual hyperfine splitting.

The origin of the extra suppression, leading to the "ultrafine" label, arises for precisely the same algebraic reason as it does for the nonrelativistic quark potential model: The same set of spin-dependent operators as in Eqs. (2)–(4) arises in NRQCD, and the spin-spin operator Eq. (2) again appears with the contact interaction coefficient $\delta^{(3)}(\mathbf{r})$ [23]. The $L > 0$ overlap integrals are again severely suppressed, and indeed only survive due to the renormalization of the α_s coefficient to Eq. (2) to include terms of the form $\ln(r)$ that are singular for $r \to 0$. One thus obtains the ultrafine suppression in effective field theory treatments of QCD.

The values obtained from Eq. (10) are remarkably small: One obtains 9.5 keV, 2.8 keV, 3.8 keV, and 1.1 keV for 1P and 2P charmonium, and 1P and 2P bottomonium, respectively; such differences are far smaller than the central values given in Table I. The positronium result is expected to be even further suppressed, to $O(\alpha^6)$ [26].

In comparison, the Schnitzer ratio [27],

$$R_1 \equiv \frac{M(^3P_2) - M(^3P_1)}{M(^3P_1) - M(^3P_0)} , \quad (11)$$

is known to be exactly $\frac{4}{5}$ at leading order in α_s, and this result is borne out in NRQCD calculations [25]. However, the $O(\alpha_s)$ corrections to this ratio have a strong μ dependence, which can be used to accommodate its rather different experimental value for $1P$ charmonium, ~ 0.47. One may anticipate a similar strong μ dependence to arise in the next (uncomputed) $O(\alpha_s^6)$ corrections to Eq. (10), but one still expects the measured values for $\Delta_{n,1}$ to remain no larger than $O(10\,\text{keV})$.

V. NONPERTURBATIVE HEAVY-QUARK LIMIT

The matrix elements that appear in the NRQCD results reported in the previous section are evaluated in the heavy-quark limit, for which nonrelativistic Coulombic meson wave functions are appropriate. This approximation breaks down as the quarks become lighter and the scale $\Lambda_{\rm QCD}$ becomes more relevant. Furthermore, the nonperturbative regime also becomes more relevant as the radial quantum number increases, because larger spatial scales are probed. Under these conditions, mass splittings that had been proportional to the heavy quark mass can scale as $\Lambda_{\rm QCD}^3/m_Q^2$. It is thus prudent to enquire into the regime of validity of the NRQCD computations presented above.

An analogous problem arises in the application of the operator-product formalism to the interaction of heavy mesons with hadronic matter. In this case, Peskin [28–31] has estimated that the method is reliable if:

$$m_Q \gg n^2 \frac{\Lambda_{\rm QCD}}{\alpha_s^2(r_Q^{-1})} . \quad (12)$$

It was subsequently argued that this expression should contain a numerical coefficient of order 10 [32]. As a result, the operator-product expansion (in this context) is never valid for physical quarks.

In view of this issue, we seek to estimate the nonperturbative behavior of the hyperfine matrix element that contributes to the hyperfine and ultrafine splittings. A formalism for examining this question was developed long ago by Eichten and Feinberg [33], who applied the heavy-quark expansion to the Wilson loop to obtain expressions for the spin-dependent interaction of heavy quarks at order $1/m_Q^2$. The result for the coefficient of the spin-spin term is proportional to the temporal integral of the matrix element of chromomagnetic fields. Subsequently, a somewhat more transparent, but equivalent, expression was obtained with a Foldy-Wouthuysen reduction of the

QCD Hamiltonian in Coulomb gauge [34]:

$$V_{\rm hyp}^{(n_R)}(\mathbf{R}=\mathbf{r}_Q-\mathbf{r}_{\bar{Q}}) = \alpha_s \frac{4\pi}{3m_Q^2}\,\mathbf{S}_f\cdot\mathbf{S}_{\bar{f}} \times \sum_{m\neq n}\frac{1}{\epsilon_n(R)-\epsilon_m(R)} \times \left\langle n_R;\mathbf{r}_Q,\mathbf{r}_{\bar{Q}}\left|\int d^3x\, h^\dagger(\mathbf{x})\mathbf{B}(\mathbf{x})h(\mathbf{x})\right| m_R;\mathbf{r}_Q,\mathbf{r}_{\bar{Q}}\right\rangle \cdot \left\langle m_R;\mathbf{r}_Q,\mathbf{r}_{\bar{Q}}\left|\int d^3y\, \chi^\dagger(\mathbf{y})\mathbf{B}(\mathbf{y})\chi(\mathbf{y})\right| n_R;\mathbf{r}_Q,\mathbf{r}_{\bar{Q}}\right\rangle + (h\leftrightarrow\chi)\,. \quad (13)$$

States are labeled with the coordinates of the static quarks, $\mathbf{r}_Q$, $\mathbf{r}_{\bar{Q}}$, and gluonic quantum numbers are denoted by m_R, n_R. Heavy-quark and -antiquark creation operators are labeled by $h^\dagger$ and $\chi^\dagger$, respectively. The operators $\mathbf{B}$ are chromomagnetic fields. One of the fields is evaluated at the position of the quark $\mathbf{r}_Q$, while the other is evaluated at the antiquark position $\mathbf{r}_{\bar{Q}}$. Evaluating both fields on a single quark line is possible, but does not yield a spin-dependent interaction.

Perturbatively, the matrix element of Eq. (13) becomes

$$V_{\rm hyp}^{(n_R)}(R) \propto \nabla_Q\cdot\nabla_{\bar{Q}}\,\langle \mathbf{A}(\mathbf{r}_Q,t)\cdot\mathbf{A}(\mathbf{r}_{\bar{Q}},t')\rangle\,.$$

The matrix element is proportional to $1/R$, and the hyperfine interaction is then proportional to $\delta^{(3)}(\mathbf{R})$. A simple nonperturbative estimate can be made by introducing a gluon effective mass term into the gluon propagator. In this case, one obtains a Yukawa-type interaction, with a range given by the inverse gluon effective mass, which is also seen to be short-ranged.

More generally, one can argue that infinitely many strongly interacting virtual gluons tend to decorrelate the chromomagnetic fields rapidly as the interquark separation is increased [34]. This expectation is confirmed in quenched lattice computations of the chromomagnetic field correlation in the presence of a Wilson loop [35], in which it is found that a Yukawa potential with a gluon mass of approximately 2.5 GeV fits the simulation well. The potential itself is zero, within statistical error, for $R > 0.2$ fm.

VI. ULTRAFINE SPLITTINGS AND EXOTICS

All of the previous discussion leads to the same conclusion: The heavy-quark hyperfine interaction is short-ranged. Thus, matrix elements of the interaction must decrease with radial and orbital quantum number. Furthermore, experiment indicates that $\Delta_{1,1}(c\bar{c})$, $\Delta_{1,1}(b\bar{b})$, and $\Delta_{2,1}(b\bar{b})$ are all small, and hence this quantity must be small for all n and L in the charmonium and bottomonium systems.

This conclusion follows from the quark model, which does not consider coupled channels; NRQCD, which only considers the short-range contribution of light quarks; heavy-quark QCD, which suppresses the effect of light quarks; or quenched lattice computations. Thus it is possible that long-distance light-quark effects – such as those manifested in meson-meson contributions to quarkonium states, or by $Q\bar{Q}q\bar{q}$ wave function components – can ruin the relationship $\Delta_{n,L} \ll \Lambda_{\rm QCD}$. But this condition can be taken as the *definition* of a crypto-exotic state that contains important light-quark degrees of freedom.

In view of this observation and the putative exotic nature of the $X(3872)$, we suggest that measuring the mass of the $h_c(2P)$ and computing $\Delta_{2,1}(c\bar{c})$ will unambiguously reveal if the $X(3872)$ contains important light-quark degrees of freedom. This conclusion, of course, relies upon assuming that $\chi_{c2}(2P)$, the new $\chi_{c0}(2P)$ candidate, and the undiscovered $h_c(2P)$ are pure $c\bar{c}$ states; at minimum, one can conclude that a substantial violation of the relation $\Delta_{n,L} \ll \Lambda_{\rm QCD}$ in heavy quarkonium points to *at least one* of the states containing a significant non-$Q\bar{Q}$ component. Indeed, even an effect giving $\Delta_{n,L} > O(\Lambda_{\rm QCD}^3/m_Q^2)$, the heavy-quark spin-symmetry expectation, will warrant close attention.

The same comments hold for the D-wave $c\bar{c}$ states: the $\psi(3770)$ and $\psi(3823)$ are believed to be 1^3D_1 and 1^3D_2, respectively; the observation of a spin-0 η_c 1^1D_1 or spin-3 ψ 1^3D_3 will allow a precise prediction of the mass of the other, while a measurement of both will allow one to test for a non-$c\bar{c}$ component in this multiplet.

The prospects for experimentally measuring the $2P$ charmonium ultrafine splitting are encouraging. For example, BESIII observed the $Z_c^0(4020)$ in the reaction $e^+e^- \to \pi\pi h_c(1P)$ [36]. A similar effort could yield a signal for $\pi\pi h_c(2P)$, with the $h_c(2P)$ being detected in its $D\bar{D}^*$ decay mode [37].

Alternatively, attempting to find $\chi_{cJ}(2P)$ with $J > 0$ in the recoil mass products X of $e^+e^- \to J/\psi X$ is not expected to be profitable, since this channel has been seen to be dominated by $\eta_c(1S)$, $\eta_c(2S)$, $\chi_{c0}(1P)$, and the $X(3940)$, with little evidence for $\chi_{c1}(1P)$, $\chi_{c2}(1P)$, or any of the expected $\chi_{cJ}(2P)$ [38].

Examining the process $B \to K\omega J/\psi$ should shed light on the enigmatic $X(3915)$, which likely plays a role in the charmonium $2P$ spectrum. Finally, collecting sufficient data in $B \to KD\bar{D}^*$ should permit observation of the $h_c(2P)$ and the $\chi_{c1}(2P)$ [39].

VII. CONCLUSIONS

We have argued that the splitting $\Delta_{n,L}$ defined in Eq. (7) is robustly "ultrafine" in the absence of explicit long-distance light-quark degrees of freedom, and therefore can serve as an unambiguous test of the "coupled-channel exoticity" of quarkonium states. Prospects for applying this test in the charmonium $2P$ sector appear to be good.

It is interesting to speculate on the applicability of this idea to quarkonium hybrid states. The chromomagnetic matrix element of Sec. V, which gives the direct quark-

antiquark spin coupling $\mathbf{S}_f \cdot \mathbf{S}_{\bar{f}}$, depends upon the gluonic state of the heavy-quark meson [which is explicit in Eq. (13) and is implicit in the Eichten-Feinberg formalism]. Naively, the ultrafine splitting could be large in states with substantial hybrid components. However, the arguments of Sec. V lead us to believe that this will not be the case, because the addition of valence gluonic degrees of freedom should not reverse the rapid decorrelation of the chromomagnetic fields. This expectation can be checked directly with lattice measurements of $V_{\rm hyp}^{(\rm hybrid)}$ [Eq. (13)], which should be readily achievable with present capabilities. The application of this result to splittings in complete hybrid multiplets will be complicated by the addition of many new spin-dependent operators in the heavy-quark expansion, but should, nevertheless, also be of interest.

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