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Search for $\wedge_{\_}\{c\}^{\wedge}\{+\} \rightarrow \phi p \pi^{\wedge}\{0\}$ and branching fraction measurement of $\wedge_{-}\{c\} \wedge\{+\} \rightarrow K^{\wedge}\{-\} \pi^{\wedge}\{+\} p \pi^{\wedge}\{0\}$<br>B. Pal et al. (Belle Collaboration)

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## Search for $\Lambda_{c}^{+} \rightarrow \phi p \pi^{0}$ and branching fraction measurement of $\Lambda_{c}^{+} \rightarrow K^{-} \pi^{+} p \pi^{0}$

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We have searched for the Cabibbo-suppressed decay $\Lambda_{c}^{+} \rightarrow \phi p \pi^{0}$ in $e^{+} e^{-}$collisions using a data sample corresponding to an integrated luminosity of $915 \mathrm{fb}^{-1}$. The data were collected by the Belle experiment at the KEKB $e^{+} e^{-}$asymmetric-energy collider running at or near the $\Upsilon(4 S)$ and $\Upsilon(5 S)$ resonances. No significant signal is observed, and we set an upper limit on the branching fraction of $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow \phi p \pi^{0}\right)<15.3 \times 10^{-5}$ at $90 \%$ confidence level. The contribution of nonresonant $\Lambda_{c}^{+} \rightarrow K^{+} K^{-} p \pi^{0}$ decays is found to be consistent with zero, and the corresponding upper limit on its branching fraction is set to be $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow K^{+} K^{-} p \pi^{0}\right)_{N R}<6.3 \times 10^{-5}$ at $90 \%$ confidence level. We also search for an intermediate hidden-strangeness pentaquark decay $P_{s}^{+} \rightarrow \phi p$. We see no evidence for this intermediate decay and set an upper limit on the product branching fraction of $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow\right.$ $\left.P_{s}^{+} \pi^{0}\right) \times \mathcal{B}\left(P_{s}^{+} \rightarrow \phi p\right)<8.3 \times 10^{-5}$ at $90 \%$ confidence level. Finally, we measure the branching fraction for the Cabibbo-favored decay $\Lambda_{c}^{+} \rightarrow K^{-} \pi^{+} p \pi^{0}$; the result is $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow K^{-} \pi^{+} p \pi^{0}\right)=$ $(4.42 \pm 0.05$ (stat.) $\pm 0.12$ (syst.) $\pm 0.16$ (norm.) $) \%$, which is the most precise measurement to date.

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The story of exotic hadron spectroscopy begins with the discovery of the $X(3872)$ by the Belle collaboration in 2003 [1]. Since then, many exotic $X Y Z$ states have been reported by Belle and other experiments [2]. Recent observations of two hidden-charm pentaquark states $P_{c}^{+}(4380)$ and $P_{c}^{+}(4450)$ by the LHCb collaboration in the $J / \psi p$ invariant mass spectrum of the $\Lambda_{b}^{0} \rightarrow J / \psi p K^{-}$ process [3] raises the question of whether a hiddenstrangeness pentaquark $P_{s}^{+}$, where the $c \bar{c}$ pair in $P_{c}^{+}$ is replaced by an $s \bar{s}$ pair, exists [4-6]. The strangeflavor analogue of the $P_{c}^{+}$discovery channel is the decay $\Lambda_{c}^{+} \rightarrow \phi p \pi^{0}[5,6]$, shown in Fig. 1 (a) [7]. The detection of a hidden-strangeness pentaquark could be possible through the $\phi p$ invariant mass spectrum within this channel [see Fig. 1 (b)] if the underlying mechanism creating the $P_{c}^{+}$states also holds for $P_{s}^{+}$, independent of the flavor [6], and only if the mass of $P_{s}^{+}$is less than $M_{\Lambda_{c}^{+}}-M_{\pi^{0}}$. In an analogous $s \bar{s}$ process of $\phi$ photoproduction ( $\gamma p \rightarrow \phi p$ ), a forward-angle bump structure at $\sqrt{s} \approx 2.0 \mathrm{GeV}$ has been observed by the LEPS [8] and CLAS collaborations [9]. However, this structure appears only at the most forward angles, which is not expected for the decay of a resonance [10].


FIG. 1. Feynman diagram for the decay (a) $\Lambda_{c}^{+} \rightarrow \phi p \pi^{0}$ and (b) $\Lambda_{c}^{+} \rightarrow P_{s}^{+} \pi^{0}$.

Previously, the decay $\Lambda_{c}^{+} \rightarrow \phi p \pi^{0}$ has not been studied by any experiment. In this paper, we report a search for this decay using a data set corresponding to an integrated luminosity of $915 \mathrm{fb}^{-1}$ collected with the Belle detector [11] recorded at or near the $\Upsilon(4 S)$ and $\Upsilon(5 S)$ resonances at the KEKB asymmetric-energy $e^{+} e^{-}$ (3.5 on 8.0 GeV ) collider [12]. In addition, we search for the nonresonant decay $\Lambda_{c}^{+} \rightarrow K^{+} K^{-} p \pi^{0}$ and measure the branching fraction of the Cabibbo-favored decay $\Lambda_{c}^{+} \rightarrow K^{-} \pi^{+} p \pi^{0}$.

The Belle detector is described in detail elsewhere [11]. To calculate the detector acceptance and reconstruction efficiencies and to study background, we use Monte Carlo (MC) simulated events. The MC events are generated uniformly in phase space with EvtGen [13] and JetSet [14]; the detector response is modeled using Geant3 [15]. Final-state radiation is taken into account using the Рнотоs [16] package.

The reconstruction of $\Lambda_{c}^{+} \rightarrow \phi p \pi^{0}$ (and $\Lambda_{c}^{+} \rightarrow$ $K^{-} \pi^{+} p \pi^{0}$ ) decays proceeds by first reconstructing $\pi^{0} \rightarrow$ $\gamma \gamma$ candidates. An electromagnetic calorimeter (ECL) cluster not matched to any track is identified as a photon candidate. Such candidates are required to have an energy greater than 50 MeV in the barrel region and 100 MeV in the endcap regions, where the barrel re-
gion covers the polar angle range $32^{\circ}<\theta<130^{\circ}$, and the endcap regions cover the ranges $12^{\circ}<\theta<32^{\circ}$ and $130^{\circ}<\theta<157^{\circ}$. To reject showers produced by neutral hadrons, the photon energy deposited in the $3 \times 3$ array of ECL crystals centered on the crystal with the highest energy must exceed $80 \%$ of the energy deposited in the corresponding $5 \times 5$ array of crystals. We require that the $\gamma \gamma$ invariant mass be within $0.020 \mathrm{GeV} / c^{2}$ (about $3.5 \sigma$ in resolution) of the known $\pi^{0}$ mass [17]. To improve the $\pi^{0}$ momentum resolution, we perform a mass-constrained fit and require that the resulting $\chi^{2}$ be less than 30 . In addition, the momentum of the $\pi^{0}$ candidates in the center-of-mass (CM) frame is required to be higher than 0.30 $\mathrm{GeV} / c$.

We subsequently combine $\pi^{0}$ candidates with three charged tracks. Such tracks are identified using requirements on the distance of closest approach with respect to the interaction point along the $z$ axis (antiparallel to the $e^{+}$beam) of $|d z|<1.0 \mathrm{~cm}$ and in the transverse plane of $d r<0.1 \mathrm{~cm}$. In addition, charged tracks are required to have a minimum number of hits in the vertex detector ( $>1$ in both the $z$ and transverse directions). Furthermore, information obtained from the central drift chamber, the time-of-flight scintillation counters and the aerogel threshold Cherenkov counters is combined to form a likelihood $\mathcal{L}$ for hadron identification. A charged track with the likelihood ratios of $\mathcal{L}_{K} /\left(\mathcal{L}_{\pi}+\mathcal{L}_{K}\right)>0.9$ and $\mathcal{L}_{K} /\left(\mathcal{L}_{p}+\mathcal{L}_{K}\right)>0.6 ; \mathcal{L}_{K} /\left(\mathcal{L}_{\pi}+\mathcal{L}_{K}\right)<0.6$ and $\mathcal{L}_{\pi} /\left(\mathcal{L}_{p}+\mathcal{L}_{\pi}\right)>0.6 ;$ and $\mathcal{L}_{p} /\left(\mathcal{L}_{p}+\mathcal{L}_{K}\right)>0.9$ and $\mathcal{L}_{p} /\left(\mathcal{L}_{p}+\mathcal{L}_{\pi}\right)>0.9$ are regarded as kaon, pion and proton, respectively. The efficiencies of these requirements for kaons, pions, and protons are $77 \%, 97 \%$, and $75 \%$, respectively. The probabilities for a kaon, pion, or proton to be misidentified are $\mathcal{P}(K \rightarrow \pi) \approx 10 \%, \mathcal{P}(K \rightarrow$ $p) \approx 1 \% ; \mathcal{P}(\pi \rightarrow K) \approx 1 \%, \mathcal{P}(\pi \rightarrow p)<1 \%$; and $\mathcal{P}(p \rightarrow K) \approx 7 \%, \mathcal{P}(p \rightarrow \pi) \approx 1 \%$. Candidate $\phi$ mesons are formed from two oppositely charged tracks that have been identified as kaons. We accept events in the wide $K^{+} K^{-}$mass range $m\left(K^{+} K^{-}\right) \in(0.99,1.13) \mathrm{GeV} / c^{2}$. To suppress combinatorial background, especially from $B$ meson decays, we require that the scaled momentum $\left(x_{p}=P c / \sqrt{E_{\mathrm{CM}}^{2} / 4-M^{2} c^{4}}\right)$ be greater than 0.45 , where $E_{\mathrm{CM}}$ is the total CM energy, and $P$ and $M$ are the momentum and invariant mass of the $\Lambda_{c}^{+}$candidates. A vertex fit is performed to the charged tracks to form a $\Lambda_{c}^{+}$vertex, and we require that the $\chi^{2}$ from the fit be less than 50. The decay $\Lambda_{c}^{+} \rightarrow \Sigma^{+} \phi$ has the same final state as the signal decay and is Cabibbo-favored. To avoid contamination from this decay, we reject candidates in which the $p \pi^{0}$ system has an invariant mass within $0.010 \mathrm{GeV} / c^{2}$ of the known $\Sigma^{+}$mass [17]. We extract the $\Lambda_{c}^{+}$yield in a signal region that spans $2.5 \sigma$ in resolution around the $\Lambda_{c}^{+}$mass [17]; this range corresponds to $0.015 \mathrm{GeV} / c^{2}$ for $\Lambda_{c} \rightarrow K^{-} \pi^{+} p \pi^{0}$ and approximately $0.010 \mathrm{GeV} / c^{2}$ for the other decays studied.

After applying all these selection criteria, about $16 \%$ of events in the signal region have multiple $\Lambda_{c}^{+}$candidates. For these events, we retain the candidate having
the smallest sum of $\chi^{2}$ values obtained from the $\pi^{0}$ massconstrained fit and the $\Lambda_{c}^{+}$vertex fit. According to MC simulation, this criterion selects the correct $\Lambda_{c}^{+}$candidate in $72 \%$ of multiple-candidate events.

In order to extract the signal yield, we perform a twodimensional (2D) unbinned extended maximum likelihood fit to the variables $m\left(K^{+} K^{-} p \pi^{0}\right)$ and $m\left(K^{+} K^{-}\right)$. Our likelihood function accounts for three components: $\phi p \pi^{0}$ signal, $K^{+} K^{-} p \pi^{0}$ nonresonant events, and combinatorial background. The likelihood function is defined as

$$
\begin{equation*}
e^{-\sum_{j} Y_{j}} \prod_{i}^{N}\left(\sum_{j} Y_{j} \mathcal{P}_{j}\left[m^{i}\left(K^{+} K^{-} p \pi^{0}\right), m^{i}\left(K^{+} K^{-}\right)\right]\right) \tag{1}
\end{equation*}
$$

where $N$ is the total number of events, $\mathcal{P}_{j}\left[m^{i}\left(K^{+} K^{-} p \pi^{0}\right), m^{i}\left(K^{+} K^{-}\right)\right] \quad$ is the probability density function (PDF) of signal or background component $j$ for event $i$, and $j$ runs over all signal and background components. The parameter $Y_{j}$ is the yield of component $j$. The $m\left(K^{+} K^{-} p \pi^{0}\right)$ for signal and nonresonant contributions are modeled with the sum of two Crystal Ball (CB) functions [18] having a common mean, whereas for the combinatorial background, a second-order Chebyshev polynomial is used. The peak positions and resolutions of the CB functions are adjusted according to data-MC differences observed in the high statistics sample of $\Lambda_{c}^{+} \rightarrow K^{-} \pi^{+} p \pi^{0}$ decays. The $m\left(K^{+} K^{-}\right)$of signal is modeled with a relativistic Breit-Wigner function convolved with a Gaussian resolution function $(\mathrm{RBW} \otimes \mathrm{G})$, with the mass and width of the resonance $\phi$ fixed to their nominal values [17]. The width of the Gaussian resolution function is fixed to the value obtained from the MC simulation. The $m\left(K^{+} K^{-}\right)$of nonresonant background is modeled with a one-dimensional nonparametric PDF [19]. The $m\left(K^{+} K^{-}\right)$of combinatorial background is modeled with the sum of a third-order Chebyshev polynomial and the same $\mathrm{RBW} \otimes \mathrm{G}$ function as used to model the signal. The floated parameters are the component yields $Y_{j}$ and, for the combinatorial background, the coefficients of the Chebyshev polynomials and the fraction of the RBW. All other parameters are fixed in the fit to the values obtained from the MC simulation. Projections of the fit result are shown in Fig. 2. From the fit, we extract $148.4 \pm 61.8$ signal events, $75.9 \pm 84.8$ nonresonant events, and $7158.4 \pm 36.4$ combinatorial background events in the $\Lambda_{c}^{+}$signal region. The statistical significance is evaluated as $\sqrt{-2 \ln \left(\mathcal{L}_{0} / \mathcal{L}_{\text {max }}\right)}$, where $\mathcal{L}_{0}$ is the likelihood value when the signal yield is fixed to zero, and $\mathcal{L}_{\text {max }}$ is the nominal likelihood value. The statistical significances are found to be 2.4 and 1.0 standard deviations for $\Lambda_{c}^{+} \rightarrow \phi p \pi^{0}$ and nonresonant $\Lambda_{c}^{+} \rightarrow K^{+} K^{-} p \pi^{0}$ decays, respectively.

We use the well-established decay $\Lambda_{c}^{+} \rightarrow p K^{-} \pi^{+}[17]$ as the normalization channel for the branching fraction measurements. The track, particle identification, and vertex selection criteria are similar to those used for the
signal decays. If there are multiple candidates present in an event, we select the candidate having the smallest value of $\chi^{2}$ from the $\Lambda_{c}^{+}$vertex fit. The resulting invariant mass distribution of the $p K^{-} \pi^{+}$candidates is shown in Fig. 3. The signal is modeled with the sum of three Gaussian functions, and the combinatorial background is modeled with a linear function. There are $1468435 \pm 4816$ signal candidates and $567855 \pm 815$ background candidates in the $\Lambda_{c}^{+}$signal region.

The ratio of branching fractions is calculated as

$$
\begin{equation*}
\frac{\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow \text { final state }\right)}{\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow p K^{-} \pi^{+}\right)}=\frac{Y_{\mathrm{Sig}} / \varepsilon_{\mathrm{Sig}}}{Y_{\text {Norm }} / \varepsilon_{\text {Norm }}} \tag{2}
\end{equation*}
$$

where $Y$ represents the observed yield in the signal region of the decay of interest and $\varepsilon$ corresponds to the reconstruction efficiency as obtained from the MC simulation. For the $\phi p \pi^{0}$ final state, we include $\mathcal{B}\left(\phi \rightarrow K^{+} K^{-}\right)=$ ( $48.9 \pm 0.5$ ) \% [17] in $\varepsilon_{\text {sig }}$ of Eq. (2). The reconstruction efficiencies are $(2.165 \pm 0.007) \%,(2.291 \pm 0.008) \%$ and $(16.564 \pm 0.023) \%$ for $\phi p \pi^{0}$, nonresonant $K^{+} K^{-} p \pi^{0}$, and $p K^{-} \pi^{+}$final states, respectively, where the errors are due to MC statistics only. The ratio $\varepsilon_{\text {Sig }} / \varepsilon_{\text {Norm }}$ is corrected by a factor $1.028 \pm 0.018$ to account for small differences in particle identification efficiencies between data and simulation. This correction is estimated from a sample of $D^{*+} \rightarrow D^{0}\left(\rightarrow K^{-} \pi^{+}\right) \pi^{+}$decays. For the $\phi p \pi^{0}$ final state, the ratio is

$$
\frac{\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow \phi p \pi^{0}\right)}{\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow p K^{-} \pi^{+}\right)}=\left(1.538 \pm 0.641_{-0.100}^{+0.077}\right) \times 10^{-3}
$$

Whenever two or more uncertainties are quoted, the first is the statistical and the second systematic. Using $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow p K^{-} \pi^{+}\right)=(6.46 \pm 0.24) \% \quad[20]$, we obtain

$$
\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow \phi p \pi^{0}\right)=\left(9.94 \pm 4.14_{-0.65}^{+0.50} \pm 0.37\right) \times 10^{-3}
$$

where the third uncertainty is that due to the branching fraction $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow p K^{-} \pi^{+}\right)$.

Since the significances are below 3.0 standard deviations both for $\phi p \pi^{0}$ signal and $K^{+} K^{-} p \pi^{0}$ nonresonant decays, we set upper limits on their branching fractions at $90 \%$ confidence level (C.L.) using a Bayesian approach. The limit is obtained by integrating the likelihood function from zero to infinity; the value that corresponds to $90 \%$ of this total area is taken as the $90 \%$ C.L. upper limit. We include the systematic uncertainty in the calculation by convolving the likelihood distribution with a Gaussian function whose width is set equal to the total systematic uncertainty. The results are

$$
\begin{aligned}
\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow \phi p \pi^{0}\right) & <15.3 \times 10^{-5} \\
\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow K^{+} K^{-} p \pi^{0}\right)_{\mathrm{NR}} & <6.3 \times 10^{-5}
\end{aligned}
$$

which are the first limits on these branching fractions.
To search for a putative $P_{s}^{+} \rightarrow \phi p$ decay, we select $\Lambda_{c}^{+} \rightarrow K^{+} K^{-} p \pi^{0}$ candidates in which $m\left(K^{+} K^{-}\right)$is within $0.020 \mathrm{GeV} / c^{2}$ of the $\phi$ meson mass [17] and plot


FIG. 2. Projections of the 2D fit: (a) $m\left(K^{+} K^{-} p \pi^{0}\right)$ and (b) $m\left(K^{+} K^{-}\right)$. The points with the error bars are the data, and the (red) dotted, (green) dashed and (brown) dot-dashed curves represent the combinatorial, signal and nonresonant candidates, respectively, and (blue) solid curves represent the total PDF. The solid curve in (b) completely overlaps the curve for the combinatorial background.


FIG. 3. Fit to the invariant mass distribution of $p K^{-} \pi^{+}$. The points with the error bars are the data, the (red) dotted and (green) dashed curves represent the combinatorial and signal candidates, respectively, and (blue) curve represents the total PDF.
the background-subtracted $m(\phi p)$ distribution (Fig. 4). This distribution is obtained by performing 2D fits as discussed above in bins of $m(\phi p)$. The data shows no clear evidence for a $P_{s}^{+}$state. We set an upper limit on the product branching fraction $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow P_{s}^{+} \pi^{0}\right) \times \mathcal{B}\left(P_{s}^{+} \rightarrow\right.$ $\phi p)$ by fitting the distribution of Fig. 4 to the sum of a RBW function and a phase space distribution determined from a sample of simulated $\Lambda_{c}^{+} \rightarrow \phi p \pi^{0}$ decays. We obtain $77.6 \pm 28.1 P_{s}^{+}$events from the fit, which gives an upper limit of

$$
\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow P_{s}^{+} \pi^{0}\right) \times \mathcal{B}\left(P_{s}^{+} \rightarrow \phi p\right)<8.3 \times 10^{-5}
$$

at $90 \%$ C.L. This limit is calculated using the same procedure as that used for our limit on $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow \phi p \pi^{0}\right)$. The
systematic uncertainties for the two cases are essentially identical except for that due to the size of the MC sample used to calculate the reconstruction efficiency. The efficiency used here $[\varepsilon=(2.438 \pm 0.026) \%$ ] corresponds to the fitted values $M_{P_{s}^{+}}=(2.025 \pm 0.005) \mathrm{GeV} / c^{2}$ and $\Gamma_{P_{s}^{+}}=(0.022 \pm 0.012) \mathrm{GeV}$.


FIG. 4. The background-subtracted distribution of $m(\phi p)$ in the $\phi p \pi^{0}$ final state. The points with error bars are data, and the (blue) solid line shows the total PDF. The (red) dotted curve shows the fitted phase space component (which has fluctuated negative).

For the $\Lambda_{c}^{+} \rightarrow K^{-} \pi^{+} p \pi^{0}$ sample, the mass distribution is plotted in Fig. 5. We fit this distribution to obtain the signal yield. We model the signal with a sum of two CB functions having a common mean and the combinatorial background with a linear function. We find $242039 \pm 2342$ signal candidates and $472729 \pm 467$ background candidates in the $\Lambda_{c}^{+}$signal region. The corresponding signal efficiency is $(3.988 \pm 0.009) \%$, obtained
from MC simulation. We measure the ratio of branching fractions

$$
\frac{\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow K^{-} \pi^{+} p \pi^{0}\right)}{\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow K^{-} \pi^{+} p\right)}=(0.685 \pm 0.007 \pm 0.018)
$$

which results in a branching fraction

$$
\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow K^{-} \pi^{+} p \pi^{0}\right)=(4.42 \pm 0.05 \pm 0.12 \pm 0.16) \%
$$

This is the most precise measurement of $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow\right.$ $K^{-} \pi^{+} p \pi^{0}$ ) to date and is consistent with the recently measured value $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow K^{-} \pi^{+} p \pi^{0}\right)=(4.53 \pm 0.23 \pm$ $0.30) \%$ by the BESIII collaboration [21].


FIG. 5. Fit to the invariant mass distribution of $m\left(K^{-} \pi^{+} p \pi^{0}\right)$. The points with the error bars are the data, the (red) dotted and (green) dashed curves represent the combinatorial and signal candidates, respectively, and (blue) curve represents the total PDF. The $\chi^{2} /$ (number of bins) of the fit is 1.43 , which indicate that the fit gives a good description of the data.

The systematic uncertainties on all branching fractions are listed in Table I. The uncertainties due to fixed parameters in the PDF shape are estimated by varying the parameters individually according to their statistical uncertainties. For each variation, the branching fraction is recalculated, and the difference with the nominal value is taken as the systematic uncertainty associated with that parameter. In order to determine the systematic uncertainty due to the $m\left(K^{+} K^{-}\right)$PDF of nonresonant $K^{+} K^{-} p \pi^{0}$, we replace the nonparametric PDF by a fourth-order polynomial and refit the data. For the $\phi p \pi^{0}$ final state, we also try including a separate PDF for an $f_{0}(980)$ intermediate state. The differences in the fit results are included as systematic uncertainties. We add all uncertainties in quadrature to obtain the overall uncertainty due to PDF parametrization. The uncertainties due to errors in the calibration factors used to account for small data-MC differences in the signal PDF are evaluated separately but in a similar manner. A systematic uncertainty of $-1.2 \%$ is assigned to account for changes associated with the choice of the $m\left(K^{+} K^{-}\right)$range in
$\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow \phi p \pi^{0}\right)$. A $2.1 \%$ systematic uncertainty is assigned due to the best candidate selection. This is evaluated by analyzing the decay channel $\Lambda_{c}^{+} \rightarrow \Sigma^{+} \phi$, which has much higher purity than the signal channels analyzed. We determine this by applying an alternative best candidate selection, i.e., the deviations of the candidate $\phi$ and $\Sigma^{+}$masses from their nominal values. The difference in the branching fraction due to the two methods of the best candidate selection is taken as the systematic uncertainty. We assign a $1.5 \%$ systematic uncertainty due to $\pi^{0}$ reconstruction; this is determined from a study of $\tau^{-} \rightarrow \pi^{-} \pi^{0} \nu_{\tau}$ decays. Since the branching fractions are measured with respect to the normalization channel $\Lambda_{c}^{+} \rightarrow p K^{-} \pi^{+}$, which has an identical number of charged tracks, the systematic uncertainty due to differences in tracking performance between signal and normalization modes is negligible. There is a $1.8 \%$ systematic uncertainty assigned for the particle identification efficiencies in the $\phi p \pi^{0}$ and nonresonant $K^{+} K^{-} p \pi^{0}$ final states relative to the $p K^{-} \pi^{+}$normalization channel. The uncertainty in acceptance due to possible resonance substructure in the decay is found to be negligible. The total of the above systematic uncertainties is calculated as their sum in quadrature. In addition, there is a $3.7 \%$ uncertainty due to the branching fraction of the normalization mode. As this large uncertainty does not arise from our analysis and will decrease with future measurements of $\Lambda_{c}^{+} \rightarrow p K^{-} \pi^{+}$, we quote it separately.
In summary, we have searched for the decays $\Lambda_{c}^{+} \rightarrow$ $\phi p \pi^{0}$ and nonresonant $\Lambda_{c}^{+} \rightarrow K^{+} K^{-} p \pi^{0}$. No significant signal is observed for either decay mode and we set $90 \%$ C.L. upper limits on their branching fractions, which are $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow \phi p \pi^{0}\right)<1.5 \times 10^{-4}$ and $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow K^{+} K^{-} p \pi^{0}\right)_{\mathrm{NR}}<0.6 \times 10^{-4}$. We see no evidence for a hidden-strangeness pentaquark decay $P_{s}^{+} \rightarrow \phi p$ and set an upper limit on the product branching fraction of $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow P_{s}^{+} \pi^{0}\right) \times \mathcal{B}\left(P_{s}^{+} \rightarrow \phi p\right)<8.3 \times 10^{-5}$ at $90 \%$ C.L. This limit is a factor of six higher than the product branching fraction measured by LHCb for an analogous hidden-charm pentaquark state: $\mathcal{B}\left(\Lambda_{b}^{0} \rightarrow\right.$ $\left.P_{c}(4450)^{+} K^{-}\right) \times \mathcal{B}\left(P_{c}(4450)^{+} \rightarrow J / \psi p\right)=(1.3 \pm 0.4) \times$ $10^{-5}[3]$. We also measure $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow K^{-} \pi^{+} p \pi^{0}\right)=$ ( $4.42 \pm 0.05 \pm 0.12 \pm 0.16) \%$. This is the world's most precise measurement of this branching fraction.

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TABLE I. Systematic uncertainties (\%) on $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow \phi p \pi^{0}\right), \mathcal{B}\left(\Lambda_{c}^{+} \rightarrow K^{+} K^{-} p \pi^{0}\right)_{\mathrm{NR}}$, and $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow K^{-} \pi^{+} p \pi^{0}\right)$.

| Source | $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow \phi p \pi^{0}\right)$ | $\mathcal{B}\left(\Lambda_{c}^{+} \rightarrow K^{+} K^{-} p \pi^{0}\right)_{\mathrm{NR}} \mathcal{B}\left(\Lambda_{c}^{+} \rightarrow K^{-} \pi^{+} p \pi^{0}\right)$ |  |
| :--- | :---: | :---: | :---: |
| PDF parametrization | ${ }_{-1.9}^{+1.0}$ | ${ }^{+1.9}$ | - |
| Calibration factor | ${ }_{-5.8}^{+3.2}$ | ${ }_{-1.5}^{+2.8}$ | - |
| Choice of $m\left(K^{+} K^{-}\right)$range | ${ }_{-1.2}^{+0.0}$ | - | - |
| Best candidate selection | 2.1 | 2.1 | 2.1 |
| MC sample size | 0.4 | 0.4 | 0.3 |
| $\pi^{0}$ reconstruction | 1.5 | 1.5 | 1.5 |
| Particle identification | 1.8 | 1.8 | - |
| $\mathcal{B}\left(\phi \rightarrow K^{+} K^{-}\right)$ | 1.0 | - | - |
| Total (without $\left.\mathcal{B}_{\text {Norm }}\right)$ | ${ }_{-6.5}^{+5.0}$ | ${ }_{-3.8}^{+4.6}$ | 2.6 |
| $\mathcal{B}_{\text {Norm }}$ | 3.7 | 3.7 | 3.7 |

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