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Tyler Corbett, O. J. P. Éboli, and M. C. Gonzalez-Garcia

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Unitarity Constraints on Dimension-six Operators II: Including Fermionic Operators

Tyler Corbett*
ARC Centre of Excellence for Particle Physics at the Terascale, School of Physics, The University of Melbourne, Victoria, Australia

O. J. P. Éboli†
Instituto de Física, Universidade de São Paulo, São Paulo – SP, Brazil

M. C. Gonzalez–Garcia‡
Institució Catalana de Recerca i Estudis Avançats (ICREA), Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona, 647 Diagonal, E-08028 Barcelona, Spain and C.N. Yang Institute for Theoretical Physics, SUNY at Stony Brook, Stony Brook, NY 11794-3840, USA

We analyze the scattering of fermions, Higgs and electroweak gauge bosons in order to obtain the partial–wave unitarity bounds on dimension–six effective operators, including those involving fermions. We also quantify whether, at the LHC energies, the dimension–six operators lead to unitarity violation after taking into account the presently available constraints on their Wilson coefficients. Our results show that for most dimension–six operators relevant for the LHC physics there is no unitarity violation at the LHC energies, and consequently, there is no need for the introduction of form factors in the experimental and phenomenological analyses, making them model independent. We also identify two operators for which unitarity violation is still an issue at the LHC Run–II.

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I. INTRODUCTION

The discovery of a scalar state with properties in agreement with those of the Standard Model (SM) Higgs boson at the Large Hadron Collider (LHC) set the final stone in establishing the validity of the model. Presently there are no high energy data that are in significant conflict with the SM predictions. In this framework, with no other new state yet observed, one can parametrize generic departures from the SM by an effective Lagrangian constructed with the SM states and respecting the SM symmetries, abandoning only the renormalizabilty condition which constrains the dimension of the operators to be of dimension four or less. In particular the established existence of a particle resembling the SM Higgs boson implies that the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry can be realized linearly in the effective theory, an assumption under which we will work in this paper. In this framework the first departures from the SM at the LHC which also respect its global symmetries appear at dimension–six [1, 2].

When using such an effective field theory (EFT) to quantify possible deviations from the SM predictions, one must be sure of its validity in the energy range probed by the experiments. As is well known, the higher–dimensional operators included in the effective Lagrangian can lead to perturbative partial–wave unitarity violation at high energies, signaling a maximum value of the center–of–mass energy for its applicability. Unitarity thus imposes a condition on the energy range of validity of the EFT additional to that defined by generic perturbativity which fails when higher-dimension operators become as important as lower-dimension ones. In this respect unitarity bounds, such as the ones we will derive in this work, represent a necessary but not sufficient condition on the validity of the EFT. If there is unitarity violation we must either modify the EFT, *e.g.* by adding form factors that take into account higher order terms [3] or restrict its use to analyze data in phase space regions where unitarity is not violated. Ultimately, another possibility is to replace the EFT by an ultraviolet (UV) complete model.

In Ref. [4] we presented a general study of unitarity violation in electroweak and/or Higgs boson pair production in boson and/or fermion collisions associated with the presence of dimension–six operators involving bosons, concentrating on those which are blind to low energy bounds. The rationale behind this choice was that not–blind operators were

*Electronic address: corbett.t.s@gmail.com
†Electronic address: eboli@fma.if.usp.br
‡Electronic address: concha@insti.physics.sunysb.edu

expected to be too constrained by electroweak precision data to be relevant. In this work we revisit this assumption and extend the study to introduce the effects of these operators sensitive to low energy observables, and in particular those involving the coupling of fermions to electroweak bosons. This is timely since the LHC has already started to be able to probe triple electroweak gauge boson couplings with a precision comparable to, or even better than, LEP [5]. With such a precision the LHC experiments are already sensitive to deviations of the couplings of electroweak gauge bosons to fermions that are of the order of the limits obtained using the electroweak precision data (EWPD) [6, 7].

In this paper we evaluate the unitarity bounds on bosonic and fermionic dimension–six operators from boson pair production amplitudes. As in Ref. [4] we take into account all coupled channels in both elastic and inelastic scattering and all possible helicity amplitudes. Moreover, we consistently work at fixed order in the effective Lagrangian expansion[1]. We also study the variation of the constraints under the assumption of the flavour dependence of the fermionic operators.

With these results we can address whether, at the LHC energies, the dimension–six operators can indeed lead to unitarity violation after taking into account the presently available constraints on the anomalous couplings. This is accomplished by substituting the present limits of the Wilson coefficients in our partial–wave unitarity bounds to extract the center–of–mass energy at which perturbative unitarity is violated. In order to do so we consistently derive the EWPD constraints on the coefficients of the non-blind operators. Our results show that for all dimension–six operators relevant for the LHC physics, except for just two ($\mathcal{O}_{\Phi,2}$ and $\mathcal{O}^{(1)}_{\Phi d}$), there is no unitarity violation at the LHC energies, and consequently, we can safely neglect the introduction of form factors in the experimental and phenomenological analyses, making them cleaner and free of ad-hoc parameters. In the case of the operator $\mathcal{O}_{\Phi,2}$ there is no unitarity violation up to subprocess center–of–mass energies of the order of 2.1 TeV, meaning that we have to be more careful in analyzing the high energy tail of processes where the Higgs boson can participate. On the other hand for $\mathcal{O}^{(1)}_{\Phi d}$ perturbative unitarity holds for diboson (VV) subprocess center–of–mass energy less than 3.5 TeV.

This paper is organized as follows. Section II contains the dimension–six operators relevant for our analyses, while we present in Section III the unitarity bounds for bosonic and fermionic operators (listing the unitarity violating amplitudes in Appendix A). We discuss the consequences of these results in Section IV taking into account the existing constraints on the Wilson coefficients of the dimension–six operators. We present the details of our fit to the EWPD in Appendix B, while for completeness we summarize in Appendix C the unitarity constraints on fermion dipole operators.

II. EFFECTIVE LAGRANGIAN

We parametrize deviations from the Standard Model (SM) in terms of higher dimension operators as

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum_{n>4,j} \frac{f_{n,j}}{\Lambda^{n-4}} \mathcal{O}_{n,j} \; . \tag{1}$$

The first operators that impact the LHC physics are of $n = 6$, or dimension–six. Their basis contains 59 independent operators, up to flavor and hermitian conjugation, where we impose the SM gauge symmetry, as well as baryon and lepton number conservation [1, 2]. Of those, 49 can be chosen to be C and P conserving and do not involve gluons. Since the S-matrix elements are unchanged by the use of the equations of motion (EOM), there is a freedom in the choice of basis [16–19]. Here we work in that of Hagiwara, Ishihara, Szalapski, and Zeppenfeld [20, 21].

A. Bosonic Operators

Assuming C and P conservation there are nine dimension–six operators in our basis involving only bosons that take part at tree level in two–to–two scattering of gauge and Higgs bosons after we employ the EOM to eliminate redundant operators [22]. We group these operators according to their field content. In the first class there is just one operator that contains exclusively gauge bosons.

$$\mathcal{O}_{WWW} = {\rm Tr}[\widehat{W}_\mu^\nu \widehat{W}_\nu^\rho \widehat{W}_\rho^\mu] \; . \tag{2}$$

[1] Other studies in the literature have been performed either considering only one non–vanishing coupling at a time, and/or they did not take into account coupled channels, or they worked in the framework of effective vertices [3, 8–15]

In the next group there are six operators that include Higgs and electroweak gauge fields.

$$\begin{aligned} &\mathcal{O}_{WW} = \Phi^\dagger \widehat{W}_{\mu\nu} \widehat{W}^{\mu\nu} \Phi \ , && \mathcal{O}_{BB} = \Phi^\dagger \widehat{B}_{\mu\nu} \widehat{B}^{\mu\nu} \Phi \ , \\ &\mathcal{O}_{BW} = \Phi^\dagger \widehat{B}_{\mu\nu} \widehat{W}^{\mu\nu} \Phi \ , && \mathcal{O}_{\Phi,1} = (D_\mu \Phi)^\dagger \Phi \Phi^\dagger (D^\mu \Phi) \ , \\ &\mathcal{O}_{W} = (D_\mu \Phi)^\dagger \widehat{W}^{\mu\nu} (D_\nu \Phi) \ , && \mathcal{O}_{B} = (D_\mu \Phi)^\dagger \widehat{B}^{\mu\nu} (D_\nu \Phi) \ . \end{aligned} \tag{3}$$

The final class contains two operators expressed solely in terms of Higgs fields

$$\mathcal{O}_{\Phi,2} = \frac{1}{2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) \quad \text{and} \quad \mathcal{O}_{\Phi,3} = \frac{1}{3} (\Phi^\dagger \Phi)^3 \ . \tag{4}$$

Here Φ stands for the Higgs doublet and we have adopted the notation $\widehat{B}_{\mu\nu} \equiv i(g'/2) B_{\mu\nu}$, $\widehat{W}_{\mu\nu} \equiv i(g/2)\sigma^a W^a_{\mu\nu}$, with g and g' being the $SU(2)_L$ and $U(1)_Y$ gauge couplings respectively, and σ^a the Pauli matrices.

The dimension–six operators given in Eqs. (2)–(4) affect the scatterings $VV \to VV$ and $f\bar{f} \to VV$, where V stands for the electroweak gauge bosons or the Higgs, through modifications of triple and quartic gauge boson couplings, Higgs couplings to fermions and gauge bosons, interactions of gauge bosons with fermion pairs, and the Higgs self-couplings; see Table I. Moreover, these anomalous couplings enter in the analyses of Higgs physics, as well as, triple gauge couplings of electroweak gauge bosons and were analyzed in [4, 22–25].

B. Operators with fermions

After requiring that the dimension–six operators containing fermions conserve C, P and baryon number, we are left with 40 independent operators (barring flavour indexes) in our basis which do not involve gluon fields. We classify them in four groups. In the first group there are three dimension–six operators that modify the Yukawa couplings of the Higgs boson, and therefore do not contribute to the processes that we study at high energies. The second class possesses 25 four–fermion contact interactions that again do not take part in our analyses.

The third group includes the operators that lead to anomalous couplings of the gauge bosons with the fermions that exhibit the same Lorentz structures as the SM vertices and are relevant for our analyses. This class contains eight dimension–six operators

$$\begin{aligned} &\mathcal{O}^{(1)}_{\Phi L,ij} = \Phi^\dagger (i \overleftrightarrow{D}_\mu \Phi)(\bar{L}_i \gamma^\mu L_j), && \mathcal{O}^{(3)}_{\Phi L,ij} = \Phi^\dagger (i \overleftrightarrow{D}^a_\mu \Phi)(\bar{L}_i \gamma^\mu T_a L_j), \\ &\mathcal{O}^{(1)}_{\Phi Q,ij} = \Phi^\dagger (i \overleftrightarrow{D}_\mu \Phi)(\bar{Q}_i \gamma^\mu Q_j), && \mathcal{O}^{(3)}_{\Phi Q,ij} = \Phi^\dagger (i \overleftrightarrow{D}^a_\mu \Phi)(\bar{Q}_i \gamma^\mu T_a Q_j), \\ &\mathcal{O}^{(1)}_{\Phi e,ij} = \Phi^\dagger (i \overleftrightarrow{D}_\mu \Phi)(\bar{e}_{R_i} \gamma^\mu e_{R_j}), && \\ &\mathcal{O}^{(1)}_{\Phi u,ij} = \Phi^\dagger (i \overleftrightarrow{D}_\mu \Phi)(\bar{u}_{R_i} \gamma^\mu u_{R_j}), && \\ &\mathcal{O}^{(1)}_{\Phi d,ij} = \Phi^\dagger (i \overleftrightarrow{D}_\mu \Phi)(\bar{d}_{R_i} \gamma^\mu d_{R_j}), && \\ &\mathcal{O}^{(1)}_{\Phi ud,ij} = \tilde{\Phi}^\dagger (i \overleftrightarrow{D}_\mu \Phi)(\bar{u}_{R_i} \gamma^\mu d_{R_j} + \text{h.c.}), && \end{aligned} \tag{5}$$

where we defined $\tilde{\Phi} = i\sigma_2 \Phi^*$, $\Phi^\dagger \overleftrightarrow{D}_\mu \Phi = \Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi$ and $\Phi^\dagger \overleftrightarrow{D}^a_\mu \Phi = \Phi^\dagger T^a D_\mu \Phi - (D_\mu \Phi)^\dagger T^a \Phi$ where $T^a = \sigma^a/2$. We have also used the notation of L for the lepton doublet, Q for the quark doublet and f_R for the $SU(2)_L$ singlet fermions, where i, j are family indices.

The set of operators in Eq. (5) is redundant as two can be removed by the use of the EOM of the electroweak gauge bosons. We chose to remove from the basis the following combinations of fermionic operators [22][2]

$$\sum_i \mathcal{O}^{(1)}_{\Phi L,ii}, \quad \text{and} \quad \sum_i \mathcal{O}^{(3)}_{\Phi L,ii} \ . \tag{6}$$

We notice that the operators in the third group not only contribute to $VV \to VV$ and $f\bar{f} \to VV$ processes, but they can also be bounded by the EWPD, in particular from Z–pole and W–pole observables; see Section IV and

[2] This is a different choice with respect to the basis in Ref. [2], there these two fermionic operators are kept in exchange of the bosonic operators $\mathcal{O}_W$ and $\mathcal{O}_B$.

	VVV	$VVVV$	HVV	$HVVV$	$HHVV$	HHH	$HHHH$	$H\bar{f}f$	$Z\bar{q}q$	$Z\bar{l}l$	$W\bar{u}d$	$W\bar{l}\nu$
$\mathcal{O}_{WWW}$	X	X										
$\mathcal{O}_{WW}$			X	X	X							
$\mathcal{O}_{BB}$			X		X							
$\mathcal{O}_{BW}$	X	X	X	X	X				X	X	X	X
$\mathcal{O}_{W}$	X	X	X	X	X							
$\mathcal{O}_{B}$	X		X	X	X							
$\mathcal{O}_{\Phi,1}$	X	X	X		X	X	X	X	X	X	X	X
$\mathcal{O}_{\Phi,2}$			X		X	X	X	X				
$\mathcal{O}_{\Phi,3}$						X	X					
$\mathcal{O}^{(1)}_{\Phi Q}$, $\mathcal{O}^{(1)}_{\Phi u}$, $\mathcal{O}^{(1)}_{\Phi d}$									X			
$\mathcal{O}^{(3)}_{\Phi Q}$,									X		X	
$\mathcal{O}^{(1)}_{\Phi ud}$											X	
$\mathcal{O}^{(1)}_{\Phi L}$,$\mathcal{O}^{(1)}_{\Phi e}$,										X		
$\mathcal{O}^{(3)}_{\Phi L}$,									X		X	X

TABLE I: Couplings relevant for our analysis that are modified by the dimension–six operators in Eqs. (2)–(5). Here, V stands for any electroweak gauge boson, H for the Higgs and f for SM fermions.

Appendix B. By using the EOM to remove the combinations in Eq. (6) we have selected the operator basis in such a way that there is a clear separation between those constrained-by and those blind-to the EWPD bounds [22].

To avoid the generation of too large flavor violation, in what follows we assume no generation mixing in these operators, that is, for any operator $\mathcal{O}_{ij} = \mathcal{O}_{ii}\delta_{ij}$.

Finally we notice that the complete basis of dimension–six operators also contains a fourth group of dipole fermionic operators (*i.e.* with tensor Lorentz structure) and that can participate in two–to–two scatterings of fermions into gauge and Higgs bosons but that do not modify the Z–pole and W–pole physics at tree level, since their contributions do not interfere with the SM ones. They are

$$\begin{array}{ll} \mathcal{O}_{eW,ij} = i\bar{L}_i\sigma^{\mu\nu}\ell_{R,j}\widehat{W}_{\mu\nu}\Phi \quad , & \mathcal{O}_{eB,ij} = i\bar{L}_i\sigma^{\mu\nu}\ell_{R,j}\widehat{B}_{\mu\nu}\Phi \quad , \\ \mathcal{O}_{uW,ij} = i\bar{Q}_i\sigma^{\mu\nu}u_{R,j}\widehat{W}_{\mu\nu}\tilde{\Phi} \quad , & \mathcal{O}_{uB,ij} = i\bar{Q}_i\sigma^{\mu\nu}u_{R,j}\widehat{B}_{\mu\nu}\tilde{\Phi} \quad , \\ \mathcal{O}_{dW,ij} = i\bar{Q}_i\sigma^{\mu\nu}d_{R,j}\widehat{W}_{\mu\nu}\Phi \quad , & \mathcal{O}_{dB,ij} = i\bar{Q}_i\sigma^{\mu\nu}u_{R,j}\widehat{B}_{\mu\nu}\Phi \quad , \end{array} \tag{7}$$

where i,j are family indices. These operators lead to partial–wave unitarity violation in different channels from the operators in Eq. (5), and therefore can be bounded independently. For completeness we present the corresponding unitarity violating amplitudes and bounds in Appendix C.

III. CONSTRAINTS FROM UNITARITY VIOLATION IN TWO–TO–TWO PROCESSES

Let us start by studying the unitarity violating amplitudes associated with the bosonic operators listed in Eqs. (2)–(4) of which all but $\mathcal{O}_{\Phi,3}$ lead to amplitudes which grow with s in the two–to–two scattering of electroweak gauge bosons and Higgs

$$V_{1\lambda_1}V_{2\lambda_2} \to V_{3\lambda_3}V_{4\lambda_4} \quad . \tag{8}$$

The helicity amplitude of these processes is then expanded in partial waves in the center–of–mass system, following the conventions of [26]

$$\mathcal{M}(V_{1\lambda_1}V_{2\lambda_2} \to V_{3\lambda_3}V_{4\lambda_4}) = 16\pi\sum_J\left(J+\frac{1}{2}\right)\sqrt{1+\delta^{V_{2\lambda_2}}_{V_{1\lambda_1}}}\sqrt{1+\delta^{V_{4\lambda_4}}_{V_{3\lambda_3}}}\, d^J_{\lambda\mu}(\theta)\; e^{iM\varphi}\; T^J(V_{1\lambda_1}V_{2\lambda_2} \to V_{3\lambda_3}V_{4\lambda_4}) \quad , \tag{9}$$

where d is the usual Wigner rotation matrix and $\lambda = \lambda_1 - \lambda_2$, $\mu = \lambda_3 - \lambda_4$, $M = \lambda_1 - \lambda_2 - \lambda_3 + \lambda_4$, and θ (φ) is the polar (azimuth) scattering angle. In the case one of the vector bosons is replaced by the Higgs we use this expression by setting the correspondent λ to zero.

The helicity scattering amplitudes for the operators $\mathcal{O}_{\Phi,1}$ and $\mathcal{O}_{BW}$ are presented in the Appendix A, while the corresponding amplitudes for the other bosonic operators can be found in Ref. [4]. Notice that the contributions of the bosonic operators to $VV \to VV$ scattering amplitudes grow with s since the gauge invariance leads to the cancellation of potential terms growing as s^2 [27]. Moreover, all bosonic operators contribute to the $J = 0$ and $J = 1$ partial waves, however, $\mathcal{O}_{WWW}$ also leads to the growth of $J \geq 2$ amplitudes. Nevertheless, the most stringent bounds come from the $J = 0$ and 1 partial waves, therefore, we restrict our attention to these channels. Furthermore unitarity violating amplitudes arise for the three possible charge channels $Q = 0, 1, 2$; see Ref. [4] for notation and a list of all the states contribution to each (Q, J) channel.

In order to obtain the strongest bounds on the coefficients of the eight operators, we diagonalize the six matrices containing the T^J_Q amplitudes for each of the (Q, J) channels and impose that all their eigenvalues (a total of 59) satisfy the constraint

$$|T^J(V_{1\lambda_1} V_{2\lambda_2} \to V_{1\lambda_1} V_{2\lambda_2})| \leq 2 \ . \tag{10}$$

Initially we obtain the unitarity bounds on the eight bosonic operators assuming that only one Wilson coefficient differs from zero at a time. This is a conservative scenario, *i.e.* leads to stringent bounds, since we do not take into account that more than one operator contributing to a given channel could lead to cancellations and therefore looser limits. For this case we obtain:

$$\begin{array}{lll} \left|\frac{f_{\phi,2}}{\Lambda^2} s\right| \leq 33 \ , & \left|\frac{f_{\phi,1}}{\Lambda^2} s\right| \leq 50 \ , & \left|\frac{f_W}{\Lambda^2} s\right| \leq 87 \ , \\ \left|\frac{f_B}{\Lambda^2} s\right| \leq 617 \ , & \left|\frac{f_{WW}}{\Lambda^2} s\right| \leq 99 \ , & \left|\frac{f_{BB}}{\Lambda^2} s\right| \leq 603 \ , \\ \left|\frac{f_{BW}}{\Lambda^2} s\right| \leq 456 \ , & \left|\frac{f_{WWW}}{\Lambda^2} s\right| \leq 85 \ . & \end{array} \tag{11}$$

Next we study the constraints on the full set of eight bosonic operators when they are all allowed to vary. In order to find closed ranges in the eight–dimensional parameter space we need to consider also the constraints from fermion annihilation into electroweak gauge bosons [3]. To do so we obtain the helicity amplitudes of all processes

$$f_{1\sigma_1} \bar{f}_{2\sigma_2} \to V_{3\lambda_3} V_{4\lambda_4} \ , \tag{12}$$

and then perform the expansion in partial waves of the center–of–mass system; for further details and conventions see Ref. [4].

We present in the Appendix A the leading order terms of the scattering amplitudes that give rise to unitarity violation at high energies taking into account the dimension–six operators in Eqs. (2)–(5). These amplitudes are proportional to $\delta_{\sigma_1,-\sigma_2}$ since we neglect the fermion masses in the high energy limit. It is interesting to notice that the dimension–six operators $\mathcal{O}_{WWW}$, $\mathcal{O}_W$ and $\mathcal{O}_B$ modify the triple electroweak gauge boson couplings (TGC), therefore, as expected, their presence would affect the SM cancellations that cut off the growth of the $f\bar{f} \to VV$ amplitudes. On the other hand, the operator $\mathcal{O}_{BW}$ also modifies the TGC, however, its effects on the Z/γ wave function renormalizations cancel the growth with the center–of–mass energy due to the anomalous TGC. Similar cancellations occur for $\mathcal{O}_{\Phi,1}$ which contributes to triple gauge vertices, as well as the coupling of gauge bosons to fermions.

In order to obtain more stringent bounds and to separate the contributions of the different operators, we consider the processes [3]

$$X \to V_{3\lambda_3} V_{4\lambda_4} \ , \tag{13}$$

where X is a linear combination of fermionic initial states:

$$|X\rangle = \sum_{f_i \sigma_i} x_{f_1\sigma_1, f_2\sigma_2} |f_{1\sigma_1} \bar{f}_{2\sigma_2}\rangle \ , \tag{14}$$

with the normalization $\sum_{f_i\sigma_i} |x_{f_1\sigma_1,f_2\sigma_2}|^2 = 1$. The corresponding bounds read [3]

$$\sum_{V_{3\lambda_3}, V_{4\lambda_4}} \left|T^J(X \to V_{3\lambda_3} V_{4\lambda_4})\right|^2 \leq 1 \ . \tag{15}$$

In particular using the linear combinations as displayed in the first three lines of Table II we are able to impose independent bounds in each of the Wilson coefficients of the three bosonic operators participating in the $f\bar{f} \to VV$ scattering amplitudes.

Fermion State	V_3V_4	$T^{J=1}$
$\frac{1}{\sqrt{2\mathrm{N_g}(1+\mathrm{N_c})}}\vert\sum_{i=1}^{\mathrm{N_g}}\left(-e^-_{-,i}\,e^+_{+,i}\,+\,\nu_{-,i}\bar{\nu}_{+,i}\,+\sum_{a=1}^{\mathrm{N_c}}\left(-d^a_{-,i}\,\bar{d}^a_{+,i}\,+\,u^a_{-,i}\,\bar{u}^a_{+,i}\right)\right)\rangle$	$W^+_-W^-_-,\,W^+_+W^-_+$	$+i\frac{1}{12\sqrt{2}\pi}\sqrt{2\mathrm{N_g}(\mathrm{N_c}+1)}\frac{3g^4}{4}\frac{f_{WWW}\,s}{\Lambda^2}$
$\frac{1}{\sqrt{2\mathrm{N_g}}}\vert\sum_{i=1}^{\mathrm{N_g}}\left(e^-_{-,i}\,e^+_{+,i}\,+\,\nu_{-,i}\bar{\nu}_{+,i}\right)\rangle$	$W^+_0W^-_0$	$+i\frac{1}{12\sqrt{2}\pi}\sqrt{\frac{\mathrm{N_g}}{2}}\frac{g^2\mathrm{s_W}^2}{4\,\mathrm{c_W}^2}\frac{f_B\,s}{\Lambda^2}$
$\frac{1}{\sqrt{2\mathrm{N_g}}}\vert\sum_{i=1}^{\mathrm{N_g}}((e^-_{-,i}\,e^+_{+,i}\,-\,\nu_{-,i}\bar{\nu}_{+,i}))\rangle$	$W^+_0W^-_0$	$+i\frac{1}{12\sqrt{2}\pi}\sqrt{\frac{\mathrm{N_g}}{2}}\frac{g^2}{4}\frac{f_W\,s}{\Lambda^2}$
$\frac{1}{\sqrt{3\mathrm{N_g}}}\vert\sum_{i=1}^{\mathrm{N_g}}\left(e^-_{-,i}\,e^+_{+,i}\,+\,\nu_{-,i}\bar{\nu}_{+,i}+e^-_{+,i}\,e^+_{-,i}\right)\rangle$	$W^+_0W^-_0$	$-i\frac{1}{12\sqrt{2}\pi}\sqrt{\frac{\mathrm{N_g}}{3}}\frac{f^{(1)}_{\Phi e}\,s}{\Lambda^2}$
$\frac{1}{\sqrt{\mathrm{N_g}(\mathrm{N_c}+8)}}\vert\sum_{i=1}^{\mathrm{N_g}}\left(2\,e^-_{-,i}\,e^+_{+,i}\,+2\,\nu_{-,i}\bar{\nu}_{+,i}-\sum_{a=1}^{\mathrm{N_c}}u^a_{+,i}\,\bar{u}^a_{-,i}\right)\rangle$	$W^+_0W^-_0$	$+i\frac{1}{12\sqrt{2}\pi}\frac{1}{\sqrt{\mathrm{N_g}(\mathrm{N_c}+8)}}\mathrm{N_g}\mathrm{N_c}\frac{f^{(1)}_{\Phi u}\,s}{\Lambda^2}$
$\frac{1}{\sqrt{\mathrm{N_g}(\mathrm{N_c}+2)}}\vert\sum_{i=1}^{\mathrm{N_g}}\left(e^-_{-,i}\,e^+_{+,i}\,+\,\nu_{-,i}\bar{\nu}_{+,i}+\sum_{a=1}^{\mathrm{N_c}}d^a_{+,i}\,\bar{d}^a_{-,i}\right)\rangle$	$W^+_0W^-_0$	$-i\frac{1}{12\sqrt{2}\pi}\frac{\mathrm{N_g}\mathrm{N_c}}{\sqrt{\mathrm{N_g}(\mathrm{N_c}+2)}}\frac{f^{(1)}_{\Phi d}\,s}{\Lambda^2}$
$\frac{1}{\sqrt{\mathrm{N_g}(2\mathrm{N_c}+2)}}\vert\sum_{i=1}^{\mathrm{N_g}}\left(e^-_{-,i}\,e^+_{+,i}\,+\,\nu_{-,i}\bar{\nu}_{+,i}\,+\sum_{a=1}^{\mathrm{N_c}}\left(d^a_{-,i}\,\bar{d}^a_{+,i}\,+\,u^a_{-,i}\,\bar{u}^a_{+,i}\right)\right)\rangle$	$W^+_0W^-_0$	$+i\frac{1}{12\sqrt{2}\pi}\frac{2\mathrm{N_g}\mathrm{N_c}}{\sqrt{\mathrm{N_g}(2\mathrm{N_c}+2)}}\frac{f^{(1)}_{\Phi Q}\,s}{\Lambda^2}$
$\frac{1}{\sqrt{4\mathrm{N_g}}}\vert\sum_{i=1}^{\mathrm{N_g}}\left(e^-_{-,i}\,e^+_{+,i}\,-\,\nu_{-,i}\bar{\nu}_{+,i}\,+u^a_{-,i}\,\bar{u}^a_{+,i}-d^a_{-,i}\,\bar{d}^a_{+,i}\right)\rangle$	$W^+_0W^-_0$	$+i\frac{1}{12\sqrt{2}\pi}\frac{\mathrm{N_g}}{\sqrt{4}}\frac{1}{2}\frac{f^{(3)}_{\Phi Q}\,s}{\Lambda^2}$
$\frac{1}{\sqrt{\mathrm{N_g}\mathrm{N_c}}}\vert\sum_{i=1}^{\mathrm{N_g}}\sum_{a=1}^{\mathrm{N_c}}d^a_{+,i}\,\bar{u}^a_{-,i}\rangle$	$W^+_0W^-_0$	$-i\frac{1}{12\sqrt{2}\pi}\sqrt{\mathrm{N_g}\mathrm{N_c}}\sqrt{2}\frac{f^{(1)}_{\Phi ud}\,s}{\Lambda^2}$

TABLE II: Initial fermionic states used to obtain bounds on the generation independent fermionic operators. We also present the high–energy dominant terms of the corresponding amplitudes. We denote by $\mathrm{N_g} = 3$ the number of generations while $\mathrm{N_c} = 3$ is the number of colors. $\sqrt{s}$ stands for the center–of–mass energy of the processes.

Combining those with the conditions from partial–wave unitarity of the 59 eigenvalues of the elastic boson scattering amplitudes discussed above we find the most general constraints in the eight–dimensional parameter space. In summary, allowing all coefficients to be nonzero, and searching for the largest allowed values for each operator coefficient while varying over the other coefficients, yields:

$$\begin{aligned}
&\left|\frac{f_{\phi,2}}{\Lambda^2}s\right| \leq 209 \quad , &&\left|\frac{f_{\phi,1}}{\Lambda^2}s\right| \leq 151 \quad , &&\left|\frac{f_W}{\Lambda^2}s\right| \leq 436 \quad , \\
&\left|\frac{f_B}{\Lambda^2}s\right| \leq 1460 \quad , &&\left|\frac{f_{WW}}{\Lambda^2}s\right| \leq 319 \quad , &&\left|\frac{f_{BB}}{\Lambda^2}s\right| \leq 1340 \quad , \\
&\left|\frac{f_{BW}}{\Lambda^2}s\right| \leq 1386 \quad , &&\left|\frac{f_{WWW}}{\Lambda^2}s\right| \leq 85 \quad .
\end{aligned} \tag{16}$$

As expected these bounds extend the region of validity of the effective theory with respect to the case where just one Wilson coefficient is allowed to be non–vanishing. This is the most general scenario because it implicitly assumes that the values of the Wilson coefficients are tuned to have the largest energy region where the effective theory is valid.

It is important to stress that both the one–dimensional bounds in Eq. (11) and the general bounds in Eq. (16) hold independently of the values of the coefficients of the fermionic operators due to the choice of initial states in Table II.

A. Bounds on Generation Independent Operators

Now we focus our attention on the operators involving fermions, what require assumptions concerning their flavour structure as we discuss next. Initially we assume that the new physics giving rise to the dimension–six operators is generation blind. In this case we can drop the generation index in all coefficients. Therefore, the constraint on the operators in Eq. (6) implies that the operators $\mathcal{O}^{(1)}_{\Phi L}$ and $\mathcal{O}^{(3)}_{\Phi L}$ are redundant.

As for the case of the bosonic operators, in order to obtain more stringent bounds and to separate the contributions of the different operators we consider specific initial states X. In particular choosing the linear combinations as displayed in Table II we are able to impose independent bounds in each of the fermionic Wilson coefficients participating in the $f\bar{f} \to VV$ scattering amplitudes.

Starting from the states defined in Table II and the scattering amplitudes given in Appendix A we obtain the partial–wave helicity amplitudes also listed in Table II. The corresponding constraints on the Wilson coefficients of

State	$T^{J=1}$
$\frac{1}{\sqrt{2\mathrm{N_g}+\mathrm{N_g}^2}}\vert\sum_{i=1}^{\mathrm{N_g}}\left(e^-_{-,i}\,e^+_{+,i}\,+\,\nu_{-,i}\bar{\nu}_{+,i}\right)+\mathrm{N_g}\,e^-_{+,j}\,e^+_{-,j}\rangle$	$-i\frac{1}{12\sqrt{2}\pi}\frac{1}{\sqrt{2\mathrm{N_g}+\mathrm{N_g}^2}}\mathrm{N_g}\frac{f^{(1)}_{\Phi e,jj}\,s}{\Lambda^2}$
$\frac{1}{\sqrt{8\mathrm{N_g}+\mathrm{N_g}^2\mathrm{N_c})}}\vert\sum_{i=1}^{\mathrm{N_g}}\left(2\,e^-_{-,i}\,e^+_{+,i}\,+2\,\nu_{-,i}\bar{\nu}_{+,i}\right)\,-\sum_{a=1}^{\mathrm{N_c}}\mathrm{N_g}\,u^a_{+,j}\,\bar{u}^a_{-,j}\rangle$	$+i\frac{1}{12\sqrt{2}\pi}\frac{1}{\sqrt{8\mathrm{N_g}+\mathrm{N_g}^2\mathrm{N_c}}}\mathrm{N_g}\mathrm{N_c}\frac{f^{(1)}_{\Phi u,jj}\,s}{\Lambda^2}$
$\frac{1}{\sqrt{2\mathrm{N_g}+\mathrm{N_g}^2\mathrm{N_c}}}\vert\sum_{i=1}^{\mathrm{N_g}}\left(e^-_{-,i}\,e^+_{+,i}\,+\,\nu_{-,i}\bar{\nu}_{+,i}\right)\,+\sum_{a=1}^{\mathrm{N_c}}\mathrm{N_g}\,d^a_{+,j}\,\bar{d}^a_{-,j}\rangle$	$-i\frac{1}{12\sqrt{2}\pi}\frac{1}{\sqrt{2\mathrm{N_g}+\mathrm{N_g}^2\mathrm{N_c}}}\mathrm{N_g}\mathrm{N_c}\frac{f^{(1)}_{\Phi d,jj}\,s}{\Lambda^2}$
$\frac{1}{\sqrt{2\mathrm{N_g}+2\mathrm{N_g}^2\mathrm{N_c}}}\vert\sum_{i=1}^{\mathrm{N_g}}\left(e^-_{-,i}\,e^+_{+,i}\,+\,\nu_{-,i}\bar{\nu}_{+,i}\right)\,+\sum_{a=1}^{\mathrm{N_c}}\mathrm{N_g}\,\left(d^a_{-,j}\,\bar{d}^a_{+,j}\,+\,u^a_{-,j}\,\bar{u}^a_{+,j}\right)\rangle$	$+i\frac{1}{12\sqrt{2}\pi}\frac{1}{\sqrt{2\mathrm{N_g}+2\mathrm{N_g}^2\mathrm{N_c}}}\mathrm{N_g}\mathrm{N_c}\,2\,\frac{f^{(1)}_{\Phi Q,jj}\,s}{\Lambda^2}$
$\frac{1}{\sqrt{2\mathrm{N_g}+2\mathrm{N_g}^2}}\vert\sum_{i=1}^{\mathrm{N_g}}\left(e^-_{-,i}\,e^+_{+,i}\,-\,\nu_{-,i}\bar{\nu}_{+,i}\right)\,+\,\mathrm{N_g}\,u^a_{-,j}\,\bar{u}^a_{+,j}\,-\,\mathrm{N_g}\,d^a_{-,j}\,\bar{d}^a_{+,j}\rangle$	$+i\frac{1}{12\sqrt{2}\pi}\frac{1}{\sqrt{2\mathrm{N_g}+2\mathrm{N_g}^2}}\frac{\mathrm{N_g}}{2}\frac{f^{(3)}_{\Phi Q,jj}\,s}{\Lambda^2}$
$\frac{1}{\sqrt{\mathrm{N_c}}}\vert\sum_{a=1}^{\mathrm{N_c}}d^a_{+,j}\,\bar{u}^a_{-,j}\rangle$	$-i\frac{1}{12\sqrt{2}\pi}\frac{1}{\sqrt{\mathrm{N_c}}}\mathrm{N_c}\sqrt{2}\frac{f^{(1)}_{\Phi ud,jj}\,s}{\Lambda^2}$
$\frac{1}{\sqrt{4}}\vert\,e^-_{-,1}\,e^+_{+,1}\,+\,\nu_{-,1}\bar{\nu}_{+,1}\,-\,e^-_{-,j}\,e^+_{+,j}\,-\,\nu_{-,j}\bar{\nu}_{+,j}\rangle$	$-i\frac{1}{12\sqrt{2}\pi}\frac{1}{2}2\frac{f^{(1)}_{\Phi L,jj-11}\,s}{\Lambda^2}$
$\frac{1}{\sqrt{4}}\vert\,e^-_{-,1}\,e^+_{+,1}\,-\,\nu_{-,1}\bar{\nu}_{+,1}\,-\,e^-_{-,j}\,e^+_{+,j}\,+\,\nu_{-,j}\bar{\nu}_{+,j}\rangle$	$+i\frac{1}{12\sqrt{2}\pi}\frac{1}{2}\,\frac{1}{2}\,\frac{f^{(3)}_{\Phi L,jj-11}\,s}{\Lambda^2}$

TABLE III: Initial fermionic states used to obtain independent bounds on the generation dependent fermionic operators. We also present the high energy dominant term of the corresponding amplitudes. In the last two lines $j = 2, 3$.

the fermionic operators are

$$\left|\frac{f^{(1)}_{\Phi e}}{\Lambda^2}s\right| < 53 \quad , \qquad \left|\frac{f^{(1)}_{\Phi u}}{\Lambda^2}s\right| < 34 \quad , \qquad \left|\frac{f^{(1)}_{\Phi d}}{\Lambda^2}s\right| < 23 \quad ,$$
$$\left|\frac{f^{(1)}_{\Phi Q}}{\Lambda^2}s\right| < 14 \quad , \qquad \left|\frac{f^{(3)}_{\Phi Q}}{\Lambda^2}s\right| < 123 \quad , \qquad \left|\frac{f^{(1)}_{\Phi ud}}{\Lambda^2}s\right| < 13 \quad . \tag{17}$$

B. Bounds on Generation Dependent Operators

In this case first we need to eliminate the redundant operators in Eq. (6). In order to do so we define four independent combinations of the six leptonic operators $\mathcal{O}^{(3)}_{\Phi L,ii}$ and $\mathcal{O}^{(1)}_{\Phi L,ii}$ which are not removed by the EOM's. They are

$$\mathcal{O}^{(1)}_{\Phi L,22-11} = \mathcal{O}^{(1)}_{\Phi L,22} - \mathcal{O}^{(1)}_{\Phi L,11}\ , \qquad \mathcal{O}^{(1)}_{\Phi L,33-11} = \mathcal{O}^{(1)}_{\Phi L,33} - \mathcal{O}^{(1)}_{\Phi L,11}\ ,$$
$$\mathcal{O}^{(3)}_{\Phi L,22-11} = \mathcal{O}^{(3)}_{\Phi L,22} - \mathcal{O}^{(3)}_{\Phi L,11}\ , \qquad \mathcal{O}^{(3)}_{\Phi L,33-11} = \mathcal{O}^{(3)}_{\Phi L,33} - \mathcal{O}^{(3)}_{\Phi L,11}\ , \tag{18}$$

and we denote the corresponding Wilson coefficients as $f^{(1)}_{\Phi L,22-11}$, $f^{(1)}_{\Phi L,33-11}$, $f^{(3)}_{\Phi L,22-11}$, and $f^{(3)}_{\Phi L,33-11}$ respectively.

It is interesting to notice that the sum over the three generations for the $Q = 0$ leptonic $+-00$ amplitudes cancel for the left–handed operators because this is the combination removed by the EOM. With this in mind we define the initial states in Table III to impose bounds on each of the fermionic operators. Using these initial states and the corresponding helicity amplitudes, the bounds coming from $f\bar{f} \to VV$ on each of the Wilson coefficients of the fermionic operators read

$$\left|\frac{f^{(1)}_{\Phi e,jj}}{\Lambda^2}s\right| < 69 \quad , \qquad \left|\frac{f^{(1)}_{\Phi u,jj}}{\Lambda^2}s\right| < 42 \quad , \qquad \left|\frac{f^{(1)}_{\Phi d,jj}}{\Lambda^2}s\right| < 34 \quad ,$$
$$\left|\frac{f^{(1)}_{\Phi Q,jj}}{\Lambda^2}s\right| < 23 \quad , \qquad \left|\frac{f^{(3)}_{\Phi Q,jj}}{\Lambda^2}s\right| < 174 \quad , \qquad \left|\frac{f^{(1)}_{\Phi ud,jj}}{\Lambda^2}s\right| < 22 \quad , \tag{19}$$
$$\left|\frac{f^{(1)}_{\Phi L,jj-11}}{\Lambda^2}s\right| < 53 \quad , \qquad \left|\frac{f^{(3)}_{\Phi L,jj-11}}{\Lambda^2}s\right| < 213 \quad ,$$

with $j = 2, 3$ in the last two inequalities. As expected, the above limits are weaker than the ones displayed in Eq. (17) for the generation independent operators.

coupling	95% allowed range for f_i/Λ^2 TeV^{-2})		
	Generation Independent	Generation Dependent	
f_{BW}	$(-0.32\,,\,1.7)$	$(-0.90\,,\,2.6)$	
$f_{\Phi 1}$	$(-0.040\,,\,0.15)$	$(-0.11\,,\,0.23)$	
f_{LLLL}	$(-0.043\,0.013)$	$(-1.3\,,\,-0.21)$	
		Case	Range
$f^{(1)}_{\Phi Q}$	$(-0.083\,,\,0.10)$	$(11)=(22)$	$(-0.33\,,\,0.29)$
$f^{(3)}_{\Phi Q}$	$(-0.60\,,\,0.12)$	$(11)=(22)$	$(-0.92\,,\,0.64)$
		$f^{(1)}_{\Phi Q,33}+\frac{1}{4}f^{(3)}_{\Phi Q,33}$	$(-0.21\,,\,0.041)$
$f^{(1)}_{\Phi u}$	$(-0.25\,,\,0.37)$	$(11)=(22)$	$(-0.19\,,\,0.50)$
$f^{(1)}_{\Phi d}$	$(-1.2\,,\,-0.13)$	$(11)=(22)$	$(-2.7\,,\,1.9)$
		(33)	$(-1.3\,,\,-0.23)$
$f^{(1)}_{\Phi L}$	—-	$(22-11)$	$(0.005\,,\,0.41)$
		$(33-11)$	$(-0.63\,,\,-0.096)$
$f^{(3)}_{\Phi L}$	—-	$(22-11)$	$(-1.62\,,\,-0.060)$
		$(33-11)$	$(0.38\,,\,2.5)$
		(11)	$(-0.11\,,\,0.049)$
$f^{(1)}_{\Phi e}$	$(-0.075\,,\,0.011)$	(22)	$(-0.15\,,\,0.063)$
		(33)	$(-0.15\,,\,0.044)$

TABLE IV: *95% C.L. allowed ranges for the Wilson coefficients of the dimension–six operators that contribute to the EWPD.*

Let us stress that the bounds on the coefficients of the fermionic operators in Eq. (17) for generation independent and in Eq. (19) for generation dependent operators hold independently of the values of the coefficients of the bosonic operators due to the choice of initial states in Tables II and III.

IV. DISCUSSION

Let us start by noticing that even in the most general case, allowing for all operators to have non–vanishing coefficients, we have obtained bounds that are closed ranges. This means that there is a bounded region of the parameter space for which the effective theory is perturbatively valid. In other words there is no combination of Wilson coefficients that can extend indefinitely the energy domain where there is no partial–wave unitarity violation.

Second we want to address whether, within that region of coefficients, violation of unitarity can be an issue at the Run II LHC energies. Our procedure to quantitatively answer this question is to determine the maximum center–of–mass energy for which the unitarity limits are not violated given our present knowledge on the Wilson coefficients of the dimension–six operators from lower energy data. For definiteness we considered the maximum allowed value of these coefficients at the 95% confidence level in our analysis. Clearly the results depend on this hypothesis and the energy range where perturbative unitarity holds is extended if we consider these coefficients at 68% confidence level.

In this respect EWPD gathered at the Z–pole and W–pole lead to stringent constraints on operators contributing at linear order to these observables and these results are model independent. These are the fermionic operators leading to Z and W couplings to fermions with the same Lorentz structure as the SM, most of the fermionic operators in Eq. (5), together with the bosonic operators $\mathcal{O}_{BW}$ and $\mathcal{O}_{\Phi,1}$. The 95% CL allowed range for their coefficients obtained from a global analysis performed in the full multi–dimensional parameter space are presented in Table IV; further details of the analysis are given in Appendix B.

With these results we have quantified the maximum center–of–mass energy for which partial–wave unitarity holds for each operator in two scenarios. In the first we do not allow for cancellations among the contributions of the bosonic operators in the s-growing terms in $VV \to VV$ scattering, and therefore, we use the constraints obtained with just one non-vanishing Wilson coefficient; see Eq. (11). In addition to that we considered generation independent fermion operators, Eq. (17), and the corresponding bounds on the Wilson coefficients in the central column in Table IV. In the second scenario we use the unitarity constraints on the bosonic operators allowing for cancellations in the $VV \to VV$ scattering amplitudes, as in Eq. (16), together with the assumption of generation–dependent fermion operators, Eq. (19), and the corresponding bounds on the Wilson coefficients in the last column in Table IV. The maximum center–of–mass energy for which partial–wave unitarity holds in either scenario is:

$$\begin{array}{llll} \mathcal{O}_{\Phi,1} & \sqrt{s}_{\rm max} = 18\,{\rm TeV} \;, & \mathcal{O}_{BW} & \sqrt{s}_{\rm max} = 16.\,{\rm TeV} \;, \\ \mathcal{O}^{(1)}_{\Phi e} & \sqrt{s}_{\rm max} = 21\,{\rm TeV} \;, & \mathcal{O}^{(1)}_{\Phi u} & \sqrt{s}_{\rm max} = 9.2\,{\rm TeV} \;, \\ \mathcal{O}^{(1)}_{\Phi d} & \sqrt{s}_{\rm max} = 3.5\,{\rm TeV} \;, & \mathcal{O}^{(1)}_{\Phi Q} & \sqrt{s}_{\rm max} = 8.3\,{\rm TeV} \;, \\ \mathcal{O}^{(3)}_{\Phi Q} & \sqrt{s}_{\rm max} = 14\,{\rm TeV} \;, & & \\ \mathcal{O}^{(1)}_{\Phi L,22-11} & \sqrt{s}_{\rm max} = 11\,{\rm TeV} \;, & \mathcal{O}^{(1)}_{\Phi L,33-11} & \sqrt{s}_{\rm max} = 9.2\,{\rm TeV} \;, \\ \mathcal{O}^{(3)}_{\Phi L,22-11} & \sqrt{s}_{\rm max} = 12\,{\rm TeV} \;, & \mathcal{O}^{(3)}_{\Phi L,33-11} & \sqrt{s}_{\rm max} = 9.2\,{\rm TeV} \;. \end{array} \tag{20}$$

Notice that the fermionic operator $\mathcal{O}^{(1)}_{\Phi ud}$ does not contribute to the observables used in the Z–pole and W–pole data analysis at the linear level as it gives a right–handed W coupling which does not interfere with the SM amplitude. It does however, contribute linearly to observables which depend on specific entries of the CKM matrix via a finite renormalization of the quark mixing, in particular to deep inelastic scattering of neutrinos off nucleons, as well as, measurements of the CKM matrix elements in hadronic decays [28, 29]. The derivation of the bounds on its coefficient from this data involves additional assumptions about its flavour structure and the presence of further four–fermion operators which also contribute to these processes, making them more model dependent. Under the assumption of generation independent couplings with no cancellation with the additional four–fermion operators one obtains the constraints in Refs. [28, 29], $-0.006 \leq \frac{f^{(1)}_{\Phi ud}}{\Lambda^2} \leq 0.01$ which imply $\sqrt{s}_{\rm max} = 25.$ TeV.

For the remaining dimension–six operators that we studied, the present bounds on their Wilson coefficients come from global fits to Higgs physics and TGC [5, 25] at the LHC Run I and the corresponding maximum center–of–mass energy for which partial–wave unitarity holds is:

$$\begin{array}{llll} \mathcal{O}_{B} & \sqrt{s}_{\rm max} = 7.2\,{\rm TeV} \;, & \mathcal{O}_{W} & \sqrt{s}_{\rm max} = 4.7\,{\rm TeV} \;, \\ \mathcal{O}_{BB} & \sqrt{s}_{\rm max} = 10.\,{\rm TeV} \;, & \mathcal{O}_{WW} & \sqrt{s}_{\rm max} = 5.2\,{\rm TeV} \;, \\ \mathcal{O}_{\Phi,2} & \sqrt{s}_{\rm max} = 2.1\,{\rm TeV} \;, & \mathcal{O}_{WWW} & \sqrt{s}_{\rm max} = 5.7\,{\rm TeV} \;. \end{array} \tag{21}$$

In order to access the importance of the above results for the LHC analyses we should keep in mind that, presently, the most energetic diboson (VV) events possess a center–of–mass energy of the order of 3 TeV; see for instance [30]. As more integrated luminosity is accumulated this maximum energy will grow to 4–5 TeV, so we consider that as long as unitarity violation occurs only above these energies, it will not be an issue within the present LHC runs. This condition, of course, will have to be revisited at higher luminosity runs, but at that point also one will have to take into account the possible more stringent bounds on the Wilson coefficients.

From the results in Eq. (20)–(21) we read that there is no need of modification of the dimension–six effective theory to perform the LHC analyses for most operators. One exception is the operator $\mathcal{O}^{(1)}_{\Phi d}$ whose relatively lower $\sqrt{s}_{\rm max} = 3.5$ TeV, however, originates from the weaker bounds on its coefficients induced by the 2.8σ discrepancy of $A^{0,b}_{\rm FB}$ in the EWPD. Notwithstanding, studies of anomalous triple gauge couplings in diboson production should analyze more carefully the high energy tail of the distributions if they include this coupling. Furthermore, there is one additional exception that is the operator $\mathcal{O}_{\Phi,2}$. Since this operator modifies the production and decay of Higgs bosons, as well as, the $VV \to VV$ scattering in vector boson fusion the high energy tails of these processes may also need a special scrutiny.

An eventual caveat of the above conclusions is that the UV completion might be strongly interacting and the lowest center–of–mass energy exhibiting perturbative unitarity violation then marks the onset of the strongly interacting region. If this were the case at the LHC we should observe new states, which has not yet been the case yet.

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Appendix A: Helicity Amplitudes

We present in this appendix the list of unitarity violating amplitudes considered in this work that must be complemented by those in Ref. [4].

	$(\times \frac{f_{\Phi,1}}{\Lambda^2} \times s)$
$W^+W^+ \to W^+W^+$	1
$W^+Z \to W^+Z$	$-\frac{X}{4}$
$W^+Z \to W^+H$	$\frac{2+Y}{4}$
$W^+H \to W^+H$	$-\frac{X}{4}$
$W^+W^- \to W^+W^-$	$-\frac{Y}{2}$
$W^+W^- \to ZZ$	$\frac{1}{2}$
$W^+W^- \to ZH$	$\frac{Y-1}{2}$
$W^+W^- \to HH$	$-\frac{1}{2}$
$ZZ \to HH$	1
$ZH \to ZH$	$\frac{X}{2}$

TABLE V: Leading unitarity violating terms of the scattering amplitudes for longitudinal gauge bosons generated by the operator $\mathcal{O}_{\Phi,1}$ where $X = 1 - \cos\theta$ and $Y = 1 + \cos\theta$ and θ is the polar scattering angle.

	$(\times e^2 \frac{f_{BW}}{\Lambda^2}) \times s$						
	0000	$00++$	$0+0-$	$0+-0$	$+00-$	$+0-0$	$++00$
$W^+W^+ \to W^+W^+$	0	0	0	0	0	0	0
$W^+Z \to W^+Z$	0	$\frac{1}{4c_W}$	$\frac{1}{4}X$	$\frac{1}{8c_W}Y$	$\frac{1}{8c_W}Y$	0	$\frac{1}{4c_W}$
$W^+\gamma \to W^+\gamma$	$-$	$-$	$-\frac{1}{4}X$	$-$	$-$	$-$	$-$
$W^+Z \to W^+\gamma$	$-$	$-\frac{1}{4s_W}$	$-\frac{1}{4}X\cot(2\theta_W)$	$-$	$-\frac{1}{8s_W}Y$	$-$	$-$
$W^+Z \to W^+H$	0	$-$	$-$	$\frac{1}{8c_W}Y$	$-$	$-$	$-\frac{1}{4c_W}$
$W^+\gamma \to W^+H$	$-$	$-$	$-$	$-\frac{1}{8s_W}Y$	$-$	$-$	$\frac{1}{4s_W}$
$W^+H \to W^+H$	0	$-$	$-$	$-$	$-$	0	$-$
$W^+W^- \to W^+W^-$	0	0	0	0	0	0	0
$W^+W^- \to ZZ$	0	$-\frac{1}{2}$	$-\frac{1}{8c_W}X$	$+\frac{1}{8c_W}Y$	$+\frac{1}{8c_W}Y$	$-\frac{1}{8c_W}X$	0
$W^+W^- \to \gamma\gamma$	$-$	$\frac{1}{2}$	$-$	$-$	$-$	$-$	$-$
$W^+W^- \to Z\gamma$	$-$	$\frac{1}{2}\cot(2\theta_W)$	$\frac{1}{8s_W}X$	$-$	$-\frac{1}{8s_W}Y$	$-$	$-$
$W^+W^- \to ZH$	0	$-$	$-$	$\frac{1}{8c_W}Y$	$-$	$\frac{1}{8c_W}X$	0
$W^+W^- \to \gamma H$	$-$	$-$	$-$	$-\frac{1}{8s_W}Y$	$-$	$-\frac{1}{8s_W}X$	$-$
$W^+W^- \to HH$	0	$-$	$-$	$-$	$-$	$-$	0
$ZZ \to ZZ$	0	$\frac{1}{2}$	$-\frac{1}{4}X$	$\frac{1}{4}Y$	$\frac{1}{4}Y$	$-\frac{1}{4}X$	$\frac{1}{2}$
$ZZ \to \gamma\gamma$	$-$	$-\frac{1}{2}$	$-$	$-$	$-$	$-$	$-$
$ZZ \to Z\gamma$	$-$	$-\frac{1}{2}\cot(2\theta_W)$	$\frac{1}{4}X\cot(2\theta_W)$	$-$	$-\frac{1}{4}Y\cot(2\theta_W)$	$-$	$-$
$ZZ \to ZH$	0	$-$	$-$	0	$-$	0	0
$ZZ \to \gamma H$	$-$	$-$	$-$	0	$-$	0	0
$ZZ \to HH$	0	$-$	$-$	$-$	$-$	$-$	$-\frac{1}{2}$
	0000	$00++$	$0+0-$	$0+-0$	$+00-$	$+0-0$	$++00$
$Z\gamma \to ZZ$	$-$	$-$	$\frac{1}{4}X\cot(2\theta_W)$	$-\frac{1}{4}Y\cot(2\theta_W)$	$-$	$-$	$-\frac{1}{2}\cot(2\theta_W)$
$Z\gamma \to \gamma\gamma$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$Z\gamma \to Z\gamma$	$-$	$-$	$\frac{1}{4}X$	$-$	$-$	$-$	$-$
$Z\gamma \to ZH$	$-$	$-$	$-$	0	$-$	$-$	0
$Z\gamma \to \gamma H$	$-$	$-$	$-$	0	$-$	$-$	$-$
$Z\gamma \to HH$	$-$	$-$	$-$	$-$	$-$	$-$	$\frac{1}{2}\cot(2\theta_W)$
$\gamma\gamma \to \gamma\gamma$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$\gamma\gamma \to HH$	$-$	$-$	$-$	$-$	$-$	$-$	$\frac{1}{2}$
$ZH \to ZH$	0	$-$	$-$	$-$	$-$	$-\frac{1}{4}X$	$-$
$ZH \to Z\gamma$	$-$	0	$-$	$-$	0	$-$	$-$
$\gamma H \to \gamma H$	$-$	$-$	$-$	$-$	$-$	$\frac{1}{4}X$	$-$
$ZH \to \gamma H$	$-$	$-$	$-$	$-$	$-$	$\frac{1}{4}X\cot(2\theta_W)$	$-$

TABLE VI: Leading unitarity violating terms of the scattering amplitudes for gauge bosons with the different helicities generated by the operator $\mathcal{O}_{BW}$. $X = 1 - \cos\theta$ and $Y = 1 + \cos\theta$ and θ is the polar scattering angle.

Process	$\sigma_1,\sigma_2,\lambda_3,\lambda_4$	Bosonic operator contribution $\left(\times\, i\, \frac{s}{\Lambda^2} \times \sin\theta\right)$	fermionic operator contribution $\left(\times\, i\, \frac{s}{\Lambda^2} \times \sin\theta\right)$
$e_i^- e_i^+ \to W^+W^-$	$-+00$	$-\frac{g^2}{8}\frac{c_W^2 f_W + s_W^2 f_B}{c_W^2}$	$\frac{1}{4}(f^{(3)}_{\phi L,ii} - 4f^{(1)}_{\phi L,ii})$
	$+-00$	$-\frac{g^2}{4}\frac{s_W^2 f_B}{c_W^2}$	$-f^{(1)}_{\phi e,ii}$
	$-+--$	$-\frac{3g^4}{8} f_{WWW}$	0
	$-+++$	$-\frac{3g^4}{8} f_{WWW}$	0
$\nu_i\bar{\nu}_i \to W^+W^-$	$-+00$	$\frac{g^2}{8}\frac{c_W^2 f_W - s_W^2 f_B}{c_W^2}$	$-\frac{1}{4}(f^{(3)}_{\phi L,ii} + 4f^{(1)}_{\phi L,ii})$
	$+-00$	0	0
	$-+--$	$\frac{3g^4}{8} f_{WWW}$	0
	$-+++$	$\frac{3g^4}{8} f_{WWW}$	0
$u_i\bar{u}_i \to W^+W^-$	$-+00$	$\frac{g^2 N_c}{8}\frac{3c_W^2 f_W + s_W^2 f_B}{3c_W^2}$	$-\frac{N_c}{4}(f^{(3)}_{\phi Q,ii} + 4f^{(1)}_{\phi Q,ii})$
	$+-00$	$\frac{g^2 N_c}{6}\frac{s_W^2}{c_W^2} f_B$	$-N_c\, f^{(1)}_{\phi u,ii}$
	$-+--$	$\frac{3g^4 N_c}{8} f_{WWW}$	0
	$-+++$	$\frac{3g^4 N_c}{8} f_{WWW}$	0
$d_i\bar{d}_i \to W^+W^-$	$-+00$	$-\frac{g^2 N_c}{8}\frac{3c_W^2 f_W - s_W^2 f_B}{3c_W^2}$	$\frac{N_c}{4}(f^{(3)}_{\phi Q,ii} - 4f^{(1)}_{\phi Q,ii})$
	$+-00$	$-\frac{g^2 N_c}{12}\frac{s_W^2 f_B}{c_W^2}$	$-N_c\, f^{(1)}_{\phi d,ii}$
	$-+--$	$-\frac{3g^4 N_c}{8} f_{WWW}$	0
	$-+++$	$-\frac{3g^4 N_c}{8} f_{WWW}$	0
$e_i^-\bar{\nu}_i \to W^-Z$	$-+00$	$\frac{g^2}{4\sqrt{2}} f_W$	$-\frac{1}{2\sqrt{2}} f^{(3)}_{\phi L,ii}$
	$+-00$	0	0
	$-+--$	$\frac{3c_W g^4}{4\sqrt{2}} f_{WWW}$	0
	$-+++$	$\frac{3c_W g^4}{4\sqrt{2}} f_{WWW}$	0
$e_i^-\bar{\nu}_i \to W^-A$	$-+00$	0	0
	$+-00$	0	0
	$-+--$	$\frac{3s_W g^4}{4\sqrt{2}} f_{WWW}$	0
	$-+++$	$\frac{3s_W g^4}{4\sqrt{2}} f_{WWW}$	0
$d_i\bar{u}_i \to W^-Z$	$-+00$	$\frac{g^2 N_c}{4\sqrt{2}} f_W$	$-\frac{N_c}{2\sqrt{2}} f^{(3)}_{\phi Q,ii}$
	$+-00$	0	$-N_c\sqrt{2}\, f^{(1)}_{\phi ud,ii}$
	$-+--$	$\frac{3c_W g^4 N_c}{4\sqrt{2}} f_{WWW}$	0
	$-+++$	$\frac{3c_W g^4 N_c}{4\sqrt{2}} f_{WWW}$	0
$d_i\bar{u}_i \to W^-A$	$-+00$	0	0
	$+-00$	0	0
	$-+--$	$\frac{3s_W g^4 N_c}{4\sqrt{2}} f_{WWW}$	0
	$-+++$	$\frac{3s_W g^4 N_c}{4\sqrt{2}} f_{WWW}$	0

TABLE VII: Leading unitarity violating terms of the scattering amplitudes $\mathcal{M}(f_{1\sigma_1}\bar{f}_{2\sigma_2} \to V_{3\lambda_3}V_{4\lambda_4})$. Notice that in writing these amplitudes we have not imposed the conditions in Eq. (6) yet. See the text for details.

Appendix B: Constraints from EWPD

We briefly summarize here the details of our analysis of EWPD. Similar analyses for different choices of operator basis can be found in [31–35]. We work on the Z–scheme where the input parameters are chosen to be $\alpha_s, G_F, \alpha_{\rm em}, M_Z$ [36], and M_h [37]. In addition to these quantities we also consider the fermion masses as input parameters. All the other quantities appearing in the Lagrangian are implicitly expressed as combinations of experimental inputs.

In our analyses we evaluated the dimension–six contributions to the observables keeping both SM contribution and the linear terms in the anomalous couplings, *i.e.* the we considered only the interference between the SM and the anomalous contributions. The predictions for the shift in the observables of the Z and W pole physics with respect to their SM values are

$$\Delta\Gamma_Z = 2\Gamma_{Z,\rm SM}\left(\frac{\sum_f (g_L^f \Delta g_L^f + g_R^f \Delta g_R^f) N_C^f}{\sum_f (|g_L^f|^2 + |g_R^f|^2) N_C^f}\right) \;, \tag{B1}$$

$$\Delta\sigma_h^0 = 2\sigma_{h,\rm SM}^0\left(\frac{(g_L^e \Delta g_L^e + g_R^e \Delta g_R^e)}{|g_L^e|^2 + |g_R^e|^2} + \frac{\sum_q (g_L^q \Delta g_L^q + g_R^q \Delta g_R^q)}{\sum_q (|g_L^q|^2 + |g_R^q|^2)} - \frac{\Delta\Gamma_Z}{\Gamma_{Z,\rm SM}}\right) \;, \tag{B2}$$

$$\Delta R_l^0 \equiv \Delta\left(\frac{\Gamma_Z^{\rm had}}{\Gamma_Z^l}\right) = 2R_{l,\rm SM}^0\left(\frac{\sum_q (g_L^q \Delta g_L^q + g_R^q \Delta g_R^q)}{\sum_q (|g_L^q|^2 + |g_R^q|^2)} - \frac{(g_L^l \Delta g_L^l + g_R^l \Delta g_R^l)}{|g_L^l|^2 + |g_R^l|^2}\right) \;, \tag{B3}$$

$$\Delta R_q^0 \equiv \Delta\left(\frac{\Gamma_Z^q}{\Gamma_Z^{\rm had}}\right) = 2R_{q,\rm SM}^0\left(\frac{(g_L^q \Delta g_L^q + g_R^q \Delta g_R^q)}{|g_L^q|^2 + |g_R^q|^2} - \frac{\sum_{q'} (g_L^{q'} \Delta g_L^{q'} + g_R^{q'} \Delta g_R^{q'})}{\sum_{q'} (|g_L^{q'}|^2 + |g_R^{q'}|^2)}\right) \;, \tag{B4}$$

$$\Delta\mathcal{A}_f = 4\mathcal{A}_{f,\rm SM}\,\frac{g_L^f g_R^f}{|g_L^f|^4 - |g_R^f|^4}\left(g_R^f \Delta g_L^f - g_L^f \Delta g_R^f\right) \;, \tag{B5}$$

$$\Delta P_\tau^{\rm pol} = \Delta\mathcal{A}_l \;, \tag{B6}$$

$$\Delta A_{\rm FB}^{0,f} = A_{\rm FB,SM}^{0,f}\left(\frac{\Delta\mathcal{A}_l}{\mathcal{A}_l} + \frac{\Delta\mathcal{A}_f}{\mathcal{A}_f}\right) \;, \tag{B7}$$

$$\Delta\Gamma_W = \Gamma_{W,\rm SM}\left(\frac{4}{3}\Delta g_{WL}^{ud} + \frac{2}{3}\Delta g_{WL}^{e\nu} + \Delta M_W\right) \;, \tag{B8}$$

$$\Delta Br_W^{e\nu} = Br_{W,\rm SM}^{e\nu}\left(-\frac{4}{3}\Delta g_{WL}^{ud} + \frac{4}{3}\Delta g_{WL}^{e\nu}\right) \,, \tag{B9}$$

where we write the induced corrections to the SM fermion couplings of the Z boson ($g_{L(R)}^f$) as

$$\Delta g_{L,R}^f = g_{L,R}^f \Delta g_1 + Q^f \Delta g_2 + \Delta\tilde{g}_{L,R}^f \,. \tag{B10}$$

The universal shifts of the fermion couplings in Eq. (B10) due to dimension–six operators are

$$\Delta g_1 = \frac{1}{2}\left(\alpha\, T - \frac{\delta G_F}{G_F}\right) \,, \quad \Delta g_2 = \frac{\rm s_W^2}{\rm c_{2\theta_W}}\left({\rm c_W^2}\left(\alpha\,{\rm T} - \frac{\delta{\rm G_F}}{\rm G_F}\right) - \frac{1}{4{\rm s_W^2}}\alpha\,{\rm S}\right) \,. \tag{B11}$$

where we denoted the sine (cosine) of the weak mixing angle by $\rm s_W$ ($\rm c_W$). The cosine and sine of twice θ_W are then denoted $\rm c_{2\theta_W}$ and $\rm s_{2\theta_W}$ respectively. The tree level contributions of the dimension–six operators to the oblique parameters [38, 39] are

$$\alpha\, S = -e^2 \tfrac{v^2}{\Lambda^2} f_{BW} \,, \;\; \alpha\, T = -\tfrac{1}{2}\tfrac{v^2}{\Lambda^2} f_{\Phi,1} \,, \;\; \alpha\, U = 0 \,, \;\; \tfrac{\delta G_F}{G_F} = -2 f_{LLLL}\tfrac{v^2}{\Lambda^2} + (f_{\Phi L,11}^{(3)} + f_{\Phi L,22}^{(3)})\tfrac{v2}{\Lambda^2} \tag{B12}$$

where for completeness we also included the effect of the dimension–six four–fermion operator contributing with a finite renormalization to the Fermi constant

$$\mathcal{O}_{LLLL} = (\bar{L}\gamma^\mu L)(\bar{L}\gamma^\mu L) \,. \tag{B13}$$

The coupling modifications that depend on the fermion flavor are given by

$$\begin{aligned}
\Delta\tilde{g}_L^u &= -\tfrac{v^2}{8\Lambda^2}(4 f_{\Phi Q}^{(1)} - f_{\Phi Q}^{(3)}) \,, & \Delta\tilde{g}_R^u &= -\tfrac{v^2}{2\Lambda^2} f_{\Phi u}^{(1)} \,, \\
\Delta\tilde{g}_L^d &= -\tfrac{v^2}{8\Lambda^2}(4 f_{\Phi Q}^{(1)} + f_{\Phi Q}^{(3)}) \,, & \Delta\tilde{g}_R^d &= -\tfrac{v^2}{2\Lambda^2} f_{\Phi d}^{(1)} \,, \\
\Delta\tilde{g}_L^\nu &= -\tfrac{v^2}{8\Lambda^2}(4 f_{\Phi L}^{(1)} - f_{\Phi L}^{(3)}) \,, & \Delta\tilde{g}_R^\nu &= 0 \,, \\
\Delta\tilde{g}_L^e &= -\tfrac{v^2}{8\Lambda^2}(4 f_{\Phi L}^{(1)} + f_{\Phi L}^{(3)}) \,, & \Delta\tilde{g}_R^e &= -\tfrac{v^2}{2\Lambda^2} f_{\Phi e}^{(1)} \,.
\end{aligned} \tag{B14}$$

As for the couplings of the W to fermions, in the SM we normalize the left (right)–handed couplings as 1 (0) and the corresponding shifts on these couplings due dimension–six operators are

$$\Delta g_{WL}^{ff'} = \Delta g_W + \Delta\tilde{g}_{WL}^{ff'}\,, \qquad \Delta g_{WR}^{ff'} = \Delta\tilde{g}_{WR}^{ff'}\,, \tag{B15}$$

with the universal shift given by

$$\Delta g_W = \frac{\Delta M_W}{M_W} - \frac{1}{2}\frac{\delta G_F}{G_F}\,, \tag{B16}$$

where the correction to the W mass coming from the dimension–six operators reads

$$\frac{\Delta M_W}{M_W} = \frac{\mathrm{c}_\mathrm{W}^2}{2\mathrm{c}_{2\theta_\mathrm{W}}}\alpha\, T - \frac{1}{4\mathrm{c}_{2\theta_\mathrm{W}}}\alpha\, S + \frac{1}{8\mathrm{s}_\mathrm{W}^2}\alpha\, U - \frac{\mathrm{s}_\mathrm{W}^2}{2\mathrm{c}_{2\theta_\mathrm{W}}}\frac{\delta G_F}{G_F}\,. \tag{B17}$$

The fermion dependent contributions of the dimension–six operators to the W-couplings are

$$\Delta\tilde{g}_{WL}^{ud} = \frac{v^2}{4\Lambda^2} f_{\Phi Q}^{(3)}\,, \quad \Delta\tilde{g}_{WR}^{ud} = \tfrac{v^2}{\Lambda^2} f_{\Phi ud}^{(1)}\,, \quad \Delta\tilde{g}_{WL}^{e\nu} = \frac{v^2}{4\Lambda^2} f_{\Phi L}^{(3)}\,, \quad \Delta\tilde{g}_{WR}^{e\nu} = 0\,. \tag{B18}$$

We notice that, as we are including the effect of the operators in the observables at linear order, the operator $\mathcal{O}_{\Phi ud,ij}^{(1)}$ does not contribute since it leads to a right-handed CC current which does not interfere with the corresponding SM amplitude.

We perform two different fits which differ on the assumptions on the generation dependence of the fermionic operators.

1. Fit with Generation Independent Operators

In the first case we assume that the fermionic operators are generation independent. In this case, as discussed above, we can drop the generation index in all coefficients. Furthermore removing the operators in Eq. (6) implies that those two operators do not appear in the fit to the EWPD. We have then 8 coefficients to be determined

$$\left\{ \frac{f_{BW}}{\Lambda^2}\,,\ \frac{f_{\Phi,1}}{\Lambda^2}\,,\ \frac{f_{LLLL}}{\Lambda^2}\,,\ \frac{f_{\Phi Q}^{(1)}}{\Lambda^2}\,,\ \frac{f_{\Phi Q}^{(3)}}{\Lambda^2}\,,\ \frac{f_{\Phi u}^{(1)}}{\Lambda^2}\,,\ \frac{f_{\Phi d}^{(1)}}{\Lambda^2}\,,\ \frac{f_{\Phi e}^{(1)}}{\Lambda^2} \right\}\,.$$

In our analyses we fitted 15 observables of which 12 are Z observables [40]:

$$\Gamma_Z\ ,\ \sigma_h^0\ ,\ \mathcal{A}_\ell(\tau^{\rm pol})\ ,\ R_\ell^0\ ,\ \mathcal{A}_\ell(\mathrm{SLD})\ ,\ A_{\rm FB}^{0,l}\ ,\ R_c^0\ ,\ R_b^0\ ,\ \mathcal{A}_c\ ,\ \mathcal{A}_b\ ,\ A_{\rm FB}^{0,c}\ ,\ \text{ and }\ A_{\rm FB}^{0,b}\ (\mathrm{SLD/LEP\text{-}I})\ ,$$

supplemented by three W observables

$$M_W\ ,\ \Gamma_W\ ,\ \text{and}\ \mathrm{Br}(W\to\ell\nu)$$

that are, respectively, its average mass from [36], its width from LEP-II/Tevatron [41], and the leptonic W branching ratio for which the average in Ref. [36] is taken. The correlations among the inputs can be found in Ref. [40] and have been taken into consideration in the analyses. The SM prediction and its uncertainty due to variations of the SM parameters are taken from [32].

When performing the fit within the context of the SM the result is $\chi^2_{\rm EWPD,SM} = 18.0$, while including the 8 new parameters it gets reduced to $\chi^2_{\rm EWPD,min} = 5.3$. The results of the analysis are shown in Table IV where we quote the 95% C.L. allowed ranges for each parameter in the center column. The range for parameter x is obtained accounting for all possible cancellations in the multiparameter space by imposing the condition $\Delta\chi^2_{\rm EWPD,marg}(x) < 4$ where by $\Delta\chi^2_{\rm EWPD,marg}(x)$ we denote the value of $\Delta\chi^2_{\rm EWPD}$ minimized with respect to the other seven parameters for each value of the parameter x. We notice that the only operator coefficient not compatible with zero at 2σ is $f_{\Phi d}^{(1)}$, a result driven by the 2.7σ discrepancy between the observed $A_{\rm FB}^{0,b}$ and the SM expectation.

2. Fit with Generation Dependent Operators

Lifting the assumption of generation independent operators we are left with seven independent leptonic operators. These are, three $\mathcal{O}^{(1)}_{\Phi e,ii}$ plus four combinations of $\mathcal{O}^{(1)}_{\Phi L,ii}$ and $\mathcal{O}^{(3)}_{\Phi L,ii}$ defined in Eq. (18). On the other hand, for operators involving quarks there is not enough information in the observables considered to resolve the contributions from the two first generations. Consequently we make the simplifying assumption that operators for the first and second generations have the same Wilson coefficients and only those from the third generation are allowed to be different. Furthermore, for the third generation of quarks only $\mathcal{O}^{(1)}_{\Phi Q,33}$ and the linear combination $f^{(1)}_{\Phi Q,33} + \frac{1}{4} f^{(3)}_{\Phi Q,33}$ contribute independently to the Z and W observables; see Eq. (B14). Altogether there are a total of 16 coefficients to be determined from the fit to the Z and W observables:

$$
\begin{array}{llll}
\frac{f_{BW}}{\Lambda^2} \;, & \frac{f_{\Phi 1}}{\Lambda^2} \;, & \frac{f_{LLLL}}{\Lambda^2} \;, & \frac{f^{(1)}_{\Phi Q,11}}{\Lambda^2} = \frac{f^{(1)}_{\Phi Q,22}}{\Lambda^2} \;, \\
\frac{f^{(1)}_{\Phi Q,33}}{\Lambda^2} + \frac{1}{4}\frac{f^{(3)}_{\Phi Q,33}}{\Lambda^2} \;, & \frac{f^{(3)}_{\Phi Q,11}}{\Lambda^2} = \frac{f^{(3)}_{\Phi Q,22}}{\Lambda^2} \;, & \frac{f^{(1)}_{\Phi u,11}}{\Lambda^2} = \frac{f^{(1)}_{\Phi u,22}}{\Lambda^2} \;, & \frac{f^{(1)}_{\Phi d,11}}{\Lambda^2} = \frac{f^{(1)}_{\Phi d,22}}{\Lambda^2} \;, \\
\frac{f^{(1)}_{\Phi d,33}}{\Lambda^2} \;, & \frac{f^{(1)}_{\Phi e,11}}{\Lambda^2} \;, & \frac{f^{(1)}_{\Phi e,22}}{\Lambda^2} \;, & \frac{f^{(1)}_{\Phi e,33}}{\Lambda^2} \;, \\
\frac{f^{(1)}_{\Phi L,22-11}}{\Lambda^2} \;, & \frac{f^{(1)}_{\Phi L,33-11}}{\Lambda^2} \;, & \frac{f^{(3)}_{\Phi L,22-11}}{\Lambda^2} \;, & \frac{f^{(3)}_{\Phi L,33-11}}{\Lambda^2} \;.
\end{array}
\tag{B19}
$$

In order to obtain the corresponding constraints on these 16 parameters a fit including 24 experimental data points is performed. These are 19 Z observables [40]:

$$
\begin{array}{lllll}
\Gamma_Z \;, & \sigma_h^0 \;, & R_e^0 \;, & R_\mu^0 \;, & R_\tau^0 \;, \\
A_{\rm FB}^{0,e} \;, & A_{\rm FB}^{0,\mu} \;, & A_{\rm FB}^{0,\tau} \;, & \mathcal{A}_e(\tau^{\rm pol}) \;, & \mathcal{A}_\tau(\tau^{\rm pol}) \;, \\
\mathcal{A}_e({\rm SLD}) \;, & \mathcal{A}_\mu({\rm SLD}) \;, & \mathcal{A}_\tau({\rm SLD}) \;, & R_c^0 \;, & R_b^0 \;, \\
\mathcal{A}_c \;, & \mathcal{A}_b \;, & A_{\rm FB}^{0,c} \;, \text{ and} & A_{\rm FB}^{0,b} \;, &
\end{array}
$$

plus five W observables:

$$
M_W \quad , \quad \Gamma_W \quad , \quad {\rm Br}(W \to e\nu) \quad , \quad {\rm Br}(W \to \mu\nu) \quad , \text{ and } {\rm Br}(W \to \tau\nu) \quad ,
$$

where the three leptonic W branching ratios were taken from Ref. [36]. These 24 observables yield 16 independent constraints on the Wilson coefficients. The correlations among the inputs can be found in Refs. [36, 40] and were considered in the analysis. As in the previous analysis, the SM prediction for these observables and its uncertainty due to variations of the SM parameters are taken from [32].

The fit within the context of the SM leads to $\chi^2_{\rm EWPD,SM} = 29$, while with the inclusion of the 16 new parameters the minimum gets reduced to $\chi^2_{\rm EWPD,min} = 8.2$. The 95% allowed ranges for each of the 16 parameters are shown in the last column in Table IV. As in the previous case, for each coupling the range is obtained after marginalization over the other 15 couplings.

As we can see from Table IV, removing the generation independence hypothesis leads to looser constraints, as could be anticipated. Moreover, flavor independent Wilson coefficients and the ones related to the first two families agree with the SM at the 2σ level, with the exception of $f^{(1)}_{\Phi L,22-11}/\Lambda^2$. On the other hand, we can see clearly the effect of the observable $A_{\rm FB}^{0,b}$ on almost all the third generation Wilson coefficients whose 2σ allowed ranges do not contain the SM.

Appendix C: Dipole operators

The leading high energy contributions of the dipole fermionic operators in Eq. (7) to the $f\bar{f} \to VV$ scattering is given in Table VIII. Neglecting fermion masses the dipole fermionic operators contribute to different helicity states to those from operators in Eq. (5) as can be seen from Tables VII and VIII, due to the presence of $\sigma^{\mu\nu}$ in Eq. (7).

Process	$\sigma_1, \sigma_2, \lambda_3, \lambda_4$	partial-wave amplitude ($\times \frac{s}{\Lambda^2} \times \sin\theta$)
$e_i^- e_i^+ \to W^+W^-$	$--0+$	$-g\, f_{eW,ii}$
	$++-0$	$g\, f_{eW,ii}$
$e_i^- e_i^+ \to ZZ$	$--0+$	$-\frac{g}{2c_W}\,(f_{eW,ii}c_W^2 + f_{eB,ii}s_W^2)$
	$--+0$	$-\frac{g}{2c_W}\,(f_{eW,ii}c_W^2 + f_{eB,ii}s_W^2)$
	$++0-$	$\frac{g}{2c_W}\,(f_{eW,ii}c_W^2 + f_{eB,ii}s_W^2)$
	$++-0$	$\frac{g}{2c_W}\,(f_{eW,ii}c_W^2 + f_{eB,ii}s_W^2)$
$u_i\bar{u}_i \to W^+W^-$	$--+0$	$-g\,N_c\, f_{uW,ii}$
	$++0-$	$g\,N_c\, f_{uW,ii}$
$u_i\bar{u}_i \to ZZ$	$--0+$	$-\frac{g}{2c_W}\,N_c\,(f_{uW,ii}c_W^2 - f_{uB,ii}s_W^2)$
	$--+0$	$-\frac{g}{2c_W}\,N_c\,(f_{uW,ii}c_W^2 - f_{uB,ii}s_W^2)$
	$++0-$	$\frac{g}{2c_W}\,N_c\,(f_{uW,ii}c_W^2 - f_{uB,ii}s_W^2)$
	$++-0$	$\frac{g}{2c_W}\,N_c\,(f_{uW,ii}c_W^2 - f_{uB,ii}s_W^2)$
$d_i\bar{d}_i \to W^+W^-$	$--0+$	$-g\,N_c\, f_{dW,ii}$
	$++-0$	$g\,N_c\, f_{dW,ii}$
$d_i\bar{d}_i \to ZZ$	$--0+$	$-\frac{g}{2c_W}\,N_c\,(f_{dW,ii}c_W^2 + f_{dB,ii}s_W^2)$
	$--+0$	$-\frac{g}{2c_W}\,N_c\,(f_{dW,ii}c_W^2 + f_{dB,ii}s_W^2)$
	$++0-$	$\frac{g}{2c_W}\,N_c\,(f_{dW,ii}c_W^2 + f_{dB,ii}s_W^2)$
	$++-0$	$\frac{g}{2c_W}\,N_c\,(f_{dW,ii}c_W^2 + f_{dB,ii}s_W^2)$
$e_i^- \bar{\nu}_i \to W^-Z$	$++-0$	$-\frac{g}{\sqrt{2}} f_{eW,ii}$
	$++0-$	$-\frac{g}{\sqrt{2}c_W}(f_{eB,ii}s_W^2 - f_{eW,ii}c_W^2)$
$e_i^- \bar{\nu}_i \to W^-A$	$++0-$	$\frac{g\,s_W}{\sqrt{2}}(f_{eB,ii} + f_{eW,ii})$
$d_i\bar{u}_i \to W^-Z$	$--0+$	$\frac{g\,N_c}{\sqrt{2}}(f_{uB,ii}s_W^2 + f_{uW,ii}c_W^2)$
	$--+0$	$-\frac{g\,N_c}{\sqrt{2}} f_{uW,ii}$
	$++-0$	$-\frac{g\,N_c}{\sqrt{2}} f_{dW,ii}$
	$++0-$	$-\frac{g\,N_c}{\sqrt{2}c_W}(f_{dB,ii}s_W^2 - f_{dW,ii}c_W^2)$
$d_i\bar{u}_i \to W^-A$	$--0+$	$-\frac{g\,s_W\,N_c}{\sqrt{2}}(f_{uB,ii} - f_{uW,ii})$
	$++0-$	$+\frac{g\,s_W\,N_c}{\sqrt{2}}(f_{dB,ii} + f_{dW,ii})$

TABLE VIII: Unitarity violating scattering amplitudes $\mathcal{M}({f_1}_{\sigma_1} {\bar{f}_2}_{\sigma_2} \to {V_3}_{\lambda_3} {V_4}_{\lambda_4})$ induced by the operators in Eq. (7).

Assuming that the Wilson coefficients of the dipole operators are generation independent, we can obtain, using Table VIII, the following unitarity bounds:

$$
\begin{aligned}
\frac{1}{\sqrt{N_g}}|\sum_{i=1}^{N_g} e^-_{-,i}\, e^+_{-,i}\rangle \to W_0^+ W_+^- \;&\Rightarrow\; \left|\frac{f_{eW}}{\Lambda^2}s\right| \leq 49 \\
\frac{1}{\sqrt{N_g}}|\sum_{i=1}^{N_g} e^-_{+,i}\, \bar{\nu}_{+,i}\rangle \to W_0^+ A_- \;&\Rightarrow\; \left|\frac{(f_{eW}+f_{eB})}{\Lambda^2}s\right| \leq 144 \Rightarrow \left|\frac{f_{eB}}{\Lambda^2}s\right| \leq 193 \\
\frac{1}{\sqrt{N_g N_c}}|\sum_{i=1}^{N_g}\sum_{a=1}^{N_c} u^a_{-,i}\, \bar{u}^a_{-,i}\rangle \to W_+^+ W_0^- \;&\Rightarrow\; \left|\frac{f_{uW}}{\Lambda^2}s\right| \leq 28 \\
\frac{1}{\sqrt{N_g N_c}}|\sum_{i=1}^{N_g}\sum_{a=1}^{N_c} d^a_{+,i}\, \bar{u}^a_{+,i}\rangle \to W_0^- A_+ \;&\Rightarrow\; \left|\frac{(f_{uW}-f_{uB})}{\Lambda^2}s\right| \leq 83 \Rightarrow \left|\frac{f_{uB}}{\Lambda^2}s\right| \leq 111 \\
\frac{1}{\sqrt{N_g N_c}}|\sum_{i=1}^{N_g}\sum_{a=1}^{N_c} d^a_{-,i}\, \bar{d}^a_{-,i}\rangle \to W_0^+ W_+^- \;&\Rightarrow\; \left|\frac{f_{dW}}{\Lambda^2}s\right| \leq 28 \\
\frac{1}{\sqrt{N_g N_c}}|\sum_{a=1}^{N_c}\sum_{i=1}^{N_g} d^a_{+,i}\, \bar{u}^a_{+,i}\rangle \to W_0^- A_- \;&\Rightarrow\; \left|\frac{(f_{dW}+f_{dB})}{\Lambda^2}s\right| \leq 83 \Rightarrow \left|\frac{f_{dB}}{\Lambda^2}s\right| \leq 111
\end{aligned}
\tag{C1}
$$

Dropping the generation independence hypothesis for the dipole operators, we can use the same set of amplitudes as in Eq. (C1) but now without summing over generations. In this case the partial–wave unitarity constraints read:

$$\left|\frac{f_{eW,ii}}{\Lambda^2}s\right| \le 85\,, \quad \left|\frac{f_{eB,ii}}{\Lambda^2}s\right| \le 334 \tag{C2}$$

$$\left|\frac{f_{qW,ii}}{\Lambda^2}s\right| \le 49\,, \quad \left|\frac{f_{qB,ii}}{\Lambda^2}s\right| \le 193 \tag{C3}$$

where the last line applies for $q = u, d$.

Because they flip the fermion chirality these operators do not interfere at tree–level with the SM amplitudes and also generically they are expected to be suppressed by the fermion Yukawa[3]. In this case only the operators involving the top quark can be sizable. There is an extensive study of the top quark properties at the LHC which includes the operators $\mathcal{O}_{uW}$ and $\mathcal{O}_{uB}$; see, for instance, Ref. [43] and references therein. In particular the measurement of W–boson helicity in top–quark decays [44] give us direct access to f_{uW}/Λ^2. Using the global fit to the top quarks properties in Ref. [43] ($|f_{uW}/\Lambda^2| < 3.8$ TeV^{-2}) we estimate that the operator $\mathcal{O}_{uW}$ does not lead to perturbative unitarity violation in top–quark processes at the LHC for maximum center–of–mass energies up to 2.7 TeV.

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