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Universal Landau Pole and Physics below the 100 TeV Scale

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Abstract

We reconsider the possibility that all standard model gauge couplings blow up at a common scale in the ultraviolet. The simplest implementation of this idea assumes supersymmetry and the addition of a single vector-like generation of matter fields around the TeV scale. We provide an up-to-date numerical study of this scenario and show that either the scale of the additional matter or the scale of the light superparticle masses falls below potentially relevant LHC bounds. We then consider minimal extensions of the extra matter sector that raise its scale above the reach of the LHC, to determine whether there are cases that might be probed at a 100 TeV collider. We also consider the possibility that the heavy matter sector involves new gauge groups constrained by the same ultraviolet boundary condition, which in some cases can provide an explanation for the multiplicity of heavy states. We comment on the relevance of this framework to theories with dark and visible sectors.

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I. INTRODUCTION

The idea that the three gauge couplings of the standard model may assume a common value at a high energy scale has motivated a vast literature on grand unified theories [1]. The particle content of the minimal supersymmetric standard model (MSSM) is consistent with such a unification, with a perturbative unified gauge coupling obtained around 2×10^{16} GeV. However, it was pointed out long ago [2, 3] that a different framework also leads to the correct predictions for the gauge couplings at observable energies, namely one in which the gauge couplings blow up at a common scale Λ in the ultraviolet (UV):

$$\alpha_1^{-1}(\Lambda) = \alpha_2^{-1}(\Lambda) = \alpha_3^{-1}(\Lambda) = 0 \quad . \quad (1.1)$$

Since the SU(3) coupling is asymptotically free, this boundary condition can only be obtained via the introduction of extra matter [3–6]. Supersymmetric models offer the simplest possibility, a single vector-like generation of mass m_V [3–5]. For a chosen value of m_V , one may fix the scale Λ by the requirement that the low-energy value of the fine structure constant α_{EM} is reproduced; the values of $\sin^2 \theta_W$ and α_3^{-1} are then predicted at any chosen renormalization scale μ , up to theoretical uncertainties. If a value of m_V can be found in which both $\sin^2 \theta_W(m_Z)$ and $\alpha_3^{-1}(m_Z)$ are consistent with the data, then a viable solution is obtained. This approach, followed in Ref. [5], found m_V around the TeV scale, assuming that m_V is also the scale of the light superparticle masses (which we call m_{susy} below).

A numerical renormalization group analysis cannot directly encode the boundary condition in Eq. (1.1) since the gauge couplings are in the non-perturbative regime, where the renormalization group equations (RGEs) cannot be trusted. In Ref. [5], the boundary condition studied was $\alpha_1(\Lambda) = \alpha_2(\Lambda) = \alpha_3(\Lambda) = 10$, values that are barely perturbative. Since the couplings are rapidly increasing as the renormalization scale is increased, one makes the reasonable assumption that the value of Λ that satisfies this boundary condition is very close to the one given by Eq. (1.1). On the other hand, as the renormalization scale is decreased, the couplings become increasingly perturbative. Of particular importance is that the results are insensitive to the precise choice of boundary condition as long as each of the couplings is large [7]. It was shown in Ref. [5], that varying the $\alpha_i(\Lambda)$ by an order of magnitude in either direction has only a small effect on the final results. We will see this explicitly in our study of the one-vector-like-generation scenario in Sec. II. The insensitivity of the predicted values of $\sin^2 \theta_W(m_Z)$ and $\alpha_3^{-1}(m_Z)$ to the choice of boundary conditions is due to the existence

of an infrared fixed point in the renormalization group equation for the ratios of the gauge couplings [8]. Note that this insensitivity includes the case where the $\alpha_i(\Lambda)$ are taken to be large but not strictly identical at a common high scale.

The possibility that the gauge couplings may have large values in the UV is interesting from a variety of perspectives. Large couplings may arise in strongly coupled heterotic string theories, which often also provide the additional vector-like states necessary to drive the gauge couplings to large values [8]. On the other hand, a universal Landau pole, as defined by Eq. (1.1), may arise in models with composite gauge bosons: compositeness implies the vanishing of the gauge fields' wave-function renormalization factors at the compositeness scale, where the gauge fields become non-dynamical [9]. Redefining fields and couplings so that the gauge fields' kinetic terms are always kept in canonical form, one finds that the vanishing wave-function renormalization factors translate into the blow-up of the gauge couplings at the same scale. Thus, the framework we study may be consistent with a wider range of possible ultraviolet completions than a conventional grand unified theory (GUT) with a large unified gauge coupling, though it is not necessary to commit ourselves to any one of them in order to study the consequences at low energies.

An additional motivation relevant to the present work is that the assumption of a universal Landau pole leads to the expectation of new physics at a calculable energy scale, m_V , that is above the weak scale but potentially within the reach of future collider experiments¹. In Sec. II, we show that the minimal scenario, involving one vector-like generation of additional matter, requires values of either m_V or m_{susy} that are below some of the current LHC bounds on vector-like quarks or colored superparticles, respectively. Although experimental bounds come with model-specific assumptions that are usually easy to evade, we pursue an alternative possibility. We show that there are small extensions of the new matter sector that successfully reproduce the correct values of the gauge couplings at m_Z while predicting values of m_V that are above the reach of the LHC, but below 100 TeV for some choices of m_{susy} . In some cases, m_V may be light enough for the vector-like states to be explored at a 100 TeV hadron collider, which makes study of this sector more interesting. Aside from the presence of the heavy matter fields, one possibility that we also discuss in the present work

¹ This, of course, assumes that the vector-like matter occurs at a single common scale. This assumption is relaxed in Ref. [6].

is that these fields may transform under an additional gauge group factor. The motivation is two-fold: (1) By placing the additional matter fields into irreducible representations of a new gauge group, we might provide an explanation for the multiplicity of states needed to achieve the desired UV boundary condition. In the case where the heavy matter remains vector-like, the new gauge group can be broken at a much lower scale. The resulting low-energy theory is that of a “dark” sector consisting of the new gauge and symmetry breaking fields; the heavy matter provides for communication between the dark and visible sectors, via a “portal” of higher-dimension operators that are induced when the heavy fields are integrated out. The gauge coupling of the dark gauge boson is predicted from a boundary condition analogous to Eq. (1.1) and the magnitude of the portal couplings are set by the value of m_V obtained in the RGE analysis. This presents a simpler framework for constraining some of the otherwise free parameters of a dark sector than, for example, attempting to embed both dark and visible sectors in a conventional GUT. (2) The heavy matter may be chiral under the new gauge group. The structure of the new sector is then more analogous to the electroweak sector of the MSSM, and the scale m_V is associated with one or more massive gauge bosons that may have observable consequences.

Our paper is organized as follows: In Sec. II, we consider the consequences of a universal Landau pole in the minimal case where the MSSM is augmented by a single vector-like generation. The study presented in this section differs from the past literature not only in our use of up-to-date experimental errors for our input parameters, but also in that we allow the scales m_V and m_{susy} to vary independently. In addition, we consider an alternative choice for the vector-like matter that contributes the same amount to the beta functions at one loop, but differs from the one-generation scenario at two loops. In Sec. III, we consider extensions of these minimal scenarios, in particular, including a small number of additional complete SU(5) multiplets of vector-like matter. We focus on finding solutions in which m_V is less than 100 TeV, with a special interest in cases where the vector-like matter is light enough to be detected at a future hadron collider. In Sec. IV we consider model building issues associated with the physics at the scale m_V , focusing on the implication of additional gauge groups. In Sec. V, we summarize our conclusions.

II. ONE VECTOR-LIKE GENERATION

In this section, we consider a minimal scenario studied in the past literature [3–5], the MSSM augmented by an additional vector-like generation of matter fields. We denote the scale of the vector-like matter m_V and we impose the same boundary conditions as in Ref. [5], namely $\alpha_1(\Lambda) = \alpha_2(\Lambda) = \alpha_3(\Lambda) = 10$ as an approximation to Eq. (1.1). Taking m_V as an input, we determine Λ by the condition that the weak scale value of the fine structure constant $\alpha_{EM}(m_Z)$ is reproduced. With Λ fixed, we are now able to determine the gauge couplings at any lower scale, as a function of our choice for m_V . Above the scale m_{susy} , we use the two-loop supersymmetric RGEs for the gauge couplings. Below m_{susy} , we do the same using the two-loop nonsupersymmetric RGEs, aside from running between the top quark mass and m_Z which we treat as a threshold correction and include at one loop. We assume the presence of the second Higgs doublet required by supersymmetry above the scale m_{susy} . Expanding on the approach of Ref. [5], we do not assume that the scales m_V and m_{susy} are the same, though the relaxation of that requirement will only be important in Sec. III.

As indicated in the introduction, the ratios of the gauge couplings are driven towards infrared fixed point values, so that predictions for $\sin^2 \theta_W$ and α_3^{-1} at m_Z are relatively insensitive to the choice of boundary conditions at the scale Λ . For example, allowing the $\alpha_i(\Lambda)$ to vary independently between 1 and 100, we find that the their weak-scale values scatter within roughly 2% for $\alpha_1(m_Z)$ and $\alpha_2(m_Z)$ and 5% for $\alpha_3(m_Z)$. Given the same variation of boundary conditions, we take the resulting scatter in the values of $\sin^2 \theta_W(m_Z)$ and $\alpha_3^{-1}(m_Z)$ as a measure of the theoretical uncertainty in our output predictions. We include these estimates with our numerical results.

The RGEs that we use above the top mass have the form

$$\frac{dg_i}{dt} = \frac{g_i}{16\pi^2} \left[b_i g_i^2 + \frac{1}{16\pi^2} \left(\sum_{j=1}^3 b_{ij} g_i^2 g_j^2 - \sum_{j=U,D,E} a_{ij} g_i^2 \text{Tr}[Y_j Y_j^\dagger] \right) \right], \quad (2.1)$$

where $t = \ln \mu$ is the log of the renormalization scale, $\alpha_i = g_i^2/4\pi$, and the Y_i are Yukawa matrices. The beta function coefficients b_i and b_{ij} can be determined using general formulae [13, 14]. For example, in the case of one vector-like generation with $m_V = m_{susy}$, one

finds for $\mu > m_V$

$$b_i = \begin{pmatrix} \frac{53}{5} \\ 5 \\ 1 \end{pmatrix} \quad \text{and} \quad b_{ij} = \begin{pmatrix} \frac{977}{75} & \frac{39}{5} & \frac{88}{3} \\ \frac{13}{5} & 53 & 40 \\ \frac{11}{3} & 15 & \frac{178}{3} \end{pmatrix}, \quad (2.2)$$

while for $m_t < \mu < m_V$ we have the nonsupersymmetric beta functions

$$b_i^{NS} = \begin{pmatrix} \frac{41}{10} \\ -\frac{19}{6} \\ -7 \end{pmatrix} \quad \text{and} \quad b_{ij}^{NS} = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix}. \quad (2.3)$$

More general forms for the one- and two-loop beta functions that take into account the possibility of additional matter are presented in Sec. III. Note that the gauge couplings for $\mu > m_{susy}$ are defined in the dimensional reduction ($\overline{\text{DR}}$) scheme, which preserves supersymmetry; the couplings are converted to the modified minimal subtraction scheme ($\overline{\text{MS}}$) at the matching scale $\mu = m_{susy}$ before they are run to lower energies. The gauge couplings in the two schemes are related by [15]

$$\frac{4\pi}{\alpha_i^{\overline{\text{MS}}}} = \frac{4\pi}{\alpha_i^{\overline{\text{DR}}}} + \frac{1}{3}(C_A)_i, \quad (2.4)$$

where $C_A = \{0, 2, 3\}$ for $i = 1, 2, 3$.

The coefficients for the terms that depend on the Yukawa couplings in Eq. (2.1) are given by

$$a_{ij} = \begin{pmatrix} \frac{26}{5} & \frac{14}{5} & \frac{18}{5} \\ 6 & 6 & 2 \\ 4 & 4 & 0 \end{pmatrix} \quad \text{and} \quad a_{ij}^{NS} = \begin{pmatrix} \frac{17}{10} & \frac{1}{2} & \frac{3}{2} \\ \frac{3}{2} & \frac{3}{2} & \frac{1}{2} \\ 2 & 2 & 0 \end{pmatrix}, \quad (2.5)$$

for $\mu > m_{susy}$ and $\mu < m_{susy}$, respectively. In practice, we only need to take the top quark Yukawa coupling y_t into account, since it is significantly larger than the other Yukawa couplings. Since y_t affects the running of the gauge couplings only through a two-loop term, we need only include its running at one-loop. For $\mu > m_{susy}$ we have [11]

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2} \left(-\sum c_i g_i^2 + 6y_t^2 \right), \quad c_i = \left(\frac{13}{15}, 3, \frac{16}{3} \right), \quad (2.6)$$

while for $\mu < m_{susy}$ [11],

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2} \left(-\sum c_i^{\text{SM}} g_i^2 + \frac{9}{2} y_t^2 \right), \quad c_i^{\text{SM}} = \left(\frac{17}{20}, \frac{9}{4}, 8 \right). \quad (2.7)$$

For definiteness, we assume $\tan \beta = 2$, and compute the weak scale value of y_t via $y_t(m_Z) = \frac{\sqrt{2} m_t}{v \sin \beta}$, using the $\overline{\text{MS}}$ value of the top quark mass, 160_{-4}^{+5} GeV [10], and $v = 246$ GeV. The value $y_t(\Lambda)$ is computed numerically so that we obtain the desired $y_t(m_Z)$ value for a given set of input parameters. While this approach is sufficient to determine the representative impact of including the top quark Yukawa coupling in our RGE analysis, it turns out to be overkill: in models where the gauge couplings blow up in the UV, the top quark Yukawa coupling is rapidly driven to zero in the same limit. Hence, its effect on the values of m_V and Λ determined in our numerical analysis turns out to be small, less than the estimates of theoretical uncertainty that we build into the analysis. Although we include it, ignoring y_t altogether does not affect our results qualitatively and can be a useful approach for speeding up numerical cross-checks.

For a given choice of m_V and m_{susy} , the blow-up scale Λ is chosen to yield the correct value of the fine structure constant at the weak scale,

$$\alpha_{\text{EM}}^{-1}(m_Z) = \frac{5}{3} \alpha_1^{-1}(m_Z) + \alpha_2^{-1}(m_Z), \quad (2.8)$$

where the factor of $5/3$ comes from the fact that we assume SU(5) normalization [16] of the U(1) gauge coupling, as in Ref. [5]. While this makes the analysis compatible with a conventional SU(5) GUT at large coupling, this normalization can also arise directly in string theory without an SU(5) GUT [12]. Other normalizations of the U(1) factor are certainly possible, depending on the UV completion. However, we do not consider other possibilities here and adopt the normalization that has been assumed almost uniformly in the past literature. For our numerical study, we take the target central value of $\alpha_{\text{EM}}^{-1}(m_Z) = 127.95$ [10]. With Λ determined in this way, we compute $\alpha_3(m_Z)^{-1}$ and the Weinberg angle $\sin^2 \theta_W(m_Z)$, which is determined by $\alpha_1(m_Z)$ and $\alpha_2(m_Z)$:

$$\sin^2 \theta_W(m_Z) = \frac{3\alpha_1(m_Z)}{3\alpha_1(m_Z) + 5\alpha_2(m_Z)}. \quad (2.9)$$

We compare the output predictions of $\alpha_3(m_Z)^{-1}$ and $\sin^2 \theta_W(m_Z)$, including the theoretical uncertainty that we discussed earlier, to the experimentally measured values [10]

$$\sin^2 \theta_W = 0.23129 \pm 5 \times 10^{-5}, \quad \alpha_3^{-1}(m_Z) = 8.4674 \pm 0.0789, \quad (2.10)$$

both given in the $\overline{\text{MS}}$ scheme. A previous study of the one vector-like generation scenario found viable solutions with $m_V = m_{\text{susy}} \approx 1$ TeV [5]. Since the time of that work, the

experimental errors in $\sin^2 \theta_W(m_Z)$ and $\alpha_3^{-1}(m_Z)$ have decreased substantially. Nevertheless, as indicated in Table I, we find $m_V = m_{\text{susy}} \approx 1.2$ TeV, assuming ± 2 standard deviation experimental error bands and using our protocol for determining theoretical error bands; those bands are both displayed in Fig. 1. To determine the theoretical error band, we

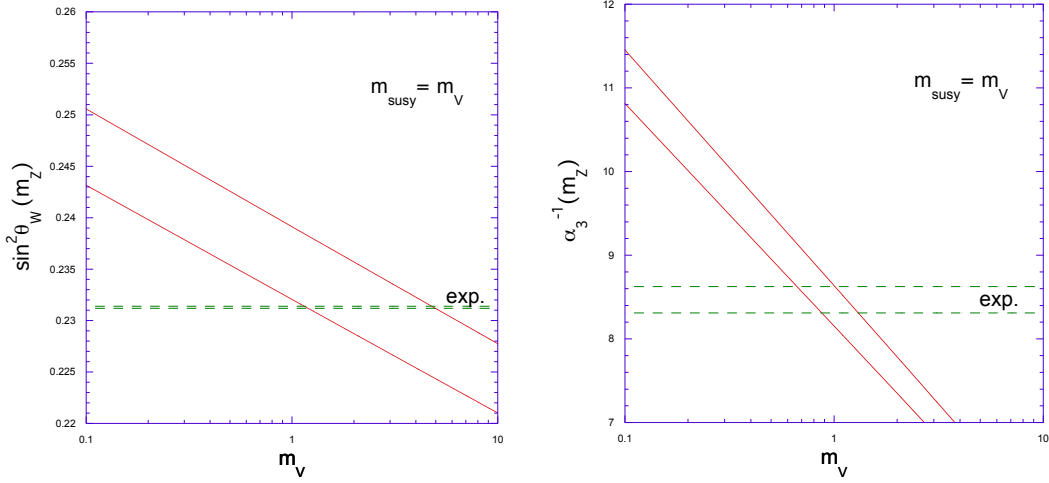


FIG. 1: The dependence of $\sin^2 \theta_W(m_Z)$ and $\alpha_3^{-1}(m_Z)$ on the mass of the vector-like generation, m_V , including theoretical uncertainties. In this example, the light superparticle mass scale m_{susy} is identified with m_V . The acceptable ranges of m_V in each of the plots have non-vanishing overlap for $1.15 \text{ TeV} < m_V < 1.31 \text{ TeV}$, indicating a viable solution.

find the maximum and minimum values of $\sin^2 \theta_W(m_Z)$ and $\alpha_3^{-1}(m_Z)$ that are obtained by varying the α_i independently between 1 and 100 at the blow-up scale. In particular, we find that $\sin^2 \theta_W(m_Z)$ is maximum when $\{\alpha_1(\Lambda), \alpha_2(\Lambda), \alpha_3(\Lambda)\} = \{100, 1, 100\}$ and minimum when the boundary condition set is $\{1, 100, 1\}$; $\alpha_3^{-1}(m_Z)$ is maximized and minimized for the sets $\{100, 100, 1\}$ and $\{1, 1, 100\}$, respectively. We quote the variation in the output predictions as a percentage relative to the value obtained when the $\alpha_i(\Lambda) = 10$, for $i = 1 \dots 3$, in Table I. For the values of m_V that yield viable predictions for $\sin^2 \theta_W(m_Z)$ and $\alpha_3^{-1}(m_Z)$, we find that the scale Λ is around 8×10^{16} GeV.

The value of m_{susy} for this solution can be compared to recent bounds on gluinos from the LHC, which now exceed 2 TeV (for example, see Ref. [17]). These bounds generally make assumptions about the supersymmetric particle spectrum (for example, light neutralinos) and one can always play the game of making model-specific adjustments to evade the assumptions of any given experimental exclusion limit. We will not pursue that approach

Model	m_{SUSY} (TeV)	m_V range (TeV)	Λ range (GeV)	$\alpha_3^{-1}(m_Z)$ % error	$\sin^2 \theta_W(m_Z)$ % error
(5, 2, 0, 0)	m_V	1.15 – 1.31	$7.8 - 8.7 \times 10^{16}$	+3.7%, –2.1%	+1.5%, –1.5%
(3, 2, 4, 0)	m_V	0.66 – 1.16	$6.9 - 11 \times 10^{16}$	+2.8%, –1.5%	+1.4%, –1.2%

TABLE I: Numerical results for m_V and Λ in the one-generation scenario, the (5,2,0,0) model, and a model whose vector-like sector consists of four $\mathbf{5} + \bar{\mathbf{5}}$ pairs, the (3,2,4,0) model. These models have the same one-loop beta functions, but differ at two-loop. Also shown are the theoretical error estimates as discussed in the text.

here. We instead consider the possibility that m_V and m_{susy} are not identical, so that m_{susy} can be raised unambiguously above the LHC reach. In this case, however, we obtain lower values of m_V , which in this model would place an entire vector-like generation below 1 TeV. As a point of comparison, current LHC bounds on a charge-2/3 vector-like quark that decays 100% of the time to bW is 1.295 TeV at the 95% CL [18]. The same comment regarding the limitations of experimental exclusion limits applies here as well; we will be content simply to point out that the one-generation model will become less plausible as time goes on given the increasing reach of LHC searches for superparticles and vector-like quarks.²

This result motivates the topic of the next section, extensions of this minimal sector that include sets of new particles that fill complete SU(5) multiplets. We find that these lead to larger values of m_V . In studies of perturbative gauge coupling unification, it is well known that adding additional matter in complete SU(5) multiplets preserves successful unification. In the present framework, we find viable solutions for m_V are also obtained when complete SU(5) multiplets are added. To study the effect on m_V and Λ , we consider adding the smallest SU(5) representations, with dimensions five and ten, allowing for multiple copies. We label models by four numbers (n_g, n_h, n_5, n_{10}) which represent the number of chiral generations, complex Higgs doublets, $\mathbf{5} + \bar{\mathbf{5}}$ pairs and $\mathbf{10} + \bar{\mathbf{10}}$ pairs.³ In this notation, the one-vector-like-generation scenario that we have discussed in this section will be called the (5, 2, 0, 0) model henceforth. We note that a model with four $\mathbf{5} + \bar{\mathbf{5}}$ pairs added to the MSSM, the (3, 2, 4, 0) model, has the same one-loop beta functions as the (5, 2, 0, 0) model,

² Unless, of course, some of these particles are discovered.

³ It is interesting to note that in level-one string theories with Wilson line symmetry breaking, extra vector-like matter will naturally appear in $\mathbf{5} + \bar{\mathbf{5}}$ and $\mathbf{10} + \bar{\mathbf{10}}$ pairs, since these are representations found in the $\mathbf{27} + \bar{\mathbf{27}}$ of E_6 [8].

and could be considered an equally minimal alternative. Results for the $(3, 2, 4, 0)$ model are also shown in Table I, and are useful for illustrating the effect of different two-loop beta functions. The preferred range of m_V in the $(3, 2, 4, 0)$ model is slightly below that of the $(5, 2, 0, 0)$ model, again pointing to the need for alternative choices for the new matter sector to avoid potential phenomenological difficulties.

III. NEXT-TO-MINIMAL POSSIBILITIES

In this section, we consider vector-like matter sectors that are consistent with values of the superparticle masses and m_V that are no smaller than 2 TeV. We look at next-to-minimal scenarios, *i.e.* ones with a small number of additional $\mathbf{5} + \overline{\mathbf{5}}$ and $\mathbf{10} + \overline{\mathbf{10}}$ pairs, for the reasons discussed at the end of the previous section. We have particular interest in solutions that may be plausible for exploration at a 100 TeV hadron collider. To proceed, we use the results for the one- and two-loop beta functions, derived from the general formulae in Refs. [13] and [14]. In the supersymmetric case, we find

$$b_i = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} n_g + \begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix} n_h + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} n_5 + \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} n_{10} + \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix}, \quad (3.1)$$

$$\begin{aligned} b_{ij} = & \begin{pmatrix} \frac{38}{15} & \frac{6}{5} & \frac{88}{15} \\ \frac{2}{5} & 14 & 8 \\ \frac{11}{15} & 3 & \frac{68}{3} \end{pmatrix} n_g + \begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_h \\ & + \begin{pmatrix} \frac{21}{45} & \frac{9}{5} & \frac{32}{15} \\ \frac{3}{5} & 7 & 0 \\ \frac{4}{15} & 0 & \frac{34}{3} \end{pmatrix} n_5 + \begin{pmatrix} \frac{23}{5} & \frac{3}{5} & \frac{48}{5} \\ \frac{1}{5} & 21 & 16 \\ \frac{6}{5} & 6 & 34 \end{pmatrix} n_{10} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -54 \end{pmatrix}, \end{aligned} \quad (3.2)$$

while in the nonsupersymmetric case,

$$b_i^{NS} = \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} n_g + \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix} n_h + \begin{pmatrix} \frac{2}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{pmatrix} n_5 + \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} n_{10} + \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix}, \quad (3.3)$$

$$\begin{aligned}
b_{ij}^{NS} = & \begin{pmatrix} \frac{19}{15} & \frac{3}{5} & \frac{44}{15} \\ \frac{1}{5} & \frac{49}{3} & 4 \\ \frac{11}{30} & \frac{3}{2} & \frac{76}{3} \end{pmatrix} n_g + \begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{13}{16} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_h + \begin{pmatrix} \frac{7}{30} & \frac{9}{10} & \frac{16}{15} \\ \frac{3}{10} & \frac{49}{6} & 0 \\ \frac{2}{15} & 0 & \frac{38}{3} \end{pmatrix} n_5 \\
& + \begin{pmatrix} \frac{23}{10} & \frac{3}{10} & \frac{24}{5} \\ \frac{1}{10} & \frac{49}{2} & 8 \\ \frac{3}{5} & 3 & 38 \end{pmatrix} n_{10} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{136}{3} & 0 \\ 0 & 0 & -102 \end{pmatrix}. \tag{3.4}
\end{aligned}$$

As indicated earlier, n_g , n_h , n_5 and n_{10} represent the number of chiral generations, Higgs doublets, $\mathbf{5} + \overline{\mathbf{5}}$ and $\mathbf{10} + \overline{\mathbf{10}}$ pairs, respectively. One can check that these formulae reduce to the expected results for the MSSM, where $n_g = 3$, $n_h = 2$, $n_5 = n_{10} = 0$ in Eqs. (3.1) and (3.2), and for the standard model, where $n_g = 3$, $n_h = 1$, $n_5 = n_{10} = 0$ in Eqs. (3.3) and (3.4).

Model	m_{SUSY} (TeV)	m_V range (TeV)	Λ range (GeV)	$\alpha_3^{-1}(m_Z)$ % error	$\sin^2 \theta_W(m_Z)$ % error
(5, 2, 1, 0)	2	95 – 260	$4.9 - 8.2 \times 10^{16}$	+4.2%, –2.8%	+1.5%, –1.4%
	m_V	13 – 28	$3.2 - 5.9 \times 10^{16}$	+4.0%, –2.7%	+1.5%, –1.4%
(3, 2, 5, 0)	2	65 – 217	$4.9 - 9.2 \times 10^{16}$	+3.4%, –2.2%	+1.4%, –1.2%
	10	17 – 32	$4.1 - 5.8 \times 10^{16}$	+3.3%, –2.2%	+1.4%, –1.2%
	m_V	13 – 17	$4.0 - 4.9 \times 10^{16}$	+3.3%, –2.2%	+1.4%, –1.2%
(3, 2, 6, 0)	2	$3.8 - 13 \times 10^3$	$4.3 - 8.6 \times 10^{16}$	+3.7%, –2.7%	+1.4%, –1.2%
	10	$1.2 - 2.6 \times 10^3$	$3.6 - 5.6 \times 10^{16}$	+3.7%, –2.7%	+1.4%, –1.2%
	30	522 – 794	$3.1 - 4.0 \times 10^{16}$	+3.6%, –2.7%	+1.4%, –1.2%
(3, 2, 0, 2)	2	$1.6 - 1.8 \times 10^4$	$7.1 - 7.6 \times 10^{16}$	+5.6%, –4.1%	+1.5%, –1.5%
	10	$3.0 - 5.3 \times 10^3$	$4.4 - 6.1 \times 10^{16}$	+5.4%, –3.9%	+1.5%, –1.5%
	100	277 – 961	$2.2 - 4.5 \times 10^{16}$	+5.1%, –3.8%	+1.5%, –1.5%
	m_V	166 – 370	$1.9 - 3.9 \times 10^{16}$	+5.0%, –3.7%	+1.5%, –1.5%

TABLE II: Solutions for m_V and Λ , for a variety of next-to-minimal heavy matter sectors, for $m_{\text{susy}} \leq m_V$.

Table II displays results analogous to those presented for the minimal scenario in Table I, for a variety of heavy matter sectors, with $m_{\text{susy}} \leq m_V$. The cases considered fall into pairs that have the same one-loop beta functions; for example, adding one additional $\mathbf{5} + \overline{\mathbf{5}}$ pair to

the one-vector-like generation scenario gives us the $(5, 2, 1, 0)$ model, which has the same b_i as a model with five $\mathbf{5} + \overline{\mathbf{5}}$ pairs, namely $(3, 2, 5, 0)$. The same can be said for the remaining two models, involving six $\mathbf{5} + \overline{\mathbf{5}}$ and two $\mathbf{10} + \overline{\mathbf{10}}$ pairs, respectively. Results are shown for values of m_{susy} ranging from 2 TeV to m_V . We see that solutions for m_V decrease as m_{susy} is increased. Holding m_{susy} fixed, heavy matter sectors that give larger contributions to the one-loop beta functions tend to have larger values of m_V . Larger collections of heavy matter do not provide additional solutions with $m_{susy} \leq m_V$ and $m_V < 100$ TeV.

Of the cases shown in Table II, the lowest values of the vector-like matter scale, $m_V \approx 13$ TeV, are obtained in the $(5, 2, 1, 0)$ and $(3, 2, 5, 0)$ scenarios, for $m_V = m_{susy}$. While vector-like quarks with this mass are within the kinematic reach of a 100 TeV hadron collider, their detectability is a separate question. Assuming that a 100 TeV collider has a discovery reach that is greater than that of the LHC by a factor of 5 [19], and that the LHC's ultimate sensitivity to vector-like quarks is just below 2 TeV [20], one might roughly expect a discovery reach for vector-like quarks at a 100 TeV hadron collider just below ~ 10 TeV. This rough estimate is consistent with the 9 TeV reach projected in Ref. [21] for fermionic top quark partners, which are also color triplet fermions. These statements are very rough, and a detailed collider study would be required to determine whether the 13 TeV vector-like quarks in the $(5, 2, 1, 0)$ and $(3, 2, 5, 0)$ models would have observable consequences at a 100 TeV machine.

Fortunately, we find that if the superparticle mass scale is raised above the scale m_V , the reduction in m_V continues. For these cases, we take m_V to refer to the mass of the fermionic components of the vector-like states, while m_{susy} represents a common mass for the superpartners in both the ordinary and the vector-like sectors, in place of our earlier definition. This spectrum is consistent with allowed choices for the soft-supersymmetry-breaking parameters of the theory and simplifies the subsequent analysis. With this assumption, we only find the correct predictions for the gauge couplings at the weak scale in the $(3, 2, 0, 2)$ model. Although a higher m_{susy} indicates that supersymmetry is less effective at addressing the hierarchy problem, one could still argue that this case has its merits: (1) supersymmetry still ameliorates the hierarchy problem between m_{susy} and Λ , which are the scales with the widest separation in the models that we consider, and (2) supersymmetry may be expected if string theory is the UV completion, whether or not supersymmetry has anything to do with solving the hierarchy problem. From a purely phenomenological perspective, taking

$m_{susy} > m_V$ brings the $(3, 2, 0, 2)$ heavy matter sector down into the range where it might be directly probed. In Table III, we present numerical results for that case. As the superparticle mass scale increases from 250 TeV to 1500 TeV, the minimum allowed values of m_V decrease from 71 TeV to 3 TeV. It seems more likely in this case that the vector-like matter could be within the discovery reach of a 100 TeV hadron collider, while all the superpartners remain undetectable. It is interesting to note that it is easiest in the $(3, 2, 0, 2)$ model to incorporate an additional gauge group that acts on the heavy matter sector, a topic we turn to in the next section.

Model	m_{SUSY} (TeV)	m_V range (TeV)	Λ range (GeV)	$\alpha_3^{-1}(m_Z)$ % error	$\sin^2 \theta_W(m_Z)$ % error
$(3, 2, 0, 2)$	250	71 – 250	$1.7 - 2.8 \times 10^{16}$	+5.0%, –3.7%	+1.5%, –1.5%
	500	22 – 216	$1.5 - 3.6 \times 10^{16}$	+4.9%, –3.6%	+1.5%, –1.5%
	1000	7 – 64	$1.3 - 3.1 \times 10^{16}$	+4.8%, –3.5%	+1.5%, –1.5%
	1500	3 – 31	$1.2 - 2.8 \times 10^{16}$	+4.7%, –3.5%	+1.5%, –1.5%

TABLE III: Solutions for m_V and Λ for $m_{susy} > m_V$. Here m_V refers to the fermionic components of the vector-like sector, while m_{susy} represents a common mass for the superparticles of the ordinary and vector-like sectors. Of the models in Table II, only the $(3, 2, 0, 2)$ case provides viable solutions.

IV. MODEL BUILDING ISSUES

The results of the previous section indicate that there are values of m_V implied by Eq. (1.1) that are beyond the reach of the LHC, but may be within the reach of future collider experiments, particularly in the case where the superparticle mass scale exceeds the scale m_V . Aside from the extra matter fields, other physics associated with this sector might also be experimentally probed. In this section, we consider two motivations for including an extra gauge group that only affects the heavy fields:⁴ (1) The heavy fields may fall in irreducible representations of the new gauge group, explaining the multiplicity of new particles required to achieve the blow up of the couplings at the scale Λ , and (2) the new sector may

⁴ In the context of strong unification, extensions of the standard model gauge group have been considered for other purposes in Ref. [22].

be chiral under the new gauge groups, rendering it more analogous in structure to the matter sector of the MSSM. Although there are a large number of ways in which either possibility might arise, we consider one example here, based on the $(3, 2, 0, 2)$ model discussed in the previous section.

Regarding the first motivation, we consider the possibility that the duplication of vector-like $\mathbf{10} + \overline{\mathbf{10}}$ pairs in the $(3, 2, 0, 2)$ model is a result of their embedding into a two-dimensional representation of an additional gauge group, which is necessarily non-Abelian. The simplest possibility for the gauge group structure of the model is $G_{SM} \times \text{SU}(2)_X$, where G_{SM} represents the standard model gauge factors. As before, we indicate the standard model charge assignments implicitly and compactly by displaying the $\text{SU}(5)$ multiplets that the heavy matter fields would occupy in a conventional unified theory, even though that is not our assumption. Hence under $\text{SU}(5) \times \text{SU}(2)_X$, we now assume that the extra matter is given by

$$\psi \sim (10, 2) \quad \text{and} \quad \overline{\psi} \sim (\overline{10}, 2) . \quad (4.1)$$

We also introduce two $\text{SU}(2)_X$ doublet Higgs fields that will be responsible for spontaneously breaking the new gauge group factor

$$\phi_1 \sim (1, 2) \quad \text{and} \quad \phi_2 \sim (1, 2) . \quad (4.2)$$

The matter fields in Eq. (4.1) and the new Higgs fields in Eq. (4.2) are separately vector-like, so that these fields may be made massive at any desired scale; it also follows that all chiral gauge anomalies are canceled. Note that the multiplicity of $\text{SU}(2)$ doublets in Eqs. (4.1) and (4.2) is even, which implies that the $\text{SU}(2)_X$ Witten anomaly is absent. Given these assignments, the one-loop beta function for the new gauge factor is positive, allowing for straightforward implementation of the UV boundary condition in Eq. (1.1).

One issue that needs to be addressed in a model like this one is the stability of the extra matter fields. Vector-like $\mathbf{5} + \overline{\mathbf{5}}$ and $\mathbf{10} + \overline{\mathbf{10}}$ pairs have the appropriate electroweak and color quantum numbers to participate in mass mixing with standard model matter fields. The amount of such mixing is arbitrary, and only a small amount is necessary so that the heavy states are rendered unstable, avoiding any cosmological complications. Assigning the matter fields of the heavy sector to multiplets of a new gauge group can have unwanted consequences if these states are rendered exactly stable (or extremely long lived). In the present model, this problem does not arise provided that the new gauge group is spontaneously broken,

since mass mixing is generated via renormalizable couplings involving ψ , the ϕ_i , and the standard model fields identified with a **10**. If embedding in an additional gauge group is used to account for the multiplicity of states in some of the other models that we have considered, the model must also provide for the decay of the heavy states; the $(3, 2, 0, 2)$ models seem to naturally avoid this problem with smallest field content and the potentially simplest symmetry-breaking sector, which is one reason why we focus on this example here.

Note that the numerical results for the $(3, 2, 0, 2)$ model described in Sec. III must be adjusted to take into account the presence of the $SU(2)_X$ gauge group, whose coupling blows up at the same scale as the other gauge couplings and affects their renormalization group running. However, since the effect is only via two-loop terms, we don't expect a dramatic change in our qualitative conclusions. To support this statement, we consider the case where $m_{susy} = m_V$ and take into account the effect of the new gauge group by modifying the supersymmetric RGEs for running between the scales Λ and m_V . In this case, the supersymmetric beta functions become

$$b_i = \begin{pmatrix} \frac{63}{5} & 7 & 3 & 5 \end{pmatrix} , \quad (4.3)$$

$$b_{ij} = \begin{pmatrix} \frac{429}{25} & \frac{33}{5} & \frac{184}{5} & 18 \\ \frac{11}{5} & 67 & 56 & 18 \\ \frac{23}{5} & 21 & 82 & 18 \\ 6 & 18 & 48 & 53 \end{pmatrix} . \quad (4.4)$$

Repeating the analysis of Sec. III, we find only a modest adjustment in the ranges for m_V and Λ , as shown in Table IV below.

Model	m_{SUSY} (TeV)	m_V range (TeV)	Λ range (GeV)	$\alpha_3^{-1}(m_Z)$ % error	$\sin^2 \theta_W(m_Z)$ % error
$(3, 2, 0, 2)$	m_V	198 – 497	$1.6 - 3.6 \times 10^{16}$	+5.6%, –4.1%	+1.6%, –1.5%

TABLE IV: Results for the $(3, 2, 0, 2)$ scenario with $m_V = m_{susy}$ taking into account the effect of the $SU(2)_X$ gauge group.

It is interesting to note that $SU(2)_X$ breaking scale is not tied to the value of m_V in this model, which means it could in principal be much lower. For example, with $\langle \phi \rangle \sim 1$ GeV, the resulting low-energy effective theory would be that of a non-Abelian dark sector with a one- or two-Higgs doublet symmetry-breaking sector. Communication between the visible and

dark sectors would follow from operators generated when the m_V -scale physics is integrated out, suggesting that this sector may have other interesting consequences besides its effect on gauge coupling running. Whether phenomenologically interesting models of this type can be constructed remains an open question.

Finally, we note that a different motivation for an extra gauge factor is to render the m_V -scale physics chiral, so that the structure of the new matter sector is more similar to the rest of the MSSM. In the previous example, we could simply change the charge assignment of $\bar{\psi}$ to

$$\bar{\psi}_1 \sim (\overline{10}, 1) \quad \text{and} \quad \bar{\psi}_2 \sim (\overline{10}, 1) . \quad (4.5)$$

Now the mass terms for the extra matter are generated via Yukawa couplings involving ψ , $\bar{\psi}$ and the ϕ_i ; the vacuum expectation value $\langle \phi \rangle$ is now associated with the scale m_V determined in the RGE analysis. We make one additional modification to the theory, which is to add an additional pair of Higgs fields

$$\phi'_1 \sim (1, 2) \quad \text{and} \quad \phi'_2 \sim (1, 2) . \quad (4.6)$$

The modification in Eq. (4.5) leads to the vanishing of the one-loop beta function for $SU(2)_X$, while Eq. (4.6) restores the desired asymptotic non-freedom. Based on our earlier observations, it is clear that the numerical values for m_V and Λ in this model will be qualitatively similar to those of the other $(3, 2, 0, 2)$ models that we have considered, and we leave further numerical study for the interested reader.

V. CONCLUSIONS

In this paper, we have revisited the possibility that the standard model gauge couplings reach a common Landau pole in the ultraviolet. This provides a predictive framework for relating the values of the gauge couplings at the weak scale, without the necessary assumption of conventional grand unification. To implement this framework, all the gauge couplings must be asymptotically non-free, which implies that new matter must be included in the theory. We have numerically explored the possibility that this new matter appears at two scales, a light superpartner mass scale m_{susy} , and the scale where additional heavier vector-like states appear, m_V . We have revisited a scenario considered in the past in which the minimal supersymmetric standard model is enlarged by a single vector-like generation and

found that either m_{susy} or m_V falls below potentially relevant LHC lower bounds on colored MSSM superparticles or vector-like quarks. Although one cannot rule out the possibility that these states are present and have evaded detection for model-specific reasons, we are motivated to consider a safer possibility: we include a relatively small additional amount of extra heavy matter, which leads to solutions for m_V that are beyond the reach of the LHC, but potentially within the reach of a higher-energy hadron collider. For example, given a heavy sector consisting in total of five $\mathbf{5} + \overline{\mathbf{5}}$ pairs, we obtain successful gauge coupling predictions for $m_{susy} = m_V \approx 13$ TeV. For a heavy sector of two $\mathbf{10} + \overline{\mathbf{10}}$ pairs, we can achieve m_V as low as 3 TeV, if we allow higher values of $m_{susy} \approx 1500$ TeV. (In this case, m_V refers to the fermionic components of the vector-like sector, while m_{susy} represents a common mass for the superparticles of the ordinary and vector-like sectors.)

We also considered whether the size of the new matter sector could be related to its embedding into the irreducible representation of an additional non-Abelian gauge group. We presented the simplest model that was consistent with our numerical solutions, a model with two $\mathbf{10} + \overline{\mathbf{10}}$ pairs, in which this duplication is due to their embedding in the fundamental representation of a new SU(2) gauge group. In the case where the heavy matter sector is vector-like under the new SU(2), the new gauge group can be broken at a much lower scale and the effective theory is that of a spontaneously broken non-Abelian dark sector. In the case where the heavy matter sector is chiral under the new SU(2), m_V is associated with the symmetry breaking scale. In this case, new heavy gauge bosons would be among the spectrum of particles that might be sought at a future collider with a suitable reach.

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