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Isospin splittings in baryons with two heavy quarks

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ABSTRACT

Isospin splittings in baryons with two heavy quarks and a u or d quark are calculated using simple methods proposed previously by the authors. The results are $M(\Xi_{cc}^{++}) - M(\Xi_{cc}^{+}) = 1.41 \pm 0.12^{+0.76}$ MeV, $M(\Xi_{bb}^{0}) - M(\Xi_{bb}^{-}) = -4.78 \pm 0.06^{+0.03}$ MeV, and $M(\Xi_{bc}^{+}) - M(\Xi_{bc}^{0}) = -1.69 \pm 0.07^{+0.39}$ MeV, where the statistical errors reflect uncertainties in input mass splittings, and the systematic errors are associated with the choice of constituent-quark masses.

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I Introduction

Baryons with more than one heavy quark have proved to be elusive. The SELEX collaboration has presented evidence for several states [1–3], but other experiments have not confirmed them [4–8]. Simple constituent-quark models incorporating effective quark masses, hyperfine interactions, and estimates of binding energies [9, 10] have proved remarkably successful in reproducing the masses of known hadrons with accuracies of several MeV. In agreement with most other estimates [11–35] including ones using lattice gauge theory [36–44], this method [45] gives masses of ccq ($q = u, d$) about 100 MeV above the SELEX values, and close to the most recent lattice estimates [44].

The capability of the LHCb experiment to identify hadrons containing heavy quarks makes it a prime instrument for determining the masses of the lowest $\Xi_{cc}^{++} = ccu$ and $\Xi_{cc}^{+} = ccd$ states. As a benchmark, Ref. [45] predicts $M(\Xi_{cc}) = 3627 \pm 12$ MeV for their isospin average. Their isospin splitting is then of interest, both as a theoretical question and as a guide to further observation. In particular, the SELEX Collaboration reports

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Table I: Experimental mass splittings between octet baryons [50].

Splitting	Symbol	Value (MeV)
$M(p) - M(n)$	N_1	-1.2933
$M(\Sigma^+) - M(\Sigma^-)$	Σ_1	-8.08 ± 0.08
$M(\Sigma^+) - 2M(\Sigma^0) + M(\Sigma^-)$	Σ_2	1.535 ± 0.090
$M(\Xi^0) - M(\Xi^-)$	Ξ_1	-6.85 ± 0.21

large splittings whose values depend on which of several bumps are assigned to the lowest Ξ_{cc} states [46]. In the present paper we apply some simple methods, used with previous success (see also [47]), to estimate isospin splittings in the ground-state Ξ_{cc}, Ξ_{bb}, and Ξ_{bc} baryons. We describe the methods in Sec. II, present an alternative set of input parameters in Sec. III, quote results in Sec. IV, and conclude in Sec. V.

II Methods

The impending improvement in the mass of the Ξ^0 baryon by the NA48 experiment at CERN [48] and the KTeV experiment at Fermilab led one of us [49] to consider improved tests of relations for baryon isomultiplet splittings. A simple model was adopted which took into account the intrinsic difference $\Delta = (1-\frac{K}{m})(m_u - m_d)$ between u and d quarks, Coulomb interactions $\Delta E_{ij\ \rm em} = \alpha Q_i Q_j \langle 1/r_{ij} \rangle$ between quarks, strong hyperfine (HF) interactions

$$\Delta E_{ij\ \rm HFs} = {\rm const} \times \frac{|\Psi_{ij}(0)|^2 \langle \sigma_i \cdot \sigma_j \rangle}{m_i m_j} ~, \tag{1}$$

and electromagnetic HF interactions

$$\Delta E_{ij\ \rm HFe} = -\frac{2\pi\alpha Q_i Q_j |\Psi_{ij}(0)|^2 \langle \sigma_i \cdot \sigma_j \rangle}{3 m_i m_j} ~, \tag{2}$$

where symbols are defined in Ref. [49]. We use the observed mass splittings among the octet baryons [50], labeled with subscripts denoting their ΔI values, summarized in Table I, to define the relative strengths of each contribution. The correction term $1 - \frac{K}{m}$ in the definition of Δ takes account of quark mass effects on kinetic energy; small corrections to singly- and doubly-strange baryons, discussed in [47], are neglected.

Each of these splittings may be expressed as a function of four unknowns: Δ (intrinsic $u - d$ mass difference including effect on kinetic energies), a (Coulomb interaction), b (strong HF interaction), and c (electromagnetic HF interaction), where we have simplified the notation of Ref. [49] and neglected effects of two-body kinetic energy operators:

$$N_1 = \Delta + \frac{a}{3} + b\left(\frac{1}{m_u^2} - \frac{1}{m_d^2}\right) + \frac{c}{9}\left(\frac{4}{m_u^2} - \frac{1}{m_d^2}\right) \tag{3}$$

$$\Sigma_1 = N_1 + \Xi_1 \tag{4}$$

$$\Sigma_2 \simeq a + \frac{c}{\bar{m}^2} \tag{5}$$

$$\Xi_1 = \Delta - \frac{2a}{3} + b\left(\frac{4}{m_d m_s} - \frac{4}{m_u m_s}\right) + \frac{c}{9}\left(\frac{4}{m_d m_s} + \frac{8}{m_u m_s}\right) ~, \tag{6}$$

Table II: Contributions to isospin splittings (MeV) using universal constituent-quark masses in mesons and baryons.

	N_1	Σ_1	Σ_2	Ξ_1	$\Xi_{cc,1}$	$\Xi_{bb,1}$	$\Xi_{bc,1}$
$m_u - m_d$	–2.68	–5.36	0.00	–2.68	–2.68	–2.68	–2.68
Coulomb	0.94	–0.94	2.83	–1.89	3.77	-1.89	0.94
StrHF	0.88	-0.24	0.00	-1.12	-0.33	–0.11	–0.22
EMHF	–0.43	–1.54	–1.30	–1.11	0.64	–0.11	0.27
Total	–1.293	–8.086	1.535	-6.793	1.409	–4.783	-1.687
					±0.116	±0.058	±0.067

where $\bar{m}$ is the average of m_u and m_d, and we have neglected a term of second order in Δ in Σ_2. We have written a shorthand for Σ_1 since under the present assumptions it satisfies the Coleman-Glashow relation $\Sigma_1 = N_1 + \Xi_1$ [51] and is not independent. Given quark masses and an estimate of strong hyperfine structure from the splitting between the Δ resonance and the nucleon (fixing b), one can determine the three free parameters Δ, a, and $\gamma \equiv c/\bar{m}^2$.

Similar methods lead to estimates for isospin splittings in baryons with two heavy quarks. The results, after neglecting terms of second order in Δ, and defining $\beta \equiv b/\bar{m}^2$, are

$$\Xi_{cc,1} \equiv M(\Xi_{cc}^{++}) - M(\Xi_{cc}^{+}) = \Delta + \frac{4a}{3} + \frac{4\beta\Delta}{m_c} - \frac{8\gamma\bar{m}}{3m_c} , \quad (7)$$

$$\Xi_{bb,1} \equiv M(\Xi_{bb}^{0}) - M(\Xi_{bb}^{-}) = \Delta - \frac{2a}{3} + \frac{4\beta\Delta}{m_b} + \frac{4\gamma\bar{m}}{3m_b} , \quad (8)$$

$$\Xi_{bc,1} \equiv M(\Xi_{bc}^{+}) - M(\Xi_{bc}^{0}) = \Delta + \frac{a}{3} + 2\beta\Delta\left(\frac{1}{m_c} + \frac{1}{m_b}\right) + \frac{\gamma\bar{m}}{3}\left(\frac{2}{m_b} - \frac{4}{m_c}\right) . \quad (9)$$

In order to specify Δ, a, and γ we must choose a set of constituent-quark masses. This was done in Ref. [52], in two models, depending on whether or not a universal set of masses was chosen for mesons and baryons. In this section we shall consider quark masses which fit both baryons and mesons simultaneously, with an added "string-junction" contribution $S = 161.5$ MeV for baryons. Such an additive constant does not affect mass *differences*, with which we are concerned here. (The alternative set is considered in the next section.) Thus we take $\bar{m} = 308.5$ MeV, $m_s = 482.2$ MeV, $\beta = 50.4$ MeV, $m_c = 1655.6$ MeV, and $m_b = 4988.6$ MeV. A fit to octet baryon masses then gives $\Delta = -2.681$ MeV, $a = 2.830$ MeV, $\gamma = -1.295$ MeV, and contributions summarized in Table II. Here we have fixed N_1 at its measured value of -1.2933 MeV, as its experimental error is negligible. The uncertainties are those generated by varying each octet-baryon splitting by 1σ and adding the errors in quadrature.

Note that the $\Delta I = 2$ mass difference is fitted *exactly*. The χ^2 for this fit is 0.083, of which 0.010 comes from Σ_1 and 0.073 comes from Ξ_1. This is just the extent to which the Coleman-Glashow relation is obeyed.

Table III: Contributions to isospin splittings (MeV) using separate constituent-quark masses in mesons and baryons.

	N_1	Σ_1	Σ_2	Ξ_1	$\Xi_{cc,1}$	$\Xi_{bb,1}$	$\Xi_{bc,1}$
$m_u - m_d$	–2.48	–4.95	0.00	–2.48	–2.48	–2.48	–2.48
Coulomb	1.02	–1.02	3.05	–2.04	4.07	–2.04	1.02
StrHF	0.67	–0.24	0.00	–0.91	–0.29	–0.10	–0.19
EMHF	–0.51	–1.88	–1.52	–1.37	0.86	–0.15	0.36
Total	–1.293	–8.086	1.535	–6.793	2.167	–4.754	–1.293
					±0.109	±0.058	±0.062

III Alternative parameters

In a model in which mesons and baryons are described by separate constituent-quark masses [52], the parameters are $\bar{m} = 363.7$ MeV, $m_s = 536.3$ MeV, $\beta = 49.3$ MeV, $m_c = 1710.5$ MeV, and $m_b = 5043.3$ MeV. The fit gives $\Delta = -2.476$ MeV, $a = 3.053$ MeV, and $\gamma = -1.518$ MeV. The results are shown in Table III. The uncertainties are those generated by varying each octet-baryon splitting by 1σ and adding the errors in quadrature.

The fit again reproduces the value of Σ_2 exactly, obtains the same values for Σ_1 and Ξ_1, and thus has the same individual and overall χ^2 values.

IV Results

A slight preference for the string-based constituent-quark masses was expressed in Ref. [52]. Hence we shall quote predictions for isospin splittings based on that model, with a systematic error associated with the possible choice of independent constituent-quark masses for mesons and baryons. The results are: $M(\Xi_{cc}^{++}) - M(\Xi_{cc}^{+}) = 1.41 \pm 0.12^{+0.76}$ MeV, $M(\Xi_{bb}^{0}) - M(\Xi_{bb}^{-}) = -4.78 \pm 0.06^{+0.03}$ MeV, and $M(\Xi_{bc}^{+}) - M(\Xi_{bc}^{0}) = -1.69 \pm 0.07^{+0.39}$ MeV. The first error is the greater of two very similar statistical errors in Tables II and III.

Some approaches give values consistent with ours. Ref. [53] finds $\Xi_{cc,1} = 2.3 \pm 1.7$ MeV, $\Xi_{bb,1} = -5.3 \pm 1.1$ MeV, and $\Xi_{bc,1} = -1.5 \pm 0.9$ MeV. Ref. [46] finds 1.5 ± 2.7 MeV, -6.3 ± 1.7 MeV, and -0.9 ± 1.8 MeV for these quantities, while a lattice-QCD-based approach [54] finds $\Xi_{cc,1} = (2.16)(11)(17)$ MeV, slightly favoring our set of independent quark masses for mesons and baryons. These results, along with some others, are compared in Table IV.

V Discussion and conclusions

We have estimated isospin mass splittings in baryons Ξ_{cc}, Ξ_{bb}, and Ξ_{bc} containing two heavy quarks. A major source of systematic error, particularly in $\Xi_{cc,1} \equiv M(\Xi_{cc}^{++}) - M(\Xi_{cc}^{+})$, is uncertainty in the choice of constituent-quark mass, giving $\Xi_{cc,1} = 1.41$ MeV for our favored model of universal quark masses in mesons and baryons, while separate quark masses for mesons and baryons yield $\Xi_{cc,1} = 2.17$ MeV.

One assumption we have made concerns the universality of the expectation value $\langle r_{ij} \rangle$ in evaluating the Coulomb self-energy. It is possible that two heavy quarks are more

Table IV: Comparison of predictions for isospin splittings (MeV) in doubly heavy baryons.

Reference	$\Xi_{cc,1}$	$\Xi_{bb,1}$	$\Xi_{bc,1}$
This work	$1.41 \pm 0.12^{+0.76}$	$-4.78 \pm 0.06^{+0.03}$	$-1.69 \pm 0.07^{+0.39}$
[23]	4.7		
[46]	1.5 ± 2.7	-6.3 ± 1.7	-0.9 ± 1.8
[53][a]	2.3 ± 1.7	-5.3 ± 1.1	-1.5 ± 0.9
[54]	$2.16 \pm 0.11 \pm 0.12$		
[55]	4.7		
[56]	1.11		
[57]	-9		

[a] Ignores EM hyperfine interactions.

tightly bound to one another than a light quark and a heavy one or two light quarks. To lowest order, this should not affect isospin splittings. However, the difference between binding of two light quarks from binding of a heavy quark with a light one remains to be tested. A start on this program was made in Sec. VI of Ref. [49]. A relation $\Sigma_{c2} \equiv M(\Sigma_c^{++}) - 2M(\Sigma_c^+) + M(\Sigma_c^+) = \Sigma_2$ was found there to be poorly obeyed, but now reads (1.92 ± 0.82) MeV $= (1.535 \pm 0.090)$ MeV, in satisfactory agreement with the predicted equality.

It is worth recalling predicted lifetimes of baryons with two heavy quarks, as the states with longer lifetimes are likely to be easier to distinguish from background in a hadron collider. Predictions by the authors are given in Table XVI of Ref. [45], including $\tau(\Xi_{cc}^{++}) =$ 185 fs and $\tau(\Xi_{cc}^+) = 53$ fs. Most other predictions quoted there are about three times as large, while preserving the ratio $\tau(\Xi_{cc}^{++})/\tau(\Xi_{cc}^+) \simeq 3$. The reason for the shorter lifetime of $\Xi_{cc}^+ = ccd$ is that the internal W exchange process $cd \to su$ is permitted, while it cannot occur for $\Xi_{cc}^{++} = ccu$. For a similar reason, one expects $\tau(\Xi_{bc}^+) > \tau(\Xi_{bc}^0)$ whereas $\tau(\Xi_{bb}^0) \simeq \tau(\Xi_{bb}^-)$.

We hope that these estimates prove of use in discovery of such states.

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