Exploring correlations in the CGC wave function: Odd azimuthal anisotropy

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Exploring correlations in the CGC wave function: odd azimuthal anisotropy.

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We extend the CGC approach to calculation of the double inclusive gluon production by including high density effect in the CGC wave function of the projectile (proton). Our main result is that these effects lead to the appearance of odd harmonics in the two particle correlation $C(k,p)$. We find that in the high momentum limit, $|k|,|p| \gg Q_s$, this results in a positive $c_1\{2\}$. Additionally when the magnitudes of the two momenta are approximately equal, $|k|/|p| \approx 1$, the density effects also generate a positive third harmonic $c_3\{2\}$, which translates into a non-vanishing v_3 when the momenta of the trigger and associated particle are in the same momentum bin. The sign of $c_3\{2\}$ becomes negative when $|k|/|p| > 1.1$ suggesting an interesting experimental signature.

I. INTRODUCTION.

Currently one of the outstanding questions of strong interactions is the origin of long range rapidity correlations observed in p-p and p-Pb collisions at LHC. Starting with the observation of the so-called ridge correlations in high multiplicity p-p collisions by CMS [1], this phenomenon generated a lot of interest. Subsequent observation of the same effect in p-Pb collisions by all three big experiments at LHC sharpened the questions even further [2–8]. In particular the observation that the triangular "flow" coefficient v_3 is practically identical in p-Pb and Pb-Pb collisions at the same total multiplicity strongly suggests the origin of the correlations is due to some collective or quasi-collective behavior. The recent experimental results are even more surprising [9, 10]; with an improved subtraction of hard component, the ridge type correlations are now seen in the minimal bias p-p data, and even in events with lower than average multiplicity.

One possibility is that the collectivity is the result of strong final state interactions, as is suggested by good hydrodynamics fits to the data [11–14]. This is surprising in view of the fact that the correlations are observed up to relatively high transverse momenta $k \sim 10$ GeV, and also in events where the number of produced particles is small. The transport model results presented in Ref. [15] show that final state interactions even with modest parton-parton cross section describe well some aspects of the data. Another possibility is that the correlated features of production are due to preexisting correlations in the wave function of the colliding hadrons. This last possibility has been intensively studied during the last several years in the framework of the Color Glass Condensate [16–23].

The CGC-based calculations have successfully described the "ridge" data [21, 22]. Nevertheless the CGC approach faces serious challenges in describing other aspects of data. Perhaps the most challenging aspect is description of multi-particle correlations, in particular $v_2\{4\}$ [1]. The mechanism utilized in the numerical work of Refs. [21, 22] is not capable of producing this quantity as has been shown in Refs. [25, 26]. This is currently an open question, and we do not have anything to add to this part of discussion.

Another enduring problem of the CGC-based approaches to correlations has been a non-vanishing value of the triangular "flow" coefficient v_3 clearly observed in the data. All the approaches in Refs. [16–23] predict that the double inclusive production is symmetric under reversal of the direction of one of the transverse momenta [2]

$$\sigma(k,p) = \sigma(k,-p) \tag{1}$$

[1] Here we follow the notation of Ref. [24]:

$$c_n\{2\} \equiv v_n^2\{2\} = \langle \exp\left[in(\phi_1-\phi_2)\right]\rangle,$$
$$c_n\{4\} \equiv -v_n^4\{4\} = \langle \exp\left[in(\phi_1+\phi_2-\phi_3-\phi_4)\right]\rangle - 2\langle \exp\left[in(\phi_1-\phi_3)\right]\rangle\langle \exp\left[in(\phi_2-\phi_4)\right]\rangle.$$

[2] Here and thereafter, in order to simplify the notation, we denote the transverse two-dimensional vectors by k, that is $k \equiv \vec{k}_\perp$ and $|k| \equiv |\vec{k}_\perp|$. For the scalar and the cross products we use $k\cdot q \equiv k_i q_i$ and $k\times q = \epsilon_{ij} k_i q_j$ correspondingly, where $1 \le i,j \le 2$.

even on the configuration-by-configuration basis. Such symmetry precludes existence of odd harmonics. Although this is not a fundamental symmetry of QCD in any way, it proved to be very stubborn and difficult to avoid in CGC without including final state interactions. It was recently shown analytically in Ref. [27] that odd harmonics are indeed generated in the double inclusive production when classical evolution of the Yang-Mills fields in the final state is accounted for. This conclusion is consistent with earlier numerical work [28], although in the latter case there is some uncertainty as to whether the results reflect evolution to asymptotically large times. [3]

The purpose of the present paper is to point out that in fact final state interactions are not essential to generate odd harmonics within the CGC approach. Our central point is that all the CGC-based calculations so far have used (some implicitly) the form of the CGC wave function which is appropriate for description of dilute projectile only. This "dilute CGC" wave function leads to the accidental symmetry alluded to earlier. Corrections to this dilute limit on the wave function level have been calculated a while ago [32, 33]. In particular it was shown in Refs. [32, 33] that these corrections are essential to reproduce the JIMWLK evolution equation (see Refs. [34–38] and [39–41]) when the evolution is generated by boosting the dense hadron. In the present paper we use this improved CGC wave function to calculate the double inclusive gluon distribution.

We were inspired to consider this departure from the commonly employed dilute CGC state by an old work by Kharzeev, Levin and McLerran (KLM) [42]. The idea suggested in Ref. [42] is that a wave function of a dense object can naturally incorporate nontrivial correlations. Consider a fluctuation in such a wave function which contains a high p parton. The transverse momentum of this parton has to be balanced in this component of a wave function. In a dilute system it is most likely to be balanced by another hard parton with momentum $-p$. However in a dense environment it is more likely that the balancing transverse momentum is shared by several semi hard partons. Although the original suggestion in Ref. [42] was that the transverse momentum is distributed between semi hard partons at different rapidities, the extent of this rapidity spread may be not too large. The momentum distribution in such a component of the wave function looks somewhat like a directed flow, with the direction defined by the momentum of the hard fluctuation. If the number of the balancing semi hard partons is large enough the two particle correlation function should exhibit a maximum for same sign transverse momenta. This picture suggests a positive $c_1\{2\}$, and possibly a non-vanishing $c_3\{2\}$ in this type of components of a dense wave function. In fact with "directional flow" of the kind described above one may expect a non-vanishing contribution to $c_2\{4\}$ as well, although this effect will be presumably smaller. Of course it is a quantitative question whether these signatures survive the averaging over all components of the wave function, and later the production process, and how large the net effect is.

In any case, it appears that the dilute CGC wave function does not encode the KLM-type correlations. It is not all that surprising, given that the effect requires the partonic system to be dense enough. Hence the motivation to consider high density corrections to the dilute CGC limit.

We find that when the improved wave function is used to calculate the double inclusive gluon production the accidental symmetry present in the dilute limit disappears. This leads to appearance of the odd harmonics in the two particle correlation function. We find (as in Ref. [27]) that the odd harmonics ($c_1\{2\}$ and $c_3\{2\}$) are suppressed relative to the even (e.g. $c_2\{2\}$) parametrically by a factor of α_s. This is roughly consistent with the experimentally observed hierarchy between v_2 and v_3. We calculate $c_1\{2\}$ and $c_3\{2\}$ in the limit of high transverse momenta.

The paper is structured as follows. In Sec. 2 we describe the improved CGC wave function, and calculate the antisymmetric (in $(k, p) \to (k, -p)$) piece in the gluon pair density in the wave function. We show that this piece does not vanish. The sign of the first harmonic is positive, while the sign of the third harmonic is negative. In Sec. 3 we calculate the double inclusive gluon production for eikonal scattering on a target. We then expand the general expressions in the limit of large transverse momentum of produced particles. Interestingly, we find that to leading power in Q_s^2/p^2 the antisymmetric part of the correlation function is saturated by a charge conjugation odd "condensate", i.e. the Odderon. Since the contribution of the Odderon is expected to be subleading at high energy, we calculate the next order in the expansion, and estimate it using a simple model for target averaging. We find that $c_1\{2\}$ is positive, while $c_3\{2\}$ is negative when $|k|/|p| > 1.1$ and $|p|/|k| > 1.1$, but is positive for $1.1 > |k|/|p| > 1/1.1$. We close by discussing our results and their implications in Sec. 4.

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[3] We mention two recent papers [29, 30], which purportedly obtain nonvanishing v_3 without invoking final state interactions. We failed to understand the theoretical framework of Ref. [29] and thus cannot comment on it. On the other hand Ref. [30] has focused on production of identical gluons (same color and same polarization). It thus did not include some of the graphs which generally contribute to total double inclusive production. In particular it does not include a subset of graphs with the amplitude in a color singlet state near the produced gluon rapidity (which appear as diffractive graphs within the rapidity bin that includes the two observed gluons, see Ref. [31]). It is well known that within the glasma graph calculations these "diffractive" contributions generate a correlation at $\Delta\phi = \pi$ identical in strength to that generated by "non-diffractive" graphs at $\Delta\phi = 0$, see for example Ref. [31]. Thus in the framework of the glasma graphs inclusion of these additional contributions renders all odd harmonics to vanish.

II. GLUON PAIR DENSITY IN THE CGC WAVE FUNCTION.

A. The CGC wave function.

We start with discussing the wave function of the vacuum of the soft gluons in the background of strong valence color charge density.

The CGC calculations so far have relied on a simplified version of this wave function valid in the situation when the color charge density is weak and perturbative. In this regime one can diagonalize the QCD Hamiltonian pertubatively with the result that the vacuum of the soft modes is a coherent state of the form [43]

$$|\mathrm{CGC}\rangle_{\mathrm{dilute}} = \mathcal{C}|0\rangle \ , \tag{2}$$

where $\mathcal{C}$ is the displacement operator defined as

$$\mathcal{C} = e^{i\sqrt{2}\int_k b_{\alpha i}(-k)\left[a^\dagger_{\alpha i}(k) \ + \ a_{\alpha i}(-k)\right]}. \tag{3}$$

Here $|0\rangle$ is the light cone vacuum of the soft modes, and the soft gluon creation and annihilation operators represent the rapidity independent (rapidity averaged) mode of the soft gluon field over the soft rapidity interval, that is

$$a_{\alpha i}(k) \equiv \frac{1}{\sqrt{Y}}\int \frac{d\eta}{2\pi} a_{\alpha i}(\eta, k) \ . \tag{4}$$

The Weizsäcker-Williams field $b_{\alpha i}$ [4] is generated by valence color charges of a hadronic projectile

$$\partial_i b_{\alpha i}(x) = \rho_\alpha(x). \tag{5}$$

The field is two-dimensional pure-gauge:

$$b_{\alpha i}(x) = -\frac{1}{g} f_{\alpha\beta\delta} U^+_{\beta\gamma}(x)\partial_i U_{\gamma\delta}(x),$$

where U is a SU(3) element in the adjoint representation.

It has been known for a while that Eq. (2) is not an appropriate wave function for a dense system. In particular it was shown by direct calculation in Refs. [32, 33] that in the soft gluon wave function the coherent state as in Eq. (2) is accompanied by a Gaussian factor, so that the state is a coherent Bogoliubov (squeezed) state. The squeezing is crucial to reproduce the JIMWLK evolution equation of a dense object at high energy, see Refs. [34–38] and [39–41].

The original presentation in Refs. [32, 33] is quite complicated. It involves calculation of the wave function of all soft gluon rapidity modes in the soft rapidity interval. On the other hand it is clear that only the rapidity independent mode is large, and it is this mode that is responsible for high energy evolution. This is obvious in Eq. (2). One therefore expects that it should be possible to "integrate out" all the modes of the gluon field perturbatively except for the rapidity independent one.

Such a first principle calculation has not been done. However there is a simplified way to infer the reduced wave function of the rapidity integrated gluon mode using the results of Refs. [32, 33]. The important point is that the general wave function derived in Refs. [32, 33] is Gaussian and integrating over part of the field modes is bound to lead to a Gaussian shape of the reduced wave function as well. Therefore in order to find the reduced wave function we ask what is the Gaussian wave function that depends only on the rapidity integrated field and which reproduces the JIMWLK evolution equation. The answer is that the following reduced wave function fits the bill [5]

$$\Psi_{\mathrm{CGC}}[\phi] = \langle\phi|\Omega|0\rangle = \ e^{i2\int_k b_{\alpha i}(-k)\phi_{\alpha i}(k)}\langle\phi|\mathcal{B}\rangle \ , \tag{6}$$

where $\Omega = \mathcal{C}\mathcal{B}$ is a unitary operator and for convenience we separately defined the Gaussian state $|\mathcal{B}\rangle$ as [6]

$$|\mathcal{B}\rangle \equiv \mathcal{B}|0\rangle; \qquad \langle\phi|\mathcal{B}\rangle = \mathcal{N}e^{-\frac{1}{2}\int_{k,p} B^{-1}_{\alpha\beta ij}(k,p)\phi_{\alpha i}(k)\phi_{\beta j}(p)} \ . \tag{7}$$

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[4] The Fourier transform of the classical field is defined as $b^\alpha_i(k) = \int d^2x e^{-ikx} b^\alpha_i(x)$.

[5] We reiterate that to tie loose ends it would be desirable to derive this wave function directly from the results of Refs. [32, 33] by integrating out the rapidity dependent modes of the soft gluon field. In the present paper we will not undertake this endeavour but will rather content ourselves with the simplified argument given above.

[6] Using the relation (16) one can recognise $\langle\phi|\mathcal{C}|0\rangle$ in the prefactor in Eq. (6).

Here the field ϕ is defined as

$$\phi_{i\alpha}(k) = \frac{1}{\sqrt{2}}\left(a^\dagger_{i\alpha}(k) + a_{i\alpha}(-k)\right), \tag{8}$$

The operator $\mathcal{B}$ is a unitary operator, exponential of a quadratic function of the creation and annihilation operators $a(k)$ and $a^\dagger(k)$. We do not write it out explicitly as the knowledge of the wave function in field space is sufficient for our purposes.

The constant $\mathcal{N}$ is the normalization factor and the operator B is given by

$$B = (1 - \mathfrak{l} - L)^2 = 1 - \mathfrak{l} - L + [\mathfrak{l}, L]_+ \ , \tag{9}$$

where the longitudinal projector in coordinate space is

$$\mathfrak{l}^{\alpha\beta}_{ij}(x,y) \equiv \delta^{\alpha\beta}\frac{\partial_i\partial_j}{\partial^2}(x,y) \tag{10}$$

and

$$L^{\alpha\beta}_{ij}(x,y) = U^{\alpha\gamma}(x)\frac{\partial_i\partial_j}{\partial^2}(x,y)U^{\dagger\,\gamma\beta}(y) = D^{\alpha\gamma}_i\left[\frac{1}{D^2}\right]^{\gamma\lambda}D^{\lambda\beta}_j \ . \tag{11}$$

Here the covariant derivative is given by

$$D^{\alpha\beta}_i(x) = \delta^{\alpha\beta}\partial_i - gf^{\alpha\beta\gamma}b^\gamma_i(x) \ . \tag{12}$$

Note that the CGC state in Eq. (6) is not specified in terms of the light cone vacuum ket , but rather the wave function is written in the "field representation", meaning that it is a function of the field $\phi(k)$. The calculation of various expectation values is performed by a functional integration over the field ϕ. In the weak field limit, where the eikonal factor is trivial $U(x) = 1$, we have $B(x-y) = \delta^2(x-y)$, or in momentum space $B(k,p) = (2\pi)^2\delta^2(k+p)$, and the Gaussian state $|\mathcal{B}\rangle$ in Eq. (6) becomes the light cone vacuum state, see also Eq. (2). In the dense regime $b \sim 1/g$ and the correction due to a nontrivial squeezing parameter is an order one effect.

B. The pair density

Our first task is to see what is the effect of the nontrivial squeezing on the gluon pair density in the CGC wave function.

The single gluon number density in the CGC wave function is given by the following formal expression

$$f(k) = \mathcal{N}\int D\rho W_{\rm P}[\rho]\ f_\rho(k) = \mathcal{N}\int D\rho W_{\rm P}[\rho]\ \langle {\rm CGC}|a^\dagger_{i\alpha}(k)a_{i\alpha}(k)|{\rm CGC}\rangle \ , \tag{13}$$

where $\mathcal{N}$ is the normalisation factor and we explicitly show the ensemble average with a weight functional $W_{\rm P}[\rho]$ characterising the distribution of the projectile color sources, ρ.

Similarly, we define the gluon pair density as

$$f(k,p) = \mathcal{N}\int D\rho W_{\rm P}[\rho]\ f_\rho(k,p) = \mathcal{N}\int D\rho W_{\rm P}[\rho]\ \langle {\rm CGC}|a^\dagger_{i\alpha}(k)a^\dagger_{j\beta}(p)a_{i\alpha}(k)a_{j\beta}(p)|{\rm CGC}\rangle \ . \tag{14}$$

Our normalisation of creation and annihilation operators is such that

$$[a_{i\alpha}(k), a^\dagger_{j\beta}(p)] = (2\pi)^2\delta_{ij}\delta_{\alpha\beta}\delta^2(k-p) \ . \tag{15}$$

We define the field variable and its conjugate momentum

$$\begin{aligned}\phi_{i\alpha}(k) &= \frac{1}{\sqrt{2}}\left(a^\dagger_{i\alpha}(k) + a_{i\alpha}(-k)\right), \\ \pi_{i\alpha}(k) &= \frac{i}{\sqrt{2}}\left(a^\dagger_{i\alpha}(k) - a_{i\alpha}(-k)\right),\end{aligned} \tag{16}$$

so that

$$[\phi_{i\alpha}(k), \pi_{j\beta}(p)] = i(2\pi)^2\delta_{ij}\delta_{\alpha\beta}\delta^2(k+p) \tag{17}$$

and the inverse

$$\begin{aligned} a_{i\alpha}(k) &= \frac{1}{\sqrt{2}}\left(\phi_{i\alpha}(-k) + i\pi_{i\alpha}(-k)\right), \\ a^\dagger_{i\alpha}(k) &= \frac{1}{\sqrt{2}}\left(\phi_{i\alpha}(k) - i\pi_{i\alpha}(k)\right). \end{aligned} \tag{18}$$

To calculate the expectation values in Eq. (14) we use the following properties of the displacement operator (3):

$$\begin{aligned} \mathcal{C}^\dagger a_{\alpha i}(q)\mathcal{C} &= a_{\alpha i}(q) + i\sqrt{2}b_{\alpha i}(-q), \\ \mathcal{C}^\dagger a^\dagger_{\alpha i}(q)\mathcal{C} &= a^\dagger_{\alpha i}(q) - i\sqrt{2}b_{\alpha i}(q). \end{aligned} \tag{19}$$

The expectation values in the Gaussian state $|\mathcal{B}\rangle$ are

$$\begin{aligned} \langle\phi_{i\alpha}(k)\phi_{j\beta}(k')\rangle_{\mathcal{B}} &= \frac{1}{2}B_{\alpha\beta ij}(k,k'), \\ \langle\phi_{i\alpha}(k)\pi_{j\beta}(k')\rangle_{\mathcal{B}} &= -i\langle\phi_{i\alpha}(k)\frac{\delta}{\delta\phi_{j\beta}(-k')}\rangle = \frac{i}{2}(2\pi)^2\delta_{ij}\delta_{\alpha\beta}\delta^2(k+k'), \\ \langle\pi_{i\alpha}(k)\phi_{j\beta}(k')\rangle_{\mathcal{B}} &= \langle\phi_{j\beta}(k')\pi_{i\alpha}(k)\rangle_{\mathcal{B}} - \langle[\phi_{j\beta}(k'), \pi_{i\alpha}(k)]\rangle = -\frac{i}{2}(2\pi)^2\delta_{ij}\delta_{\alpha\beta}\delta^2(k+k'), \\ \langle\pi_{i\alpha}(k)\pi_{j\beta}(k')\rangle_{\mathcal{B}} &= \frac{1}{2}B^{-1}_{\alpha\beta ij}(k,k'). \end{aligned} \tag{20}$$

In Eq. (14) we will keep only terms of order $1/g^4$ and $1/g^2$, neglecting all other subleading terms

$$\begin{aligned} f_\rho(k,p) &\approx 4b_{i\alpha}(-k)b_{i\alpha}(k)b_{j\beta}(-p)b_{j\beta}(p) + 2\Big(b_{i\alpha}(-k)b_{i\alpha}(k)\langle a^\dagger_{j\beta}(p)a_{j\beta}(p)\rangle_{\mathcal{B}} + b_{j\beta}(-p)b_{j\beta}(p)\langle a^\dagger_{i\alpha}(k)a_{i\alpha}(k)\rangle_{\mathcal{B}}\Big) \\ &+ 2\Big(b_{i\alpha}(k)b_{j\beta}(-p)\langle a^\dagger_{j\beta}(p)a_{i\alpha}(k)\rangle_{\mathcal{B}} + b_{j\beta}(p)b_{i\alpha}(-k)\langle a^\dagger_{i\alpha}(k)a_{j\beta}(p)\rangle_{\mathcal{B}}\Big) \\ &- 2\Big(b_{i\alpha}(k)b_{j\beta}(p)\langle a_{i\alpha}(k)a_{j\beta}(p)\rangle_{\mathcal{B}} + b_{i\alpha}(-k)b_{j\beta}(-p)\langle a^\dagger_{i\alpha}(k)a^\dagger_{j\beta}(p)\rangle_{\mathcal{B}}\Big). \end{aligned} \tag{21}$$

Recall that we are interested in the odd moments of the correlation function. The only piece that can give a non-vanishing contribution to this quantity is:

$$\begin{aligned} \tilde{f}_\rho(k,p) &= 2\Big(b_{i\alpha}(k)b_{j\beta}(-p)\langle a^\dagger_{j\beta}(p)a_{i\alpha}(k)\rangle_{\mathcal{B}} + b_{j\beta}(p)b_{i\alpha}(-k)\langle a^\dagger_{i\alpha}(k)a_{j\beta}(p)\rangle_{\mathcal{B}}\Big) \\ &- 2\Big(b_{i\alpha}(k)b_{j\beta}(p)\langle a_{i\alpha}(k)a_{j\beta}(p)\rangle_{\mathcal{B}} + b_{i\alpha}(-k)b_{j\beta}(-p)\langle a^\dagger_{i\alpha}(k)a^\dagger_{j\beta}(p)\rangle_{\mathcal{B}}\Big) \\ &= \frac{1}{2}b_{i\alpha}(k)\Big[B_{\alpha\beta ij}(-k,p) + B^{-1}_{\alpha\beta ij}(-k,p) - 2(2\pi)^2\delta_{ij}\delta_{\alpha\beta}\delta^2(p-k)\Big]b_{j\beta}(-p) \\ &+ \frac{1}{2}b_{i\alpha}(-k)\Big[B_{\alpha\beta ij}(k,-p) + B^{-1}_{\alpha\beta ij}(k,-p) - 2(2\pi)^2\delta_{ij}\delta_{\alpha\beta}\delta^2(p-k)\Big]b_{j\beta}(p) \\ &+ \frac{1}{2}b_{i\alpha}(k)\Big[-B_{\alpha\beta ij}(-k,-p) + B^{-1}_{\alpha\beta ij}(-k,-p)\Big]b_{j\beta}(p) \\ &+ \frac{1}{2}b_{i\alpha}(-k)\Big[-B_{\alpha\beta ij}(k,p) + B^{-1}_{\alpha\beta ij}(k,p)\Big]b_{j\beta}(-p), \end{aligned} \tag{22}$$

where $f_\rho(k,p) = \tilde{f}_\rho(k,p) + \{\text{explicitly even part}\}$. Note that the purely "classical" term - the first term in Eq. (21) is of order $1/g^4$, the antisymmetric piece Eq. (22) is of order $1/g^2$. Since the classical term contributes to v_2 (see e.g. Ref. [18]), we may expect the ratio v_3/v_2 to be of order α_s, which is phenomenologically reasonable.

It is easy to see that terms with B^{-1} are even under $p \to -p$. The potential contribution to odd cumulants will be given by the asymmetric part of $\tilde{f}$, i.e.

$$\begin{aligned} \frac{1}{2}\Big(\tilde{f}_\rho(k,p) - \tilde{f}_\rho(k,-p)\Big) &= \frac{1}{2}\Big(b(k)\tilde{B}(-k,p)b(-p) + b(-k)\tilde{B}(k,-p)b(p)\Big) \\ &- \frac{1}{2}\Big(b(k)\tilde{B}(-k,-p)b(p) + b(-k)\tilde{B}(k,p)b(-p)\Big), \end{aligned} \tag{23}$$

where the matrix convolution is implied and $\tilde{B}_{\alpha\beta ij}(k,p) = B_{\alpha\beta ij}(k,p) - (2\pi)^2\delta_{ij}\delta_{\alpha\beta}\delta^2(p+k)$.

Equation (23) is fairly complicated, as the squeezing parameter B is a non-local function of the classical field b. To get some insight into this expression we will evaluate it in the limit of large k and p.

C. The high momentum limit.

Henceforth we will assume that the color charge density is averaged over using the McLerran-Venugopalan (MV) model [44, 45] with the width μ, that is

$$W_{\rm P}[\rho] = \bar{\mathcal{N}} \exp\left\{-\frac{1}{2}\int_k \frac{\rho^a(k)\rho^a(-k)}{\mu^2(k)}\right\} . \tag{24}$$

where $\bar{\mathcal{N}}$ is a normalization factor ensuring the normalization of W_P as the probability density distribution. For the most part we take $\mu^2(k)=\mu^2$, but whenever necessary we will assume that $\mu^2(k)_{k^2/Q_s^2\to 0}\to 0$, which implements the global color neutrality constraint on the MV ensemble.

The high momentum limit then corresponds to the limit $\mu^2/k^2 \ll 1$ and $\mu^2/p^2 \ll 1$. Therefore we need to expand our expression to leading order in μ^2.

First we find the leading contribution to $\tilde{B}(k,p)$ in the expansion in powers of ρ. We start with the key ingredients for the operator L. The square of the covariant derivative is given by

$$\left[D^2\right]^{\alpha\beta} = \delta^{\alpha\beta}\partial^2 - g f^{\alpha\beta\gamma}[\partial_i, b_i^\gamma]_+ + g^2 f^{\alpha\lambda\gamma} f^{\lambda\beta\gamma'} b_i^\gamma b_i^{\gamma'} = \delta^{\alpha\beta}\partial^2 - g[\partial_i, b_i^{\alpha\beta}]_+ + g^2 b_i^{\alpha\gamma} b_i^{\gamma\beta} , \tag{25}$$

where we introduced the adjoint representation for the field b_i: $b_i^{\alpha\beta}\equiv f^{\alpha\beta\gamma}b_i^\gamma$. To second order in the field we get

$$\frac{1}{D^2}\approx\left[1+g\frac{1}{\partial^2}[\partial_i,b_i]_+ - g^2\frac{1}{\partial^2}b^2 + g^2\frac{1}{\partial^2}[\partial_i,b_i]_+\frac{1}{\partial^2}[\partial_i,b_i]_+\right]\frac{1}{\partial^2} . \tag{26}$$

Thus

$$\tilde{B}_{ij} = g^2\left\{-b_i\frac{1}{\partial^2}b_j - \partial_i\frac{1}{\partial^2}b^2\frac{1}{\partial^2}\partial_j + \frac{\partial_i}{\partial^2}(\vec{\partial}\cdot\vec{b})\frac{1}{\partial^2}b_j + b_i\frac{1}{\partial^2}(\vec{b}\cdot\vec{\partial})\frac{\partial_j}{\partial^2} + \frac{\partial_i}{\partial^2}\left[(\vec{b}\cdot\vec{\partial})\frac{1}{\partial^2}(\vec{\partial}\cdot\vec{b}) - (\vec{\partial}\cdot\vec{b})\frac{1}{\partial^2}(\vec{b}\cdot\vec{\partial})\right]\frac{\partial_j}{\partial^2}\right\} . \tag{27}$$

Note that there are no linear terms in b_i. In momentum space this gives

$$\tilde{B}_{ij}(k,p)\equiv\int_{x,y}e^{-ikx}\tilde{B}_{ij}(x,y)e^{-ipy} = g^2\int_q b_n(k-q)\left[\mathfrak{t}_{in}(k)\frac{1}{q^2}\mathfrak{t}_{mj}(p) - \frac{k_i}{k^2}\mathfrak{t}_{nm}(q)\frac{p_j}{p^2}\right]b_m(p+q), \tag{28}$$

where the transverse projector in momentum space reads

$$\mathfrak{t}_{ij}(q) = \delta_{ij} - \frac{q_i q_j}{q^2}. \tag{29}$$

We also have to expand the classical field b in powers of ρ. To leading order we have

$$b_i(k) = k_i c(k) \tag{30}$$

with $c(k)\sim\rho(k)/k^2$. Then

$$\tilde{B}_{ij}(k,p) = -g^2\int_q c(k-q)\left[\mathfrak{t}_{in}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p) + \mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(p)\right]c(p+q), \tag{31}$$

where the longitudinal projector of Eq. (10) in momentum space

$$\mathfrak{l}_{ij}(q) = \frac{q_i q_j}{q^2}. \tag{32}$$

Note that under the approximation (30), we will be interested in the combination $k_i\tilde{B}_{ij}(k,p)p_j$, to which the first term of Eq. (31) does not contribute:

$$k_i\tilde{B}_{ij}(k,p)p_j = -g^2\int_q c(k-q)\left[k_n\mathfrak{t}_{nm}(q)p_m\right]c(p+q). \tag{33}$$

Therefore we obtain the following for the antisymmetric part

$$
\begin{aligned}
&\frac{1}{2}\left(\tilde{f}_\rho(k,p)-\tilde{f}_\rho(k,-p)\right) = \\
&g^2\left[\frac{1}{2}c^\alpha(k)\int_q c^{\alpha\gamma}(-k-q)\left[k_n \mathfrak{t}_{nm}(q)p_m\right]c^{\gamma\beta}(p+q)c^\beta(-p)\right.\\
&+\frac{1}{2}c^\alpha(-k)\int_q c^{\alpha\gamma}(k-q)\left[k_n \mathfrak{t}_{nm}(q)p_m\right]c^{\gamma\beta}(-p+q)c^\beta(p)\\
&+\frac{1}{2}c^\alpha(k)\int_q c^{\alpha\gamma}(-k-q)\left[k_n \mathfrak{t}_{nm}(q)p_m\right]c^{\gamma\beta}(-p+q)c^\beta(p)\\
&\left.+\frac{1}{2}c^\alpha(-k)\int_q c^{\alpha\gamma}(k-q)\left[k_n \mathfrak{t}_{nm}(q)p_m\right]c^{\gamma\beta}(p+q)c^\beta(-p)\right].
\end{aligned}
\tag{34}
$$

Here to avoid confusion the color indices were explicitly shown. As before $c^{\alpha\beta} = f^{\alpha\beta\gamma}c^\gamma$.

The above expression does not vanish configuration-by-configuration in contrast to what one would get by using Eq. (1) of the dilute limit. This is encouraging enough to continue by computing an average with respect to the projectile field using the Gaussian ensemble

$$
\langle c^\alpha(k)c^\beta(k')\rangle_\rho = -\delta^{\alpha\beta}(2\pi)^2\delta^2(k+k')\gamma(k),
\tag{35}
$$

where γ is defined by usual

$$
\langle \frac{1}{-\partial^2}\rho^\alpha(x)\frac{1}{-\partial^2}\rho^\beta(y)\rangle_\rho = \delta^{\alpha\beta}\gamma(x-y).
\tag{36}
$$

In the MV model, we have $\gamma(k) = \frac{\mu^2}{k^4}$.

We finally get

$$
\begin{aligned}
\frac{1}{2}\langle\tilde{f}_\rho(k,p)-\tilde{f}_\rho(k,-p)\rangle_\rho &= -g^2N_c(N_c^2-1)(2\pi)^2S_\perp\gamma(k)\int_q\gamma(q)p_n\mathfrak{t}_{nm}(q-p)p_m\left(\delta^2(p-k)-\delta^2(p+k)\right)\\
&+\; g^2N_c(N_c^2-1)S_\perp\gamma(k)\gamma(p)k_n\left[\mathfrak{t}_{nm}(k-p)+\mathfrak{t}_{nm}(k+p)\right]p_m,
\end{aligned}
\tag{37}
$$

where $S_\perp$ is the transverse area of the projectile.

The first term describes the back-to-back component of the gluon correlation at exactly the same momentum. It has to be treated as the "hard" component to be subtracted from the correlation and does not represent any particular interest for the current study. The combination appearing in the last term can be simplified

$$
k_n\left[\mathfrak{t}_{nm}(k-p)+\mathfrak{t}_{nm}(k+p)\right]p_m = \frac{4k\cdot p}{|k-p|^2|k+p|^2}\left(k\times p\right)^2 = \frac{4k\cdot p}{|k-p|^2|k+p|^2}\left(k^2p^2-(k\cdot p)^2\right).
\tag{38}
$$

Note that for $k=p$

$$
k_n\left[\mathfrak{t}_{nm}(k-p)+\mathfrak{t}_{nm}(k+p)\right]p_m|_{p=k} = k^2\cos(\phi),
\tag{39}
$$

where ϕ is the angle between k and p.

In general we get

$$
k_n\left[\mathfrak{t}_{nm}(k-p)+\mathfrak{t}_{nm}(k+p)\right]p_m = \frac{4k^3p^3\cos(\phi)\sin^2(\phi)}{k^4+p^4-2k^2p^2\cos(2\phi)} = k^2C(z,\phi),
\tag{40}
$$

where

$$
C(z,\phi) = \frac{2z^3\sin(\phi)\sin(2\phi)}{z^4-2z^2\cos(2\phi)+1}; \qquad z\equiv\frac{|p|}{|k|}.
\tag{41}
$$

We plot this function in Fig. 2.

As a proxy to $c_3\{2\}$, our goal is to evaluate

$$
\int d\phi e^{i3\phi}C(z,\phi) = \frac{z^3}{2i}\oint_{|\zeta|=1}d\zeta\frac{\zeta(1+\zeta^2)(1-\zeta^2)^2}{z^2\zeta^4-(1+z^4)\zeta^2+z^2},
\tag{42}
$$

where $\zeta = e^{i\phi}$ was introduced. The poles are located at $\pm z$ and $\pm 1/z$. Therefore for $z < 1$,

$$\int d\phi e^{i3\phi} f(z,\phi) = \pi z^3(z^2-1) \tag{43}$$

and for $z > 1$

$$\int d\phi e^{i3\phi} f(z,\phi) = \pi \frac{1-z^2}{z^3}. \tag{44}$$

We plot the third harmonic in Fig. 3. We also plot the first harmonic $c_1\{2\}$, which can be readily evaluated. We will apply (but not explicitly detail) a similar procedure for the case of particle production in the next section.

We conclude that in the high momentum limit, the first correction to the dilute limit introduces a non-trivial odd azimuthal anisotropy. The first harmonic is positive, but the third harmonic is negative. Since the flow coefficient v_3 is defined as $v_3 = \sqrt{c_3\{2\}}$, our calculation does not yield a real v_3.

We note however, that the calculation in this section is not a proper calculation of particle production, but rather of correlations in the projectile wave function. In the next section we perform a more appropriate calculation, namely that of particle production.

III. CORRELATIONS IN PARTICLE PRODUCTION.

In this section we calculate the double inclusive production in scattering of the CGC state Eq. (6).

Like in the previous section we will be forced into a high momentum approximation to be able to extract some usable information from the general formulae. For the moment we start with the formal derivation.

A. The double inclusive production.

According to Refs. [46, 47] the single inclusive gluon production is given by

$$\frac{d^3N}{d^2kd\eta} = \int D\rho W_{\rm P}[\rho] \int D\alpha_{\rm T} W_{\rm T}[\alpha_{\rm T}] \; \langle 0|\Omega^\dagger \hat{S}^\dagger \Omega \left[a^\dagger(k)a(k)\right] \Omega^\dagger \hat{S}\Omega|0\rangle. \tag{45}$$

and similarly for the double inclusive

$$\frac{d^6N}{d^2kd^2pd\eta_k d\eta_p} = \int D\rho W_{\rm P}[\rho] \int D\alpha_{\rm T} W_{\rm T}[\alpha] \; \langle 0|\Omega^\dagger \hat{S}^\dagger \Omega \left[a^\dagger(k)a^\dagger(p)a(p)a(k)\right] \Omega^\dagger \hat{S}\Omega|0\rangle. \tag{46}$$

Here $W_{\rm T}[\alpha_{\rm T}]$ is a probability density distribution for the target color field. We remind that Ω is the unitary operator which diagonalizes the QCD Hamiltonian (6), and $\hat{S}$ is the second-quantized eikonal S-matrix

$$\hat{S} = \mathcal{P}\exp\left\{ i\int d^2x \hat{\rho}^a(x)\hat{\alpha}_{\rm T}(x)\right\}. \tag{47}$$

Since $\Omega(\phi,\pi)$ is a Gaussian operator, we can find the transformation of the fields and momenta under the action of Ω. We read off these expressions from the expectation values of quadratic operators Eq. (20) calculated in the previous section [7]

$$\begin{aligned} \Omega\phi(k)\Omega^\dagger &= (1-\mathfrak{l}-L)^{-1}(\mathfrak{t}-\mathfrak{l})(k,p)\phi(-p), \\ \Omega\pi(k)\Omega^\dagger &= (1-\mathfrak{l}-L)(\mathfrak{t}-\mathfrak{l})(k,p)\{\pi(-p)-2b(-p)\}. \end{aligned} \tag{48}$$

[7] In the following we discard the terms involving commutators of the field b which can be calculated at relevant order[32, 33]. In principle these terms are not suppressed by α_s relative to the terms we keep. However they correspond to processes where a gluon rescatters several times on the same valence parton, which do not seem to be physically important. Including these terms would be very cumbersome and not particularly illuminating.

Let us simplify our notations by using the index i cumulatively for the rotational, color indices and spatial coordinate, and by defining

$$\Gamma \equiv (\mathfrak{t} - \mathfrak{l})(1 - \mathfrak{l} - L); \quad \bar{b} \equiv b[S\rho]; \qquad \bar{L} = L[\bar{b}] \,. \tag{49}$$

All the matrix products in the following expressions are understood in coordinate space. Transition to momentum space is done eventually by the Fourier transform. We then have

$$\begin{aligned}
&\mathcal{C} b_i \mathcal{C}^\dagger = b_i; \quad \mathcal{C}\Gamma[b]\mathcal{C}^\dagger = \Gamma[b];\\
&\mathcal{C}^\dagger b_i \mathcal{C} = b_i; \quad \mathcal{C}^\dagger\Gamma[b]\mathcal{C} = \Gamma[b];\\
&\mathcal{C}\pi_i\mathcal{C}^\dagger = \pi_i - 2b_i; \quad \mathcal{C}^\dagger\pi_i\mathcal{C} = \pi_i + 2b_i;\\
&\mathcal{B}^\dagger\phi_i\mathcal{B} = \Gamma_{ij}\phi_j; \quad \mathcal{B}^\dagger\pi_i\mathcal{B} = \Gamma^{-1T}_{ij}\pi_j; \quad \mathcal{B}\phi_i\mathcal{B}^\dagger = \Gamma^{-1}_{ij}\phi_j; \quad \mathcal{B}\pi_i\mathcal{B}^\dagger = \Gamma^{T}_{ij}\pi_j;
\end{aligned} \tag{50}$$

and finally

$$\begin{aligned}
\Omega^\dagger\hat{S}^\dagger\Omega\phi_i\Omega^\dagger\hat{S}\Omega &= \bar{\Gamma}^{-1}_{ij} S_{jk}\Gamma_{kl}\phi_l \,, \\
\Omega^\dagger\hat{S}^\dagger\Omega\pi_i\Omega^\dagger\hat{S}\Omega &= 2\bar{\Gamma}^{T}_{ij}(S_{jk}b_k - \bar{b}_j) + \bar{\Gamma}^{T}_{ij}S_{jk}\Gamma^{-1T}_{kl}\pi_l \,,
\end{aligned} \tag{51}$$

where $\bar{\Gamma}[b] \equiv \Gamma[\bar{b}]$. Only the following operators will be of relevance

$$\begin{aligned}
\mathbb{A} &\stackrel{\text{def}}{=} \bar{\Gamma}^{-1}S\Gamma \,, \\
\mathbb{B} &\stackrel{\text{def}}{=} \bar{\Gamma}^{T}S\Gamma^{-1T} \,, \\
\mathbb{C} &\stackrel{\text{def}}{=} 2\bar{\Gamma}^{T}(Sb - \bar{b}) \,.
\end{aligned} \tag{52}$$

More explicitly in momentum space

$$\begin{aligned}
\mathbb{A}^{ad}_{im}(k,l) &\stackrel{\text{def}}{=} \bar{\Gamma}^{-1\,ab}_{ij}(k,p)S^{bc}(q-p)\Gamma^{cd}_{jm}(-q,l) \,, \\
\mathbb{B}^{ad}_{im}(k,l) &\stackrel{\text{def}}{=} \bar{\Gamma}^{Tab}_{ij}(k,p)S^{bc}(q-p)\Gamma^{-1T\,cd}_{jm}(-q,l) \,, \\
\mathbb{C}^{a}_{i}(k) &\stackrel{\text{def}}{=} 2\bar{\Gamma}^{Tab}_{ij}(k,p)[S^{bc}(q-p)b^c_j(-q) - \bar{b}^b_j(-p)]
\end{aligned} \tag{53}$$

with

$$S^{ab}(p-q) = \int_x S^{ab}(x)e^{-i(p-q)x} \,. \tag{54}$$

For the defined combinations the following property obviously holds

$$\mathbb{A}\mathbb{B}^T = 1. \tag{55}$$

Using this definitions we can find the transformation of the creation/annihilation operators under the unitary operator $\Omega^\dagger\hat{S}\Omega$:

$$(\Omega^\dagger\hat{S}\Omega)^\dagger a(k)(\Omega^\dagger\hat{S}\Omega) = \frac{1}{\sqrt{2}}\left((\mathbb{A}\phi)(-k) + i(\mathbb{B}\pi)(-k) + i\mathbb{C}(-k)\right) \,, \tag{56}$$

$$(\Omega^\dagger\hat{S}\Omega)^\dagger a^\dagger(k)(\Omega^\dagger\hat{S}\Omega) = \frac{1}{\sqrt{2}}\left((\mathbb{A}\phi)(k) - i(\mathbb{B}\pi)(k) - i\mathbb{C}(k)\right) . \tag{57}$$

Now it is easy to find the expression for the single inclusive gluon production, see Eq. (45),

$$\begin{aligned}
&\langle 0|(\Omega^\dagger\hat{S}\Omega)^\dagger a^\dagger(k)a(k)(\Omega^\dagger\hat{S}\Omega)|0\rangle = \\
&\frac{1}{4}\left\{\mathrm{tr}_{\mathrm{L,C}}[\mathbb{A}(k,p)\mathbb{A}^T(-k,-p)] + \mathrm{tr}_{\mathrm{L,C}}[\mathbb{B}(k,p)\mathbb{B}^T(-k,-p)] - \mathrm{tr}_{\mathrm{L,C}}[\mathbb{A}(k,p)\mathbb{B}^T(-k,-p)]\right.\\
&\left.-\ \mathrm{tr}_{\mathrm{L,C}}[\mathbb{B}(k,p)\mathbb{A}^T(-k,-p)]\right\} + \frac{1}{2}\mathbb{C}(k)\mathbb{C}(-k)\\
&= \frac{1}{4}\left\{\mathrm{tr}_{\mathrm{L,C}}[\mathbb{A}(k,p)\mathbb{A}^T(-k,-p)] + \mathrm{tr}_{\mathrm{L,C}}[\mathbb{B}(k,p)\mathbb{B}^T(-k,-p)]\right\} - \frac{1}{2}C_A\delta(0) + \frac{1}{2}\mathbb{C}(k)\mathbb{C}(-k) \,,
\end{aligned} \tag{58}$$

where $\mathrm{tr}_{L,C}$ represents trace over both color and Lorentz indices. In the above the transposition refers only to color and Lorentz indices. The result is obviously symmetric under $k \to -k$ without any reference to averaging over the projectile or target fields. The $\mathbb{C}(k)\mathbb{C}(-k)$ agrees with the corresponding expression in Ref. [48]. Note that although the structure of this term is that of the production off a classical field, the "field" $\mathbb{C}$ is not of entirely classical origin. It is affected by the Bogoliubov part of the CGC vacuum wave function through the factor $\bar{\Gamma}$ in Eq. (52). In the high density limit, $\rho \sim 1/g$, this is an order one correction to the classical result.

Now consider the double inclusive production (46) before averaging with respect to projectile/target configurations

$$F(k,p) \equiv \langle 0|(\Omega^\dagger \hat{S}\Omega)^\dagger a^\dagger(k)a^\dagger(p)a(p)a(k)(\Omega^\dagger \hat{S}\Omega)|0\rangle. \tag{59}$$

By inserting $1 = (\Omega^\dagger \hat{S}\Omega)(\Omega^\dagger \hat{S}\Omega)^\dagger$ after each creation and anihilation operator and using Eqs. (56) and (57) the expectation value can be readily found.

The leading term is $\mathbb{C}^4$. As the analogous term in the single gluon inclusive production, this term is affected at $O(1)$ by the Bogoliubov component of the CGC wave function. Without the Bogoliubov correction this is just the "glasma graph" term frequently discussed in the literature. Thus we see that the high density corrections affect the glasma graph term at leading order.

However this term is symmetric under $k \to -k$. As before we are interested only in extracting the piece that is antisymmetric under this transformation. We will systematically neglect all symmetric terms in our analysis. We thus keep only the NLO (in b) term only. For this operation instead of the equality sign we will use "$\models$".

$$\begin{aligned}\frac{1}{2}\left(F(k,p) - F(k,-p)\right) \;\models\; & \frac{\mathbb{C}(k)}{2}\,\frac{\mathbb{A}\mathbb{A}^T(-k,p) - (2\pi)^2\delta^2(p-k)}{2}\,\frac{\mathbb{C}(-p)}{2} \\ + \; & \frac{\mathbb{C}(-k)}{2}\,\frac{\mathbb{A}\mathbb{A}^T(k,-p) - (2\pi)^2\delta^2(p-k)}{2}\,\frac{\mathbb{C}(p)}{2} \\ - \; & \frac{\mathbb{C}(k)}{2}\,\frac{\mathbb{A}\mathbb{A}^T(-k,-p) - (2\pi)^2\delta^2(p+k)}{2}\,\frac{\mathbb{C}(p)}{2} \\ - \; & \frac{\mathbb{C}(-k)}{2}\,\frac{\mathbb{A}\mathbb{A}^T(k,p) - (2\pi)^2\delta^2(p+k)}{2}\,\frac{\mathbb{C}(-p)}{2}\,. \end{aligned} \tag{60}$$

Note that this term is entirely absent from the dilute limit of the glasma graph contribution. In the dense limit discussed here it is nonvanishing, albeit suppressed with respect to the leading symmetric term by a single power of α_s.

B. High momentum expansion.

We now expand this expression in μ^2/p^2. First, expanding to order b^2 (see Appendix B for details) we get:

$$\begin{aligned} &\left[\mathbb{A}\mathbb{A}^T - 1\right]^{ab}_{ij}(k,p) \approx \\ &g^2 \int_{q,l,m} \Bigg[\left[f^{acd} S^{de}(k-l)\rho^e(l-q)\right]\left[f^{cbf} S^{fg}(p-m)\rho^g(m+q)\right] \times \\ &\frac{1}{(k-q)^2(p+q)^2}\left[\mathfrak{t}_{in}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p) + \mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(p)\right] \\ &- \left[S^{ac}(k-l) f^{cde}\rho^e(l-q)\right]\left[S^{bf}(p-m) f^{dfg}\rho^g(m+q)\right] \times \\ &\frac{1}{(l-q)^2(m+q)^2}\left[\mathfrak{t}_{in}(l)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(m) + \mathfrak{l}_{in}(l)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(m)\right]\Bigg]. \end{aligned} \tag{61}$$

Although we do not favor this classification, we nevertheless comment that the first term of Eq. (61) in the square brackets corresponds to the so-called final state interaction – emission and possible rescattering of gluons after the scattering of the valence charges off the target.

Also to leading order we have

$$\mathbb{C}^a_i(k) \approx i\int_u \left[\frac{u_i}{u^2} - \frac{k_i}{k^2}\right] S^{as}(k-u)\rho^s(u)\,. \tag{62}$$

Here the second term also represents the final state interactions, as defined above.

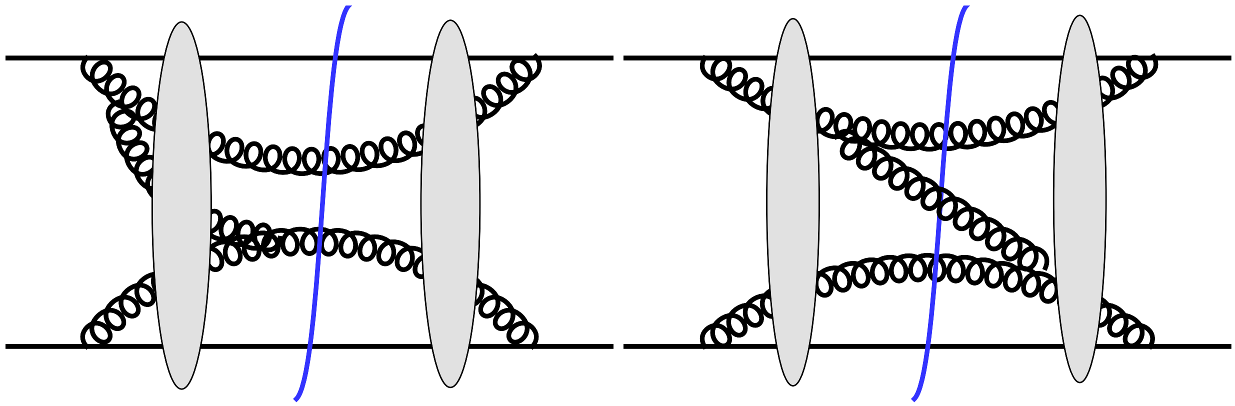

FIG. 1: Schematic representation of the types of diagrams that contribute to the double inclusive production at order $g^6\rho^4$. The horizontal lines symbolize the valence gluons constituting the valence color charge density ρ. The vertical line denotes the final state. Momenta of two final sate gluons are fixed. In the case of the three gluon final state, one of the gluons is summed over inclusively. The blobs symbolize the scatterings on the target. The final state gluons can be emitted either before or after these scatterings, and all the combinations have to be accounted for.

Now we have to put it all together. We will consider one term, the rest will be restored by inspection.

$$\frac{\mathbb{C}(-k)}{2}\left[\mathbb{A}\mathbb{A}^T(k,p)-(2\pi)^2\delta^2(k+p)\right]\frac{\mathbb{C}(-p)}{2} = g^2\int_{q,l,m,u,v} \tag{63}$$

$$\Bigg[\left[f^{acd}S^{de}(k-l)\,\underbrace{\rho^e(l-q)}_{1}\right]\left[f^{cbf}S^{fg}(p-m)\,\underbrace{\rho^g(m+q)}_{2}\right]S^{as}(-k-u)\,\underbrace{\rho^s(u)}_{3}\,S^{bt}(-p-v)\,\underbrace{\rho^t(v)}_{4}\times$$

$$\frac{1}{(k-q)^2(p+q)^2}\left[\frac{u_i}{u^2}+\frac{k_i}{k^2}\right]\left[\mathfrak{t}_{in}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p)+\mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(p)\right]\left[\frac{v_j}{v^2}+\frac{p_j}{p^2}\right]$$

$$-\left[S^{ac}(k-l)f^{cde}\,\underbrace{\rho^e(l-q)}_{1}\right]\left[S^{bf}(p-m)f^{dfg}\,\underbrace{\rho^g(m+q)}_{2}\right]S^{as}(-k-u)\,\underbrace{\rho^s(u)}_{3}\,S^{bt}(-p-v)\,\underbrace{\rho^t(v)}_{4}\times$$

$$\frac{1}{(l-q)^2(m+q)^2}\left[\frac{u_i}{u^2}+\frac{k_i}{k^2}\right]\left[\mathfrak{t}_{in}(l)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(m)+\mathfrak{l}_{in}(l)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(m)\right]\left[\frac{v_j}{v^2}+\frac{p_j}{p^2}\right]\Bigg].$$

Note that although our original expressions for production resum infinite number of diagrams in the external field, any fixed order in expansion in powers of ρ corresponds to a finite number of diagrams. Although it is not entirely straightforward to identify directly the diagrams that correspond to eq.(63), the systematics of expansion in powers of ρ and the discussion in Appendix A suggest that those are the diagrams illustrated schematically in Fig. 1. Thus it should be possible to check the results of this section directly by computing the corresponding diagrams in the framework of the light cone perturbation theory.

The next step is to contract the charge densities using the MV model. This generates three distinct contractions in each one of the two terms

$$\left\langle\frac{\mathbb{C}(-k)}{2}\left[\mathbb{A}\mathbb{A}^T(k,p)-(2\pi)^2\delta^2(k+p)\right]\frac{\mathbb{C}(-p)}{2}\right\rangle_\rho = g^2\left[\langle 12\rangle\langle 34\rangle+\langle 13\rangle\langle 24\rangle+\langle 14\rangle\langle 23\rangle\right],$$

where the integers in the angular brackets denote the corresponding contractions of the sources identified in Eq. (63)

by the numbers located below the rectangular brackets, so that

$$\langle 12\rangle\langle 34\rangle = S^{\dagger sa}(-k-u)f^{acd}S^{de}(k-l)S^{\dagger ef}(p+l)f^{fcb}S^{bs}(-p+u)\mu^2(u)\mu^2(q-l)\times \tag{64}$$
$$\frac{1}{(k-q)^2(p+q)^2}\left[\frac{u_i}{u^2}+\frac{k_i}{k^2}\right]\left[\mathfrak{t}_{in}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p)+\mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(p)\right]\left[-\frac{u_j}{u^2}+\frac{p_j}{p^2}\right],$$
$$+N_c\mathrm{Tr}\left[S^\dagger(-k-u)S(k-l)S^\dagger(p+l)S(-p+u)\right]\mu^2(u)\mu^2(q-l)\times$$
$$\frac{1}{(l-q)^4}\left[\frac{u_i}{u^2}+\frac{k_i}{k^2}\right]\left[\mathfrak{t}_{in}(l)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(l)+\mathfrak{l}_{in}(l)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(l)\right]\left[-\frac{u_j}{u^2}+\frac{p_j}{p^2}\right],$$
$$\langle 13\rangle\langle 24\rangle = f^{acd}S^{de}(k-l)S^{\dagger ea}(-k+l-q)f^{cbf}S^{fg}(p-m)S^{\dagger gb}(-p+m+q)\mu^2(q-l)\mu^2(m+q)\times$$
$$\frac{1}{(k-q)^2(p+q)^2}\left[\frac{(q-l)_i}{(q-l)^2}+\frac{k_i}{k^2}\right]\left[\mathfrak{t}_{in}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p)+\mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(p)\right]\left[-\frac{(m+q)_j}{(m+q)^2}+\frac{p_j}{p^2}\right]$$
$$-S^{ac}(k-l)f^{cde}S^{\dagger ea}(-k+l-q)S^{bf}(p-m)f^{dfg}S^{\dagger gb}(q-p+m)\mu^2(q-l)\mu^2(m+q)\times$$
$$\frac{1}{(l-q)^2(m+q)^2}\left[\frac{(q-l)_i}{(q-l)^2}+\frac{k_i}{k^2}\right]\left[\mathfrak{t}_{in}(l)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(m)+\mathfrak{l}_{in}(l)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(m)\right]\left[-\frac{(m+q)_j}{(m+q)^2}+\frac{p_j}{p^2}\right],$$
$$\langle 14\rangle\langle 23\rangle = f^{acd}S^{de}(k-l)S^{\dagger eb}(-p+l-q)f^{cbf}S^{fg}(p-m)S^{\dagger ga}(-k+q+m)\mu^2(l-q)\mu^2(m+q)\times$$
$$\frac{1}{(k-q)^2(p+q)^2}\left[-\frac{(q+m)_i}{(q+m)^2}+\frac{k_i}{k^2}\right]\left[\mathfrak{t}_{in}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p)+\mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(p)\right]\left[\frac{(q-l)_j}{(q-l)^2}+\frac{p_j}{p^2}\right]$$
$$-S^{ac}(k-l)f^{cde}S^{\dagger eb}(-p+l-q)S^{bf}(p-m)f^{dfg}S^{\dagger ga}(-k+q+m)\mu^2(l-q)\mu^2(m+q)\times$$
$$\frac{1}{(l-q)^2(m+q)^2}\left[-\frac{(q+m)_i}{(q+m)^2}+\frac{k_i}{k^2}\right]\left[\mathfrak{t}_{in}(l)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(m)+\mathfrak{l}_{in}(l)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(m)\right]\left[\frac{(q-l)_j}{(q-l)^2}+\frac{p_j}{p^2}\right].$$

C. Operator product expansion.

The last step in the calculation of double inclusive production is the averaging over the target fields. Before restricting ourselves to an explicit model for this averaging, we consider the case where the target fields are much softer than $p,\ k$, that is $Q_s/p \ll 1$ and $Q_s/k \ll 1$. In this case we can perform what can be called an operator product expansion in the general model-independent framework. The technical details are presented in Appendix B.

1. Odderon

To the leading order of the operator product expansion of Eqs. (64) we get

$$\langle 13\rangle\langle 24\rangle = -\int_q \frac{\mu^4}{p^4k^4}\left[\mathfrak{t}_{sn}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p)+\mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{tm}(p)\right]\times$$
$$\left[\left[f^{acd}S^{de}\partial_sS^{\dagger ea}\right](-q)\left[f^{cbf}S^{fg}\partial_tS^{\dagger gb}\right](q)-\left[S^{ac}f^{cde}\partial_sS^{\dagger ea}\right](-q)\left[S^{bf}f^{dfg}\partial_tS^{\dagger gb}\right](q)\right], \tag{65}$$
$$\langle 14\rangle\langle 23\rangle = -\int_q \frac{\mu^4}{p^4k^4}\left[\mathfrak{t}_{sn}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p)+\mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{tm}(p)\right]\times$$
$$\left[\left[f^{acd}S^{de}\partial_tS^{\dagger eb}\right](-q)\left[f^{cbf}S^{fg}\partial_sS^{\dagger ga}\right](q)-\left[S^{ac}f^{cde}\partial_tS^{\dagger eb}\right](-q)\left[S^{bf}f^{dfg}\partial_sS^{\dagger ga}\right](q)\right]. \tag{66}$$

The sum of these expressions is odd under the charge conjugation transformation $S\to S^\dagger$ and thus is due to the Odderon exchange. In principle this contribution does not have to vanish. However we do not have a well motivated model how to incorporate the Odderon in the target probability distribution.[8] We therefore have no way of determining the sign of this contribution without a specific model. Additionally, the Odderon contribution becomes subleading

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[8] The Odderon contribution averages to zero in the MV model.

at high energy, see e.g. [49, 50]. We therefore consider the next order term in the operator product expansion which must dominate at high enough energy.

2. *Non-vanishing contribution in the charge conjugation even ensemble.*

The next order expressions analogous to Eqs. (65) and (66) are long and cumbersome. We therefore will not present them here in full generality. Instead we directly present the result of averaging using a simple averaging model.

The averaging over the target fields is performed in the following way. We first note that since we are probing the correlators of the eikonal factors S on short distance scales, we can write without any loss of generality[9]

$$S(x) = \exp\{iT^a E^a_i x_i\}\,. \tag{67}$$

To calculate averages of local products we need to take a maximum of three derivatives on each factor of S, and then set $x = 0$. Under this procedure

$$\partial_s S(x) \to iT^a E^a_s; \quad \partial_r\partial_s S(x) \to -\frac{1}{2}\{T^a, T^b\}E^a_s E^b_t; \quad \partial_t\partial_r\partial_s S(x) \to -i\frac{1}{6}\{T^a, T^b, T^c\}E^a_s E^b_t E^c_r; \tag{68}$$

where $\{T^a, T^b, T^c\}$ denotes the sum of all permutations of a, b, c. To average over the color electric field E we then use a simple Gaussian ensemble [26] with the basic two point function of the form

$$\langle E^a_i E^b_j\rangle_\alpha = \lambda^2\delta^{ab}\delta_{ij}\,. \tag{69}$$

The details of the calculation are presented in Appendix C. The final expressions are

$$A_1(k,p) \equiv \langle\langle 12\rangle\langle 34\rangle\rangle_\alpha = 7S_\perp N_c^3(N_c^2-1)\frac{\lambda^4\mu^4}{k^6}\delta^2(p+k)\int_q \frac{1}{(k-q)^4}\left(\frac{3}{2} - 2\frac{(k\cdot q)^2}{k^2q^2} - 5\frac{(k\times q)^2 k\cdot(k-q)}{(k-q)^2q^2k^2}\right), \tag{70}$$

$$A_2(k,p) \equiv \langle\langle 13\rangle\langle 24\rangle\rangle_\alpha = -\frac{S_\perp}{4}N_c^3(N_c^2-1)\frac{\mu^4\lambda^4}{p^6k^6}(k\cdot p), \tag{71}$$

and finally

$$A_3 \equiv \langle\langle 14\rangle\langle 23\rangle\rangle_\alpha = N_c^3(N_c^2-1)S_\perp\frac{\mu^4\lambda^4}{k^4p^4}\Bigg\{ \tag{72}$$
$$-7\frac{k\cdot p}{k^2p^2} - 3\frac{(k\cdot p)^3}{k^4p^4} + \frac{15}{2}\frac{k\cdot(k-p)p\cdot(k-p)}{k^2p^2(k-p)^2} + \frac{k\cdot p(p\cdot(k-p))^2}{k^2p^4(k-p)^2} + \frac{k\cdot p(k\cdot(k-p))^2}{k^4p^2(k-p)^2}$$
$$+\frac{1}{4}\frac{(k\cdot(k-p))^2}{k^2(k-p)^2}\left(\frac{5}{k^2}-\frac{7}{p^2}\right) + \frac{1}{4}\frac{(p\cdot(k-p))^2}{p^2(k-p)^2}\left(\frac{5}{p^2}-\frac{7}{k^2}\right) + \frac{7}{2}\left(\frac{1}{k^2}+\frac{1}{p^2}\right)\frac{k\cdot pk\cdot(k-p)p\cdot(k-p)}{k^2p^2(k-p)^2}$$
$$+\frac{3}{8}\left[\frac{k\cdot(k-p)}{k^2(k-p)^2} - \frac{p\cdot(k-p)}{p^2(k-p)^2}\right]\Bigg\}.$$

[9] The reasoning behind this is the following. We will need to calculate averages of products for the eikonal factors of the type $\langle S(x_1)...S(x_n)\rangle$ where all distances are smaller than the inverse saturation momentum of the target, $|x_i - x_j| < 1/Q_s$. For translationally invariant target, which we assume here, we can shift all the coordinates to vicinity of zero, so that $|x_i| < 1/Q_s$. We can now fix the convenient gauge $S(x=0) = 1$. This can always be done by using a residual x^- independent gauge transformation without leaving the light cone gauge. Eq. (67) is just expansion of the phase of the eikonal factor to leading order in x, which is a good approximation since in the regime of interest x is smaller than the correlation length of the color electric fields. We note that this form of the eikonal factor yields the Golec-Biernat–Wusthoff model [51, 52] of the dipole cross section. The McLerran-Venugopalan model at short distances corresponds to an additional slow dependence of the field E on x in Eq. (67). This can be easily incorporated in our formulae by considering λ^2 to be a slowly (logarithmically) varying function of the external momenta p and k. Although this may be important for quantitative comparisons to the data, it does not affect qualitative features of our results and at the level of accuracy of the present paper is clearly irrelevant.

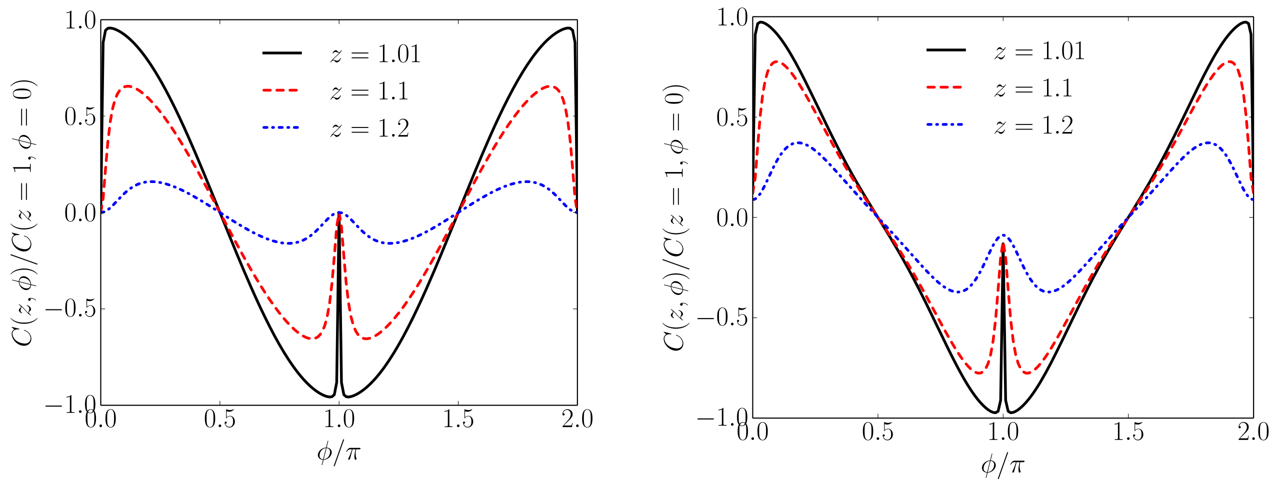

FIG. 2: The correlation function as a function of the azimuthal angle, ϕ for different values of $z = p/k$. The correlation functions are defined in the text and normalized by $C(z = 1, \phi = 0)$. Left panel - correlation in the projectile wave function. Right panel - correlation in particle production.

The expression A_1 yields "hard" back to back production. This is the analog of a similar term in our calculation of pair density in the projectile wave function, and we neglect it for the same reason. We now combine all the terms to obtain

$$\frac{1}{2}\left[\frac{d^6N}{d^2kd^2pd\eta_kd\eta_p}(k,p) - \frac{d^6N}{d^2kd^2pd\eta_kd\eta_p}(k,-p)\right] = 2g^2\left[A_2(k,-p) + A_3(k,-p) - A_2(k,p) - A_3(k,p)\right] . \tag{73}$$

In Fig. 2 we show the correlation functions for produced gluons (right) and for gluons in the projectile wave function (left). For the former, we defined the correlation function using Eq. (60) and normalizing by the uncorrelated piece $S_\perp^2\mu^4\lambda^4/(k^4p^4)$

$$\frac{1}{2}\frac{\frac{d^6N}{d^2kd^2pd\eta_kd\eta_p}(k,p) - \frac{d^6N}{d^2kd^2pd\eta_kd\eta_p}(k,-p)}{\frac{N_c^4S_\perp^2\mu^4\lambda^4}{k^4p^4}} = \frac{\alpha_sN_c}{S_\perp p^2}C(z = p/k, \phi). \tag{74}$$

For the latter, the correlations function is defined similarly.

The first and the third harmonics of the correlations functions are shown in Fig. 3.

IV. DISCUSSION AND CONCLUSIONS.

Note that the odd part of the production cross section is proportional to $\mu^4\lambda^4$. This means that if either the projectile or the target is dilute, this contribution to the correlated production vanishes. In this respect it is similar to the even contribution from "glasma graphs". As is clear from Eq. (74) the leading contribution to the odd part of the correlation function is the same order in the color charge density as that to the even part, and is just suppressed by one power of α_s.

Another important point is that just like the glasma graph contribution, the odd contribution is long range in rapidity. In fact our result Eq. (74) is rapidity independent. Some dependence on rapidity separation between the gluons will undoubtedly appear once the rapidity difference is large enough $|\eta_k - \eta_p| \sim 1/\alpha_s$. This effect is not accounted for in our calculation.

The present calculation does not include contributions to two particle production arising from a single Pomeron exchange. This contribution (at the leading order in weak field expansion) is proportional to $\alpha_s\mu^2\lambda^2$ and obviously does not appear in our formulae. For strong fields these contributions are subleading and for that reason they are not contained in the CGC wave function. This single Pomeron mechanism leads predominantly to back-to-back minijet production which can in principle contribute to nonvanishing odd azimuthal anisotropy. Such back-to-back jets are however subtracted in the experimental analysis and are of no interest to us here.

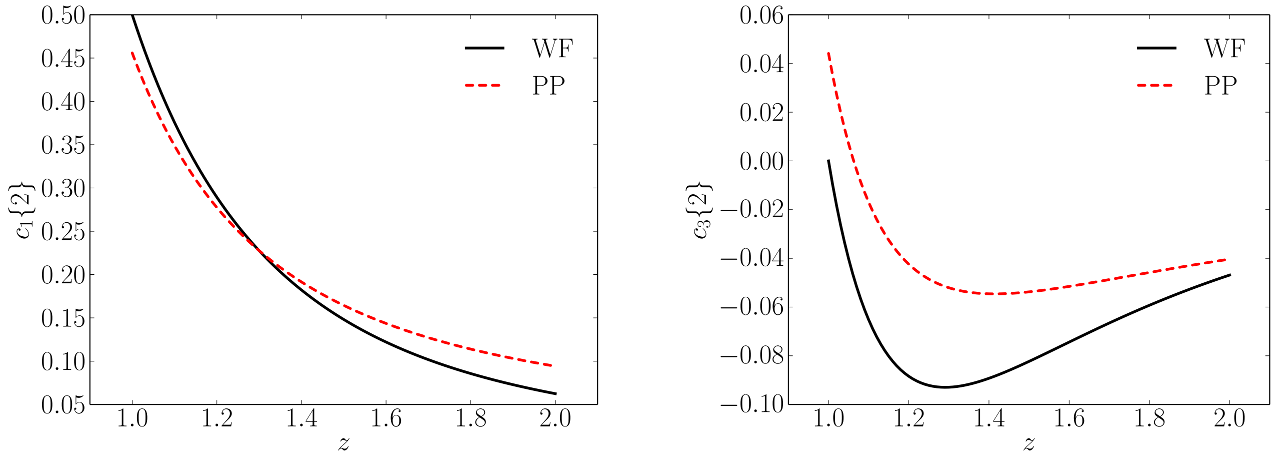

FIG. 3: The first and the third cumulants as a function of $z = p/k$ obtained from the correlation function the projectile wave function (WF) and from double inclusive particle production (PP).

Our calculation is based on the CGC wave function derived in the limit of the dense projectile. Recall that in Ref. [33] this wave function was obtained in the leading order of light cone perturbation theory in strong background with $\rho \sim 1/g$. This wave function differs from the "dilute CGC" coherent state by the Bogoliubov squeezing prefactor which is an $O(1)$ correction in this parametric regime. In this sense the squeezing provides the most important correction to the wave function. All other corrections not included in this Bogoliubov factor lead to terms suppressed by powers of the QCD coupling on the level of the wave function.

We have indeed shown that the full account of the squeezing leads to an $O(1)$ correction to the "glasma graph" results both in the single inclusive and the double inclusive production cross sections. This $O(1)$ correction unfortunately preserves the accidental symmetry $(k, p) \to (k, -p)$ observed in the dilute regime. However the expression for the double inclusive production also contains a term odd under this transformation. This term is suppressed by a single power of α_s relative to the glasma graphs contribution. Our strategy in this paper was to take this odd contribution at its face value and explore its consequences.

In order to get our numerical estimates we had to expand the production cross section to leading order in powers of ρ. Excluding the odderon, the leading contribution is of order $g^6\rho^4$. In this order our calculation should be interpretable in terms of a finite number of Feynman diagrams. We have tentatively identified the relevant graphs in Fig. 1. We note that the procedure employed here does not sum all the Feynman diagram contribution to the double inclusive cross section at order $g^6\rho^4$. For example the running coupling correction to the glasma graphs is absent. However the physical feature that allows for appearance of the odd contribution in our calculation is the nonfactorizable production of the two gluons (configuration by configuration at fixed color charge density). We believe that including of the additional factorizable terms at the same order in α_s (like the running coupling correction) will have no effect on the calculation of v_3.

We now discuss qualitative features of our results.

Consider first the shape of the (scaled, see Eq. (74)) correlation function in Fig. 2. Qualitatively, the gluon pair density in the projectile CGC wave function is consistent with the expectations based on KLM argument outlined in the introduction. Indeed the pair density has a strong peak in forward direction. At $z \approx 1$ the peak is close to $\phi = 0$. As z grows the peak decreases in hight and moves to larger angles, but always stays at $\cos\phi > 1/\sqrt{3}$. These properties remain practically unaltered in the double inclusive gluon production. The overall shape of the production cross section resembles closely the form of the gluon pair density in the projectile wave function.

Note the overall normalization of the double inclusive production amplitude. As expected, it is suppressed by the factor of $S_\perp p^2$, reflecting the fact that the sources of correlation are local in the coordinate space. If we were able to calculate production for $p^2, k^2 \sim Q_s^2$, the suppression factor would presumably be $S_\perp Q_s^2$. This is exactly the same as that of the local anisotropy [18, 19, 23] and the "glasma graph" [16], or Bose enhancement [53] contributions. Our result has an additional suppression by a factor of α_s relative to those contributions, however it is leading at large N_c whereas both the glasma graphs and the local anisotropy are order $1/N_c^2$ [16, 54]. At $N_c = 3$ and $\alpha_s \sim .2$ the relative importance of these contributions is determined by a numerical factors of order one, which may well be model-dependent. It does however raise an interesting possibility that additional contribution to v_2 that should arise from using the amended CGC wave function can be competitive with the contributions hitherto considered, or even larger than those at least close to the saturation momentum. We have not calculated here the even part of the correlation function which gives rise to v_2, but such calculation is certainly feasible and will be reported elsewhere.

Next consider Fig. 3 which shows the coefficients of the first and third harmonics in the correlation function. The coefficient $c_1\{2\}$ is positive both for the pair density in the wave function and for pair production in scattering, consistent with the KLM expectation. There is no significant difference between the two, although it drops slower with z for production. The situation is different for $c_3\{2\}$. It is always negative for the pair density in the wave function. For particle production $c_3\{2\}$ is negative for most of z interval, but it changes sign and is positive for z close to one, approximately for $.9 < z < 1.1$.

We do not have a good understanding why $c_3\{2\}$ changes sign due to scattering. We can nevertheless argue why the form of the pair density should be distorted the most when the two momenta are close to each other in magnitude. Remember that we are working in the regime where the momenta of produced particles are much larger than the momentum transfer from the target. In this regime most of the momentum of the produced gluons is inherited from the projectile wave functions. The role of the scattering is then mostly to decohere these gluons from the incoming wave function and put them on shell. If the relative momentum of the gluons is large, they sit close to each other (in coordinate space) in the projectile wave function, and it is difficult to decohere them. This is always the case if $z \gg 1$ (or $z \ll 1$). However the "bin" with $z \approx 1$, contains two gluon configurations which have small relative momentum and thus are well separated in the coordinate space. Such gluons probe widely separated regions of the target and acquire significantly different eikonal phase during the propagation. The scattering is therefore much more efficient in decohering such configurations from the incoming wave function. Thus we expect that in the $z \approx 1$ "bin" the scattering skews the distribution of pairs towards the ones which have same sign transverse momenta. This may be the mechanism that produces a positive $c_3\{2\}$ in this transverse momentum bin.

The fact that we obtain positive $c_3\{2\}$ in a narrow range of z is rather interesting. When v_3 is determined by taking the square root of $c_3\{2\}$ keeping both the "trigger" and the "associated" particle in the same momentum bin, our results yield a positive $c_3\{2\}$ and therefore real v_3. Increasing the momentum of the bin p as per our results would lead to a rather slow decrease of v_3 as $1/p$. However if one samples the trigger and associated particles from different momentum bins, the v_3 should decrease much faster as seen in experiment, see e.g. Ref. [55]. In fact it should cease being real when the two bins are separated by about $10 - 20\%$ of the central value. It would be very interesting to see if such trend can be traced in the data at lower multiplicities and higher trigger momentum.

Finally we want to comment on the range of applicability of the improved CGC wave function that we have utilized in the current calculation. In Refs. [32, 33] it was derived as the wave function containing the bulk of the probability density in the field space. The tails of the wave function at large values of the field ϕ however are not described well by this Gaussian shape. For that reason it should be used with caution. For example one cannot use this wave function to describe nucleus-nucleus scattering. In the latter case the scattering modifies the field b by a "factor" of order one. To calculate the scattering amplitude one would then need to take an overlap of two Gaussians with central values displaced by an amount of order $\phi_1 - \phi_2 \sim 1/\alpha_s$. Such an overlap is dominated by the tails of the two wave functions which are not under control in our approximation. Similarly, by a straightforward counting of powers of α_s on can see that the accuracy of the present approximation is not sufficient to calculate particle production when the number of particles produced in the collision is $O(1/\alpha_s)$. Even though the wave function Eq .(6) is more accurate than the dilute limit Eq. (2), it is not accurate enough for processes where the number of produced particles is parametrically large. The natural habitat of Eq. (6) is in situations where the number of produced particles in the collision is $O(1)$. Note that this is still parametric improvement over Eq. (2) which is strictly speaking valid only in the dense-dilute regime where the number of produced particles is $O(\alpha_s)$. Moreover, this regime is of relevance for minimal bias p-p collisions where the correlations have been recently observed [9, 10]. With some measure of optimism we can hope that at least qualitatively we can trust our results also in p-p the events with higher than average multiplicity, although probably not for very high multiplicity events.

Acknowledgments

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Appendix A: The CGC wave-function

In this appendix we sketch the derivation of the Gaussian CGC ground state. For details of the derivation the reader should consult the original work [32, 33], while here we only provide the main logical steps of the derivation.

First, we note that the CGC ground state is the ground state of the soft gluons in the presence of eikonal coupling to valence modes represented by the color charge density $\rho(x)$. We thus have to diagonalize the light cone Hamiltonian

$$H_{\rm soft} = H_{\rm LFQCD} + \int d^2x \rho^a(x) \int d\eta \partial^i A^a_i(x,\eta)\,, \tag{A1}$$

where $H_{\rm LFQCD}$ is the QCD Hamiltonian on the light front, and η is rapidity. It is obvious that shifting the field one can get rid of the last term in the Hamiltonian since it is linear in the field A_i. This shift is affected by the coherent operator $\mathcal{C} = e^{i2\int_k b_{\alpha i}(-k)\phi_{\alpha i}(k)}$ so that

$$\mathcal{C} H_{\rm soft} \mathcal{C}^\dagger = H_0 + \delta H_2 + \delta \tilde{H}\,. \tag{A2}$$

Here δH_2 is quadratic in the soft gluon operators, while $\delta\tilde{H}$ contains third and fourth power of A_i.

When the color source ρ is small, i.e. $\rho^a = O(g)$, this shift diagonalizes the Hamiltonian up to terms of order g, i.e. $\delta H_2 = O(g^2)$ and $\delta\tilde{H} = O(g^3)$. Thus in the weak filed limit the CGC ground state (up to higher order perturbative corrections) is the coherent state

$$|{\rm CGC}\rangle_{\rm dilute} = \mathcal{C}|0\rangle\,, \tag{A3}$$

where $|0\rangle$ is the light cone vacuum of the soft gluons.

However in the strong field limit, where $\rho = O(1/g)$, the situation is different, since $\delta H_2 = O(1)$ while $\delta\tilde{H} = O(g)$. Thus in order to diagonalize the total Hamiltonian at order one, it is still necessary to diagonalize its nontrivial quadratic part $H_0 + \delta H_2$. It can be achieved by a Bogoliubov transformation owing to quadratic dependence on the field. The action of this Bogoliubov transformation on the soft gluon creation and annihilation operators was found in Ref. [32]. The transformation is generated by unitary operator of the Gaussian form, schematically

$$\mathcal{B} = \exp\{-\frac{1}{2}(a+a^\dagger)\Lambda(a+a^\dagger)\}\,. \tag{A4}$$

Thus at the end of the day in the strong field limit the diagonalizing operator can be written in the form $\Omega = \mathcal{C}\mathcal{B}$ and the wave function of the CGC ground state

$$\Psi_{\rm CGC}[\phi] = \langle\phi|\Omega|0\rangle = e^{i2\int_k b_{\alpha i}(-k)\phi_{\alpha i}(k)}\langle\phi|\mathcal{B}\rangle\,, \tag{A5}$$

where for convenience we separately defined the Gaussian state $|\mathcal{B}\rangle$ as

$$|\mathcal{B}\rangle \equiv \mathcal{B}|0\rangle; \qquad \langle\phi|\mathcal{B}\rangle = \mathcal{N} e^{-\frac{1}{2}\int_{k,p} B^{-1}_{\alpha\beta ij}(k,p)\phi_{\alpha i}(k)\phi_{\beta j}(p)}\,. \tag{A6}$$

As discussed in Ref. [33] this calculation can be given a diagrammatic interpretation. In particular the action of the coherent operator on the soft gluon vacuum is represented by (exponentiating) the sum of the tree level diagrams and corresponding virtual corrections, see Fig.3.

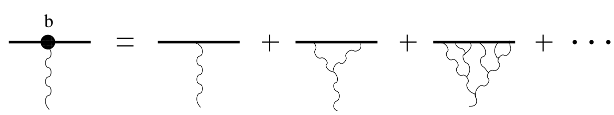

FIG. 4: The tree level diagrams representing the action of the coherent operator $\mathcal{C}$ on the soft gluon vacuum.

The action of the Bogoliubov operator is represented by (exponentiating) the sum in Fig. 4, including appropriate virtual corrections.

Thus the diagrammatics of the CGC vacuum wave function Eq. (A6) is that of independent emission of single gluons and pairs of gluons with subsequent rescattering of these gluons on the valence charge density.

It was explicitly shown in Ref. [32, 33] that the presence of the Bogoliubov operator $\mathcal{B}$ is crucial for derivation of the JIMWLK evolution equation. Although the derivation in Ref. [32, 33] is fairly complicated, as it deals explicitly with the contributions of all the rapidity modes of the soft gluon field, it was recognized in those papers that only

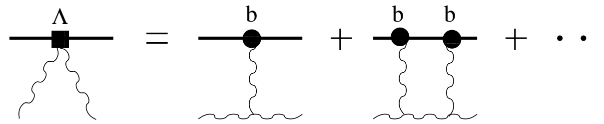

FIG. 5: The diagrams representing the action of the Bogoliubov operator on the soft gluon vacuum.

limited amount of information is necessary in order to reproduce the JIMWLK equation. In particular it was shown that all that is required of the Gaussian state $|\mathcal{B}\rangle$ is that it correctly encodes the correlation function

$$\langle\mathcal{B}|\phi(x)\phi(y)|\mathcal{B}\rangle = (1-\mathfrak{l}-L)^2(x,y)\,, \tag{A7}$$

where $\phi(x)$ is the integrated over rapidity field Eq. (8). In the present paper, this property of the Gaussian state was used to infer the form of the state after integration over all the rapidity modes except for the constant mode $\phi(x)$.

Appendix B: Expansion in the classical field.

In this Appendix we derive Eq. (61) - the expansion of the main calculational block for the double inclusive production cross section to leading order in the projectile charge density (or equivalently b). For notational simplicity we rescale the classical field b via $b \to \frac{1}{g}b$. Using Eq. (26) we have

$$1-\mathfrak{l}-L \approx \mathfrak{t}-\mathfrak{l}+b\frac{1}{\partial^2}\partial+\partial\frac{1}{\partial^2}b-\frac{\partial}{\partial^2}[\partial b]_+\frac{\partial}{\partial^2}-b\frac{1}{\partial^2}b+b\frac{1}{\partial^2}[\partial b]_+\frac{1}{\partial^2}\partial+\frac{\partial}{\partial^2}[\partial b]_+\frac{1}{\partial^2}b+\frac{\partial}{\partial^2}b^2\frac{\partial}{\partial^2}-\frac{\partial}{\partial^2}[\partial b]_+\frac{1}{\partial^2}[\partial b]_+\frac{\partial}{\partial^2} \tag{B1}$$

and

$$\Gamma = 1+b\frac{1}{\partial^2}\partial-\partial\frac{1}{\partial^2}b-\frac{\partial}{\partial^2}[\partial b]_-\frac{\partial}{\partial^2}-b\frac{1}{\partial^2}b+b\frac{1}{\partial^2}[\partial b]_+\frac{1}{\partial^2}\partial+\frac{\partial}{\partial^2}[\partial b]_-\frac{1}{\partial^2}b-\frac{\partial}{\partial^2}b^2\frac{\partial}{\partial^2}-\frac{\partial}{\partial^2}[\partial b]_-\frac{1}{\partial^2}[\partial b]_+\frac{\partial}{\partial^2} \tag{B2}$$

with the inverse

$$\Gamma^{-1} = 1-b\frac{1}{\partial^2}\partial+\partial\frac{1}{\partial^2}b+\frac{\partial}{\partial^2}[\partial b]_-\frac{\partial}{\partial^2}-b\frac{1}{\partial^2}\partial b\frac{1}{\partial^2}\partial+\frac{\partial}{\partial^2}b\partial\frac{1}{\partial^2}b-\frac{\partial}{\partial^2}b\partial\frac{1}{\partial^2}b\partial\frac{\partial}{\partial^2}+\frac{\partial}{\partial^2}\partial b\frac{1}{\partial^2}\partial b\frac{\partial}{\partial^2}. \tag{B3}$$

The expression we need is

$$\mathbb{A}\mathbb{A}^T = \bar\Gamma^{-1}S\Gamma\Gamma^TS^T\bar\Gamma^{-1} = \bar\Gamma^{-1}S(\mathfrak{t}-\mathfrak{l})B(\mathfrak{t}-\mathfrak{l})S^T\bar\Gamma^{-1T} \tag{B4}$$

so

$$(\mathfrak{t}-\mathfrak{l})B(\mathfrak{t}-\mathfrak{l}) = 1-b_i\frac{1}{\partial^2}b_j-\partial_i\frac{1}{\partial^2}b^2\frac{1}{\partial^2}\partial_j+\frac{\partial_i}{\partial^2}(\vec\partial\cdot\vec b)\frac{1}{\partial^2}b_j+b_i\frac{1}{\partial^2}(\vec b\cdot\vec\partial)\frac{\partial_j}{\partial^2}+\frac{\partial_i}{\partial^2}\left[(\vec b\cdot\vec\partial)\frac{1}{\partial^2}(\vec\partial\cdot\vec b)-(\vec\partial\cdot\vec b)\frac{1}{\partial^2}(\vec b\cdot\vec\partial)\right]\frac{\partial_j}{\partial^2}. \tag{B5}$$

We only need expression in Eq. (B4) to order b^2

$$\begin{aligned}\mathbb{A}\mathbb{A}^T-1 &\approx \bar\Gamma^{-1}\bar\Gamma^{-1T}+S(\mathfrak{t}-\mathfrak{l})\tilde B(\mathfrak{t}-\mathfrak{l})S^\dagger-1 \\ &= \bar b_i\frac{1}{\partial^2}\bar b_j+\partial_i\frac{1}{\partial^2}\bar b^2\frac{1}{\partial^2}\partial_j-\frac{\partial_i}{\partial^2}(\vec\partial\cdot\vec{\bar b})\frac{1}{\partial^2}\bar b_j-\bar b_i\frac{1}{\partial^2}(\vec{\bar b}\cdot\vec\partial)\frac{\partial_j}{\partial^2}-\frac{\partial_i}{\partial^2}\left[(\vec{\bar b}\cdot\vec\partial)\frac{1}{\partial^2}(\vec\partial\cdot\vec{\bar b})-(\vec\partial\cdot\vec{\bar b})\frac{1}{\partial^2}(\vec{\bar b}\cdot\vec\partial)\right]\frac{\partial_j}{\partial^2} \\ &-Sb_i\frac{1}{\partial^2}b_jS^\dagger-S\partial_i\frac{1}{\partial^2}b^2\frac{1}{\partial^2}\partial_jS^\dagger+S\frac{\partial_i}{\partial^2}(\vec\partial\cdot\vec b)\frac{1}{\partial^2}b_jS^\dagger+Sb_i\frac{1}{\partial^2}(\vec b\cdot\vec\partial)\frac{\partial_j}{\partial^2}S^\dagger \\ &+S\frac{\partial_i}{\partial^2}\left[(\vec b\cdot\vec\partial)\frac{1}{\partial^2}(\vec\partial\cdot\vec b)-(\vec\partial\cdot\vec b)\frac{1}{\partial^2}(\vec b\cdot\vec\partial)\right]\frac{\partial_j}{\partial^2}S^\dagger.\end{aligned} \tag{B6}$$

In order to perform Fourier transformation we use Eq. (28)

$$\begin{aligned}\left[\mathbb{A}\mathbb{A}^T-1\right](k,p) &\approx \int_q\bar b_n(k-q)\left[-\mathfrak{t}_{in}(k)\frac{1}{q^2}\mathfrak{t}_{mj}(p)+\frac{k_i}{k^2}\mathfrak{t}_{nm}(q)\frac{p_j}{p^2}\right]\bar b_m(p+q) \\ &+\int_{q,l,m}S(k-l)b_n(l-q)\left[\mathfrak{t}_{in}(l)\frac{1}{q^2}\mathfrak{t}_{mj}(m)-\frac{l_i}{l^2}\mathfrak{t}_{nm}(q)\frac{m_j}{m^2}\right]b_m(m+q)S^\dagger(p-m).\end{aligned} \tag{B7}$$

Using Eq. (31) we get

$$\left[\mathbb{A}\mathbb{A}^T - 1\right](k,p) \approx \int_q \bar{c}(k-q)\left[\mathfrak{t}_{in}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p) + \mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(p)\right]\bar{c}(p+q) \tag{B8}$$

$$- \int_{q,l,m} S(k-l)c(l-q)\left[\mathfrak{t}_{in}(l)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(m) + \mathfrak{l}_{in}(l)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(m)\right]c(m+q)S^\dagger(p-m).$$

Finally we have to substitute $c(k) = \frac{1}{k^2}\rho(k)$ and $\bar{c}(k) = \frac{1}{k^2}\int_l S(k-l)\rho(l)$ to obtain

$$\left[\mathbb{A}\mathbb{A}^T - 1\right]^{ab}_{ij}(k,p) \approx \int_{q,l,m} \tag{B9}$$

$$\Bigg[\frac{1}{(k-q)^2(p+q)^2}\left[fS(k-q-l)\rho(l)\right]\left[\mathfrak{t}_{in}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p) + \mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(p)\right]\left[fS(p+q-m)\rho(m)\right] -$$

$$\frac{1}{(l-q)^2(m+q)^2}\left[S(k-l)f\rho(l-q)\right]\left[\mathfrak{t}_{in}(l)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(m) + \mathfrak{l}_{in}(l)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(m)\right]\left[f\rho(m+q)\right]S^\dagger(p-m)\Bigg],$$

where we have indicated explicitly the position of the structure constant tensor f for the purposes of color algebra. After shifting the momentum integration variable in the first term, and restoring the original normalization of b and ρ used in the main text we can finally write

$$\left[\mathbb{A}\mathbb{A}^T - 1\right]^{ab}_{ij}(k,p) \approx g^2 \int_{q,l,m} \tag{B10}$$

$$\Bigg[\left[f^{acd}S^{de}(k-l)\rho^e(l-q)\right]\left[f^{cbf}S^{fg}(p-m)\rho^g(m+q)\right] \times$$

$$\frac{1}{(k-q)^2(p+q)^2}\left[\mathfrak{t}_{in}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p) + \mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(p)\right]$$

$$-\left[S^{ac}(k-l)f^{cde}\rho^e(l-q)\right]\left[S^{bf}(p-m)f^{dfg}\rho^g(m+q)\right] \times$$

$$\frac{1}{(l-q)^2(m+q)^2}\left[\mathfrak{t}_{in}(l)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(m) + \mathfrak{l}_{in}(l)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(m)\right]\Bigg].$$

Appendix C: Operator product expansion.

In this Appendix we derive the operator product expansion, i.e. expansion of the double inclusive cross section in powers of the derivatives ∂S. The physical parameter of this expansion is Q_s^2/k^2 or Q_s^2/p^2 where Q_s is the saturation momentum of the target.

In order to organize the expansion we will use the following notation $X_{[n,m,l]}$ where n, m and l are integers corresponding to the orders in the expansion of the terms originating from $\mathbb{C}(k)$, $\mathbb{A}\mathbb{A}^T$, and $\mathbb{C}(p)$ correspondingly. For example, the notation $[1,0,2]$ means the expansion of $\mathbb{C}(k)$ to the first order, $\mathbb{A}\mathbb{A}^T$ to the leading order, $\mathbb{C}(p)$ to the second order.

Some preliminary formulae are in order. Define $w = -k - u$, then the expansion of $\mathbb{C}(k)$ involves the following or, similar modulo redefinition of the variables, expansion

$$\frac{u_i}{u^2} + \frac{k_i}{k^2} = \frac{(-w-k)_i}{(k+w)^2} + \frac{k_i}{k^2} \;=\; -\left[\frac{w_i}{k^2} - 2\frac{k_i(k\cdot w)}{k^4}\right] \tag{C1}$$

$$+\left[\frac{k_i w^2}{k^4} - 4\frac{k_i(k\cdot w)^2}{k^6} + 2\frac{w_i(k\cdot w)}{k^4}\right] \tag{C2}$$

$$-\left[-\frac{w_i w^2}{k^4} + 4\frac{w_i(k\cdot w)^2}{k^6} + 4\frac{k_i w^2(k\cdot w)}{k^6} - 8\frac{k_i(k\cdot w)^3}{k^8}\right] \tag{C3}$$

$$\equiv \frac{1}{k^2}\left[-w_s C_{si}(k) + w_s w_t D_{sti}(k) - w_s w_t w_r E_{stri}(k)\right]. \tag{C4}$$

Similarly for $\mathbb{C}(p)$ define $v = -p + u$. Then

$$-\frac{u_i}{u^2}+\frac{p_i}{p^2} = -\frac{(p+v)_i}{(p+v)^2}+\frac{p_i}{p^2} = -\left[\frac{v_i}{p^2}-2\frac{p_i(p\cdot v)}{p^4}\right] \tag{C5}$$

$$+\left[\frac{p_i v^2}{p^4}-4\frac{p_i(p\cdot v)^2}{p^6}+2\frac{v_i(p\cdot v)}{p^4}\right] \tag{C6}$$

$$-\left[-\frac{v_i v^2}{p^4}+4\frac{v_i(p\cdot v)^2}{p^6}+4\frac{p_i v^2(p\cdot v)}{p^6}-8\frac{p_i(p\cdot v)^3}{p^8}\right] \tag{C7}$$

$$\equiv \frac{1}{p^2}\left[-v_s C_{si}(p)+v_s v_t D_{sti}(p)-v_s v_t v_r E_{stri}(p)\right]. \tag{C8}$$

Here

$$C_{si}(k) = (\mathfrak{t}-\mathfrak{l})_{si}(k) \tag{C9}$$

$$D_{sti}(k) = \left[\delta_{st}-4\mathfrak{l}_{st}(k)\right]\frac{k_i}{k^2}+2\delta_{si}\frac{k_t}{k^2} = \delta_{st}\frac{k_i}{k^2}+2(\mathfrak{t}-\mathfrak{l})_{si}(k)\frac{k_t}{k^2}, \tag{C10}$$

$$E_{stri}(k) = \frac{1}{k^2}\Big[-\delta_{si}[\delta_{tr}-4\mathfrak{l}_{tr}(k)]+4\mathfrak{l}_{si}(k)(\mathfrak{t}-\mathfrak{l})_{tr}(k)\Big]. \tag{C11}$$

Now the expansion of $\mathbb{A}\mathbb{A}^T$ will involve

$$\frac{1}{(l-q)^4} = \left[\frac{1}{(k-q-a)^2}\right]^2 = \frac{1}{(k-q)^4}+4\frac{a\cdot(k-q)}{(k-q)^6}-2\frac{a^2}{(k-q)^6}+12\frac{[a\cdot(k-q)]^2}{(k-q)^8} \tag{C12}$$

and

$$\Big[\mathfrak{t}(l)\mathfrak{l}(q)\mathfrak{t}(l)+\mathfrak{l}(l)\mathfrak{t}(q)\mathfrak{l}(l)\Big]_{ij} = \Big[\mathfrak{l}(q)+\mathfrak{l}(l)-\mathfrak{l}(l)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(l)\Big]_{ij} \tag{C13}$$

$$\begin{aligned}
&= \Big[\mathfrak{l}(q)+\mathfrak{l}(k)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(k)\Big]_{ij} \\
&-\frac{1}{k^2}\left[a_i k_j + a_j k_i - 2(a\cdot k)\frac{k_i k_j}{k^2}\right] \\
&+\frac{1}{k^2 q^2}\left[(q\cdot a)\left[q_i k_j + q_j k_i\right]+(q\cdot k)\left[q_i a_j + q_j a_i\right]-2(a\cdot k)(q\cdot k)\frac{q_i k_j + k_i q_j}{k^2}\right] \\
&+\frac{1}{k^2}\left[a_i a_j - 2(a\cdot k)\frac{k_i a_j + k_j a_i}{k^2}+k_i k_j\frac{4(a\cdot k)^2-k^2a^2}{k^4}\right] \\
&-\frac{1}{k^2q^2}\Bigg[(a\cdot q)[a_i q_j + q_i a_j]-\frac{2(a\cdot k)(k\cdot q)}{k^2}[q_i a_j + a_i q_j] \\
&-\frac{2(a\cdot k)(a\cdot q)}{k^2}[q_i k_j + k_i q_j]+\frac{[4(a\cdot k)^2-k^2a^2](q\cdot k)}{k^4}[q_i k_j + k_i q_j]\Bigg],
\end{aligned}$$

where we used

$$\mathfrak{l}_{in}(l) = \mathfrak{l}_{in}(k-a) = \mathfrak{l}_{in}(k)-\frac{1}{k^2}\left[a_i k_n + a_n k_i - 2(a\cdot k)\frac{k_i k_n}{k^2}\right]$$

$$+\frac{1}{k^2}\left[a_i a_n - 2(a\cdot k)\frac{k_i a_n + k_n a_i}{k^2}+k_i k_n\frac{4(a\cdot k)^2-k^2a^2}{k^4}\right]. \tag{C14}$$

Combining these together we get

$$\frac{1}{(l-q)^4}\Big[\mathfrak{t}(l)\mathfrak{l}(q)\mathfrak{t}(l)+\mathfrak{l}(l)\mathfrak{t}(q)\mathfrak{l}(l)\Big]_{ij} =$$

$$= \frac{1}{(k-q)^4}A^0_{ij}(k,q)+\frac{1}{k^2(k-q)^4}a_s A^1_{sij}(k,q)+\frac{1}{k^2(k-q)^4}a_s a_t A^2_{stij}(k,q), \tag{C15}$$

where we defined

$$A^0_{ij}(k,q) = \Big[\mathfrak{l}(q)+\mathfrak{l}(k)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(k)\Big]_{ij}, \tag{C16}$$

$$A^1_{sij}(k,q) \;=\; \mathfrak{t}_{si}(k)k_j\mathfrak{t}_{sj}(k)k_i + (\mathfrak{t}-\mathfrak{l})_{sk}(k)\Big[\mathfrak{l}_{ki}(q)k_j + \mathfrak{l}_{kj}(q)k_i\Big] + \delta_{si}\mathfrak{l}_{jk}(q)k_k + \delta_{sj}\mathfrak{l}_{ik}(q)k_k$$
$$+\; 4\frac{k^2}{(k-q)^2}(k-q)_s\Big[\mathfrak{l}(q)+\mathfrak{l}(k)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(k)\Big]_{ij}, \tag{C17}$$

$$A^2_{stij}(k,q) = -2\frac{k^2}{(k-q)^2}(1-6l)_{st}(k-q)\Big[\mathfrak{l}(q)+\mathfrak{l}(k)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(k)\Big]_{ij}$$
$$+\; [(\mathfrak{t}-\mathfrak{l})_{si}(k)(\mathfrak{t}-\mathfrak{l})_{tj}(k)-\delta_{st}\mathfrak{l}_{ij}(k)]$$
$$-\; [(\mathfrak{t}-\mathfrak{l})_{si}(k)(\mathfrak{t}-\mathfrak{l})_{tn}(k)-\delta_{st}\mathfrak{l}_{in}(k)]\mathfrak{l}_{nj}(q) - [(\mathfrak{t}-\mathfrak{l})_{sn}(k)(\mathfrak{t}-\mathfrak{l})_{tj}(k)-\delta_{st}\mathfrak{l}_{jn}(k)]\mathfrak{l}_{ni}(q)$$
$$+\; 4\frac{(k-q)_t}{(k-q)^2}\Big(\mathfrak{t}_{si}(k)k_j\mathfrak{t}_{sj}(k)k_i + (\mathfrak{t}-\mathfrak{l})_{sk}(k)\Big[\mathfrak{l}_{ki}(q)k_j + \mathfrak{l}_{kj}(q)k_i\Big] + \delta_{si}\mathfrak{l}_{jk}(q)k_k + \delta_{sj}\mathfrak{l}_{ik}(q)k_k\Big). \tag{C18}$$

With these expressions we can now proceed to reduce term by term Eq. (64). We start with the first term.

1. $\langle 12\rangle\langle 34\rangle$.

Here we know that the terms that do not involve expansion of l cancel between the two lines. Thus we only write the terms that involve the coefficients A and B.

$$\langle 12\rangle\langle 34\rangle_{[1,1,1]} \;=\; N_c\mathrm{Tr}\left[S^\dagger(w)S(a)S^\dagger(p+k-a)S(v)\right]\mu^4 w_s v_t a_r\frac{1}{p^2k^4(k-q)^4}C_{si}(k)A^1_{rij}(k,q)C_{tj}(p)$$
$$=\; iN_c\int_{a,w}\mathrm{Tr}\left[\partial_sS^\dagger(w)\partial_rS(a)S^\dagger(p+k-a)\partial_tS(-p-k-w)\right]\mu^4\frac{1}{p^2k^4(k-q)^4}C_{si}(k)A^1_{rij}(k,q)C_{tj}(p)$$
$$=\; iN_c\mathrm{Tr}\left[\{\partial_rSS^\dagger\}(p+k)\{\partial_tS\partial_sS^\dagger\}(-p-k)\right]\mu^4\frac{1}{p^2k^4(k-q)^4}C_{si}(k)A^1_{rij}(k,q)C_{tj}(p). \tag{C19}$$

The quantity under trace is a function of one momentum $(p+k)$, which is sharply peaked around $p+k=0$. We therefore can expand it in derivatives of a delta function. In general we have:

$$f(p)g(-p) \;=\; \delta^2(p)\int_k f(k)g(-k) - \partial_i\delta^2(p)\int_k k_if(k)g(-k) + \frac{1}{2}\partial_i\partial_j\delta^2(p)\int_k k_ik_jf(k)g(-k)$$
$$=\; \delta^2(p)\int_x f(x)g(x) + i\partial_i\delta^2(p)\int_x[\partial_if(x)]g(x) - \frac{1}{2}\partial_i\partial_j\delta^2(p)\int_x[\partial_i\partial_jf(x)]g(x). \tag{C20}$$

In Eq. (C19) only the derivative of the delta function survives if we assume rotational invariance of the target averages, and we therefore obtain

$$\langle 12\rangle\langle 34\rangle_{[1,1,1]} = -\partial_n\delta^2(p+k)N_c\int_x\mathrm{Tr}\left[\{\partial_n[\partial_rSS^\dagger]\partial_tS\partial_sS^\dagger\}(x)\right]\int_q\frac{\mu^4}{p^2k^4(k-q)^4}C_{si}(k)A^1_{rij}(k,q)C_{tj}(p), \tag{C21}$$

$$\langle 12\rangle\langle 34\rangle_{[1,2,1]} = \delta^2(p+k)N_c\int_x\mathrm{Tr}\left[\{[\partial_n\partial_rS]S^\dagger\partial_tS\partial_sS^\dagger\}(x)\right]\int_q\frac{\mu^4}{p^2k^4(k-q)^4}C_{si}(k)A^2_{nrij}(k,q)C_{tj}(p), \tag{C22}$$

$$\langle 12\rangle\langle 34\rangle_{[2,1,1]} = -\delta^2(p+k)N_c\int_x\mathrm{Tr}\left[\{[\partial_rS]S^\dagger\partial_tS[\partial_n\partial_sS^\dagger]\}(x)\right]\int_q\frac{\mu^4}{p^2k^4(k-q)^4}D_{nsi}(k)A^1_{rij}(k,q)C_{tj}(p), \tag{C23}$$

$$\langle 12\rangle\langle 34\rangle_{[1,1,2]} = -\delta^2(p+k)N_c\int_x\mathrm{Tr}\left[\{[\partial_rS]S^\dagger[\partial_n\partial_tS]\partial_sS^\dagger\}(x)\right]\int_q\frac{\mu^4}{p^2k^4(k-q)^4}C_{si}(k)A^1_{rij}(k,q)D_{ntj}(p). \tag{C24}$$

Substituting the condensates (see Appendix D) we get

$$\langle\langle 12\rangle\langle 34\rangle_{\mathrm{sum}}\rangle_\alpha = 7N_c^3(N_c^2-1)\frac{\lambda^4\mu^4}{k^6}\delta^2(p+k)\int_q\frac{1}{(k-q)^4}\left(\frac{3}{2}-2\frac{(k\cdot q)^2}{k^2q^2}-5\frac{(k\times q)^2k\cdot(k-q)}{(k-q)^2q^2k^2}\right). \tag{C25}$$

2. $\langle 13\rangle\langle 24\rangle$.

To calculate this term we define

$$-k+l-q=w;\quad k-l=a;\quad p-m=b;\quad -p+m+q=v \tag{C26}$$

which gives

$$q-l=-(k+w);\quad m+q=p+v;\quad q=-(w+a)=v+b. \tag{C27}$$

Now

$$\begin{aligned}
&\langle 13\rangle\langle 24\rangle_{[1,0,1]}=\frac{\mu^4}{p^4k^4}C_{si}(k)C_{tj}(p)(-k+l-q)_s(-p+m+q)_t\left[\mathfrak{t}_{in}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p)+\mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(p)\right]\times\\
&\Bigg[f^{acd}S^{de}(k-l)S^{\dagger ea}(-k+l-q)f^{cbf}S^{fg}(p-m)S^{\dagger gb}(-p+m+q)\\
&-S^{ac}(k-l)f^{cde}S^{\dagger ea}(-k+l-q)S^{bf}(p-m)f^{dfg}S^{\dagger gb}(q-p+m)\Bigg]\\
&=-\frac{\mu^4}{p^4k^4}C_{si}(k)C_{tj}(p)\left[\mathfrak{t}_{in}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p)+\mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(p)\right]\times\\
&\Bigg[\Big[f^{acd}S^{de}\partial_sS^{\dagger ea}\Big](-q)\Big[f^{cbf}S^{fg}\partial_tS^{\dagger gb}\Big](q)-\Big[S^{ac}f^{cde}\partial_sS^{\dagger ea}\Big](-q)\Big[S^{bf}f^{dfg}\partial_tS^{\dagger gb}\Big](q)\Bigg]\\
&=-\frac{\mu^4}{p^4k^4}\left[\mathfrak{t}_{sn}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p)+\mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{tm}(p)\right]\times\\
&\Bigg[\Big[f^{acd}S^{de}\partial_sS^{\dagger ea}\Big](-q)\Big[f^{cbf}S^{fg}\partial_tS^{\dagger gb}\Big](q)-\Big[S^{ac}f^{cde}\partial_sS^{\dagger ea}\Big](-q)\Big[S^{bf}f^{dfg}\partial_tS^{\dagger gb}\Big](q)\Bigg].
\end{aligned} \tag{C28}$$

It is clear that this expression is odd under the charge conjugation transformation $S\to S^\dagger$, and therefore vanishes in any MV-like model. In principle this contribution is nonzero, however it is impossible to determine its sign without a specific model. We therefore will not consider it any further.

$$\begin{aligned}
&\langle 13\rangle\langle 24\rangle_{[2,0,1]}=-\frac{\mu^4}{p^4k^4}D_{rsi}(k)C_{tj}(p)(-k+l-q)_r(-k+l-q)_s(-p+m+q)_t\times\\
&\left[\mathfrak{t}_{in}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p)+\mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(p)\right]\times\\
&\Bigg[f^{acd}S^{de}(k-l)S^{\dagger ea}(-k+l-q)f^{cbf}S^{fg}(p-m)S^{\dagger gb}(-p+m+q)\\
&-S^{ac}(k-l)f^{cde}S^{\dagger ea}(-k+l-q)S^{bf}(p-m)f^{dfg}S^{\dagger gb}(q-p+m)\Bigg]\\
&=i\frac{\mu^4}{p^4k^4}D_{rsi}(k)C_{tj}(p)\left[\mathfrak{t}_{in}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p)+\mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(p)\right]\times\\
&\Bigg[\Big[f^{acd}S^{de}[\partial_r\partial_sS^{\dagger ea}]\Big](-q)\Big[f^{cbf}S^{fg}\partial_tS^{\dagger gb}\Big](q)-\Big[S^{ac}f^{cde}[\partial_r\partial_sS^{\dagger ea}]\Big](-q)\Big[S^{bf}f^{dfg}\partial_tS^{\dagger gb}\Big](q)\Bigg],
\end{aligned} \tag{C29}$$

$$\begin{aligned}
&\langle 13\rangle\langle 24\rangle_{[1,0,2]}=i\frac{\mu^4}{p^4k^4}C_{si}(k)D_{rtj}(p)\left[\mathfrak{t}_{in}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p)+\mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(p)\right]\times\\
&\Bigg[\Big[f^{acd}S^{de}\partial_sS^{\dagger ea}\Big](-q)\Big[f^{cbf}S^{fg}[\partial_r\partial_tS^{\dagger gb}]\Big](q)-\Big[S^{ac}f^{cde}\partial_sS^{\dagger ea}\Big](-q)\Big[S^{bf}f^{dfg}[\partial_r\partial_tS^{\dagger gb}]\Big](q)\Bigg].
\end{aligned} \tag{C30}$$

Adding the two together we get

$$\langle 13\rangle\langle 24\rangle_{[2,0,1]} + \langle 13\rangle\langle 24\rangle_{[1,0,2]} = i\frac{\mu^4}{p^4k^4}\Big[D_{rsi}(k)C_{tj}(p) + C_{ti}(k)D_{rsj}(p)\Big]\left[\mathfrak{t}_{in}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p) + \mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(p)\right]\times$$
$$\Bigg[\Big[f^{acd}S^{de}[\partial_r\partial_s S^{\dagger ea}]\Big](-q)\Big[f^{cbf}S^{fg}\partial_t S^{\dagger gb}\Big](q) - \Big[S^{ac}f^{cde}[\partial_r\partial_s S^{\dagger ea}]\Big](-q)\Big[S^{bf}f^{dfg}\partial_t S^{\dagger gb}\Big](q)\Bigg]. \tag{C31}$$

This expression is also odd under charge conjugation. Additionally the integral over q results in an object with five rotational indices. There is no invariant tensor with five indices in 2 dimensions, therefore for a rotationally invariant target charge distribution this term vanishes.

$$\langle 13\rangle\langle 24\rangle_{[2,0,2]} = -\frac{\mu^4}{p^4k^4}D_{lsi}(k)D_{rtj}(p)\left[\mathfrak{t}_{in}(k)\mathfrak{l}_{nm}(q)\mathfrak{t}_{mj}(p) + \mathfrak{l}_{in}(k)\mathfrak{t}_{nm}(q)\mathfrak{l}_{jm}(p)\right]\times$$
$$\Bigg[\Big[f^{acd}S^{de}[\partial_l\partial_s S^{\dagger ea}]\Big](-q)\Big[f^{cbf}S^{fg}[\partial_r\partial_t S^{\dagger gb}]\Big](q) - \Big[S^{ac}f^{cde}[\partial_l\partial_s S^{\dagger ea}]\Big](-q)\Big[S^{bf}f^{dfg}[\partial_r\partial_t S^{\dagger gb}]\Big](q)\Bigg]. \tag{C32}$$

Writing $S^{ab} = S^{\dagger ba}$ in the second term, we see that this expression again is antisymmetric under $S \to S^\dagger$. Therefore again, it vanishes in a MV-like model.

Going back to Eq. (64) it is obvious that if we set $l = k,\ p = m$ in the second chain of projectors and in the "propagators", both the expressions in $\langle 13\rangle\langle 24\rangle$ and $\langle 14\rangle\langle 23\rangle$ are odd under the charge conjugation. Thus in both these expressions we only need consider terms where at least one of the momenta l or m is expanded around k and p respectively.

Again we expand

$$\frac{1}{(k+w)^2(p+v)^2} = \frac{1}{k^2p^2}\left[1 - \frac{2p\cdot v}{p^2} - \frac{2k\cdot w}{k^2} - \frac{w^2}{k^2} - \frac{v^2}{p^2} + \frac{4(k\cdot w)^2}{k^4} + \frac{4(p\cdot v)^2}{p^4} + \frac{4(k\cdot w)(p\cdot v)}{k^2p^2}\right], \tag{C33}$$

$$\frac{1}{(k-q)^2(p+q)^2} = \frac{1}{k^2p^2}\times \tag{C34}$$
$$\left[1 - \frac{2p\cdot(v+b)}{p^2} - \frac{2k\cdot(w+a)}{k^2} - \frac{(w+a)^2}{k^2} - \frac{(v+b)^2}{p^2}\right.$$
$$\left. + \frac{4(k\cdot(w+a))^2}{k^4} + \frac{4(p\cdot(v+b))^2}{p^4} + \frac{4(k\cdot(w+a))(p\cdot(v+b))}{k^2p^2}\right]$$

and

$$\Big[\mathfrak{l}(q) - \mathfrak{l}(k-a)\mathfrak{l}(q) - \mathfrak{l}(q)\mathfrak{l}(p-b) + \mathfrak{l}(k-a)\mathfrak{l}(p-b)\Big]_{ij} = \Big[\mathfrak{l}(q) - \mathfrak{l}(k)\mathfrak{l}(q) - \mathfrak{l}(q)\mathfrak{l}(p) + \mathfrak{l}(k)\mathfrak{l}(p)\Big]_{ij} \tag{C35}$$
$$-\frac{1}{k^2}\left[a_ik_n + a_nk_i - 2(a\cdot k)\frac{k_ik_n}{k^2}\right]\Big[\mathfrak{l}(p) - \mathfrak{l}(q)\Big]_{nj} - \frac{1}{p^2}\Big[\mathfrak{l}(k) - \mathfrak{l}(q)\Big]_{in}\left[b_np_j + p_nb_j - 2(b\cdot p)\frac{p_np_j}{p^2}\right]$$
$$+\frac{1}{k^2}\left[a_ia_n - 2(a\cdot k)\frac{k_ia_n + k_na_i}{k^2} + k_ik_n\frac{4(a\cdot k)^2 - k^2a^2}{k^4}\right]\Big[\mathfrak{l}(p) - \mathfrak{l}(q)\Big]_{nj}$$
$$+\frac{1}{p^2}\Big[\mathfrak{l}(k) - \mathfrak{l}(q)\Big]_{in}\left[b_nb_j - 2(b\cdot p)\frac{p_jb_n + p_nb_j}{p^2} + p_jp_n\frac{4(b\cdot p)^2 - p^2b^2}{p^4}\right]$$
$$+\frac{1}{k^2p^2}\left[a_ik_n + a_nk_i - 2(a\cdot k)\frac{k_ik_n}{k^2}\right]\left[b_np_j + p_nb_j - 2(b\cdot p)\frac{p_np_j}{p^2}\right]. \tag{C36}$$

All of this will generate different terms where the derivatives will act on different factors of S. It thus seems prudent to organize things according to different powers of the four momenta. The overall factor which will multiply all the following expressions is

$$-S^{ac}(a)f^{cde}S^{\dagger ea}(w)S^{bf}(b)f^{dfg}S^{\dagger gb}(v)\frac{\mu^4}{k^2p^2} = \mathrm{Tr}[S(a)f^dS^\dagger(w)]\mathrm{Tr}[S(b)f^dS^\dagger(v)]\frac{\mu^4}{k^2p^2}. \tag{C37}$$

These are the contributions that arise from using only Eq. (C33):

$$\frac{1}{k^4p^2}w_sw_rw_ov_t\Big[C_{si}(k)[-\delta_{ro}+4\mathfrak{l}_{ro}(k)]+2D_{sri}(k)k_0\Big]\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{ij}C_{jt}(p), \tag{C38}$$

$$\frac{1}{p^4k^2}w_sv_rv_ov_tC_{si}(k)\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{ij}\Big[C_{tj}(p)(-\delta_{ro}+4\mathfrak{l}_{ro}(p))+2D_{trj}(p)p_0\Big], \tag{C39}$$

$$\begin{aligned}&\frac{2}{k^2p^2}w_sw_rv_ov_t\left\{\Big[D_{sri}(k)+C_{si}(k)\frac{k_r}{k^2}\Big]C_{tj}(p)\frac{p_o}{p^2}+C_{si}(k)\frac{k_r}{k^2}\Big[D_{toj}(p)+C_{tj}(p)\frac{p_o}{p^2}\Big]\right\}\\&\times\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{ij},\end{aligned} \tag{C40}$$

$$\frac{1}{k^4p^2}w_sw_ra_ov_t\Big[D_{sri}(k)+2C_{si}(k)\frac{k_r}{k^2}\Big]\Big[\mathfrak{t}_{io}(k)k_n+\mathfrak{t}_{no}(k)k_i\Big]\big[\mathfrak{l}(p)-\mathfrak{l}(q)\big]_{nj}C_{tj}(p), \tag{C41}$$

$$\frac{1}{k^4p^2}w_sa_rv_ov_tC_{si}(k)\Big[\mathfrak{t}_{ir}(k)k_n+\mathfrak{t}_{nr}(k)k_i\Big]\big[\mathfrak{l}(p)-\mathfrak{l}(q)\big]_{nj}\Big[D_{toj}(p)+2C_{tj}(p)\frac{p_o}{p^2}\Big], \tag{C42}$$

$$\frac{1}{2k^4p^2}w_sa_ra_ov_tC_{si}(k)\Big[\delta_{on}\big[\delta_{ri}-4\mathfrak{l}_{ri}(k)\big]+\delta_{oi}\big[\delta_{rn}-4\mathfrak{l}_{rn}(k)\big]-2\mathfrak{l}_{in}(k)\big[\delta_{ro}-4\mathfrak{l}_{ro}(k)\big]\big[\mathfrak{l}(p)-\mathfrak{l}(q)\big]_{nj}\Big]C_{tj}(p), \tag{C43}$$

$$\frac{1}{k^2p^4}w_sw_rb_ov_t\Big[D_{sri}(k)+2C_{si}(k)\frac{k_r}{k^2}\Big]\big[\mathfrak{l}(k)-\mathfrak{l}(q)\big]_{in}\Big[\mathfrak{t}_{jo}(p)p_n+\mathfrak{t}_{no}(p)p_j\Big]C_{tj}(p), \tag{C44}$$

$$\frac{1}{k^2p^4}w_sb_rv_ov_tC_{si}(k)\big[\mathfrak{l}(k)-\mathfrak{l}(q)\big]_{ni}\Big[\mathfrak{t}_{jr}(p)p_n+\mathfrak{t}_{nr}(p)p_j\Big]\Big[D_{toj}(p)+2C_{tj}(p)\frac{p_o}{p^2}\Big], \tag{C45}$$

$$\frac{1}{2k^2p^4}w_sb_rb_ov_tC_{si}(k)\big[\mathfrak{l}(k)-\mathfrak{l}(q)\big]_{in}\Big[\delta_{on}\big[\delta_{rj}-4\mathfrak{l}_{rj}(p)\big]+\delta_{oj}\big[\delta_{rn}-4\mathfrak{l}_{rn}(p)\big]-2\mathfrak{l}_{jn}(p)\big[\delta_{ro}-4\mathfrak{l}_{ro}(p)\big]\Big]C_{tj}(p), \tag{C46}$$

$$\frac{1}{k^4p^4}w_sa_rb_ov_tC_{si}(k)\Big[\mathfrak{t}_{ir}(k)k_n+\mathfrak{t}_{nr}(k)k_i\Big]\Big[\mathfrak{t}_{jo}(p)p_n+\mathfrak{t}_{no}(p)p_j\Big]C_{tj}(p). \tag{C47}$$

From the above we have to subtract terms that arise from Eq. (C34), but only when they are not accompanied by expansion of the projector part. In other words, all the terms above that contain at least one factor of a or b remain as they are, but the rest get modified by the subtraction. The additional terms that have to be added are

$$\begin{aligned}&-\frac{1}{k^4p^2}\Big[w_s(w+a)_r(w+a)_ov_tC_{si}(k)[-\delta_{ro}+4\mathfrak{l}_{ro}(k)]+w_sw_r(w+a)_ov_t2D_{sri}(k)k_0\Big]\\&\times\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{ij}C_{jt}(p),\end{aligned} \tag{C48}$$

$$\begin{aligned}&-\frac{1}{p^4k^2}C_{si}(k)\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{ij}\\&\times\Big[w_s(v+b)_r(v+b)_ov_tC_{tj}(p)(-\delta_{ro}+4\mathfrak{l}_{ro}(p))+w_sv_r(v+b)_ov_t2D_{trj}(p)p_0\Big],\end{aligned} \tag{C49}$$

$$\begin{aligned}&-\frac{2}{k^2p^2}\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{ij}\times\\&\left\{\Big[w_sw_rD_{sri}(k)+w_s(w+a)_rC_{si}(k)\frac{k_r}{k^2}\Big](v+b)_ov_tC_{tj}(p)\frac{p_o}{p^2}\right.\\&\left.+w_s(w+a)_rC_{si}(k)\frac{k_r}{k^2}\Big[v_ov_tD_{toj}(p)+(v+b)_ov_tC_{tj}(p)\frac{p_o}{p^2}\Big]\right\}.\end{aligned} \tag{C50}$$

In other words Eqs. (C38), (C39) and (C40) have to be exchanged for

$$\begin{aligned}&-\frac{1}{k^4p^2}w_sw_ra_ov_t\Big[C_{si}(k)[-\delta_{ro}+4\mathfrak{l}_{ro}(k)]+2D_{sri}(k)k_0\Big]\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{ij}C_{jt}(p)\\&-\frac{1}{k^4p^2}\Big[w_sw_oa_rv_t+w_sa_oa_rv_t\Big]C_{si}(k)[-\delta_{ro}+4\mathfrak{l}_{ro}(k)]\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{ij}C_{jt}(p),\end{aligned}\tag{C51}$$

$$\begin{aligned}&-\frac{1}{p^4k^2}w_sv_rb_ov_tC_{si}(k)\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{ij}\Big[C_{tj}(p)(-\delta_{ro}+4\mathfrak{l}_{ro}(p))+2D_{trj}(p)p_0\Big]\\&-\frac{1}{p^4k^2}\Big[w_sb_rv_ov_t+w_sb_rb_ov_t\Big]C_{si}(k)\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{ij}C_{tj}(p)(-\delta_{ro}+4\mathfrak{l}_{ro}(p)),\end{aligned}\tag{C52}$$

$$\begin{aligned}&-\frac{2}{k^2p^2}\Bigg\{w_sw_rb_ov_t\Big[D_{sri}(k)+C_{si}(k)\frac{k_r}{k^2}\Big]C_{tj}(p)\frac{p_o}{p^2}+w_sa_rv_ov_tC_{si}(k)\frac{k_r}{k^2}\Big[D_{toj}(p)+C_{tj}(p)\frac{p_o}{p^2}\Big]\\&+\Big[w_sa_rv_ov_t+w_sa_rb_ov_t+w_sw_rb_ov_t+w_sa_rb_ov_t\Big]C_{si}(k)\frac{k_r}{k^2}C_{tj}(p)\frac{p_o}{p^2}\Big]\Bigg\}\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{ij}.\end{aligned}\tag{C53}$$

We can simplify these expressions by using the following identities

$$\begin{aligned}&(\mathfrak{t}-\mathfrak{l})(k)\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big](\mathfrak{t}-\mathfrak{l})(p)=\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p),\\&k\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big](\mathfrak{t}-\mathfrak{l})(p)=k\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\Big]\mathfrak{l}(p).\end{aligned}\tag{C54}$$

Thus we have

$$\begin{aligned}&-\frac{1}{k^4p^2}w_sw_ra_ov_t\Big[[-2\delta_{ro}+12\mathfrak{l}_{ro}(k)]\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}+2\delta_{sr}\Big[\mathfrak{l}(k)\mathfrak{l}(q)\mathfrak{l}(p)-\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{ot}\Big]\\&-\frac{1}{k^4p^2}\Big[w_sa_oa_rv_t\Big][-\delta_{ro}+4\mathfrak{l}_{ro}(k)]\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st},\end{aligned}\tag{C55}$$

$$\begin{aligned}&-\frac{1}{p^4k^2}w_sv_rb_ov_t\Bigg[\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}[-2\delta_{ro}+12\mathfrak{l}_{ro}(p)]+2[\mathfrak{l}(k)\mathfrak{l}(q)\mathfrak{l}(p)-\mathfrak{l}(k)\mathfrak{l}(p)]_{so}\delta_{rt}\Bigg]\\&-\frac{1}{p^4k^2}\Big[w_sb_rb_ov_t\Big]\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}[-\delta_{ro}+4\mathfrak{l}_{ro}(p)],\end{aligned}\tag{C56}$$

$$\begin{aligned}&-\frac{2}{k^4p^4}\Bigg\{w_sw_rb_ov_t\delta_{sr}\Big[k[\mathfrak{l}(q)-\mathfrak{l}(k)]\mathfrak{l}(p)\Big]_tp_o+w_sa_rv_ov_t\delta_{to}k_r\Big[\mathfrak{l}(k)[\mathfrak{l}(q)-\mathfrak{l}(p)]p\Big]_s\\&+\Big[4w_sa_rv_ov_t+w_sa_rb_ov_t+4w_sw_rb_ov_t+w_sa_rb_ov_t\Big]k_rp_o\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}\Bigg\}.\end{aligned}\tag{C57}$$

Now collecting terms we find

$$\begin{aligned}&-\mathrm{Tr}[\partial_oSf^d\partial_s\partial_rS^\dagger](q)\mathrm{Tr}[Sf^d\partial_tS^\dagger](-q)\frac{\mu^4}{k^6p^4}\times\\&\Big[[-2\delta_{ro}+12\mathfrak{l}_{ro}(k)]\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}+2\delta_{sr}\Big[\mathfrak{l}(k)\mathfrak{l}(q)\mathfrak{l}(p)-\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{ot}\Big]\\&-\mathrm{Tr}[\partial_o\partial_rSf^d\partial_sS^\dagger](q)\mathrm{Tr}[Sf^d\partial_tS^\dagger](-q)\frac{\mu^4}{k^6p^4}[-\delta_{ro}+4\mathfrak{l}_{ro}(k)]\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st},\end{aligned}\tag{C58}$$

$$\begin{aligned}&-\mathrm{Tr}[Sf^d\partial_sS^\dagger](q)\mathrm{Tr}[\partial_oSf^d\partial_r\partial_tS^\dagger](-q)\frac{\mu^4}{k^4p^6}\times\\&\Bigg[\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}[-2\delta_{ro}+12\mathfrak{l}_{ro}(p)]+2[\mathfrak{l}(k)\mathfrak{l}(q)\mathfrak{l}(p)-\mathfrak{l}(k)\mathfrak{l}(p)]_{so}\delta_{rt}\Bigg]\\&-\mathrm{Tr}[Sf^d\partial_sS^\dagger](q)\mathrm{Tr}[\partial_o\partial_rSf^d\partial_tS^\dagger](-q)\frac{\mu^4}{k^4p^6}\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}[-\delta_{ro}+4\mathfrak{l}_{ro}(p)],\end{aligned}\tag{C59}$$

$$-\frac{2\mu^4}{k^6p^6}\Big\{\mathrm{Tr}[Sf^d\partial_s\partial_r S^\dagger](q)\mathrm{Tr}[\partial_o Sf^d\partial_t S^\dagger](-q)$$

$$\times\Big[\delta_{sr}\Big[k[\mathfrak{l}(q)-\mathfrak{l}(k)]\mathfrak{l}(p)\Big]_t p_o+4k_rp_o\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}\Big]$$

$$+\mathrm{Tr}[\partial_r Sf^d\partial_s S^\dagger](q)\mathrm{Tr}[Sf^d\partial_o\partial_t S^\dagger](-q)$$

$$\times\Big[\delta_{to}k_r\Big[\mathfrak{l}(k)[\mathfrak{l}(q)-\mathfrak{l}(p)]p\Big]_s+4k_rp_o\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}\Big]$$

$$+2\Big[\mathrm{Tr}[\partial_r Sf^d\partial_s S^\dagger](q)\mathrm{Tr}[\partial_o Sf^d\partial_t S^\dagger](-q)k_rp_o\Big[\mathfrak{l}(q)-\mathfrak{l}(k)\mathfrak{l}(q)-\mathfrak{l}(q)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}\Big\}, \tag{C60}$$

$$\mathrm{Tr}[\partial_o Sf^d\partial_s\partial_r S^\dagger](q)\mathrm{Tr}[Sf^d\partial_t S^\dagger](-q)\frac{\mu^4}{k^4p^4}\times \tag{C61}$$
$$\Big[\frac{1}{k^2}\Big[D_{sri}(k)+2C_{si}(k)\frac{k_r}{k^2}\Big]\Big[\mathfrak{t}_{io}(k)k_n+\mathfrak{t}_{no}(k)k_i\Big]\big[\mathfrak{l}(p)-\mathfrak{l}(q)\big]_{nj}C_{tj}(p)+(p\leftrightarrow k)\Big],$$

$$\mathrm{Tr}[\partial_r Sf^d\partial_s S^\dagger](q)\mathrm{Tr}[Sf^d\partial_o\partial_t S^\dagger](-q)\frac{\mu^4}{k^4p^4}\times \tag{C62}$$
$$\Big[\frac{1}{k^2}C_{si}(k)\Big[\mathfrak{t}_{ir}(k)k_n+\mathfrak{t}_{nr}(k)k_i\Big]\big[\mathfrak{l}(p)-\mathfrak{l}(q)\big]_{nj}\Big[D_{toj}(p)+2C_{tj}(p)\frac{p_o}{p^2}\Big]+(p\leftrightarrow k)\Big],$$

$$\mathrm{Tr}[\partial_o\partial_r Sf^d\partial_s S^\dagger](q)\mathrm{Tr}[Sf^d\partial_t S^\dagger](-q)\frac{\mu^4}{k^4p^4}\times \tag{C63}$$
$$\Big[\frac{1}{2k^2}C_{si}(k)\Big[\delta_{on}\big[\delta_{ri}-4\mathfrak{l}_{ri}(k)\big]+\delta_{oi}\big[\delta_{rn}-4\mathfrak{l}_{rn}(k)\big]-2\mathfrak{l}_{in}(k)\big[\delta_{ro}-4\mathfrak{l}_{ro}(k)\big]\Big]\big[\mathfrak{l}(p)-\mathfrak{l}(q)\big]_{nj}C_{tj}(p)+(p\leftrightarrow k)\Big],$$

$$\mathrm{Tr}[\partial_r Sf^d\partial_s S^\dagger](q)\mathrm{Tr}[\partial_o Sf^d\partial_t S^\dagger](-q)\frac{\mu^4}{k^6p^6}\times \tag{C64}$$
$$C_{si}(k)\Big[\mathfrak{t}_{ir}(k)k_n+\mathfrak{t}_{nr}(k)k_i\Big]\Big[\mathfrak{t}_{jo}(p)p_n+\mathfrak{t}_{no}(p)p_j\Big]C_{tj}(p).$$

We can simplify these expressions by using the following

$$D_{sri}(k)+2C_{si}\frac{k_r}{k^2}=4(\mathfrak{t}-\mathfrak{l})_{si}(k)\frac{k_r}{k^2}+\delta_{sr}\frac{k_i}{k^2},\qquad D_{sri}(k)+C_{si}\frac{k_r}{k^2}=3(\mathfrak{t}-\mathfrak{l})_{si}(k)\frac{k_r}{k^2}+\delta_{sr}\frac{k_i}{k^2}, \tag{C65}$$

$$C_{si}(k)[-\delta_{ro}+4\mathfrak{l}_{ro}(k)]+2D_{sri}(k)k_o=-\delta_{si}\delta_{ro}+8\delta_{si}(\mathfrak{t}-\mathfrak{l})_{ro}(k)+[\text{antisymmetric under}(s\leftrightarrow o)], \tag{C66}$$

$$C_{si}(k)\Big[\mathfrak{t}_{ir}(k)k_n+\mathfrak{t}_{nr}(k)k_i\Big]=\mathfrak{t}_{sr}(k)k_n\mathfrak{t}_{nr}(k)k_s=\delta_{sr}k_n-\delta_{nr}k_s, \tag{C67}$$

$$C_{si}(k)\Big[\delta_{on}\big[\delta_{ri}-4\mathfrak{l}_{ri}(k)\big]+\delta_{oi}\big[\delta_{rn}-4\mathfrak{l}_{rn}(k)\big]-2\mathfrak{l}_{in}(k)\big[\delta_{ro}-4\mathfrak{l}_{ro}(k)\big]\Big]=2\delta_{rs}(\mathfrak{t}-\mathfrak{l})_{on}(k)+2\delta_{ro}\mathfrak{l}_{sn}(k)$$
$$+\text{antisymmetric under(o}\leftrightarrow\text{r)}. \tag{C68}$$

There is an additional significant simplification. Recall that at the end of the day we need to subtract from these expressions the same expressions with $p\to -p$. Thus any term which is even under such transformation will cancel in the final answer.

We will further use the model for the calculating the target correlations, as explained in Appendix D. With this averaging procedure denotes by $\langle\dots\rangle_\alpha$, the integral involving q^iq^j is always proportional to δ^{ij}. This means that it can be calculating considering the trace over the rotational indices, and thus we can substitute in all expressions $\mathfrak{l}_{ij}(q)\to 1/2\delta_{ij}$. With this substitution all the integrals over q simply yield local condensates of the products of the target S-matrices. Given this, the expressions become quite simple:

$$\langle\langle 13\rangle\langle 24\rangle\rangle_\alpha = \int_x \mathrm{Tr}[Sf^d\partial_s\partial_r S^\dagger]\mathrm{Tr}[Sf^d\partial_t\partial_o S^\dagger](x)\frac{2\mu^4}{k^6p^6}\times X + \tag{C69}$$

$$\int_x \mathrm{Tr}[\partial_r Sf^d\partial_s S^\dagger]\mathrm{Tr}[Sf^d\partial_o\partial_t S^\dagger](x)\frac{\mu^4}{k^4p^4}\times Y + \tag{C70}$$

$$\int_x \mathrm{Tr}[\partial_r Sf^d\partial_s S^\dagger]\mathrm{Tr}[\partial_o Sf^d\partial_t S^\dagger](x)\frac{\mu^4}{k^6p^6}\times \tag{C71}$$

$$\Big[\delta_{sr}\delta_{to}k\cdot p - \delta_{to}p_rk_s - \delta_{sr}p_tk_o + \delta_{ro}k_sp_t - 2k_rp_o\Big[1 - \mathfrak{l}(k) - \mathfrak{l}(p) + 2\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}\Big] \tag{C72}$$

$$+(\text{symmetric under p} \to -\text{p})$$

$$= -\frac{1}{4}N_c^3(N_c^2-1)\frac{\mu^4\lambda^4}{p^6k^6}(k\cdot p) \tag{C73}$$

We did not bother to calculate the first two terms involving X and Y, since the condensates that multiply them vanish in our model (see Appendix D).

3. $\langle 14\rangle\langle 23\rangle$.

For this term of the calculation we introduce

$$k - l = a; \quad p - m = b; \quad -p + l - q = v; \quad -k + q + m = w \tag{C74}$$

or

$$q - l = -(p + v); \quad q + m = k + w; \quad q = k - p + (w + b). \tag{C75}$$

With these definitions the expansion now is exactly the same as for the calculation of $\langle 13\rangle\langle 24\rangle$. The only thing that changes is the common factor, which now is

$$\mathrm{Tr}[S(a)f^dS^\dagger(v)S(b)f^dS^\dagger(w)]\frac{\mu^4}{k^2p^2}. \tag{C76}$$

The effect of this is that all the expansion is the same, but in the final expression the derivatives acting on the two factors $S^\dagger$ have to be interchanged. Another major difference is that the momentum of the Fourier transform of the composite operators is not q, but $q + p - k$. It also means that we need to consider one extra term, of the type $[1, 1, 1]$, which will give a contribution proportional to the derivative of the delta function. This last contribution we will calculate last. Meanwhile we can recycle our results.

$$\mathrm{Tr}\Big[[Sf^d\partial_s\partial_r\partial_o S^\dagger](q+p-k)[Sf^d\partial_t S^\dagger](-q-p+k)\Big]\frac{\mu^4}{k^4p^4}\times \tag{C77}$$

$$\Big[\frac{1}{k^2}\Big[C_{si}(k)[-\delta_{ro} + 4\mathfrak{l}_{ro}(k)] + 2D_{sri}(k)k_0\Big]\Big[\mathfrak{l}(q) - \mathfrak{l}(k)\mathfrak{l}(q) - \mathfrak{l}(q)\mathfrak{l}(p) + \mathfrak{l}(k)\mathfrak{l}(p)\Big]_{ij}C_{jt}(p) + (p\leftrightarrow k)\Big],$$

$$\mathrm{Tr}\Big[[Sf^d\partial_s\partial_r S^\dagger](q+p-k)[Sf^d\partial_o\partial_t S^\dagger](-q-p+k)\Big]\frac{2\mu^4}{k^4p^4}\times \tag{C78}$$

$$\Big\{\Big[D_{sri}(k) + C_{si}(k)\frac{k_r}{k^2}\Big]C_{tj}(p)\frac{p_o}{p^2} + C_{si}(k)\frac{k_r}{k^2}\Big[D_{toj}(p) + C_{tj}(p)\frac{p_o}{p^2}\Big]\Big\}\Big[\mathfrak{l}(q) - \mathfrak{l}(k)\mathfrak{l}(q) - \mathfrak{l}(q)\mathfrak{l}(p) + \mathfrak{l}(k)\mathfrak{l}(p)\Big]_{ij},$$

$$\mathrm{Tr}\Big[[\partial_o Sf^d\partial_t S^\dagger](q+p-k)[Sf^d\partial_s\partial_r S^\dagger](-q-p+k)\Big]\frac{\mu^4}{k^4p^4}\times \tag{C79}$$

$$\Big[\frac{1}{k^2}\Big[D_{sri}(k) + 2C_{si}(k)\frac{k_r}{k^2}\Big]\Big[\mathfrak{t}_{io}(k)k_n + \mathfrak{t}_{no}(k)k_i\Big]\Big[\mathfrak{l}(p) - \mathfrak{l}(q)\Big]_{nj}C_{tj}(p) + (p\leftrightarrow k)\Big],$$

$$\mathrm{Tr}\Big[[\partial_r S f^d \partial_o \partial_t S^\dagger](q+p-k)[S f^d \partial_s S^\dagger](-q-p+k)\Big]\frac{\mu^4}{k^4p^4}\times$$
$$\Big[\frac{1}{k^2}C_{si}(k)\Big[\mathfrak{t}_{ir}(k)k_n+\mathfrak{t}_{nr}(k)k_i\Big]\big[\mathfrak{l}(p)-\mathfrak{l}(q)\big]_{nj}\Big[D_{toj}(p)+2C_{tj}(p)\frac{p_o}{p^2}\Big]+(p\leftrightarrow k)\Big], \tag{C80}$$

$$\mathrm{Tr}\Big[[\partial_o\partial_r S f^d \partial_t S^\dagger](q+p-k)[S f^d \partial_s S^\dagger](-q-p+k)\Big]\frac{\mu^4}{k^4p^4}\times$$
$$\Big[\frac{1}{2k^2}C_{si}(k)\Big[\delta_{on}\big[\delta_{ri}-4\mathfrak{l}_{ri}(k)\big]+\delta_{oi}\big[\delta_{rn}-4\mathfrak{l}_{rn}(k)\big]-2\mathfrak{l}_{in}(k)\big[\delta_{ro}-4\mathfrak{l}_{ro}(k)\big]\Big]\big[\mathfrak{l}(p)-\mathfrak{l}(q)\big]_{nj}C_{tj}(p)+(p\leftrightarrow k)\Big], \tag{C81}$$

$$\mathrm{Tr}\Big[[\partial_r S f^d \partial_t S^\dagger](q+p-k)[\partial_o S f^d \partial_s S^\dagger](-q-p+k)\Big]\frac{\mu^4}{k^6p^6}\times$$
$$C_{si}(k)\Big[\mathfrak{t}_{ir}(k)k_n+\mathfrak{t}_{nr}(k)k_i\Big]\Big[\mathfrak{t}_{jo}(p)p_n+\mathfrak{t}_{no}(p)p_j\Big]C_{tj}(p). \tag{C82}$$

Now for the $[1,1,1]$ contribution. Since this is going to end up contributing to a derivative with respect to q, we only need to keep here the terms that depend on q. Thus we have

$$w_s a_r v_t\frac{1}{k^4p^2}C_{si}(k)C_{tj}(p)\big[\mathfrak{t}_{ri}(k)k_n+\mathfrak{t}_{rn}(k)k_i\big]\mathfrak{l}_{nj}(q)+w_s b_r v_t\frac{1}{k^2p^4}C_{si}(k)C_{tj}(p)\mathfrak{l}_{in}(q)\big[\mathfrak{t}_{rj}(p)p_n+\mathfrak{t}_{rn}(p)p_j\big]. \tag{C83}$$

Therefore

$$i\mathrm{Tr}\Big[[\partial_r S f^d \partial_t S^\dagger](q+p-k)[S f^d \partial_s S^\dagger](-q-p+k)\Big]\frac{\mu^4}{k^6p^4}C_{si}(k)C_{tj}(p)\big[\mathfrak{t}_{ri}(k)k_n+\mathfrak{t}_{rn}(k)k_i\big]\mathfrak{l}_{nj}(q)+(p\leftrightarrow k)$$
$$=-\partial_o\delta(q+p-k)\int_x\mathrm{Tr}\Big[\partial_o\left[\partial_r S f^d\partial_t S^\dagger\right]S f^d\partial_s S^\dagger\Big]\frac{\mu^4}{k^6p^4}C_{si}(k)C_{tj}(p)\big[\mathfrak{t}_{ri}(k)k_n+\mathfrak{t}_{rn}(k)k_i\big]\mathfrak{l}_{nj}(q)+(p\leftrightarrow k)$$
$$=\int_x\mathrm{Tr}\Big[\partial_o\left[\partial_r S f^d\partial_t S^\dagger\right]S f^d\partial_s S^\dagger\Big]\frac{\mu^4}{k^6p^4(k-p)^2}$$
$$\times C_{si}(k)C_{tj}(p)\big[\mathfrak{t}_{ri}(k)k_n+\mathfrak{t}_{rn}(k)k_i\big]\big[\mathfrak{t}_{no}(k-p)(k-p)_j+\mathfrak{t}_{oj}(k-p)(k-p)_n\big]+(p\leftrightarrow k). \tag{C84}$$

Finally we implement the δ-function approximation and also use Eqs. (C65):

$$\langle 14\rangle\langle 23\rangle = -\int_x \mathrm{Tr}[\partial_o S f^d \partial_t S^\dagger S f^d \partial_s \partial_r S^\dagger]\frac{\mu^4}{k^6 p^4}\times$$
$$\Big[[-2\delta_{ro}+12\mathfrak{l}_{ro}(k)]\Big[\mathfrak{l}(k-p)-\mathfrak{l}(k)\mathfrak{l}(k-p)-\mathfrak{l}(k-p)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}+2\delta_{sr}\Big[\mathfrak{l}(k)\mathfrak{l}(k-p)\mathfrak{l}(p)-\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{ot}\Big]$$
$$-\int_x \mathrm{Tr}[\partial_o\partial_r S f^d \partial_t S^\dagger S f^d \partial_s S^\dagger]\frac{\mu^4}{k^6p^4}[-\delta_{ro}+4\mathfrak{l}_{ro}(k)]\Big[\mathfrak{l}(k-p)-\mathfrak{l}(k)\mathfrak{l}(k-p)-\mathfrak{l}(k-p)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}$$
$$-\int_x \mathrm{Tr}[S f^d \partial_r\partial_t S^\dagger \partial_o S f^d \partial_s S^\dagger]\frac{\mu^4}{k^4p^6}\times$$
$$\Big[\Big[\mathfrak{l}(k-p)-\mathfrak{l}(k)\mathfrak{l}(k-p)-\mathfrak{l}(k-p)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}[-2\delta_{ro}+12\mathfrak{l}_{ro}(p)]+2[\mathfrak{l}(k)\mathfrak{l}(k-p)\mathfrak{l}(p)-\mathfrak{l}(k)\mathfrak{l}(p)]_{so}\delta_{rt}\Big]$$
$$-\int_x \mathrm{Tr}[S f^d \partial_t S^\dagger \partial_o\partial_r S f^d \partial_s S^\dagger]\frac{\mu^4}{k^4p^6}\Big[\mathfrak{l}(k-p)-\mathfrak{l}(k)\mathfrak{l}(k-p)-\mathfrak{l}(k-p)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}[-\delta_{ro}+4\mathfrak{l}_{ro}(p)]$$
$$-\int_x \frac{2\mu^4}{k^6p^6}\Big\{\mathrm{Tr}[S f^d \partial_t S^\dagger \partial_o S f^d \partial_s\partial_r S^\dagger]\Big[\delta_{sr}\Big[k[\mathfrak{l}(k-p)-\mathfrak{l}(k)]\mathfrak{l}(p)\Big]_t p_o$$
$$+4k_r p_o\Big[\mathfrak{l}(k-p)-\mathfrak{l}(k)\mathfrak{l}(k-p)-\mathfrak{l}(k-p)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}\Big]$$
$$+\mathrm{Tr}[\partial_r S f^d \partial_o\partial_t S^\dagger S f^d \partial_s S^\dagger]\Big[\delta_{to}k_r\Big[\mathfrak{l}(k)[\mathfrak{l}(k-p)-\mathfrak{l}(p)]p\Big]_s$$
$$+4k_r p_o\Big[\mathfrak{l}(k-p)-\mathfrak{l}(k)\mathfrak{l}(k-p)-\mathfrak{l}(k-p)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}\Big]$$
$$+2\Big[\mathrm{Tr}[\partial_r S f^d \partial_t S^\dagger \partial_o S f^d \partial_s S^\dagger]k_r p_o\Big[\mathfrak{l}(k-p)-\mathfrak{l}(k)\mathfrak{l}(k-p)-\mathfrak{l}(k-p)\mathfrak{l}(p)+\mathfrak{l}(k)\mathfrak{l}(p)\Big]_{st}\Big\}$$
$$+\int_x \mathrm{Tr}\Big[S f^d \partial_s\partial_r S^\dagger S f^d \partial_t\partial_o S^\dagger\Big]\frac{\mu^4}{k^4p^4}\Big[6[\mathfrak{l}(k-p)(\mathfrak{t}-\mathfrak{l})(p)+(\mathfrak{t}-\mathfrak{l})(k)\mathfrak{l}(k-p)]_{st}\frac{k_r p_o}{k^2p^2}+$$
$$2\delta_{sr}[\mathfrak{l}(k-p)\mathfrak{l}(p)]_{it}\frac{k_i p_o}{k^2p^2}+2\delta_{to}[\mathfrak{l}(k)\mathfrak{l}(k-p)]_{si}\frac{p_i k_r}{k^2p^2}+12\frac{(k\cdot p)k_s k_r p_t p_o}{k^4p^4}-2\delta_{sr}\frac{(k\cdot p)p_t p_o}{k^2p^4}-2\delta_{to}\frac{(k\cdot p)k_s k_r}{k^4p^2}\Big]$$
$$+\mathrm{Tr}\Big[\partial_o S f^d \partial_t S^\dagger S f^d \partial_s\partial_r S^\dagger\Big]\frac{\mu^4}{k^4p^4}\Big[\frac{1}{k^2}\Big[4\delta_{so}\frac{k_r k_n}{k^2}-4\delta_{no}\frac{k_s k_r}{k^2}+\delta_{sr}\mathfrak{t}_{no}(k)\Big]\mathfrak{l}_{nj}(k-p)(\mathfrak{t}-\mathfrak{l})_{jt}(p)+(k\to p)\Big]$$
$$+\mathrm{Tr}\Big[\partial_r S f^d \partial_o\partial_t S^\dagger S f^d \partial_s S^\dagger\Big]\frac{\mu^4}{k^4p^4}\Big[\frac{1}{k^2}\Big[\delta_{sr}k_n-\delta_{nr}k_s\Big]\Big[\mathfrak{l}(p)-\mathfrak{l}(k-p)\Big]_{nj}\Big[4(\mathfrak{t}-\mathfrak{l})_{tj}(p)\frac{p_o}{p^2}+\delta_{to}\frac{p_j}{p^2}\Big]+(p\leftrightarrow k)\Big]$$
$$-\mathrm{Tr}\Big[\partial_o\partial_r S f^d \partial_t S^\dagger S f^d \partial_s S^\dagger\Big]\frac{\mu^4}{k^4p^4}\times$$
$$\Big[\frac{1}{2k^2}\Big[\delta_{os}\delta_{rn}+\delta_{on}\delta_{rs}+2\delta_{on}\mathfrak{l}_{rs}(k)+2\delta_{ro}\mathfrak{l}_{sn}(k)-2\delta_{rn}\mathfrak{l}_{so}(k)-4\delta_{so}\mathfrak{l}_{rn}(k)\Big]\mathfrak{l}(k-p)_{nj}(\mathfrak{t}-\mathfrak{l})_{tj}(p)+(p\leftrightarrow k)\Big]$$
$$+\mathrm{Tr}\Big[\partial_r S f^d \partial_t S^\dagger \partial_o S f^d \partial_s S^\dagger\Big]\frac{\mu^4}{k^6p^6}\Big[\delta_{sr}\delta_{to}(k\cdot p)-\delta_{sr}k_o p_t-\delta_{to}k_s p_r+\delta_{or}k_s p_t\Big]$$
$$+\int_x \mathrm{Tr}\Big[\partial_o\left[\partial_r S f^d \partial_t S^\dagger\right] S f^d \partial_s S^\dagger\Big]\Big]\frac{\mu^4}{k^4p^4(k-p)^2}\times$$
$$\Big[\frac{1}{k^2}\Big[\delta_{rs}k_n-\delta_{rn}k_s\Big]\Big[\mathfrak{t}_{no}(k-p)(k-p)_j+\mathfrak{t}_{oj}(k-p)(k-p)_n\Big](\mathfrak{t}-\mathfrak{l})_{jt}(p)+(p\leftrightarrow k)\Big]. \quad \text{(C85)}$$

Now, using the averages from Appendix D we have:

$$\langle\langle 14\rangle\langle 23\rangle\rangle_\alpha = N_c^3(N_c^2-1)S_\perp\frac{\mu^4\lambda^4}{k^4p^4}\Bigg\{ \tag{C86}$$

$$-7\frac{k\cdot p}{k^2p^2} - 3\frac{(k\cdot p)^3}{k^4p^4} + \frac{15}{2}\frac{k\cdot(k-p)p\cdot(k-p)}{k^2p^2(k-p)^2} + \frac{k\cdot p(p\cdot(k-p))^2}{k^2p^4(k-p)^2} + \frac{k\cdot p(k\cdot(k-p))^2}{k^4p^2(k-p)^2}$$

$$+\frac{1}{4}\frac{(k\cdot(k-p))^2}{k^2(k-p)^2}\left(\frac{5}{k^2}-\frac{7}{p^2}\right) + \frac{1}{4}\frac{(p\cdot(k-p))^2}{p^2(k-p)^2}\left(\frac{5}{p^2}-\frac{7}{k^2}\right) + \frac{7}{2}\left(\frac{1}{k^2}+\frac{1}{p^2}\right)\frac{k\cdot pk\cdot(k-p)p\cdot(k-p)}{k^2p^2(k-p)^2}$$

$$+\frac{3}{8}\left[\frac{k\cdot(k-p)}{k^2(k-p)^2} - \frac{p\cdot(k-p)}{p^2(k-p)^2}\right]\Bigg\}.$$

Finally the sum is given by

$$\langle\langle 13\rangle\langle 24\rangle + \langle 14\rangle\langle 23\rangle\rangle_\alpha = N_c^3(N_c^2-1)S_\perp\frac{\mu^4\lambda^4}{k^4p^4}\Bigg\{ \tag{C87}$$

$$-\frac{29}{4}\frac{k\cdot p}{k^2p^2} - 3\frac{(k\cdot p)^3}{k^4p^4} + \frac{15}{2}\frac{k\cdot(k-p)p\cdot(k-p)}{k^2p^2(k-p)^2} + \frac{k\cdot p(p\cdot(k-p))^2}{k^2p^4(k-p)^2} + \frac{k\cdot p(k\cdot(k-p))^2}{k^4p^2(k-p)^2}$$

$$+\frac{1}{4}\frac{(k\cdot(k-p))^2}{k^2(k-p)^2}\left(\frac{5}{k^2}-\frac{7}{p^2}\right) + \frac{1}{4}\frac{(p\cdot(k-p))^2}{p^2(k-p)^2}\left(\frac{5}{p^2}-\frac{7}{k^2}\right) + \frac{7}{2}\left(\frac{1}{k^2}+\frac{1}{p^2}\right)\frac{k\cdot pk\cdot(k-p)p\cdot(k-p)}{k^2p^2(k-p)^2}$$

$$+\frac{3}{8}\left[\frac{k\cdot(k-p)}{k^2(k-p)^2} - \frac{p\cdot(k-p)}{p^2(k-p)^2}\right]\Bigg\}.$$

Appendix D: Target averaging.

In this appendix, we compute averages over the target fields using the averaging procedure explained in the text defined by the two point function in Eq. (69). First lets consider

$$\langle \mathrm{Tr}[Sf^d\partial_s\partial_rS^\dagger]\mathrm{Tr}[Sf^d\partial_t\partial_oS^\dagger]\rangle_\alpha = \frac{1}{4}\langle E_s^aE_r^bE_t^cE_o^e\rangle_\alpha\mathrm{Tr}[f^d\{f^a,f^b\}]\mathrm{Tr}[f^d\{f^c,f^e\}] = 0 \tag{D1}$$

due to $\mathrm{Tr}[f^d\{f^a,f^b\}]=0$. Analogously we have

$$\langle \mathrm{Tr}[\partial_rSf^d\partial_sS^\dagger]\mathrm{Tr}[Sf^d\partial_t\partial_oS^\dagger]\rangle_\alpha = 0. \tag{D2}$$

Proceeding further we get

$$\langle \mathrm{Tr}[\partial_rSf^d\partial_sS^\dagger]\mathrm{Tr}[\partial_oSf^d\partial_tS^\dagger]\rangle_\alpha = \frac{1}{4}N_c^3(N_c^2-1)\lambda^4[\delta_{ro}\delta_{st}-\delta_{rt}\delta_{so}], \tag{D3}$$

$$\langle \mathrm{Tr}\Big[Sf^d\partial_s\partial_o\partial_rS^\dagger Sf^d\partial_tS^\dagger\Big]\rangle_\alpha = -\frac{5}{12}N_c^3(N_c^2-1)\lambda^4[\delta_{st}\delta_{ro}+\delta_{sr}\delta_{to}+\delta_{so}\delta_{tr}], \tag{D4}$$

$$\langle \mathrm{Tr}\Big[Sf^d\partial_s\partial_rS^\dagger Sf^d\partial_t\partial_oS^\dagger\Big]\rangle_\alpha = -N_c^3(N_c^2-1)\lambda^4\Big[\delta_{sr}\delta_{to}+\frac{1}{8}[\delta_{st}\delta_{ro}+\delta_{so}\delta_{rt}]\Big], \tag{D5}$$

$$\langle \mathrm{Tr}\Big[\partial_oSf^d\partial_tS^\dagger Sf^d\partial_s\partial_rS^\dagger\Big]\rangle_\alpha = \frac{1}{2}N_c^3(N_c^2-1)\lambda^4\Big[\delta_{ot}\delta_{sr}+\frac{3}{4}[\delta_{os}\delta_{tr}+\delta_{or}\delta_{ts}]\Big], \tag{D6}$$

$$\langle \mathrm{Tr}\Big[\partial_rSf^d\partial_t\partial_oS^\dagger Sf^d\partial_sS^\dagger\Big]\rangle_\alpha = N_c^3(N_c^2-1)\lambda^4\Big[\delta_{rs}\delta_{ot}+\frac{1}{8}[\delta_{rt}\delta_{os}+\delta_{ro}\delta_{ts}]\Big], \tag{D7}$$

$$\langle \mathrm{Tr}\Big[\partial_o\partial_rSf^d\partial_tS^\dagger Sf^d\partial_sS^\dagger\Big]\rangle_\alpha = -\frac{1}{2}N_c^3(N_c^2-1)\lambda^4\left[\delta_{or}\delta_{ts}+\frac{3}{4}(\delta_{os}\delta_{rt}+\delta_{ot}\delta_{sr})\right], \tag{D8}$$

$$\langle \mathrm{Tr}\Big[\partial_r S f^d \partial_t S^\dagger \partial_o S f^d \partial_s S^\dagger\Big]\rangle_\alpha = -N_c^3(N_c^2-1)\lambda^4\left[\delta_{rs}\delta_{to}+\frac{1}{4}\delta_{rt}\delta_{os}\right], \tag{D9}$$

$$\begin{aligned}\langle \mathrm{Tr}\Big[\partial_o\left[\partial_r S f^d \partial_t S^\dagger\right] S f^d \partial_s S^\dagger\Big]\rangle_\alpha &= \langle \mathrm{Tr}\Big[\partial_o\partial_r S f^d \partial_t S^\dagger S f^d \partial_s S^\dagger\Big]\rangle + \langle \mathrm{Tr}\Big[\partial_r S f^d \partial_o\partial_t S^\dagger S f^d \partial_s S^\dagger\Big]\rangle \\ &= -\frac{1}{2}N_c^3(N_c^2-1)\lambda^4\left[\delta_{or}\delta_{ts}+\frac{3}{4}(\delta_{os}\delta_{rt}+\delta_{ot}\delta_{sr})\right] \\ &\quad +\frac{1}{8}N_c^3(N_c^2-1)\lambda^4\left[\delta_{ro}\delta_{ts}+\delta_{rt}\delta_{os}+8\delta_{rs}\delta_{ot}\right] \\ &= -\frac{1}{8}N_c^3(N_c^2-1)\lambda^4\left[3\delta_{or}\delta_{ts}+2\delta_{os}\delta_{rt}-5\delta_{ot}\delta_{sr}\right],\end{aligned} \tag{D10}$$

$$\langle \mathrm{Tr}\left[[\partial_n\partial_r S]S^\dagger\partial_t S\partial_s S^\dagger\right]\rangle_\alpha = -N_c^2(N_c^2-1)\lambda^4\left[\delta_{rn}\delta_{ts}+\frac{3}{4}\left(\delta_{nt}\delta_{rs}+\delta_{ns}\delta_{rt}\right)\right], \tag{D11}$$

$$\langle \mathrm{Tr}\left[[\partial_r S]S^\dagger\partial_t S[\partial_n\partial_s S^\dagger]\right]\rangle_\alpha = N_c^2(N_c^2-1)\lambda^4\left[\delta_{sn}\delta_{tr}+\frac{3}{4}\left(\delta_{nr}\delta_{ts}+\delta_{nt}\delta_{rs}\right)\right], \tag{D12}$$

$$\langle \mathrm{Tr}\left[[\partial_r S]S^\dagger[\partial_n\partial_t S]\partial_s S^\dagger\right]\rangle_\alpha = -N_c^2(N_c^2-1)\lambda^4\left[\delta_{tn}\delta_{rs}+\frac{3}{4}\left(\delta_{ns}\delta_{rt}+\delta_{nr}\delta_{st}\right)\right], \tag{D13}$$

$$\begin{aligned}\langle \mathrm{Tr}\left[\partial_n[\partial_r S S^\dagger]\partial_t S\partial_s S^\dagger\right]\rangle_\alpha &= \langle \mathrm{Tr}\left[[\partial_n\partial_r S]S^\dagger\partial_t S\partial_s S^\dagger\right]\rangle + \langle \mathrm{Tr}\left[\partial_r S\partial_n S^\dagger\partial_t S\partial_s S^\dagger\right]\rangle_\alpha \\ &= -N_c^2(N_c^2-1)\lambda^4\left[\delta_{rn}\delta_{ts}+\frac{3}{4}\left(\delta_{nt}\delta_{rs}+\delta_{ns}\delta_{rt}\right)\right] \\ &\quad +N_c^2(N_c^2-1)\lambda^4\left[\delta_{rn}\delta_{ts}+\delta_{nt}\delta_{rs}+\frac{1}{2}\delta_{ns}\delta_{rt}\right] \\ &= \frac{1}{4}N_c^2(N_c^2-1)\lambda^4\left[\delta_{nt}\delta_{rs}-\delta_{ns}\delta_{rt}\right].\end{aligned} \tag{D14}$$

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