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Invariant Measure of the One Loop Quantum Gravitational Back-Reaction on Inflation

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ABSTRACT

We use dimensional regularization in pure quantum gravity on de Sitter background to evaluate the one loop expectation value of an invariant operator which gives the local expansion rate. We show that the renormalization of this nonlocal composite operator can be accomplished using the counterterms of a simple local theory of gravity plus matter, at least at one loop order. This renormalization completely absorbs the one loop correction, which accords with the prediction that the lowest secular back-reaction should be a 2-loop effect.

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1 Introduction

The quantum gravitational back-reaction on accelerated expansion has a special importance because of its potential to simultaneously provide a resolution for the (old) problem of the cosmological constant [1, 2] and a predictive model for primordial inflation [3]. The idea is easy to sketch. We posit that the bare cosmological constant is not absurdly small, but rather large and positive, and that this triggered primordial inflation [4, 5]. Accelerated expansion rips virtual scalars and gravitons out of the vacuum [6, 7]; this is what causes the primordial power spectra [8, 9]. The self-gravitation of these particles must tend to slow the expansion rate, and their contribution to the vacuum energy must grow with time as more and more of them come into causal contact through the continual increase in the volume of the past light-cone. Λ-driven inflation is based on the assumption that this effect eventually stops inflation [5].

Quantum instabilities of de Sitter have been proposed for decades [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. The difficult part has been to properly quantify the effect so as to establish its reality. In 1996 a fixed-gauge computation by Mukhanov, Abramo and Brandenberger seemed to show secular slowing at one loop in scalar-driven inflation [22, 23]. However, Unruh correctly questioned the validity of treating the expectation value of the gauge-fixed metric as one would a classical metric [24]. Although the result persisted in a different gauge [25, 26], the introduction of a truly invariant measure for the local expansion rate, with the time fixed by the value of the inflaton [27], revealed the absence of any secular slowing [28, 29]. The apparent effect in a fixed gauge "time" arose from quantum fluctuations tending to push the inflaton down its potential a little faster than it would have gone classically. This is apparent from using different clocks [30, 31, 32], but the inflaton potential suffers no secular corrections at one loop order.

True quantum gravitational back-reaction is predicted to occur at two loop order because inflationary particle production is a one loop effect so the quantum gravitational response to it must occur one loop higher [4, 5]. Although the reasoning is solid, the conclusion is frustrating because two loop computations in nontrivial geometries are so difficult. There seems to be no advantage to working in scalar-driven inflation; that would only have paid off if the scalar-metric mixing had permitted a one loop effect. In the absence of a reduction in the loop order, the presence of a scalar inflaton merely complicates the problem through the evolving background and the more complex propagators and vertices. The simplest venue is therefore pure gravity on de Sitter background, *provided* a suitable invariant expansion observable can be constructed. A proposal for this has been made based on using a nonlocal scalar functional of the metric in the same way one would quantify the expansion rate using a scalar inflaton [33]. However, this observable can only be used at two loop order if it can be successfully renormalized at one loop order. That is the purpose of this paper.

In section 2 we review how the expansion operator is defined, and we give its expansion to second order in metric perturbations about de Sitter background. Section 3 evaluates the expectation value of the observable at one loop order using dimensional regularization. Its renormalization is accomplished in section 4. Section 5 discusses our results and the prospects for pushing on to two loop order.

2 The Expansion Observable

The purpose of this section is to precisely define the expansion operator and give its expansion in powers of the graviton field [33]. We also present the local gravity + matter theory from which it descends. To facilitate the application of dimensional regularization we work in D spacetime dimensions with a single time coordinate $-\infty < \eta < 0$, a $(D-1)$-dimensional space vector $\vec{x}$, and a spacelike metric.

Our nonlocal scalar $\Phi[g](x)$ is constructed to obey the equation,

$$\square \Phi \equiv \frac{1}{\sqrt{-g}} \partial_\mu \Big[\sqrt{-g}\, g^{\mu\nu} \partial_\nu \Phi \Big] = (D-1) H \; , \tag{1}$$

subject to the initial conditions (at $\eta = \eta_i \equiv -1/H$),

$$\Phi(\eta_i, \vec{x}) = 0 \qquad , \qquad -g^{\alpha\beta}(\eta_i, \vec{x}) \partial_\alpha \Phi(\eta_i, \vec{x}) \partial_\beta \Phi(\eta_i, \vec{x}) = 1 \; . \tag{2}$$

The important thing about (minus) $\Phi[g](x)$ is that it grows in the timelike direction, not just for de Sitter but for an arbitrary metric. Hence its gradient produces a timelike 4-vector. By normalizing this vector and then taking the divergence we can construct a scalar measure of the local expansion rate, just as is done with the inflaton in scalar-driven inflation [29],

$$\mathcal{H}[g](x) = \frac{1}{(D-1)\sqrt{-g(x)}} \partial_\mu \Bigg[\frac{\sqrt{-g(x)}\, g^{\mu\nu}(x) \partial_\nu \Phi[g](x)}{\sqrt{-g^{\alpha\beta}(x) \partial_\alpha \Phi[g](x) \partial_\beta \Phi[g](x)}} \Bigg] \; . \tag{3}$$

Expression (3) is a scalar, not an invariant, because the observation point $x^\mu = (\eta, \vec{x})$ has not been invariantly fixed. Recall that doing this was the key step in demonstrating that there is no one loop secular back-reaction in scalar-driven inflation [28, 29]. Just like the case of scalar-driven inflation, we use the value of the scalar to invariantly fix the surface of simultaneity on which the observation is made. Also like scalar-driven inflation, a homogeneous and isotropic state provides no reference structure with which we can fix the spatial coordinates on this surface. So we only define a metric-dependent time $\theta[g](x)$ by the condition that it makes the full scalar agree with its value on de Sitter background ($g_{\mu\nu} = a^2 \eta_{\mu\nu}$, with $a(\eta) = -1/H\eta$),

$$\Phi[g]\Big(\theta[g](x), \vec{x}\Big) \equiv \Phi_0(\eta) \equiv -\frac{1}{H} \ln\Big[a(\eta)\Big] \; . \tag{4}$$

(Making the full inflaton agree with its background value was also the time condition for scalar-driven inflation [28, 29].) Evaluating the scalar (3) at this time defines the expansion observable [33],

$$\mathbf{H}[g](x) \equiv \mathcal{H}[g]\Big(\theta[g](x), \vec{x}\Big) \; . \tag{5}$$

We define the graviton field $h_{\mu\nu}(x)$ as the perturbation of the conformally rescaled metric about de Sitter background,

$$g_{\mu\nu}(x) \equiv a^2(\eta)\Big[\eta_{\mu\nu} + \kappa h_{\mu\nu}(x)\Big] \equiv a^2(\eta) \widetilde{g}_{\mu\nu}(x) \qquad , \qquad \kappa^2 \equiv 16\pi G \; . \tag{6}$$

We adhere to the usual convention that graviton indices are raised and lowered with the Lorentz metric, for example, $h^{\mu\nu} \equiv \eta^{\mu\rho}\eta^{\nu\sigma} h_{\rho\sigma}$. The Feynman rules are given in terms of the graviton field [34, 35], so we must expand $\mathbf{H}[g](x)$ in powers of it in order to evaluate its expectation value. It is useful to also expand the scalar $\Phi[g](x)$ [33],

$$\Phi[g](x) = \Phi_0(\eta) + \kappa \Phi_1(x) + \kappa^2 \Phi_2(x) + \dots \; , \tag{7}$$

$$\Phi_1 = \frac{1}{D_A}\left[-(D-1) H h_{00} - \frac{h_{00}'}{2a} + \frac{h_{0j,j}}{a} - \frac{h_{jj}'}{2a}\right] , \tag{8}$$

where Latin letters from the middle of the alphabet denote spatial indices, $h \equiv \eta^{\mu\nu} h_{\mu\nu}$, a prime stands for differentiation with respect to η and $D_A \equiv \frac{1}{a^D}\partial_\mu (a^{D-2}\eta^{\mu\nu}\partial_\nu)$. The expansion observable has a similar expansion [33],

$$\mathbf{H}[g](x) = H + \kappa \mathbf{H}_1(x) + \kappa^2 \mathbf{H}_2(x) + \dots \; , \tag{9}$$

$$\mathbf{H}_1 = \frac{1}{2} H h_{00} + \frac{h_{ii}'}{2(D-1)a} + \partial_i \left[\frac{-h_{0i}}{(D-1)a} + \frac{\partial_i \Phi_1}{(D-1)a^2}\right] . \tag{10}$$

The homogeneity and isotropy of our state and our gauge (see section 3.1) mean that we must get zero for the expectation value of a total spatial derivative such as the final term of (10). We will therefore not bother about giving such terms for $\mathbf{H}_2$ although they have been worked out [33],

$$\mathbf{H}_2 = \frac{3}{8} H h_{00} h_{00} - \frac{1}{2} H h_{0i} h_{0i} + \frac{[-h_{ij} h'_{ij} + h_{00,i} h_{0i} + \frac{1}{2} h_{00} h'_{ii} - h_{jj,i} h_{0i}]}{2(D-1)a}$$
$$+ \frac{[(D-1)Hah'_{00} + (\partial_0^2 - Ha\partial_0 - \nabla^2)h_{ii} - 2(\partial_0 - Ha)h_{0i,i} + h_{00}\nabla^2]\Phi_1}{2(D-1)a^2}$$
$$+ \Bigl(\frac{D+1}{D-1}\Bigr) \frac{H\partial_i\Phi_1\partial_i\Phi_1}{2a^2} + \Bigl(\text{Spatial Derivative Terms}\Bigr) . \qquad (11)$$

Our expansion observable $\mathbf{H}[g](x)$ is a nonlocal composite operator functional of the metric, which makes its renormalization problematic. The normal BPHZ (Bogoliubov and Parasiuk [36], Hepp [37] and Zimmerman [38, 39]) renormalization technique only suffices to remove ultraviolet divergences from noncoincident 1PI (one-particle-irreducible) functions. It is known how to perform additional renormalizations to remove the divergences of local composite operators [40, 41]. However, the only nonlocal composite operator whose renormalization we now understand is the Wilson loop of non-Abelian gauge theory [42].

A way forward may be the observation that our expansion observable can be considered as descending from a local composite operator in the scalar-metric theory whose Lagrangian is,

$$\mathcal{L} = \frac{1}{16\pi G}\Bigl[R - (D-2)\Lambda\Bigr]\sqrt{-g} - \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} - (D-1)H\varphi\sqrt{-g} . \qquad (12)$$

Here the full cosmological constant is $\Lambda \equiv (D-1)H^2 + \delta\Lambda$. Although the scalar φ obeys the same equation (1) as $\Phi[g]$, it possesses its own independent initial value data instead of being completely fixed by the initial conditions (2). Nonetheless, we conjecture that the composite operator renormalization of $\mathbf{H}[g](x)$ may be the same as the composite operator renormalization of the corresponding operator in (12). We will see in section 4 that this conjecture is correct, at least at one loop order.

3 One Loop Expectation Value

The purpose of this section is to evaluate the expectation value of $\mathbf{H}[g](x)$ at one loop order. We begin with the Feynman rules. Next the 1-point contribution is inferred from previous work and the various 2-point contributions are reduced to convolutions of propagators. These convolutions are then reduced to the coincidence limits of integrated propagators whose evaluation is explained in Appendices A-C.

3.1 Feynman Rules

The invariant part of the pure gravitational action can be expressed in terms of the fields $h_{\mu\nu}$ and $\widetilde{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ defined in expression (6) as [34],

$$\begin{aligned}\mathcal{L} = & \frac{1}{16\pi G}\Big[R-(D-2)\Lambda\Big]\sqrt{-g} + \text{Counterterms} \;, && (13)\\ = & \Big(\text{Surface Terms}\Big) + \frac{1}{2}(D-2)Ha^{D-1}\sqrt{-\widetilde{g}}\,\widetilde{g}^{\rho\sigma}\widetilde{g}^{\mu\nu}h_{\rho\sigma,\mu}h_{\nu 0}\\ & +a^{D-2}\sqrt{-\widetilde{g}}\,\widetilde{g}^{\alpha\beta}\widetilde{g}^{\rho\sigma}\widetilde{g}^{\mu\nu}\Big\{\frac{1}{2}h_{\alpha\rho,\mu}h_{\nu\sigma,\beta}-\frac{1}{2}h_{\alpha\beta,\rho}h_{\sigma\mu,\nu}\\ & +\frac{1}{4}h_{\alpha\beta,\rho}h_{\mu\nu,\sigma}-\frac{1}{4}h_{\alpha\rho,\mu}h_{\beta\sigma,\nu}\Big\} + \text{Counterterms} \;. && (14)\end{aligned}$$

Because the expansion observable $\mathbf{H}[g](x)$ is gauge invariant it should not matter how we fix the gauge, so we make the choice which gives the simplest propagator. That choice is defined by adding the gauge fixing term [34, 35],

$$\mathcal{L}_{\rm GF} = -\frac{1}{2}a^{D-2}\eta^{\mu\nu}F_\mu F_\nu \quad , \quad F_\mu \equiv \eta^{\rho\sigma}\Big[h_{\mu\rho,\sigma} - \frac{1}{2}h_{\rho\sigma,\mu} + (D-2)aHh^{\mu\rho}\delta^0_\sigma\Big] \;. \tag{15}$$

The associated ghost Lagrangian (with anti-ghost field $\gamma^\mu(x)$ and ghost field $\epsilon_\sigma(x)$) is [34],

$$\begin{aligned}\mathcal{L}_{gh} = & -2a^{D-2}\gamma^\mu{}_{,\alpha}\eta^{\alpha\beta}\eta^{\rho\sigma}\Big[\widetilde{g}_{\rho(\mu}\partial_{\beta)} + \frac{1}{2}\widetilde{g}_{\mu\beta,\rho} + Ha\widetilde{g}_{\mu\beta}\delta^0{}_\rho\Big]\epsilon_\sigma\\ & +(a^{D-2}\gamma^\mu)_{,\mu}\eta^{\alpha\beta}\eta^{\rho\sigma}\Big[\widetilde{g}_{\rho\alpha}\partial_\beta + \frac{1}{2}\widetilde{g}_{\alpha\beta,\rho} + Ha\widetilde{g}_{\alpha\beta}\delta^0{}_\rho\Big]\epsilon_\sigma \;, && (16)\end{aligned}$$

where parenthesized indices are symmetrized.

In the gauge (15) both the graviton and ghost propagators are simple because they consist of sums of known scalar propagators, each multiplied by an index factor which is constant in space and time [34, 35],

$$\begin{aligned} i\Big[{}_{\mu\nu}\Delta_{\rho\sigma}\Big](x;x') &= \sum_{I=A,B,C} i\Delta_I(x;x') \times \Big[{}_{\mu\nu}T^I_{\rho\sigma}\Big] \;, && (17)\\ i\Big[{}_{\mu}\Delta_{\nu}\Big](x;x') &= i\Delta_A(x;x')\times\overline{\eta}_{\mu\nu} - i\Delta_B(x;x')\times\delta^0{}_\mu\delta^0{}_\nu \;. && (18)\end{aligned}$$

Here $\eta_{\mu\nu}$ is the Lorentz metric and $\overline{\eta}_{\mu\nu} \equiv \eta_{\mu\nu} + \delta^0{}_\mu \delta^0{}_\nu$ is its purely spatial part. The same constant tensors suffice to give the three index factors of the graviton propagator,

$$\Big[{}_{\mu\nu}T^A_{\rho\sigma}\Big] = 2\overline{\eta}_{\mu(\rho}\overline{\eta}_{\sigma)\nu} - \frac{2}{D-3}\,\overline{\eta}_{\mu\nu}\overline{\eta}_{\rho\sigma} \; , \tag{19}$$

$$\Big[{}_{\mu\nu}T^B_{\rho\sigma}\Big] = -4\delta^0_{(\mu}\overline{\eta}_{\nu)(\rho}\delta^0_{\sigma)} \; , \tag{20}$$

$$\Big[{}_{\mu\nu}T^C_{\rho\sigma}\Big] = \frac{2}{(D-2)(D-3)}\Big[(D-3)\delta^0_\mu\delta^0_\nu + \overline{\eta}_{\mu\nu}\Big]\Big[(D-3)\delta^0_\rho\delta^0_\sigma + \overline{\eta}_{\rho\sigma}\Big] \; . \tag{21}$$

The three scalar propagators have masses $M_A^2 = 0$, $M_B^2 = (D-2)H^2$ and $M_C^2 = 2(D-3)H^2$. They are most easily represented in terms of the de Sitter length function $y(x;x')$,

$$y(x;x') \equiv H^2 a(\eta) a(\eta') \Big[\Big\| \vec{x} - \vec{x}' \Big\|^2 - \Big(|\eta - \eta'| - i\epsilon\Big)^2\Big] \; . \tag{22}$$

The A-type scalar is well known to break de Sitter invariance [43, 44],

$$i\Delta_A = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\Bigg\{\frac{\Gamma(\frac{D}{2})}{\frac{D}{2}-1}\Big(\frac{4}{y}\Big)^{\frac{D}{2}-1} + \frac{\Gamma(\frac{D}{2}+1)}{\frac{D}{2}-2}\Big(\frac{4}{y}\Big)^{\frac{D}{2}-2} - \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\Big[\pi\cot\Big(\pi\frac{D}{2}\Big) - \ln(aa')\Big]$$
$$+ \sum_{n=1}^{\infty}\Big[\frac{1}{n}\frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})}\Big(\frac{y}{4}\Big)^n - \frac{1}{n-\frac{D}{2}+2}\frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+2)}\Big(\frac{y}{4}\Big)^{n-\frac{D}{2}+2}\Big]\Bigg\} . \tag{23}$$

On the other hand, the B-type and C-type propagators are de Sitter invariant functions of $y(x;x')$,

$$i\Delta_B = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\Bigg\{\frac{\Gamma(\frac{D}{2})}{\frac{D}{2}-1}\Big(\frac{4}{y}\Big)^{\frac{D}{2}-1} + \sum_{n=0}^{\infty}\Big[\frac{\Gamma(n+\frac{D}{2})}{\Gamma(n+2)}\Big(\frac{y}{4}\Big)^{n-\frac{D}{2}+2} - \frac{\Gamma(n+D-2)}{\Gamma(n+\frac{D}{2})}\Big(\frac{y}{4}\Big)^n\Big]\Bigg\}, \tag{24}$$

$$i\Delta_C = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\Bigg\{\frac{\Gamma(\frac{D}{2})}{\frac{D}{2}-1}\Big(\frac{4}{y}\Big)^{\frac{D}{2}-1} - \sum_{n=0}^{\infty}\Big[\Big(n-\frac{D}{2}+3\Big)\frac{\Gamma(n+\frac{D}{2}-1)}{\Gamma(n+2)}\Big(\frac{y}{4}\Big)^{n-\frac{D}{2}+2}$$
$$-(n+1)\frac{\Gamma(n+D-3)}{\Gamma(n+\frac{D}{2})}\Big(\frac{y}{4}\Big)^n\Big]\Bigg\}. \tag{25}$$

Although the infinite summations may appear daunting, each of the scalar propagators takes a simple form for $D = 4$ dimensions,

$$D = 4 \quad \Longrightarrow \quad i\Delta_A = \frac{H^2}{4\pi^2}\Big[\frac{1}{y} - \frac{1}{2}\ln\Big(\frac{y}{4aa'}\Big)\Big] \; , \; i\Delta_B = i\Delta_C = \frac{H^2}{4\pi^2 y} \; . \tag{26}$$

This means we only need the sums when there is a divergence, and then only a few of the lowest values of n are required.

We close by giving a unified treatment of differential operators and propagators on de Sitter background. The inverse of the massless scalar d'Alembertian D_A has already appeared in expression (8) for $\Phi_1(x)$,

$$D_A \equiv \frac{1}{a^D}\,\partial_\mu\Big[a^{D-2}\,\eta^{\mu\nu}\partial_\nu\Big] = \frac{1}{a^2}\Big[\partial^2 - (D-2)Ha\partial_0\Big] \; . \qquad (27)$$

We denote a general massive scalar kinetic operator with a subscript ν,

$$D_\nu \equiv D_A + (\nu^2 - \nu_A^2)H^2 \qquad , \qquad \nu_A \equiv \Big(\frac{D-1}{2}\Big) \; . \qquad (28)$$

The various propagators are obtained by acting the inverse differential operators on a delta function,

$$i\Delta_\nu(x;x') = \frac{1}{D_\nu}\Big[i\delta^D(x-x')\Big] = \frac{1}{D'_\nu}\Big[i\delta^D(x-x')\Big] \; . \qquad (29)$$

In addition to propagators we also require some integrated propagators whose evaluation is explained in Appendix C,

$$I_{\alpha\beta}(x;x') \equiv \frac{1}{D_\alpha}\Big[i\Delta_\beta(x;x')\Big] \quad , \quad I_{\alpha\beta\gamma}(x;x') \equiv \frac{1}{D_\alpha}\frac{1}{D'_\gamma}\Big[i\Delta_\beta(x;x')\Big] \; , \qquad (30)$$

$$J_{\alpha\beta}(x;x') \equiv \frac{1}{D_\alpha}\Big[\frac{i\Delta_\beta(x;x')}{a(\eta)}\Big] \quad , \quad J_{\alpha\beta\gamma}(x;x') \equiv \frac{1}{D_\alpha}\frac{1}{D'_\gamma}\Big[\frac{i\Delta_\beta(x;x')}{a(\eta)a(\eta')}\Big] \; , \qquad (31)$$

$$K_{\alpha\beta\gamma}(x;x') \equiv \frac{1}{D_\alpha}\frac{1}{D'_\gamma}\Big[\frac{i\Delta_\beta(x;x')}{a(\eta')}\Big] \; . \qquad (32)$$

For the special case where the index corresponds to a particularly useful propagator we have found it convenient to employ an alternate, alphabetical, representation according to the scheme,

$$\nu_B \equiv \nu_A - 1 \quad , \quad \nu_C \equiv \nu_A - 2 \quad , \quad \nu_D \equiv \nu_A - 3 \; . \qquad (33)$$

The last case, ν_D, corresponds to a scalar of mass $M_D^2 = 3(D-4)H^2$ which does not appear in the graviton or ghost propagators. It nonetheless occurs in our reductions of the 2-point contributions (see section 3.3) when using identities of Appendix B to reflect a time derivative from one side of a C-type propagator to the other,

$$\Big[\partial'_0 + 2Ha'\Big]i\Delta_C(x;x') = -\Big[\partial_0 + (D-4)Ha\Big]i\Delta_D(x;x') \; . \qquad (34)$$

3.2 1-Point Contributions

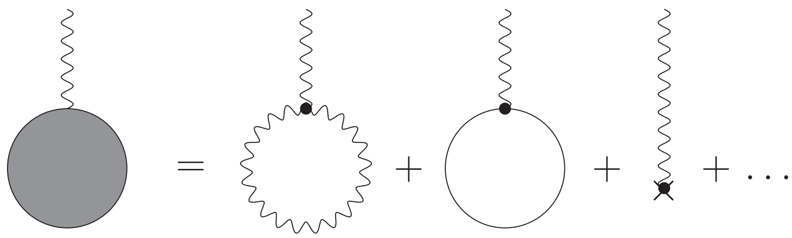

Figure 1: Diagrammatic representation of $\langle\Omega|\kappa\mathbf{H}_1(x)|\Omega\rangle$. Graviton lines are wavy and ghost lines are straight.

Figure 1 shows the diagrams which contribute to the one loop expectation value of $\kappa\mathbf{H}_1(x)$. The first two diagrams display the primitive contribution while the third diagram gives the contribution from the cosmological counterterm. The primitive diagrams have been evaluated previously using dimensional regularization and in the same gauge (15) that we employ. The result implies,[1]

$$\Big\langle\Omega\Big|\kappa\mathbf{H}_1(x)\Big|\Omega\Big\rangle = \frac{2H}{D-2}\frac{1}{D_C}\Big[-K+\frac{1}{2}(D-2)\delta\Lambda\Big] + \frac{2\partial_0}{(D-2)(D-3)a}\Big[\frac{1}{D_C}-\frac{D-2}{D_A}\Big]\Big[-K+\frac{1}{2}(D-2)\delta\Lambda\Big]\,, \tag{35}$$

where the constant K is [45],

$$K = \frac{\kappa^2 H^D}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\left[\frac{1}{D-3}-\frac{1}{2}(D-2)(D+1)+\frac{1}{8}(D-4)(D-1)\right]. \tag{36}$$

The correct renormalization condition for $\delta\Lambda$ seems to be to make the trace of the graviton 1PI 1-point function vanish on the initial value surface. Otherwise, the constant we call "H" does not represent the initial expansion

[1] The inverses of D_A and D_C act on constants with the homogeneous solutions (a^0 and a^{1-D} for D_A and a^{-2} and a^{3-D} for D_C) chosen so that expression (35) is constant,

$$\frac{1}{D_A}C = -\frac{C\ln(a)}{(D-1)H^2} \quad , \quad \frac{1}{D_C}C = -\frac{C}{2(D-3)H^2}\,.$$

rate. If this condition is adopted then expression (35) vanishes,

$$\delta\Lambda = \frac{2K}{D-2} \qquad \Longrightarrow \qquad \Big\langle \Omega \Big| \kappa \mathbf{H}_1(x) \Big| \Omega \Big\rangle = 0 \; . \tag{37}$$

Condition (37) provides for the simplest development of perturbation theory, however, it is worth examining what would happen if a different renormalization condition were adopted. In this case the expectation value of the graviton field would not vanish at one loop order, and its spatial components would suffer secular growth in our gauge. Both the failure to vanish and the secular growth follow from having declined to make the parameter H in the background metric agree with the true (initial) expansion rate which we might write as $H + \delta H$. To see this, suppose we change (37) to,

$$-K + \frac{1}{2}(D-2)\delta\Lambda = (D-2)(D-1) H \delta H \; . \tag{38}$$

Then the two nonzero terms of expression (35) become,

$$\frac{2H}{D-2} \frac{1}{D_C} \Big[(D-2)(D-1) H \delta H \Big] \;\; = \;\; -\Big(\frac{D-1}{D-3} \Big) \delta H \; , \tag{39}$$

$$-\frac{2 a^{-1} \partial_0}{D-3} \frac{1}{D_A} \Big[(D-2)(D-1) H \delta H \Big] \;\; = \;\; 2 \Big(\frac{D-2}{D-3} \Big) \delta H \; . \tag{40}$$

The sum of (39) and (40) gives precisely δH, which makes for a nice check on the consistency of our expansion observable. Of course most researchers would at this stage absorb δH into H so as to make condition (37) pertain. Persisting with a nonzero value of δH would be like working in flat space QED (quantum electrodynamics) with the parameter m failing to stand for the actual electron mass.

3.3 2-Point Contributions

The simple diagrammatic structure of the one loop 2-point contributions which is shown in Figure 2 conceals the enormous complexity of our observable. One can see from expression (11) that $\mathbf{H}_2$ contains three distinct classes of terms: those with both graviton fields at the point x^μ, those with one graviton at x^μ and the other acted upon by a factor of $\frac{1}{D_A}$, and those with both gravitons acted upon by (different) factors of $\frac{1}{D_A}$. In each case

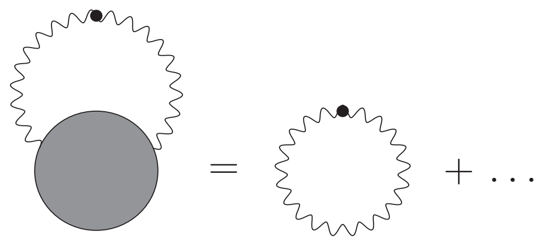

Figure 2: Diagrammatic representation of $\langle\Omega|\kappa^2\mathbf{H}_2(x)|\Omega\rangle$. Graviton lines are wavy and ghost lines are straight.

we substitute expression (17) and then perform the indicated tensor contractions and differentiations. However, factors of $\frac{1}{D_A}$ require special treatment to reflect all the derivatives outside the inverse differential operators. We will explicitly work out a sample reduction from each of the three classes, and then give the full result for that class.

To illustrate the reduction of the first class we have selected the third term on the first line of expression (11). Multiplying by the factor of κ^2 which all parts of $\mathbf{H}_2$ carry gives,

$$\Big\langle\Omega\Big|-\frac{\kappa^2 h_{ij}(x)h'_{ij}(x)}{2(D-1)a}\Big|\Omega\Big\rangle = -\frac{\kappa^2}{2(D-1)a}\lim_{x'\to x}\Bigg\{\Big[\delta_{ii}\delta_{jj}+\delta_{ij}\delta_{ij} \\ -\frac{2}{D-3}\,\delta_{ij}\delta_{ij}\Big]\partial'_0 i\Delta_A(x;x') + \frac{2}{(D-3)(D-2)}\,\delta_{ij}\delta_{ij}\partial'_0 i\Delta_C(x;x')\Bigg\}, \quad (41)$$

$$= -\frac{\kappa^2}{2a}\lim_{x'\to x}\Bigg\{\Big[D-\frac{2}{D-3}\Big]\partial'_0 i\Delta_A(x;x') + \frac{2}{(D-3)(D-2)}\,\partial'_0 i\Delta_C(x;x')\Bigg\}, \quad (42)$$

$$= -\frac{\kappa^2}{2}\Big[D-\frac{2}{D-3}\Big]\frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\,\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\,H = -\frac{\kappa^2 H^3}{8\pi^2} + O(D-4)\;. \quad (43)$$

The full result for this class of terms is,

$$\Big\langle\Omega\Big|\Big[\kappa^2\mathbf{H}_2(x)\Big]_{hh}\Big|\Omega\Big\rangle = \frac{\kappa^2 H^{D-1}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-2)}{\Gamma(\frac{D}{2})}\Bigg\{\frac{3}{4}\frac{1}{D-2}-\frac{1}{2}(D-1) \\ -\frac{1}{2}(D-2)\Big(D-\frac{2}{D-3}\Big)\Bigg\} \longrightarrow -\frac{25}{8}\times\frac{\kappa^2 H^3}{16\pi^2} + O(D-4)\;. \quad (44)$$

The second class consists of one local $h_{\mu\nu}(x)$ and the other inside a first order correction of the scalar (8). For example, consider the second term on the second line of expression (11) with the last of the four terms of $\Phi_1(x)$,

$$\frac{\kappa^2}{2(D\!-\!1)a^2}\times\Bigl(\partial_0^2-Ha\partial_0-\nabla^2\Bigr)h_{ii}(x)\times\frac{1}{D_A}\Bigl[-\frac{h'_{jj}(x)}{2a}\Bigr]\;. \tag{45}$$

We write the expectation value of any such term as the coincidence limit of $\frac{1}{D_A}$ acting on a differentiated propagator. Then we employ the reflection identities of Appendix B to move derivatives outside the factor of $\frac{1}{D_A}$, and the result is expressed in terms of the integrated scalar propagators (30-31). For expression (45) the reduction is,

$$\Bigl\langle\Omega\Bigl|\mathrm{Exp}(45)\Bigr|\Omega\Bigr\rangle=-\kappa^2\lim_{x'\to x}\frac{(\partial_0^2-Ha\partial_0-\nabla^2)}{4(D\!-\!1)a^2}\frac{1}{D'_A}\Biggl\{\frac{1}{a'}\partial'_0 i\Bigl[{}_{ii}\Delta_{jj}\Bigr](x;x')\Biggr\}, \tag{46}$$

$$=\kappa^2\lim_{x'\to x}\frac{(\partial_0^2-Ha\partial_0-\nabla^2)}{(D\!-\!3)a^2}\frac{1}{D'_A}\Biggl\{\frac{\partial'_0 i\Delta_A(x;x')}{a'}-\Bigl(\frac{D\!-\!1}{D\!-\!2}\Bigr)\frac{\partial'_0 i\Delta_C(x;x')}{2a'}\Biggr\}, \tag{47}$$

$$=\kappa^2\lim_{x'\to x}\frac{(\partial_0^2-Ha\partial_0-\nabla^2)}{(D\!-\!3)a^2}\frac{1}{D'_A}\Biggl\{-\Bigl[\partial_0+(D\!-\!2)Ha\Bigr]\frac{i\Delta_B(x;x')}{a'}$$
$$+\Bigl(\frac{D\!-\!1}{D\!-\!2}\Bigr)Hi\Delta_C(x;x')+\frac{1}{2}\Bigl(\frac{D\!-\!1}{D\!-\!2}\Bigr)\Bigl[\partial_0+(D\!-\!4)Ha\Bigr]\frac{i\Delta_D(x;x')}{a'}\Biggr\}, \tag{48}$$

$$=\kappa^2\lim_{x'\to x}\frac{(\partial_0^2-Ha\partial_0-\nabla^2)}{(D\!-\!3)a^2}\Biggl\{-\Bigl[\partial_0+(D\!-\!2)Ha\Bigr]J_{AB}(x';x)$$
$$+\Bigl(\frac{D\!-\!1}{D\!-\!2}\Bigr)HI_{AC}(x';x)+\frac{1}{2}\Bigl(\frac{D\!-\!1}{D\!-\!2}\Bigr)\Bigl[\partial_0+(D\!-\!4)Ha\Bigr]J_{AD}(x';x)\Biggr\}. \tag{49}$$

Using Appendix C the full result for this class of terms is,

$$\Bigl\langle\Omega\Bigl|\Bigl[\kappa^2\mathbf{H}_2(x)\Bigr]_{h\Phi_1}\Bigr|\Omega\Bigr\rangle=\frac{\kappa^2}{a^2}\lim_{x'\to x}\Biggl\{\Bigl[\partial_0^2+(D\!-\!4)Ha\partial_0-\frac{2\nabla^2}{D\!-\!1}\Bigr]$$
$$\times\Biggl[-\frac{(D^2\!-\!6D\!+\!7)}{(D\!-\!2)(D\!-\!3)}HI_{AC}(x';x)+\Bigl[\partial_0+(D\!-\!4)Ha\Bigr]\frac{J_{AD}(x';x)}{D\!-\!3}\Biggr]$$
$$-\Biggl[\frac{(\partial_0^2-Ha\partial_0-\nabla^2)[\partial_0+(D\!-\!2)Ha]}{D\!-\!3}+\frac{\nabla^2(\partial_0-Ha)}{D\!-\!1}\Biggr]J_{AB}(x';x)\Biggr\}$$
$$\longrightarrow\frac{\kappa^2H^{D-1}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D\!-\!1)}{\Gamma(\frac{D}{2})}\Biggl\{\frac{-\frac{132193}{5005}}{D\!-\!4}+O\Bigl((D\!-\!4)^0\Bigr)\Biggr\}. \tag{50}$$

The final class of terms consists of a graviton from one factor of $\Phi_1(x)$ multiplied by a graviton from another factor of $\Phi_1(x)$. As an example, consider the case where it is the second graviton of expression (8) which is contributed by each $\Phi_1(x)$,

$$\Big(\frac{D+1}{D-1}\Big)\frac{\kappa^2 H}{2a^2}\frac{\partial_i}{D_A}\Big[-\frac{h'_{00}(x)}{2a}\Big] \times \frac{\partial_i}{D_A}\Big[-\frac{h'_{00}(x)}{2a}\Big] \; . \tag{51}$$

The reduction of expression (51) proceeds similarly to that of (45),

$$\Big\langle \Omega \Big| \mathrm{Exp}(51) \Big| \Omega \Big\rangle = \frac{(D+1)(D-3)}{(D-1)(D-2)}\frac{\kappa^2 H}{4a^2}\lim_{x'\to x}\frac{\partial_i}{D_A}\frac{\partial'_i}{D'_A}\Big\{\frac{\partial_0\partial'_0 i\Delta_C(x;x')}{aa'}\Big\}, \tag{52}$$

$$= -\frac{(D+1)(D-3)}{(D-1)(D-2)}\frac{\kappa^2 H}{4a^2}\lim_{x'\to x}\frac{\nabla^2}{D_A D'_A}\Big\{\Big(\partial_0+Ha\Big)\Big(\partial'_0+Ha'\Big)\frac{i\Delta_C(x;x')}{aa'}\Big\}, \tag{53}$$

$$= -\frac{(D+1)(D-3)}{(D-1)(D-2)}\frac{\kappa^2 H}{4a^2}\lim_{x'\to x}\Big\{2H\Big[\partial'_0 + (D-2)Ha'\Big]\nabla^2 K_{ACB}(x;x')$$
$$+\Big[\partial_0+(D-2)Ha\Big]\Big[\partial'_0+(D-2)Ha'\Big]\nabla^2 J_{BCB}(x;x')+H^2\nabla^2 I_{ACA}(x;x')\Big\}. \tag{54}$$

With Appendix C we find that the full result for this class of terms is,

$$\Big\langle \Omega \Big| \Big[\kappa^2 \mathbf{H}_2(x)\Big]_{\Phi_1\Phi_1} \Big| \Omega \Big\rangle = \Big(\frac{D+1}{D-1}\Big)\frac{\kappa^2 H}{2a^2}\lim_{x'\to x}\nabla^2\Big\{\Big(\frac{D-1}{D-3}\Big)H^2 I_{AAA}(x;x')$$
$$-2\frac{(D^2-3D+1)^2}{(D-3)(D-2)}H^2 I_{ACA}(x;x') - \nabla^2 J_{ABA}(x;x') + \Big[\partial_0+(D-2)Ha\Big]$$
$$\times\Big[\partial'_0+(D-2)Ha'\Big]\Big[\Big(\frac{D-1}{D-3}\Big)J_{BAB}(x;x') - 2\Big(\frac{D-2}{D-3}\Big)J_{BCB}(x;x')\Big]$$
$$+H\Big[\partial'_0+(D-2)Ha'\Big]\Big[2\Big(\frac{D-1}{D-3}\Big)K_{AAB}(x;x')-4\Big(\frac{D^2-3D+1}{D-3}\Big)K_{ACB}(x;x')\Big]\Big\}$$
$$\longrightarrow \frac{\kappa^2 H^{D-1}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\Big\{\frac{-\frac{2669}{288}}{D-4} + O\Big((D-4)^0\Big)\Big\}. \tag{55}$$

Combining the results of expressions (44), (50) and (55) gives the full one loop result,

$$\Big\langle \Omega \Big| \kappa^2 \mathbf{H}_2(x) \Big| \Omega \Big\rangle = \frac{\kappa^2 H^{D-1}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\Big\{\frac{-\frac{3\cdot 853\cdot 60293}{32\cdot 13!!}}{D-4} + O\Big((D-4)^0\Big)\Big\}. \tag{56}$$

The divergent part of expression (56) is what chiefly concerns us but we take note of the fact that the finite part is also independent of time.

4 Renormalization

The expansion observable $\mathbf{H}[g](x)$ is a nonlocal composite operator whose divergences are not automatically absorbed by the BPHZ renormalization of non-coincident 1PI functions. Indeed, we saw in section 3.2 that the natural renormalization condition for the cosmological counterterm $\delta\Lambda$ is to cancel the initial value of the trace of the graviton 1PI 1-point function. Because the graviton 1PI 1-point function is a pure trace [45] this renormalization condition has the effect of completely cancelling the 1-point contributions of section 3.2, leaving the 2-point contributions of section 3.3 unaffected. From expression (56) we see that these terms diverge at one loop order.

It should be noted that no other BPHZ renormalizations can affect the expectation value of $\mathbf{H}[g](x)$ at one loop order. Renormalizing R and R^2 is degenerate with Λ for de Sitter background at this order, and the Weyl-squared term makes no contribution at all for de Sitter background at this order. The additional divergences of expression (56) derive from the fact that $\mathbf{H}[g](x)$ is a composite operator, and they require composite operator renormalization. Because $\mathbf{H}(x)$ goes like an inverse length, and the loop counting parameter κ^2 goes like a length squared, we require operators of dimension length^{-3} (times κ^2) which can mix with $\mathbf{H}[g](x)$. Had we been dealing with a *local* composite operator the list of candidates would be short, but the number of *nonlocal* candidates is infinite.

We propose that the conundrum should be resolved by limiting candidates to those which are local in the scalar + gravity theory (12) from which $\mathbf{H}[g](x)$ descends. With this conjecture there are just two candidate mixing operators of the required dimension,[2]

$$\mathbf{O}_1[g](x) \equiv \kappa^2 R\Big(\theta[g](x), \vec{x}\Big) \times \mathbf{H}[g](x) \quad , \quad \mathbf{O}_2[g](x) \equiv \kappa^2 \mathbf{H}^3[g](x) \; , \qquad (57)$$

where we recall that $\theta[g](x)$ defines the surface of simultaneity on which the scalar $\Phi[g]$ takes its background value (4). Both operators are proportional to $\kappa^2 H^3$ for de Sitter, so either can be used to completely cancel the one loop correction (56) to the expectation value of $\mathbf{H}[g](x)$. That is, we think of the renormalized expansion operator as,

$$\mathbf{H}[g] + \delta\mathbf{H}[g] = \mathbf{H}[g] + \mu^{D-4}\Big(\alpha_1 \mathbf{O}_1[g] + \alpha_2 \mathbf{O}_2[g]\Big) + \text{higher loops} \; , \qquad (58)$$

where α_1 and α_2 are functions of D.

[2] We have omitted $\dot{\mathbf{H}}[g](x)$ from the list (57) because it vanishes on de Sitter background.

It remains to discuss two issues, the first of which is the finite part of the expectation value of (58) at one loop order. We *must* choose the coefficients α_1 and α_2 to cancel the divergent part, but it might be that the finite part remains nonzero and represents an interesting prediction of quantum gravity. This is not so for three reasons. First, there is no unambiguous definition of "the finite part" of the primitive expectation value (56). For example, had the multiplicative factors of $(4\pi)^{-\frac{D}{2}}$ and $\Gamma(D-1)/\Gamma(\frac{D}{2})$ been evaluated at $D = 4$, what we call the finite part would change. Second, the finite parts of α_1 and α_2 are equally ambiguous for the same reason. Finally, the point of $\mathbf{H}[g](x) + \delta\mathbf{H}[g](x)$ is to measure the spacetime expansion rate. If a completely arbitrary choice makes this rate fail to agree with H, even on the initial value surface, *and after we have made the graviton 1-point function vanish* (at one loop order), then we have failed to properly define $\mathbf{H}[g](x) + \delta\mathbf{H}[g](x)$. We must make its initial expectation value agree with H, just as we must make what we call "the physical electron mass" agree with its observed value. The legitimate prediction of quantum gravity is how the expectation value of $\mathbf{H}[g](x) + \delta\mathbf{H}[g](x)$ changes with time. Because the result (56) of the primitive one loop diagrams is constant, as are the mixing operators (57), there is no change at one loop order. We do not expect that to remain true at two loop order, but this is contingent on the primitive two loop contributions showing secular growth.

The second issue is how to renormalize $\mathbf{H}[g](x)$ on more general backgrounds than de Sitter. One must first understand that $\mathbf{H}[g](x)$ was defined to apply for a homogeneous and isotropic background. One can see this from the fact that the spatial position has not been invariantly fixed [33]. Had the initial state possessed spatial structure this could have — and would have — been used to modify $\mathbf{H}[g](x)$ so as to invariantly fix the spatial position.

For pure gravity with a positive cosmological constant, de Sitter is the unique homogeneous and isotropic solution. However, it is simple to add a scalar whose background evolution supports a more general (FRW) homogeneous and isotropic background. The propagators and vertices for this theory are known [47, 26] and the computation we have just completed could be repeated for a general $H(t) \equiv a'/a^2$. Because ultraviolet divergences are local, we can be confident that the result would be divergences proportional to two terms: $H(t)\dot{H}(t)$ and $H^3(t)$. As it happens, the two one loop mixing operators (57) span this 2-dimensional space of possible divergences,

$$\mathbf{O}_1[\mathrm{FRW}](x) = \kappa^2\Big[2(D{-}1)H\dot{H}+D(D{-}1)H^3\Big] \quad , \quad \mathbf{O}_2[\mathrm{FRW}](x) = \kappa^2 H^3 \; . \quad (59)$$

It therefore seems inevitable that we can not only renomormalize the one loop expectation value of $\mathbf{H}[g](x)$ on de Sitter background but also on an arbitrary homogeneous and isotropic background.

5 Epilogue

Our task has been to give an invariant quantification of the prediction that there is no one loop back-reaction in pure quantum gravity on de Sitter background [4, 5]. In section 2 we reviewed the nonlocal invariant $\mathbf{H}[g](x)$ which has been proposed to quantify inflationary back-reaction [33]. In section 3 we computed the one loop expectation value of $\mathbf{H}[g](x)$, obtaining (35) for the contributions from that part of $\mathbf{H}[g](x)$ which is linear in the graviton field, and (56) for the contributions from the part of $\mathbf{H}[g](x)$ which is quadratic in the graviton field.

Section 4 dealt with the crucial issue of renormalization. The natural renormalization condition for the cosmological counterterm $\delta\Lambda$ is to entirely cancel the trace of the 1PI 1-point function, which makes the 1-point contribution (35) vanish. That leaves the divergent 2-point contribution (56) uncontrolled. These composite operator divergences require composite operator renormalization. We identified two candidate operators (57) which could be used to entirely cancel the 2-point contribution (56), at one loop order and on de Sitter background (and probably other homogeneous and isotropic backgrounds). We have therefore confirmed the prediction that there is no back-reaction at one loop order, *and* we have a plausible conjecture for controlling ultraviolet divergences at any order and on general expanding spacetime backgrounds.

These are solid accomplishments which place the extension to two loop order within reach. It is at this order that one expects secular back-reaction, which cannot be absorbed by renormalization. Some of the additional work required for this project is mechanical:

- Extend the expansion (9-11) of $\mathbf{H}[g](x)$ to include terms with three and four powers of the graviton field;
- Re-do the old 2-loop computation of the 1PI 1-point function [46] using dimensional regularization;
- Reduce the 2-loop 2-point contributions to either a single 4-point vertex with three propagators or two 3-point vertices with four propagators;

- Reduce the 2-loop 3-point contributions to a single 3-point vertex with three propagators; and
- Reduce the 2-loop 4-point contributions to four propagators.

Less mechanical is the task of including perturbative corrections to the initial state wave function [48]. It all seems doable now, although the labor involved is certainly daunting.

Before closing we should comment on the possibility that the conjecture of section 4 might represent a new insight on how to renormalize nonlocal, composite operators. This has great significance for quantum gravity because the only gauge invariant operators in that theory are nonlocal. Recall that the problem with nonlocal composite operators is limiting the list of other operators with which they can mix. Our conjecture deals with the class of nonlocal composite operators that descend from a larger parent theory in which they are local, just as our expansion observable $\mathbf{H}[g](x)$ becomes local in the scalar + gravity theory (12). We propose that the list of mixing operators be restricted to those which are local in the parent theory. The only other nonlocal composite operator whose renormalization is currently understood is the Wilson loop of non-Abelian gauge theory. They are multiplicatively renormalized [42], and that can indeed be viewed as a coupling constant renormalization in a parent theory which consists of a non-Abelian charged particle + Yang-Mills. It would be interesting to see if a similar result pertains for the quantum gravitational analogue whose puzzling ultraviolet divergences are not currently understood [49].

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6 Appendix A: General Scalar Propagator

Recall the general scalar kinetic operator D_ν which was defined in expression (28). The spatial plane wave mode functions for D_ν are,

$$u_\nu(\eta,k) = \sqrt{\frac{\pi}{4Ha^{D-1}}}\, H^{(1)}_\nu(-k\eta)\ . \tag{60}$$

Up to a possible infrared cutoff the associated propagator is,

$$i\Delta_\nu(x;x') = \int\!\frac{d^{D-1}k}{(2\pi)^{D-1}}\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}\Bigl\{\theta(\eta-\eta')u_\nu(\eta,k)u^*_\nu(\eta',k) + \theta(\eta'-\eta)u^*_\nu(\eta,k)u_\nu(\eta',k)\Bigr\}\ , \tag{61}$$

Except for a handful of de Sitter breaking terms (for which see section 3 of [50]) the result is,

$$i\Delta_\nu(x;x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\Biggl\{\Gamma\Bigl(\frac{D}{2}-1\Bigr)\Bigl(\frac{4}{y}\Bigr)^{\frac{D}{2}-1} - \frac{\Gamma(\frac{D}{2})\Gamma(1-\frac{D}{2})}{\Gamma(\frac12+\nu)\Gamma(\frac12-\nu)}\sum_{n=0}^{\infty} \times\Biggl[\frac{\Gamma(\frac32+\nu+n)\Gamma(\frac32-\nu+n)}{\Gamma(3-\frac{D}{2}+n)(n+1)!}\Bigl(\frac{y}{4}\Bigr)^{n-\frac{D}{2}+2} - \frac{\Gamma(\nu_A+\nu+n)\Gamma(\nu_A-\nu+n)}{\Gamma(\frac{D}{2}+n)n!}\Bigl(\frac{y}{4}\Bigr)^{n}\Biggr]\Biggr\}. \tag{62}$$

The special case of $\nu = \nu_A - N$ has great importance for us,

$$i\Delta_{\nu_A-N}(x;x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\Biggl\{\Gamma\Bigl(\frac{D}{2}-1\Bigr)\Bigl(\frac{4}{y}\Bigr)^{\frac{D}{2}-1} + (-1)^N\sum_{n=0}^{\infty}\Biggl[\frac{\Gamma(n-N+D-1)\Gamma(n+N)}{\Gamma(n+\frac{D}{2})\,n!}\Bigl(\frac{y}{4}\Bigr)^{n} - \frac{\Gamma(n-N+\frac{D}{2}+1)\Gamma(n+N-\frac{D}{2}+2)}{\Gamma(n-\frac{D}{2}+3)\,(n+1)!}\Bigl(\frac{y}{4}\Bigr)^{n-\frac{D}{2}+2}\Biggr]\Biggr\}. \tag{63}$$

Because all our results can be reduced to coincidence limits of differentiated propagators it is worth pointing out that the potential ultraviolet divergences in expression (63) derive from the Gamma function $\Gamma(n-N+D-1)$ which multiplies the factor of y^n. Because $y(x;x')$ vanishes at coincidence, nonzero results can only come from low powers of y. For certain integrated propagators such as $I_{ACA}(x;x')$ and $I_{AAA}(x;x')$ in expression (55) there can also be ultraviolet divergences from differentiating the multiplicative factor of $1/\Gamma(\frac12-\nu)$ in expression (62).

7 Appendix B: Reflection Identities

In flat space background all components of the graviton and ghost propagators are the same, and all depend only on the Lorentz invariant difference of the two points, $\eta_{\mu\nu}(x-x')^\mu(x-x')^\nu$. It is therefore straightforward to reflect derivatives from one coordinate of a propagator to the other, and from one side of an inverse differential operator to the other,

$$\text{Flat Space} \quad \Longrightarrow \quad \partial_\mu i\Delta(x;x') = -\partial'_\mu i\Delta(x;x') \quad , \quad \partial_\mu \frac{1}{\partial^2} = \frac{1}{\partial^2}\partial_\mu \; . \tag{64}$$

Expressions (17-18) and (23-25) show that things are considerably more complicated on de Sitter background! However, it is still possible to reflect derivatives by extending some older relations [51].

All propagators and inverse differential operators involve integrals of the function (and its conjugate),

$$f_\nu\Bigl(\eta,\eta',\Delta\vec{x}\Bigr) \equiv u_\nu(\eta,k)u^*_\nu(\eta,k)e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \; . \tag{65}$$

Using the Bessel function recursion relation $J'_\nu(z) \pm \frac{\nu}{z}J_\nu(z) = \pm J_{\nu\mp 1}(z)$ we can reflect derivatives from one argument to the other,

$$\partial_i f_\nu\Bigl(\eta,\eta',\Delta\vec{x}\Bigr) = -\partial'_i f_\nu\Bigl(\eta,\eta',\Delta\vec{x}\Bigr), \tag{66}$$

$$\Bigl[\partial_0 + (\nu_A-\nu)Ha\Bigr] f_\nu\Bigl(\eta,\eta',\Delta\vec{x}\Bigr) = -\Bigl[\partial'_0 + (\nu_A+\nu-1)Ha'\Bigr] f_{\nu-1}\Bigl(\eta,\eta',\Delta\vec{x}\Bigr), \tag{67}$$

$$\Bigl[\partial_0 + (\nu_A+\nu)Ha\Bigr] f_\nu\Bigl(\eta,\eta',\Delta\vec{x}\Bigr) = -\Bigl[\partial'_0 + (\nu_A-\nu-1)Ha'\Bigr] f_{\nu+1}\Bigl(\eta,\eta',\Delta\vec{x}\Bigr). \tag{68}$$

Applying these identities to the propagator implies,

$$\partial_i i\Delta_\nu\Bigl(x;x'\Bigr) = -\partial'_i i\Delta_\nu(x;x') \; , \tag{69}$$

$$\Bigl[\partial_0 + (\nu_A-\nu)Ha\Bigr] i\Delta_\nu(x;x') = -\Bigl[\partial'_0 + (\nu_A+\nu-1)Ha'\Bigr] i\Delta_{\nu-1}(x;x') \; , \tag{70}$$

$$\Bigl[\partial_0 + (\nu_A+\nu)Ha\Bigr] i\Delta_\nu(x;x') = -\Bigl[\partial'_0 + (\nu_A-\nu-1)Ha'\Bigr] i\Delta_{\nu+1}(x;x') \; . \tag{71}$$

The analogous relations for inverse differential operators are,

$$\frac{1}{D_\nu}\partial_i = \partial_i \frac{1}{D_\nu} \; , \tag{72}$$

$$\frac{1}{D_\nu}\Bigl[\partial_0 - (\nu_A-\nu)Ha\Bigr] = \Bigl[\partial_0 + (\nu_A+\nu-1)Ha\Bigr]\frac{1}{D_{\nu-1}} \; , \tag{73}$$

$$\frac{1}{D_\nu}\Bigl[\partial_0 - (\nu_A+\nu)Ha\Bigr] = \Bigl[\partial_0 + (\nu_A-\nu-1)Ha\Bigr]\frac{1}{D_{\nu+1}} \; . \tag{74}$$

8 Appendix C: Integrated Propagators

The integrated propagators $I_{\alpha\beta}$ and $I_{\alpha\beta\gamma}$ of expression (30) are symmetric,

$$I_{\alpha\beta}(x;x') = I_{\beta\alpha}(x;x') \qquad , \qquad I_{\alpha\beta\gamma}(x;x') = I_{\beta\alpha\gamma}(x;x') = I_{\alpha\gamma\beta}(x;x') \; . \quad (75)$$

They can also be generalized to any number of integrations,

$$I_{\alpha\beta\cdots\psi\omega}(x;x') = \frac{1}{D_\alpha}\frac{1}{D_\beta}\cdots\frac{1}{D_\psi}\, i\Delta_\omega(x;x') \; . \quad (76)$$

By counting inverse derivatives one can easily infer the leading behavior of these integrated propagators near coincidence,

$$I_{\alpha_1\cdots\alpha_n}(x;x') \sim \Delta x^{2n-4}\ln(\Delta x^2) \qquad , \qquad \Delta x^2 \equiv \eta_{\mu\nu}(x-x')^\mu(x-x')^\nu \; . \quad (77)$$

So the integrated propagators $\nabla^2 I_{AAA}(x;x')$ and $\nabla^2 I_{ACA}(x;x')$ in expression (55) are only logarithmically divergent at coincidence.

The $(n+1)$-th integrated propagator can be written simply in terms of differences of the n-th integrated propagators [50],

$$I_{\alpha\beta}(x;x') = \frac{i\Delta_\alpha(x;x') - i\Delta_\beta(x;x')}{(\beta^2-\alpha^2)H^2} \; , \quad (78)$$

$$I_{\alpha\beta\gamma}(x;x') = \frac{I_{\alpha\gamma}(x;x') - I_{\beta\gamma}(x;x')}{(\beta^2-\alpha^2)H^2} \; , \quad (79)$$

and so on. It follows that the coincidence limits of derivatives of integrated propagators which are given in expressions (50) and (55) are really coincidence limits of differences of differentiated propagators. In dimensional regularization these coincidence limits come entirely from the first few y^n terms. For example, the contributions from $I_{AC} = -(i\Delta_A - i\Delta_C)/[2(D-3)H^2]$ in expression (50) derive from just the de Sitter breaking factor of $\ln(aa')$ and the y^1 terms of the two propagators (23) and (25),

$$Ha\partial_0 I_{AC}\Big|_{x'=x} = \frac{-H^{D-4}}{2(D-3)(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\Big\{1+0\Big\}\times H^2a^2 \; , \quad (80)$$

$$\partial_0^2 I_{AC}\Big|_{x'=x} = \frac{-H^{D-4}}{2(D-3)(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\Big\{1-\Big(\frac{D-3}{D-2}\Big)\Big\}\times H^2a^2 \; , \quad (81)$$

$$\nabla^2 I_{AC}\Big|_{x'=x} = \frac{-H^{D-4}}{2(D-3)(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\Big\{0+\frac{(D-1)(D-3)}{D-2}\Big\}\times H^2a^2 \; . \quad (82)$$

Repeated subscripts, such as those in I_{ACA} and I_{AAA} of expression (55), follow from expressions (78-79) by differentiation with respect to the subscript,

$$\begin{aligned}
I_{\alpha\alpha}(x;x') &= -\frac{1}{2\alpha H^2}\,\frac{\partial i\Delta_\nu(x;x')}{\partial\nu}\Big|_{\nu=\alpha}\,, &(83)\\
I_{\alpha\alpha\beta}(x;x') &= -\frac{I_{\alpha\beta}(x;x')}{(\beta^2-\alpha^2)H^2}-\frac{1}{2\alpha(\beta^2-\alpha^2)H^4}\,\frac{\partial i\Delta_\nu(x;x')}{\partial\nu}\Big|_{\nu=\alpha}\,, &(84)\\
I_{\alpha\alpha\alpha}(x;x') &= \frac{I_{\alpha\alpha}(x;x')}{4\alpha^2H^2}+\frac{1}{8\alpha^2H^4}\,\frac{\partial^2 i\Delta_\nu(x;x')}{\partial\nu^2}\Big|_{\nu=\alpha}\,. &(85)
\end{aligned}$$

All the triple subscript integrated propagators in expression (55) involve $\frac{\nabla^2}{a^2}$. This has the effect of eliminating the purely time dependent, de Sitter breaking terms. In view of relations (84-85) the result we need is,

$$\begin{aligned}
&\frac{\nabla^2}{a^2}\,i\Delta_\nu(x;x')\Big|_{x'=x}\\
&\qquad=\frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(\frac{D}{2})\Gamma(1-\frac{D}{2})}{\Gamma(\frac12+\nu)\Gamma(\frac12-\nu)}\frac{\Gamma(\nu_A+\nu+1)\Gamma(\nu_A-\nu+1)}{\Gamma(\frac{D}{2}+1)}\times\frac12(D-1)H^2\,. \qquad(86)
\end{aligned}$$

Differentiating expression (86) with respect to ν and setting $\nu=\nu_A$ gives,

$$\begin{aligned}
\frac{\nabla^2}{a^2}\frac{\partial i\Delta_\nu}{\partial\nu}\Big|_{\substack{x'=x\\ \nu=\nu_A}} &= \frac{H^D}{(4\pi)^{\frac{D}{2}}}\frac{(D-1)\Gamma(D)}{D\Gamma(\frac{D}{2})}\Big\{\psi(D)-\psi(1)-\psi\Big(\frac{D}{2}\Big)+\psi\Big(1-\frac{D}{2}\Big)\Big\}, &(87)\\
\frac{\nabla^2}{a^2}\frac{\partial^2 i\Delta_\nu}{\partial\nu^2}\Big|_{\substack{x'=x\\ \nu=\nu_A}} &= \frac{H^D}{(4\pi)^{\frac{D}{2}}}\frac{(D-1)\Gamma(D)}{\Gamma(D\frac{D}{2})}\Big\{\psi'(D)+\psi'(1)-\psi'\Big(\frac{D}{2}\Big)-\psi'\Big(1-\frac{D}{2}\Big)\\
&\qquad+\Big[\psi(D)-\psi(1)-\psi\Big(\frac{D}{2}\Big)+\psi\Big(1-\frac{D}{2}\Big)\Big]^2\Big\}. &(88)
\end{aligned}$$

Retaining the full D-dependence becomes extremely tedious so we report only the divergent contributions,

$$\begin{aligned}
\frac{\nabla^2}{a^2}\frac{\partial i\Delta_\nu}{\partial\nu}\Big|_{\substack{x'=x\\ \nu=\nu_A}} &= \frac{H^D}{(4\pi)^{\frac{D}{2}}}\frac{(\Gamma(D-1)}{\Gamma(\frac{D}{2})}\Big\{\frac{\frac92}{D-4}+O\Big((D-4)^0\Big)\Big\}, &(89)\\
\frac{\nabla^2}{a^2}\frac{\partial^2 i\Delta_\nu}{\partial\nu^2}\Big|_{\substack{x'=x\\ \nu=\nu_A}} &= \frac{H^D}{(4\pi)^{\frac{D}{2}}}\frac{(\Gamma(D-1)}{\Gamma(\frac{D}{2})}\Big\{\frac{\frac{33}{2}}{D-4}+O\Big((D-4)^0\Big)\Big\}. &(90)
\end{aligned}$$

Employing relations (89-90) and expressions (84-85) gives,

$$\frac{\nabla^2}{a^2} I_{ACA}(x;x')\Big|_{x'=x} = \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \frac{(\Gamma(D-1)}{\Gamma(\frac{D}{2})} \Biggl\{ \frac{\frac{3}{4}}{D-4} + O\Bigl((D-4)^0\Bigr) \Biggr\}, \quad (91)$$

$$\frac{\nabla^2}{a^2} I_{AAA}(x;x')\Big|_{x'=x} = \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \frac{(\Gamma(D-1)}{\Gamma(\frac{D}{2})} \Biggl\{ \frac{\frac{3}{4}}{D-4} + O\Bigl((D-4)^0\Bigr) \Biggr\}, \quad (92)$$

The inverse factors of a and a' in the integrated propagators of expressions (31-32) require additional labor. By inserting unity in the form $1 = D_\alpha \times \frac{1}{D_\alpha}$, partially integrating, and then using the reflection identities of Appendix B we can expand $J_{\mu\nu}(x;x')$ in terms of ever more highly integrated propagators,

$$\begin{aligned} J_{\mu\nu}(x;x') &= \sum_{K=0}^{\infty} K!(-2H)^K \Bigl[\partial_0 + \Bigl(\nu_A + \mu - 1\Bigr) Ha\Bigr] \cdots \\ &\qquad \cdots \Bigl[\partial_0 + \Bigl(\nu_A + \mu - K\Bigr) Ha\Bigr] \Biggl\{ \frac{I_{\nu\, \mu-1 \cdots \mu-2K-1}(x;x')}{a^{K+1}} \Biggr\}, \quad (93) \\ &= \sum_{K=0}^{\infty} K!(-2H)^K \Bigl[\partial_0' + \Bigl(\nu_A + \nu - 1\Bigr) Ha'\Bigr] \cdots \\ &\qquad \cdots \Bigl[\partial_0' + \Bigl(\nu_A + \nu - K\Bigr) Ha'\Bigr] \Biggl\{ \frac{I_{\mu\, \nu-1 \cdots \nu-2K-1}(x;x')}{a'^{K+1}} \Biggr\}. \quad (94) \end{aligned}$$

Because either argument can be chosen for the expansion it is possible to avoid repeated indices in the expansions for J_{AD} and J_{AB}, for example,

$$\begin{aligned} J_{AD}(x';x) &= \frac{I_{AE}}{a} - 2H\Bigl[\partial_0 + (D-5)Ha\Bigr] \frac{I_{AEG}}{a^2} \\ &+ 8H^2 \Bigl[\partial_0 + (D-5)Ha\Bigr]\Bigl[\partial_0 + (D-6)Ha\Bigr] \frac{I_{AEGI}}{a^3} \\ &- 48H^3 \Bigl[\partial_0 + (D-5)Ha\Bigr]\Bigl[\partial_0 + (D-6)Ha\Bigr]\Bigl[\partial_0 + (D-7)Ha\Bigr] \frac{I_{AEGIK}}{a^4} + \ldots \quad (95) \end{aligned}$$

The integrated propagators $J_{AD}(x';x)$ and $J_{AB}(x';x)$ appear in expression (50) with certain external derivatives,

$$\frac{1}{a^2}\Biggl[\partial_0^2 + (D-4)Ha\partial_0 - \frac{2\nabla^2}{D-1}\Biggr]\Bigl[\partial_0 + (D-4)Ha\Bigr] \frac{J_{AD}(x';x)}{D-3}\Big|_{x'=x}, \quad (96)$$

$$-\frac{1}{a^2}\Biggl[\frac{[\partial_0^2 - Ha\partial_0 - \nabla^2][\partial_0 + (D-2)Ha]}{D-3} + \frac{\nabla^2(\partial_0 - Ha)}{D-1}\Biggr] J_{AB}(x';x)\Big|_{x'=x}. \quad (97)$$

K	n	Terms for J_{AD}	Terms for J_{AB}
0	0	0	0
0	1	0	$-\frac{9}{4}$
1	0	0	0
1	1	0	0
1	2	-5	-2
2	0	0	0
2	1	0	0
2	2	-144	-6
3	0	0	0
3	1	0	0
3	2	-7460	-216
3	3	756	324

Table 1: The results of acting the external derivatives of expressions (96-97) on the relevant internal derivatives (98), and then taking the coincidence limit for $D = 4$. For J_{AD} we used $N = 3$, with $N = 1$ for J_{AB}.

In view of relation (77) the only possible divergences can arise for $K \leq 3$. Because the various integrated propagators $I_{\alpha\cdots\beta}$ involve differences of propagators (63), the result for (96-97) derives from acting the appropriate external derivatives on expressions of the form,

$$\frac{K!(-2H)^K}{H^{2K-2}} \Bigl[\partial_0 + (D\!-\!2\!-\!N)Ha\Bigr] \cdots \Bigl[\partial_0 + (D\!-\!2\!-\!N\!-\!K)Ha\Bigr] \Bigl(\frac{y}{4}\Bigr)^n \; . \quad (98)$$

Table 1 gives the results of doing this for the few values of K and n which are required. These factors are then multiplied by the ratios of Gamma functions from (63) for each value of n and N ($N = 3$ for J_{AD} and $N = 1$ for J_{AB}), and finally divided by the numerical factors implied by expressions (78-79). Putting everything together gives,

$$\Bigl(96\Bigr) = \frac{H^{D-1}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D\!-\!1)}{\Gamma(\frac{D}{2})} \left\{ \frac{\frac{15227}{780}}{D\!-\!4} + O\Bigl((D\!-\!4)\Bigr) \right\}, \quad (99)$$

$$\Bigl(97\Bigr) = \frac{H^{D-1}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D\!-\!1)}{\Gamma(\frac{D}{2})} \left\{ \frac{-\frac{42443}{924}}{D\!-\!4} + O\Bigl((D\!-\!4)\Bigr) \right\}. \quad (100)$$

Because the I_{AC} contributions (80-82) are all finite, the divergent part of (50) comes from adding (99) to (100) and multiplying by κ^2.

Doubly integrated propagators with inverse factors of a and a' require a separate treatment. One first writes them as a single inverse differential operator acting on a singly integrated propagator, then the singly integrated propagator is expanded according to expressions (78) or (93-94). For example, consider $K_{ACB}(x;x')$ from expression (55),

$$K_{ACB}(x;x') = \frac{1}{D'_B} \Bigl[\frac{I_{AC}(x;x')}{a'}\Bigr] = \frac{J_{BC}(x';x) \!-\! J_{BA}(x';x)}{2(D\!-\!3)H^2} \; , \quad (101)$$

$$= \frac{1}{2(D\!-\!3)H^2} \left\{ \frac{I_{BD}}{a} - 2H\Bigl[\partial_0 + (D\!-\!4)Ha\Bigr] \frac{I_{BDF}}{a^2} + \dots \right.$$
$$\left. - \frac{I_{CA}}{a'} + 2H\Bigl[\partial'_0 + (D\!-\!3)Ha'\Bigr] \frac{I_{CEA}}{a'^2} - \dots \right\}. \quad (102)$$

Because $K_{ACB}(x;x')$ is only differentiated three times in expression (55), we do not need to go any higher than the terms shown in (102). The three J integrals in expression (55) are differentiated four times so they must be expanded to one higher order. Our final results for the J and K integrals in expression (55) are,

$$\lim_{x'=x} \frac{\nabla^2}{a^2} \Big[\partial_0' + (D-2)Ha'\Big] H K_{AAB}(x;x') \\ = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \Bigg\{ \frac{\frac{7}{4}}{D-4} + O\Big((D-4)\Big)\Bigg\}, \qquad (103)$$

$$\lim_{x'=x} \frac{\nabla^2}{a^2} \Big[\partial_0' + (D-2)Ha'\Big] H K_{ACB}(x;x') \\ = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \Bigg\{ \frac{\frac{3}{320}}{D-4} + O\Big((D-4)\Big)\Bigg\}, \qquad (104)$$

$$\lim_{x'=x} \frac{\nabla^4}{a^2} J_{ABA}(x;x') \\ = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \Bigg\{ \frac{\frac{145}{16}}{D-4} + O\Big((D-4)\Big)\Bigg\}, \qquad (105)$$

$$\lim_{x'=x} \frac{\nabla^2}{a^2} \Big[\partial_0 + (D-2)Ha\Big]\Big[\partial_0' + (D-2)Ha'\Big] J_{BAB}(x;x') \\ = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \Bigg\{ \frac{\frac{51}{16}}{D-4} + O\Big((D-4)\Big)\Bigg\}, \qquad (106)$$

$$\lim_{x'=x} \frac{\nabla^2}{a^2} \Big[\partial_0 + (D-2)Ha\Big]\Big[\partial_0' + (D-2)Ha'\Big] J_{BCB}(x;x') \\ = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \Bigg\{ \frac{\frac{163}{120}}{D-4} + O\Big((D-4)\Big)\Bigg\}. \qquad (107)$$

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