Signature change in loop quantum gravity: Two-dimensional midisuperspace models and dilaton gravity

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Signature change in loop quantum gravity: 2-dimensional midisuperspace models and dilaton gravity

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Models of loop quantum gravity based on real connections have a deformed notion of general covariance, which leads to the phenomenon of signature change. This result is confirmed here in a general analysis of all midisuperspace models without local degrees of freedom. As a subclass of models, 2-dimensional theories of dilaton gravity appear, but a larger set of examples is possible based only on the condition of anomaly freedom. While the classical dilaton gravity models are the only such systems without deformed covariance, they do give rise to signature change when holonomy modifications are included.

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I. INTRODUCTION

In canonical formulations of gravitational theories, covariance is ensured by gauge transformations generated by the constraints rather than by coordinate transformations. Poisson brackets of the constraint functions on phase space must then obey a certain form that reduces to the hypersurface deformations of general relativity in the classical (or low-curvature) limit. Anomaly freedom, or the fact that the constraints in modified or quantized gravity models must remain first class, imposes strong conditions on the possible forms of constraints and on structure functions in their brackets. Signature change is the most characteristic and apparently generic consequence of these conditions.

Conditions that ensure covariance of a canonical quantum theory of gravity have been formulated in [1]. It has been shown that not only (i) the classical Hamiltonian and diffeomorphism constraints, on quantisation, must still satisfy a first-class system and have a closed algebra; but also (ii) that this algebra must have a classical limit whereby it reduces to the familiar hypersurface-deformation brackets [2, 3] of general relativity. This statement holds also for effective or modified theories in which certain quantum corrections are included while working in a semiclassical approximation. Covariance therefore poses an important consistency question for canonical quantum-gravity theories, which goes beyond the requirement that constraints be anomaly free. Brackets (or commutators) of the constraints not only have to lead to a closed system, they must also close in such a way that a specific classical limit is obtained.

The examples discussed in detail in [1] show that anomaly freedom of gravitational models does not necessarily imply covariance. In particular, constraint brackets in midisuperpace models can often be simplified by redefining the classical constrained system, sometimes eliminating structure functions. The resulting Lie algebras are then easier to quantize in an anomaly-free way. However, after quantization, it is not guaranteed that the redefinitions can still be inverted such that a closed set of hypersurface-deformation generators is obtained. The main example given in [1] is a partially Abelianized redefinition along the lines of [4], which can be made covariant in the presence of holonomy modifications but only if there is no matter coupled to the system. Moreover, signature change is realized also in the partially Abelianized system if holonomy modifications are present.

Recently, several other models have been analyzed by partial Abelianization, together with proposed quantizations. In [5], a locally rotationally symmetric Gowdy model has been introduced and quantized in this way. In [6], the class of 2-dimensional dilaton gravity models has been studied, with a special discussion of the vacuum CGHS model [7] given in [8]. These models do not have local degrees of freedom and therefore do not encounter the obstructions found in [1, 9] for covariant holonomy-modified models with local degrees of freedom. Nevertheless, the question of covariance has not been addressed in [5, 6, 8]. In the present paper, we will fill in this lacuna. At the same time, we construct the most general covariant $1 + 1$-dimensional midisuperspace model without local degrees of freedom with spatial derivatives of the metric (or dyad and dilaton) up to second order. We compute the modified structure functions of all these models and conclude that the class of all classical 2-dimensional dilaton gravity models, with an arbitrary dilaton potential but the same form of extrinsic-curvature type components as in general relativity, is the only set with undeformed covariance. However, a large class of covariant models exists with deformed covariance, which includes quantum versions of these dilaton models with effects from loop quantum gravity.

In a specific applications of these new constructions, we are particularly interested in the phenomenon of signature change in models of loop quantum gravity. Such models are based on modified Hamiltonian constraints in which the quadratic dependence on the connection or extrinsic curvature has been replaced by a bounded function, motivated by the use of holonomies taking values in compact groups instead of connection components in the kinematical Hilbert space of the full theory. This replacement implies, rather directly, that curvature or the energy density remains bounded during gravitational collapse in simple cosmological or black-hole models. Curvature singularities may therefore be avoided, but a less direct implication of holonomy modifications in these models, given by signature change [10, 11], implies that high-curvature regions remain limits of causal space-time because they do not allow deterministic evolution. Whenever we comment on signature change in this paper, we will assume that the specific constraints of the theory arise from holonomy modifications in the specified sense: replacements of the quadratic dependence of the Hamiltonian constraint on connection or extrinsic-curvature components by bounded functions. With this understanding, most of the new models we find here have signature change if modification functions are such that they mimic holonomy modifications of loop quantum gravity. In this context, the classical limit of our models will be a low-curvature limit in which holonomy modifications become small.

II. SIGNATURE CHANGE IN THE POLARIZED GOWDY MODEL WITH LOCAL ROTATIONAL SYMMETRY

We first look at the specific model studied in [5]: the polarized Gowdy model on a three-torus with local rotational symmetry (LRS). The last condition eliminates local degrees of freedom. As usual, we identify the two homogeneous directions, x and y, with each other while keeping the inhomogeneous direction θ unchanged. We have an inhomogeneous midisuperspace model without local physical degrees of freedom.

In keeping with the conventions of [5], we work with the two triad components (E^x,ε) and the extrinsic-curvature components conjugate to them, $(K_x,\mathcal{A})$. In the reduced 1-dimensional manifold with coordinate θ, E^x and $\mathcal{A}$ have density weight one. The Poisson brackets between the canonical variables are $\{K_x(\theta_1),E^x(\theta_2)\} = G\delta(\theta_1,\theta_2) = \{\mathcal{A}(\theta_1),\varepsilon(\theta_2)\}$. We will assume $\varepsilon>0$. Derivatives with respect to the inhomogeneous coordinate are labelled by primes in the following.

As in the well-known case of spherical symmetry, there is only one global degree of freedom. However, the form of the Hamiltonian constraint in the Gowdy LRS case is distinct from that of spherical symmetry due to a different internal curvature term. For the latter model, the constraint is given by

$$H[N] = -\frac{1}{2G}\int \mathrm{d}\theta N(\theta)\left[\varepsilon^{-1/2}K_x^2E^x + 4\varepsilon^{1/2}\mathcal{A}K_x - \frac{1}{4}\varepsilon^{-1/2}\left(E^x\right)^{-1}\left(\varepsilon'\right)^2 \right.$$
$$\left. -\varepsilon^{1/2}\varepsilon''\left(E^x\right)^{-1} + \left(E^x\right)^{-2}\varepsilon^{1/2}\varepsilon'\left(E^x\right)'\right]\,, \tag{1}$$

while the diffeomorphism constraint

$$D[N^x] = \frac{1}{G}\int \mathrm{d}\theta N^x(\theta)\left[K_x'E^x - \varepsilon'\mathcal{A}\right] \tag{2}$$

takes the same form as in spherically symmetric models. The classical constraint brackets are

$$\{D[N_1^\theta], D[N_1^\theta]\} = D\left[\mathcal{L}_{N_1^\theta}N_2^\theta\right] \tag{3}$$
$$\{H[N], D[N^\theta]\} = -H\left[\mathcal{L}_{N^\theta}N\right] \tag{4}$$
$$\{H[N_1], H[N_1]\} = D\left[q^{\theta\theta}\left(N_1N_2' - N_2N_1'\right)\right]\,. \tag{5}$$

The inverse-metric component $q^{\theta\theta} = \varepsilon/(E^x)^2$ appears in the classical brackets as the only non-constant structure function, while the other non-zero components of the inverse spatial metric are $q^{xx} = q^{yy} = \varepsilon^{-1}$. (It follows from the results of [12] that the Hamiltonian constraint (1) is the same as what is obtained for a $1+1$-dimensional dilaton gravity model with zero dilaton potential, when expressed in connection variables after a canonical transformation. The LRS Gowdy model of [5] is therefore nothing but a CGHS model with zero cosmological constant.)

We introduce holonomy modifications in the Hamiltonian constraint

$$H[N] = -\frac{1}{2G}\int \mathrm{d}\theta N(\theta)\left[\varepsilon^{-1/2}f_1\left(K_x\right)E^x + 4\varepsilon^{1/2}\mathcal{A}f_2(K_x) - \frac{1}{4}\varepsilon^{-1/2}\left(E^x\right)^{-1}\left(\varepsilon'\right)^2 \right.$$
$$\left. -\varepsilon^{1/2}\varepsilon''\left(E^x\right)^{-1} + \left(E^x\right)^{-2}\varepsilon^{1/2}\varepsilon'\left(E^x\right)'\right]\,, \tag{6}$$

while keeping the diffeomorphism constraint unmodified. In the classical case, $f_1\left(K_x\right) = K_x^2$ and $f_2(K_x) = K_x$. Here, we assume pointwise holonomy corrections along the homogeneous directions while working in an effective formalism. However, as shown in [13], adding additional quantum moment terms does not change the structure of the constraint brackets (while the constraints themselves usually do have moment corrections). By keeping these modification functions general, we are able to examine the restrictions imposed on them such that the modified constraints still have closed brackets.

It is straightforward to see that the brackets between two diffeomorphism constraints and between a Hamiltonian and a diffeomorphism constraint have the same form as in the classical case. The only complicated Poisson bracket is thus the one between two Hamiltonian constraints, which gives

$$\{H[N_1], H[N_2]\} = \frac{1}{2G}\int \mathrm{d}\theta (N_1 N_2' - N_2 N_1') \left(2\frac{\varepsilon}{(E^x)^2}\left(\frac{\mathrm{d}f_2}{\mathrm{d}K_x}\right)\left[K_x (E^x)' - \varepsilon' \mathcal{A}\right] \right.$$
$$\left. \frac{\varepsilon'}{E^x}\left[\left(\frac{\mathrm{d}f_1}{\mathrm{d}K_x}\right) - 2f_2\right]\right) , \tag{7}$$

where we have integrated by parts several times. On analyzing this result, we note two features:

1. The closure of the algebra is ensured only if we have $\mathrm{d}f_1/\mathrm{d}K_x - 2f_2 = 0$, implying restrictions on the modification functions which have been kept free in the discussion so far. The coefficient of this term is neither the Hamiltonian nor the diffeomorphism constraint and thus would give rise to an anomaly unless the whole term vanishes.

2. Although closure can be ensured in this model by making the above restriction on the form of the holonomy modification functions, we obtain a structure function in the modified theory which is deformed by a factor of $\mathrm{d}f_2/\mathrm{d}K_x$ as compared with the classical case. Using the consistency condition between f_1 and f_2, the factor takes the form $\mathrm{d}f_2/\mathrm{d}K_x = \frac{1}{2}\mathrm{d}^2 f_1/\mathrm{d}^2 K_x$.

We thus have a modification in the bracket

$$\begin{aligned}\{H[N_1], H[N_1]\} &= \frac{1}{G}\int \mathrm{d}\theta (N_1 N_2' - N_2 N_1') \frac{\varepsilon}{(E^x)^2}\frac{1}{2}\left(\frac{\mathrm{d}^2 f_1}{\mathrm{d}K_x^2}\right)\left[K_x (E^x)' - \varepsilon' \mathcal{A}\right] \\ &= D\left[\frac{1}{2} q^{\theta\theta}\frac{\mathrm{d}^2 f}{\mathrm{d}K_x^2}(N_1 N_2' - N_2 N_1')\right] .\end{aligned} \tag{8}$$

Signature change can be understood from this relation as follows: In models of loop quantum gravity, holonomy modifications replace quadratic appearances of extrinsic-curvature components in the Hamiltonian constraint by some bounded functions which reach their maximum value near the Planck scale. The bounded nature of these modification functions is a crucial ingredient in claims of singularity resolution in these models. Near a local maximum of a function such as f_1, the second derivative is negative, making the coefficient on the right-hand side of (8) change its sign. The same change of sign happens if one switches the signature of the theory to Euclidean, and indeed the form of the brackets has a close relationship with the hyperbolic or elliptic nature of equations of motion consistent with the brackets [10, 11]. A negative correction factor in structure functions of (8) can therefore be interpreted as indicating signature change. For $f_1(K_x) = K_x^2$, on the other hand, we recover the classical result where the modification in the structure function goes to one. Thus, in addition to having a closed algebra for the modified constraints, we also recover the hypersurface-deformation brackets in the classical (low-curvature) limit. The model is covariant provided our conditions are fulfilled. Only one free function, $f_1(K_x)$, then remains, which is unrestricted by anomaly freedom and covariance.

In [5], a loop quantisation of the LRS Gowdy model has been proposed. To this end, the authors first Abelianize the classical bracket of two normal deformations while leaving the other two relations unchanged. Following [4], the new, Abelianized constraint is defined as a linear combination of the old Hamiltonian constraint and the diffeomorphism constraints, while the diffeomorphism constraint remains unchanged. (This partial Abelinization can also be applied to the full polarized Gowdy model [9] without local rotational symmetry.) The new constraint used in this context, Eq. (1) of [5], is

$$H_{\mathrm{new}}[N] = -\frac{1}{2G}\int \mathrm{d}\theta \, \frac{N}{\varepsilon'}\left[2\sqrt{\varepsilon}K_x^2 - \frac{\sqrt{\varepsilon}\varepsilon'}{2(E^x)^2}\right]' , \tag{9}$$

while $D[N^x]$ follows from (2), as before.

The authors then adopt the holonomy modification scheme for models of loop quantum gravity and substitute $K_x \to \sin(\gamma K_x)/\gamma$ in (9). The K_x^2 term in (9) is therefore replaced by $(\sin(\gamma K_x))^2/\gamma^2$. The new constraint commutes with itself, which is easy to see if we integrate by parts in (9) (after absorbing the denominator ε' in the lapse function) and notice that there are no spatial derivatives of E^x anymore. Although the resulting theory is consistent in the sense of being anomaly-free, it is not guaranteed to be covariant. In order to show covariance, one must be able to recover suitable generators of gauge transformations such that their brackets lead to the hypersurface-deformation brackets in the classical (low-curvature) limit. (That is, there must be such generators for any phase-space point in the modified theory, which usually take non-classical forms but reduce to the classical versions in a certain subset of the phase space. This condition of covariance is therefore non-trivial, in contrast to a simple requirement that the classical limit of the modified theory have suitable gauge generators. In a non-covariant theory, such as some of the Abelianized holonomy-modified models, there may be the right number of gauge generators for the entire theory, but they would not be continuously connected with the usual hypersurface-deformation generators in the classical or low-curvature limit [1, 9].) This important conceptual step is missing in [5], but will be completed here.

We can start from (6), having incorporated the holonomy modification functions, and try to partially Abelianize this bracket. Thus, we first holonomy-modify and then Abelianize. It is important to emphasise that we do not impose any restrictions on either of the functions f_1 or f_2 at this point. Proceeding as in the classical case, the new constraint is defined as

$$\mathcal{H}_{\rm new} = \mathcal{H}_{\rm old} - \frac{\mathrm{d}f_1}{\mathrm{d}K_x}\frac{\varepsilon'}{\sqrt{\varepsilon}}\mathcal{D}\,, \tag{10}$$

where $\mathcal{H}, \mathcal{D}$ stand for the unsmeared versions of the constraints. With this step, we arrive at the same form of the new, holonomy-modified constraint as proposed in [5], *provided the two modification functions obey the condition* $\mathrm{d}f_1/\mathrm{d}K_x = 2f_2$. (This condition ensures that the $\mathcal{A}$-term in (6) cancels out if combined with (2) as in (10).) We have the same restriction on the modification functions as found before by an analysis of anomaly freedom of hypersurface-deformation brackets. Thus, requiring the new system of constraints to be (partially) Abelian is equivalent to imposing that the old system of constraints form a closed system. The closed hypersurface-deformation brackets with modified structure functions then again indicate signature change.

We can arrive at this result from another perspective as well. Starting with the newly defined classical constraint (9), one can introduce a quantum theory as in [5]. However, to ensure covariance we must be able to define constraints which have hypersurface-deformation brackets with the correct classical limit. This condition translates to recovering a Hamiltonian constraint from the Abelianized constraint by inverting the linear transformation used above, which can be equivalently thought of as transforming the lapse function and the shift vector as

$$N = \frac{\tilde{N}}{E^x} \tag{11}$$

$$N^\theta = \tilde{N}^\theta - \tilde{N}\frac{\sqrt{\varepsilon}\,\mathrm{d}f_1/\mathrm{d}K_x}{\varepsilon'}\,. \tag{12}$$

This step puts the system of constraints in the form of our ansatz (6) and (2), with the specific choice of $f_1(K_x) = \left(\sin\left(\gamma K_x\right)\right)^2/\gamma^2$. As expected, for the holonomy-modified LRS Gowdy system in [5], signature change occurs in high curvature regions: The second derivative of f_1 in this case is proportional to $2\cos(2\gamma K_x)$, which has a negative sign near a local maximum of f_1.

It is remarkable that our result and signature change are robust even when different equivalent systems of classical constraints are used as the starting point of a loop quantization. As demonstrated earlier with spherical symmetry [1], the restrictions on holonomy modification functions are the same, no matter whether they are derived by requiring closure of the algebra or by requiring that it be possible to define new constraints which have partially Abelian brackets. The present section shows that this conclusion is also true for another model of loop quantum gravity, namely the LRS Gowdy model. We have shown that signature change is an unavoidable consequence of holonomy modifications in this model, irrespective of how one defines the system of constraints as long as one forces the resulting quantum theory to be covariant. This result may be taken as an indication that these conclusions hold more generally in midisuperspace models of loop quantum gravity without local physical degrees of freedom. The remainder of this paper confirms this expectation.

III. GENERAL CASE

A theory without local degrees of freedom should have as many pairs of canonical variables as there are first-class constraints. For hypersurface-deformation covariant systems in two space-time dimensions, there should therefore be two pairs of canonical fields, which we continue to denote as in the LRS Gowdy model of the preceding section. A generic form of a Hamiltonian constraint is

$$H[N] = -\frac{1}{2G}\int \mathrm{d}\theta N\left(\theta\right)\left\{f\left(\mathcal{A}, K_x, E^x, \varepsilon\right) + g_1(\varepsilon)\frac{(\varepsilon')^2}{E^x} + g_2(\varepsilon)\frac{\varepsilon''}{E^x}\right.$$
$$\left. + g_3(\varepsilon)\frac{\varepsilon'(E^x)'}{(E^x)^2} + g_4(\varepsilon)E^x\right\}\,, \tag{13}$$

whereas the diffeomorphism constraint again has the usual form

$$D[N^\theta] = \frac{1}{G}\int \mathrm{d}\theta N^\theta(\theta)\{K_x' E^x - \varepsilon' \mathcal{A}\} \tag{14}$$

if the spatial structure remains unchanged. The Poisson brackets between the canonical variables remain the standard ones $\{K_x(x), E^x(y)\} = G\delta(x,y) = \{\mathcal{A}(x), \varepsilon(y)\}$. One might expect quantum corrections in the Poisson structure, but by Darboux' theorem one can always transform back to canonical variables. All such corrections are then contained in the modification functions already introduced. (The structure of the diffeomorphism constraint is strongly restricted for canonical variables and would not change by such a transformation.)

The assumptions for our general form are:

1. The diffeomorphism constraint does not have modifications. For models of loop quantum gravity, this assumption is made because one usually quantizes the diffeomorphism constraint, or rather the finite action it generates, without taking recourse to holonomies around loops. (For an exception see [14].)

2. All curvature dependence is contained in a generic function f, while spatial derivatives of the triad components have separate correction functions. One could include the last term $g_4(\epsilon)E^x$ in the function f, but it is more convenient to keep it separate.

3. Every term in (13) has the correct density weight as required. (See [15] for a discussion of density weights in midisuperspace models.) In particular, this condition implies that the modification functions g_i can depend only on ε (but not on E^x), and that any spatial derivative has to be accompanied by a factor of $1/E^x$. We do not consider terms with spatial derivatives (or extrinsic curvature) in the denominator because they would not be guaranteed to be finite everywhere.

4. There are no terms of higher than second spatial derivatives to the order considered here. Such terms would require a derivative expansion as in [16].

5. There are no terms linear in spatial derivatives in order to ensure local parity invariance (transforming $\theta \mapsto -\theta$ in a local chart).

6. In midisuperspace models of general relativity, terms proportional to the second order derivatives of E^x are absent due to the fact that spatial derivatives come from the curvature tensor which cannot have two radial derivatives of the radial components owing to its antisymmetry properties. Thus we do not have terms proportional to $(E^x)''$ or $(E^{x\prime})^2$. In Sec. III C, we will show that such terms are, in fact, impossible in an anomaly-free system extending (1) and (2).

If these conditions hold, the system (13) with (2) is generic. Our goal is to start with this ansatz and try to impose conditions on the arbitrary functions by requiring closure of the constraint algebra. We will also impose that the Hamiltonian constraint has the correct classical (low-curvature) limit for small curvature components and large ε. Both conditions taken together then ensure covariance.

A. Brackets

Looking at the $\{H, H\}$ bracket, we know that the only non-zero contributions come from the first term with the rest of the terms in the Hamiltonian constraint. We write each of these contributions from $\{H[N_1], H[N_2]\}$ individually. From now on, we are going to suppress the functional dependence of each of these arbitrary functions on the canonical variables. Referring to the sum in (13), the Poisson bracket of term 1 with term 5 gives a vanishing contribution. Term 1 with term 2 gives

$$-\frac{1}{2G}\int \mathrm{d}\theta\, (N_1 N_2' - N_1' N_2)\left(\frac{\partial f}{\partial \mathcal{A}} g_1 \frac{\varepsilon'}{E^x}\right). \tag{15}$$

Term 1 with term 4 gives

$$-\frac{1}{4G}\int \mathrm{d}\theta\, (N_1 N_2' - N_1' N_2)\left(\frac{\partial f}{\partial K_x} g_3 \frac{\varepsilon'}{(E^x)^2} + \frac{\partial f}{\partial \mathcal{A}} g_3 \frac{(E^x)'}{(E^x)^2}\right). \tag{16}$$

Finally, term 1 with term 3 gives

$$\frac{1}{4G}\int \mathrm{d}\theta\,(N_1N_2'-N_1'N_2)\left(\frac{\partial f}{\partial\mathcal{A}}\dot{g}_2\frac{\varepsilon'}{E^x}-\frac{\partial f}{\partial\mathcal{A}}g_2\frac{(E^x)'}{(E^x)^2}\right.$$
$$\left.-\frac{\partial^2 f}{\partial\mathcal{A}\partial K_x}g_2\frac{K_x'}{E^x}-\frac{\partial^2 f}{\partial\mathcal{A}^2}g_2\frac{\mathcal{A}'}{E^x}-\frac{\partial^2 f}{\partial\mathcal{A}\partial E^x}g_2\frac{(E^x)'}{E^x}-\frac{\partial^2 f}{\partial\mathcal{A}\partial\varepsilon}g_2\frac{\varepsilon'}{E^x}\right)\,. \tag{17}$$

Here, a dot above any function dependent on a single variable refers to its derivative with respect to its variable.

The requirement for the algebra to be closed implies that any bracket between two constraints must be another constraint. This means that the right-hand side of $\{H,H\}$ can, in addition to the diffeomorphism constraint, also include a Hamiltonian constraint, provided its coefficient goes to zero in the classical (low-curvature) limit.

Since there is no $\mathcal{A}'$-term which can appear on the right-hand side, we infer $\partial^2 f/\partial\mathcal{A}^2=0$ from (17). Thus f is linear in $\mathcal{A}$ and can be written as

$$f(\mathcal{A},K_x,E^x,\varepsilon)=f_2(K_x,E^x,\varepsilon)\mathcal{A}+f_3(K_x,E^x,\varepsilon)\,. \tag{18}$$

Similarly, there is no $(E^x)'$-term (without a factor of ε') on the right-hand side, implying

$$f_2g_3(E^x)^{-2}+f_2g_2(E^x)^{-2}+\left(\frac{\partial f_2}{\partial E^x}\right)g_2(E^x)^{-1}=0 \tag{19}$$

or, equivalently,

$$g_2+g_3+\frac{g_2}{f_2}\left(\frac{\partial f_2}{\partial E^x}E^x\right)=0\,. \tag{20}$$

This expression can be rearranged to bring it to the form

$$-\frac{g_2+g_3}{g_2}=\frac{E^x}{f_2}\left(\frac{\partial f_2}{\partial E^x}\right)\,, \tag{21}$$

where now the left-hand side depends only on ε whereas the right-hand side depends on ε, E^x and K_x. Therefore both sides must each be equal to the same function of ε, which we call $g_5(\varepsilon)$. We have

$$1+g_3/g_2=-g_5 \tag{22}$$

and

$$\frac{E^x}{f_2}\left(\frac{\partial f_2}{\partial E^x}\right)=g_5\,. \tag{23}$$

We conclude that

$$f_2\left(K_x,E^x,\varepsilon\right)=\tilde{f}_2\left(K_x,\varepsilon\right)\left(E^x\right)^{-(1+g_3/g_2)}\,, \tag{24}$$

where we restore the explicit definition of g_5 in the final line.

Going back to the expressions (15), (17) and (16), we notice that any term proportional to just ε' (without a multiplicative factor of $\mathcal{A}$) must also be set equal to zero since there is no such term in the diffeomorphism constraint, and any ε' in the Hamiltonian constraint would be multiplied with another ε' or an $E^{x\prime}$:

$$2f_2g_1(E^x)^{-1}+g_3\frac{\partial f_3}{\partial K_x}(E^x)^{-2}-f_2\dot{g}_2(E^x)^{-1}+g_2\frac{\partial f_2}{\partial\varepsilon}(E^x)^{-1}=0 \tag{25}$$

or

$$\left(2f_2g_1-f_2\dot{g}_2+g_2\frac{\partial f_2}{\partial\varepsilon}\right)E^x=-g_3\frac{\partial f_3}{\partial K_x}\,. \tag{26}$$

On the left-hand side, we can use (24) to write out the dependence of the expression on E^x. Since the right-hand side involves g_3, which is a function of ε alone, and the derivative of f_3 with respect to K_x, we can then deduce that the dependence of f_3 on E^x is

$$f_3(K_x,E^x,\varepsilon)=\tilde{f}_3(K_x,\varepsilon)(E^x)^{-g_3/g_2}\,. \tag{27}$$

Inserting (24) and (27) in (26),

$$2\tilde{f}_2 g_1 - \tilde{f}_2 \dot{g}_2 + g_2 \frac{\partial \tilde{f}_2}{\partial \varepsilon} = -g_3 \frac{\partial \tilde{f}_3}{\partial K_x} \,. \tag{28}$$

Looking at the remaining two terms left, one of which is proportional to $\mathcal{A}\varepsilon'$ and the other to $K_x' E^x$, we have

$$\frac{\partial f_2}{\partial K_x} \left[g_2 (E^x)^{-2} K_x' E^x + g_3 (E^x)^{-2} \varepsilon' \mathcal{A} \right] \,. \tag{29}$$

For this to be proportional to the diffeomorphism constraint, we require that the prefactor of both the $K_x' E^x$ and the $\mathcal{A}\varepsilon'$ be the same. This implies

$$g_2 = -g_3 \,. \tag{30}$$

We can use this relation in (24) and (27),

$$f_2(K_x, E^x, \varepsilon) = \tilde{f}_2(K_x, \varepsilon) \,, \tag{31}$$

$$f_3(K_x, E^x, \varepsilon) = \tilde{f}_3(K_x, \varepsilon) E^x \,. \tag{32}$$

From (28) and (30),

$$2\tilde{f}_2 g_1 - \tilde{f}_2 \dot{g}_2 + g_2 \left(\frac{\partial \tilde{f}_2}{\partial \varepsilon} \right) = g_2 \left(\frac{\partial \tilde{f}_3}{\partial K_x} \right) \,. \tag{33}$$

B. Implications and special cases

Some of our new relations have interesting interpretations, which we collect in this subsection.

Equation (30) implies that the two terms

$$g_2(\varepsilon) \frac{\varepsilon''}{E^x} + g_3(\varepsilon) \frac{\varepsilon' (E^x)'}{(E^x)^2} = g_2(\varepsilon) \left(\frac{\varepsilon''}{E^x} - \frac{\varepsilon' (E^x)'}{(E^x)^2} \right) = -2 g_2(\varepsilon) \Gamma' \tag{34}$$

can always be written in terms of the function

$$\Gamma = -\frac{\varepsilon'}{2E^x} \tag{35}$$

which has the same form as the only non-trivial spin-connection component in dilaton models obtained by symmetry reduction from classical general relativity [15].

Equation (29) shows that the structure function of the modified system is equal to

$$\frac{\partial f_2}{\partial K_x} \frac{g_2}{(E^x)^2} = \beta \frac{\varepsilon}{(E^x)^2} = \beta q^{\theta\theta} \tag{36}$$

with the modification function

$$\beta = \frac{\partial f_2}{\partial K_x} \frac{g_2}{\varepsilon} \,, \tag{37}$$

comparing with (3). Using (33), $\partial f_2 / \partial K_x$ is proportional to $\partial^2 f_3 / \partial K_x^2$ if the dependence of f_2 on ε is weak. Around a local maximum of f_3 in K_x, the modification function β is therefore negative and we obtain signature change.

The modification function β does not introduce a dependence of structure functions on g_4, and there is no restriction on g_4 from anomaly freedom. There should therefore be classical gravity models for any choice of $g_4(\varepsilon)$. Indeed, as the canonical transformation derived in [12] shows, if g_4 is the only modification function that differs from the LRS Gowdy model, (13) is nothing but a 2-dimensional dilaton model with potential $V(\varepsilon) = g_4(\varepsilon)$, expressed in connection variables as used in models of loop quantum gravity. (The function g_1 does not appear explicitly in the expression of β, but unlike g_4 it cannot be chosen independently because it is related to f_2, f_3 and g_2 by (33).)

It is not easy to analyze Eq. (33) in general form, but a few special cases are of interest. First, we can see that it is not compatible with power-law lattice refinement [17, 18] which would require a dependence of modification functions on extrinsic curvature via the combination $\epsilon^q K_x$ with some real number q. If we assume two different such dependences in $\tilde{f}_2(\epsilon^p K_x)$ and $\tilde{f}_3(\epsilon^q K_x)$, (33) implies

$$2\tilde{f}_2 g_1 - \tilde{f}_2 \dot{g}_2 + p\epsilon^{p-1} g_2 K_x \dot{\tilde{f}}_2 = g_2 \epsilon^q \dot{\tilde{f}}_3 \,. \tag{38}$$

The third term, with a factor of K_x, is incompatible with almost-periodic functions $\tilde{f}_2$ and $\tilde{f}_3$ as assumed in models of loop quantum gravity.

Another special case is given by a factorizable ansatz for the modification functions:

$$\tilde{f}_2\left(K_x, \varepsilon\right) = f_4(K_x) g_6(\varepsilon) \tag{39}$$

$$\tilde{f}_3\left(K_x, \varepsilon\right) = f_5(K_x) g_7(\varepsilon) \,. \tag{40}$$

Inserting this form in (33), we find

$$2\frac{g_6 g_1}{g_7 g_2} - \frac{g_6 \dot{g}_2}{g_7 g_2} + \frac{\dot{g}_6}{g_7} = \frac{\dot{f}_5}{f_4} \,. \tag{41}$$

The left-hand side depends only on ε while the right-hand side depends solely on K_x. Thus, each of the two sides must be equal to a constant.

$$\frac{\mathrm{d}f_5}{\mathrm{d}K_x} = c f_4 \,, \tag{42}$$

and

$$2\frac{g_6 g_1}{g_7 g_2} - \frac{g_6 \dot{g}_2}{g_7 g_2} + \frac{\dot{g}_6}{g_7} = c \,. \tag{43}$$

The form of our generalized Hamiltonian constraint is now restricted to be

$$\begin{aligned} H[N] = -\frac{1}{2G} \int \mathrm{d}\theta N(\theta) \Big(& g_7 f_5(K_x) E^x + g_6 f_4(K_x) \mathcal{A} \\ & + g_1 \frac{(\varepsilon')^2}{E^x} + g_2 \frac{\varepsilon''}{E^x} - g_2 \frac{\varepsilon'(E^x)'}{(E^x)^2} + g_4 E^x \Big) \,. \end{aligned} \tag{44}$$

All of the g_i are functions of ε, with their functional dependence suppressed. However, not all of the remaining functions are unconstrained. We have the additional conditions given in (42) and (43). We can also absorb g_7 in the lapse function and rescale the rest of the g-functions accordingly. In other words, we can set $g_7 = 1$ without any loss of generality. We call the new lapse function $\tilde{N}$.

For our generalized midisuperspace model, closure of two Hamiltonian constraints, including modifications, implies the condition (42) for the modification functions. Given this condition, the deformed structure function takes the form

$$\frac{1}{c} \frac{\mathrm{d}^2 f_5}{\mathrm{d}K_x^2} g_2 g_6 (E^x)^{-2} \,, \tag{45}$$

while the final form of the Hamiltonian constraint is

$$\begin{aligned} H[N] = -\frac{1}{2G} \int \mathrm{d}\theta \tilde{N}(\theta) \Bigg(& f_5(K_x) E^x + \frac{g_6}{c} \frac{\mathrm{d}f_5}{\mathrm{d}K_x} \mathcal{A} \\ & + g_1 \frac{(\varepsilon')^2}{E^x} + g_2 \frac{\varepsilon''}{E^x} - g_2 \frac{\varepsilon'(E^x)'}{(E^x)^2} + g_4 E^x \Bigg) \,. \end{aligned} \tag{46}$$

The classical (low-curvature) limit is given by $g_2(\varepsilon) g_6(\varepsilon) = \varepsilon$, while $f_5(K_x) = K_x^2$, with $c = 2$. The function g_4 then labels different classical models with undeformed covariance, including all 2-dimensional dilaton gravity models, or the spherically symmetric model as well as Gowdy LRS.

C. Second-order spatial derivatives beyond general relativity

In our analysis so far, we did not consider two possible terms proportional to second-order spatial derivatives of triad components, namely $(E^x)''$ and $(E^{x\prime})^2$. These terms are not present in midisuperspace models of general relativity due to antisymmetry properties of the Riemann curvature tensor since derivatives with respect to the radial coordinate cannot appear on the radial component of the triads. However, such terms could conceivably arise if there is some modification to general relativity. Here, we show that the presence of such terms is incompatible with anomaly-free constraints.

Taking into account density weights, the general form of the Hamiltonian constraint with the additional terms is given by

$$H[N] = -\frac{1}{2G}\int \mathrm{d}\theta N\left(\theta\right)\left(f\left(\mathcal{A},K_x,E^x,\varepsilon\right)+g_1(\varepsilon)\frac{(\varepsilon')^2}{E^x}+g_2(\varepsilon)\frac{\varepsilon''}{E^x}\right.$$
$$\left.+g_3(\varepsilon)\frac{\varepsilon'(E^x)'}{(E^x)^2}+g_4(\varepsilon)E^x+h_1(\varepsilon)\frac{(E^x)''}{(E^x)^2}+h_2(\varepsilon)\frac{(E^{x\prime})^2}{(E^x)^3}\right) \tag{47}$$

with two new functions $h_1(\varepsilon)$ and $h_2(\varepsilon)$. The new terms arising from the Poisson bracket of two such Hamiltonian constraints are

$$-\frac{1}{2G}\int \mathrm{d}\theta(N_1N_2'-N_1'N_2)\frac{\partial f}{\partial K_x}h_2E^{x\prime}(E^x)^{-3} \tag{48}$$

and

$$-\frac{1}{4G}\int \mathrm{d}\theta(N_1N_2'-N_1'N_2)\left(2\frac{\partial f}{\partial K_x}h_1E^{x\prime}(E^x)^{-3}-\frac{\partial f}{\partial K_x}\dot{h}_1\varepsilon'(E^x)^{-2}\right.$$
$$+\frac{\partial^2 f}{\partial K_x^2}h_1K_x'(E^x)^{-2}+\frac{\partial^2 f}{\partial K_x\partial\mathcal{A}}h_1\mathcal{A}'(E^x)^{-2}$$
$$\left.+\frac{\partial^2 f}{\partial K_x\partial E^x}h_1E^{x\prime}(E^x)^{-2}+\frac{\partial^2 f}{\partial K_x\partial\varepsilon}h_1\varepsilon'(E^x)^{-2}\right)\,. \tag{49}$$

They contribute to all the conditions we had before. Starting with the requirement that there be no terms proportional to $\mathcal{A}'$ on the right-hand side, we now have

$$g_2E^x\frac{\partial^2 f}{\partial\mathcal{A}^2}+h_1\frac{\partial^2 f}{\partial\mathcal{A}\partial K_x}=0\,. \tag{50}$$

Defining $f_1(\mathcal{A},K_x,\varepsilon,E^x):=\partial f/\partial\mathcal{A}$,

$$g_2E^x\frac{\partial f_1}{\partial\mathcal{A}}=-h_1\frac{\partial f_1}{\partial K_x}\,. \tag{51}$$

This equation has the general solution

$$f_1(\mathcal{A},K_x,E^x,\varepsilon)=F(K_x-\mathcal{A}h_1/(g_2E^x),E^x,\varepsilon) \tag{52}$$

with an arbitrary function F of three variables. Since $f_1=\partial f/\partial\mathcal{A}$, we have

$$f(\mathcal{A},K_x,E^x,\varepsilon)=G(K_x-\mathcal{A}h_1/(g_2E^x),E^x,\varepsilon)+H(K_x,E_x,\varepsilon) \tag{53}$$

with $\partial G/\partial\mathcal{A}=F$ and another free function H of three arguments.

We can already see that the new terms are likely to lead to problematic conditions on the modification functions: The component $\mathcal{A}$ can only appear in the specific combination $K_x-\mathcal{A}h_1/(g_2E^x)$ with K_x, but finding anomaly-free modifications of the $\mathcal{A}$-dependence has proven difficult [16]. (Holonomy modifications of the $\mathcal{A}$-dependence would result from holonomies along curves in the θ-direction which remains inhomogeneous in the midisuperspace models considered here. Such modifications would therefore be non-local or, in a derivative expansion of an effective theory, include higher spatial derivatives of $\mathcal{A}$ which we are not considering here. The K_x-dependence, by contrast, remains local even if holonomy modifications are used in a midisuperspace model.) The function G could then only be a linear function in its first argument.

In fact, the new terms are ruled out if we use (53) and evaluate all contributions to the bracket that could give rise to the term $\mathcal{A}\varepsilon'$ in the diffeomorphism constraint. In particular, we have to make sure that we have a factor of $\mathcal{A}$ but no factor of K_x multiplying ε'. Two such terms,

$$\frac{\partial^2 f}{\partial\mathcal{A}\partial\varepsilon}\frac{g_2}{E^x}+\frac{\partial^2 f}{\partial K_x\partial\varepsilon}\frac{h_1}{(E^x)^2}=\frac{\partial^2 H}{\partial K_x\partial\varepsilon}\frac{h_1}{(E^x)^2} \tag{54}$$

do not contribute a factor of $\mathcal{A}$ as coefficients of ε'. The remaining terms are

$$-\frac{\partial f}{\partial K_x}\frac{g_3}{(E^x)^2}+\frac{\partial f}{\partial\mathcal{A}}\frac{\dot{g}_2}{E^x}+\frac{\partial f}{\partial K_x}\frac{\dot{h}_1}{(E^x)^2}=-\frac{g_3+h_1\dot{g}_2/g_2-\dot{h}_1}{(E^x)^2}G_1 \tag{55}$$

plus terms (partial derivatives of H) that do not depend on $\mathcal{A}$, where G_1 is the partial derivative of G by its first argument. We obtain a coefficient with linear dependence on $\mathcal{A}$ only if G is quadratic in $K_x-\mathcal{A}h_1/(g_2E^x)$, but even if this is the case, there will be additional terms depending on K_x which do not all cancel out. It is therefore impossible to gather all the new terms in coefficients of the diffeomorphism constraint, and no anomaly-free formulation is possible unless $h_1=0$.

With this result, we can follow the previous steps up to Eq. (20). There is now a new term $h_2(E^x)^{-3}\partial f/\partial K_x$ in the resulting equation

$$\begin{aligned}&\frac{\partial f}{\partial\mathcal{A}}g_3(E^x)^{-2}+\frac{\partial f}{\partial\mathcal{A}}g_2(E^x)^{-2}+\frac{\partial^2 f}{\partial\mathcal{A}\partial E^x}g_2(E^x)^{-1}\\&+2\frac{\partial f}{\partial K_x}h_2(E^x)^{-3}=0\end{aligned} \tag{56}$$

which, for f of the form (18), contains a factor of $\mathcal{A}$. However, all other terms in (56) are independent of $\mathcal{A}$, which is compatible with the new term only if $\partial f_2/\partial K_x=0$. But in this case there is no term of the form $K_x\mathcal{A}$ in the Hamiltonian, and the model is not compatible with the classical (low-curvature) limit. Therefore, $h_2=0$ and both new terms are ruled out.

IV. CONCLUSIONS

We have analyzed a general canonical form of $1+1$-dimensional covariant models without local physical degrees of freedom. A large subclass of such models has been recognized as *classical* 2-dimensional dilaton gravity models with an arbitrary potential. Another large class of models, most of which have not been encountered before, has a deformed notion of covariance and includes models of loop quantum gravity. Holonomy-modified versions of the 2-dimensional dilaton gravity models, as studied for instance in [6, 8], fall within the latter group. In this class, signature change is a generic consequence of modifications that introduce a bounded dependence of the Hamiltonian constraint on extrinsic curvature.

Our results unify several recent investigations of midisuperspace models of loop quantum gravity, including [5, 6, 8]. They also provide further support for the genericness of signature change in models of loop quantum gravity. So far, signature change has been avoided only by following three distinct procedures: (i) Using classical assumptions on the structure of space-time and foregoing an analysis of anomaly freedom. (ii) Implementing modifications via canonical transformations [19]. (iii) Using complex connections [20, 21]. The first option is problematic because it does not guarantee anomaly freedom. The second option is problematic as well, as discussed in the appendix. The third option needs to be explored further, in particular regarding the implementation of reality conditions. Furthermore, for complex variables, the quantization scheme becomes rather important since depending on how one implements holonomy corrections, one can still get signature change [22].

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Appendix A: An alternative modification scheme

A new "modification" scheme with bounded functions of curvature has been proposed in [19], using spherically symmetric models. Instead of modifying the Hamiltonian constraint, the authors use a canonical transformation

$K_x \to f(K_x)$ and $E^x \to E^x/\dot{f}(K_x)$, where the dot again represents the derivative of the function with respect to its argument, here K_x. (We have translated the relations of [19] to the notation used in the main body of the present paper. Instead of K_φ in [19], we therefore write K_x.) In the specific case of [19], the function is chosen as the usual sine function of models of loop quantum gravity, but we choose to keep the analysis more general. The Poisson brackets indeed remain unchanged:

$$\left\{ f(K_x(x))\,,\, \frac{E^x}{\dot{f}(K_x)}(y) \right\} = \{K_x(x)\,,\, E^x(y)\} = G\delta(x,y)\,. \tag{A1}$$

1. Constraints

Starting with the classical Hamiltonian and diffeomorphism constraints, (1) and (2), the canonical transformation takes us to

$$\begin{aligned} H[N] = -\frac{1}{2G}\int \mathrm{d}x\ N\Bigg[& (\varepsilon)^{-1/2}\frac{E^x}{\dot{f}(K_x)} f(K_x)^2 + 4(\varepsilon)^{1/2}\mathcal{A}f(K_x) \\ & -\frac{1}{4}(\varepsilon)^{-1/2}(\varepsilon')^2\left(\frac{E^x}{\dot{f}(K_x)}\right)^{-1} - (\varepsilon)^{1/2}\varepsilon''\left(\frac{E^x}{\dot{f}(K_x)}\right)^{-1} \\ & +(\varepsilon)^{1/2}\varepsilon'\left(\frac{E^x}{\dot{f}(K_x)}\right)^{-2}\left(\frac{E^{x\prime}}{f(\dot{K}_x)} - \frac{E^x f(\ddot{K}_x)K_x'}{(f(\dot{K}_x))^2}\right)\Bigg]. \end{aligned} \tag{A2}$$

$$D[N^x] = \frac{1}{2G}\int \mathrm{d}x\ N^x\left[2E^xK_x' - \mathcal{A}\varepsilon'\right]. \tag{A3}$$

(A dilaton potential could be included in (A2) but would not change the following arguments.) Viewed as a modified expression, this $H[N]$ has not been included in our main analysis because it would require modification functions $g_i(K_\varphi)$ that do not just depend on ε.

Following the procedure outlined in the previous sections, we can calculate the Poisson bracket between these constraints and find that the constraint algebra takes the form

$$\{D[N^x], D[M^x]\} = D[\mathcal{L}_{N^x}M^x], \tag{A4}$$

$$\{H[N], D[N^x]\} = -H[\mathcal{L}_{N^x}N], \tag{A5}$$

$$\{H[N], H[M]\} = D\left[(NM' - MN')\varepsilon\left(\frac{E^x}{\dot{f}(K_x)}\right)^{-2}\right]. \tag{A6}$$

(More details of the derivation are given in the following subsection.) As expected, the new structure function agrees with the usual one after applying the canonical transformation. One could interpret the last bracket as a hypersurface-deformation bracket with structure function modified by a factor of $1/\dot{f}^2$. This function is positive and therefore does not lead to signature change. According to the general results of [23], it can therefore be absorbed by a field redefinition, which would just be the inverse of the canonical transformation.

Once one (partially) Abelianizes the system of (modified) constraints, following [4], the Abelianized constraints remain Abelianized in spite of the modifications [19]. In fact, even in the presence of matter, the total constraints (gravitational plus the matter parts) form a (partially) Abelianized algebra. However, if we go back to the original hypersurface-deformation generators, the structure functions are deformed, as shown here.

This "modification" procedure suffers from several drawbacks. By applying a canonical transformation to the classical constraints, one cannot arrive at modified dynamics. (There is then no actual modification at all.) It is surprising how [19] can nevertheless make claims about singularity resolution. In fact, the canonical transformation is one-to-one only in a range of K_x where $f(K_x)$ is monotonic. For the common functions used in models of loop quantum gravity, this excludes all values of K_x greater than a certain finite threshold. The classical singularity (infinite K_x) is eliminated from these models only because the canonical transformation is valid in a limited part of phase space.

Moreover, the form of the modification is in contradiction with the usual guiding principles followed in models of loop quantum gravity, which suggest modifications of curvature terms in the Hamiltonian constraint, but no inverses of $\dot{f}(K_x)$ in triad terms.

2. Derivation of the structure function

The modification procedure introduced in [19] is of the form $K_x \to f(K_x)$, $E^x \to E^x/\dot{f}(K_x)$. Instead of showing the entire derivation of the constraint brackets after this transformation, we give a brief sketch. There are three types of terms which we shall be confronting during this calculation. Of course, the brackets of the modified variables are the same as for the classical ones, by construction. But one can also see this by an explicit calculation not based on the fact that a canonical transformation has been performed.

In particular, we have terms of the form

$$\left\{ g\frac{E^x}{\dot{f}(K_x)}\,,\, h\left(\frac{E^x}{\dot{f}(K_x)}\right)' \right\} \tag{A7}$$

which would be trivially zero in the original variables (where no K_x-terms are present in the corresponding bracket) but could be expected to be non-zero here if the nature of the transformation were not known. However, an explicit calculation, first writing

$$\left(\frac{E^x}{\dot{f}(K_x)}\right)' = \frac{E^{x\prime}}{\dot{f}(K_x)} - \frac{E^x\ddot{f}}{\dot{f}^2}K_x' \tag{A8}$$

and then using the full dependence on $E^{x\prime}$ and K_x, results in

$$\begin{aligned}
\left\{ g\frac{E^x}{\dot{f}(K_x)}\,,\, h\left(\frac{E^x}{\dot{f}(K_x)}\right)' \right\} &= gh\left(\left\{\frac{E^x}{\dot{f}}, \frac{E^{x\prime}}{\dot{f}}\right\} - \{\frac{E^x}{\dot{f}}, \frac{E^x\ddot{f}}{\dot{f}^2}K_x'\}\right) \\
&= gh\left(E^x\{\dot{f}^{-1}, E^{x\prime}\}\frac{1}{\dot{f}} - \frac{1}{\dot{f}}\{E^x, K_x'\}\frac{E^x\ddot{f}}{\dot{f}^2}\right) + \cdots \\
&= gh\left(\frac{E^x}{\dot{f}}\left(-\frac{\ddot{f}}{\dot{f}^2}\right) - \frac{1}{\dot{f}}\left(-\frac{E^x\ddot{f}}{\dot{f}^2}\right)\right)\frac{\partial\delta(x,y)}{\partial y} + \cdots \\
&= 0 + \cdots
\end{aligned}$$

where dots indicate additional terms proportional to delta functions but not their derivatives. Only derivatives of delta functions contribute to the antisymmetric bracket of smeared Hamiltonian constraints, but the only coefficient of such a term in (A9) is identically zero. A similar treatment of all other terms reveals that only brackets which are non-zero in the original variables are non-zero after the transformation, and those non-zero terms differ only by a factor of $\dot{f}^2$. In the specific case of Gowdy LRS, just as in spherical symmetry, this implies $\varepsilon/\left(E^x\right)^2 \to \varepsilon\left(\dot{f}(K_x)\right)^2/\left(E^x\right)^2$. Similar arguments work even when a matter contribution (say, in the form of a minimally coupled scalar field) is taken into account. Once again, the structure functions appearing in the brackets of the total constraints (gravitational plus the matter contributions) have the same deformation as above.

[1] M. Bojowald, S. Brahma, and J. D. Reyes, Covariance in models of loop quantum gravity: Spherical symmetry, *Phys. Rev. D* 92 (2015) 045043, [arXiv:1507.00329]

[2] P. A. M. Dirac, The theory of gravitation in Hamiltonian form, *Proc. Roy. Soc. A* 246 (1958) 333–343

[3] R. Arnowitt, S. Deser, and C. W. Misner, The Dynamics of General Relativity, *Gen. Rel. Grav.* 40 (2008) 1997–2027

[4] R. Gambini and J. Pullin, Loop quantization of the Schwarzschild black hole, *Phys. Rev. Lett.* 110 (2013) 211301, [arXiv:1302.5265]

[5] D. Martín-de Blas, J. Olmedo, and T. Pawlowski, Loop quantization of the Gowdy model with local rotational symmetry, [arXiv:1509.09197]

[6] A. Corichi, A. Karami, S. Rastgoo, and T. Vukašinac, Constraint Lie algebra and local physical Hamiltonian for a generic 2D dilatonic model, *Class. Quantum Grav.* 33 (2016) 035011, [arXiv:1508.03036]

[7] C. Callan, S. Giddings, J. Harvey, and A. Strominger, Evanescent Black Holes, *Phys. Rev. D* 45 (1992) 1005–1009, [hep-th/9111056]

[8] A. Corichi, J. Olmedo, and S. Rastgoo, Callan-Giddings-Harvey-Strominger vacuum in loop quantum gravity and singularity resolution, *Phys. Rev. D* 94 (2016) 084050, [arXiv:1608.06246]

[9] M. Bojowald and S. Brahma, Covariance in models of loop quantum gravity: Gowdy systems, *Phys. Rev. D* 92 (2015) 065002, [arXiv:1507.00679]

[10] M. Bojowald and G. M. Paily, Deformed General Relativity and Effective Actions from Loop Quantum Gravity, *Phys. Rev. D* 86 (2012) 104018, [arXiv:1112.1899]

[11] J. Mielczarek, Signature change in loop quantum cosmology, *Springer Proc. Phys.* 157 (2014) 555, [arXiv:1207.4657]

[12] M. Bojowald and J. D. Reyes, Dilaton Gravity, Poisson Sigma Models and Loop Quantum Gravity, *Class. Quantum Grav.* 26 (2009) 035018, [arXiv:0810.5119]

[13] M. Bojowald and S. Brahma, Effective constraint algebras with structure functions, *J. Phys. A: Math. Theor.* 49 (2016) 125301, [arXiv:1407.4444]

[14] A. Laddha and M. Varadarajan, The Diffeomorphism Constraint Operator in Loop Quantum Gravity, *Class. Quant. Grav.* 28 (2011) 195010, [arXiv:1105.0636]

[15] M. Bojowald, Spherically Symmetric Quantum Geometry: States and Basic Operators, *Class. Quantum Grav.* 21 (2004) 3733–3753, [gr-qc/0407017]

[16] M. Bojowald, G. M. Paily, and J. D. Reyes, Discreteness corrections and higher spatial derivatives in effective canonical quantum gravity, *Phys. Rev. D* 90 (2014) 025025, [arXiv:1402.5130]

[17] M. Bojowald, Loop quantum cosmology and inhomogeneities, *Gen. Rel. Grav.* 38 (2006) 1771–1795, [gr-qc/0609034]

[18] M. Bojowald, The dark side of a patchwork universe, *Gen. Rel. Grav.* 40 (2008) 639–660, [arXiv:0705.4398]

[19] M. Campiglia, R. Gambini, J. Olmedo, and J. Pullin, Quantum self-gravitating collapsing matter in a quantum geometry, *Class. Quantum Grav.* 33 (2016) 18, [arXiv:1601.05688]

[20] J. Ben Achour, S. Brahma, and A. Marciano, Spherically symmetric sector of self dual Ashtekar gravity coupled to matter: Anomaly-free algebra of constraints with holonomy corrections, [arXiv:1608.07314]

[21] J. Ben Achour, S. Brahma, J. Grain, and A. Marciano, A new look at scalar perturbations in loop quantum cosmology: (un)deformed algebra approach using self dual variables, [arXiv:1610.07467]

[22] S. Brahma, M. Ronco, G. Amelino-Camelia, and A. Marciano, Linking loop quantum gravity quantization ambiguities with phenomenology, *Phys. Rev. D* 95 (2017) 044005, [arXiv:1610.07865]

[23] M. Bojowald, S. Brahma, U. Büyükçam, and F. D'Ambrosio, Hypersurface-deformation algebroids and effective space-time models, *Phys. Rev. D* 94 (2016) 104032, [arXiv:1610.08355]