Halo squeezed-limit bispectrum with primordial non-Gaussianity: A power spectrum response approach

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Modeling the nonlinearity of the halo bispectrum remains a major challenge in modern cosmology, in particular for ongoing and upcoming large-scale structure observations that are performed to study the inflationary physics. The "power spectrum response" offers a solution for bispectrum in the so-called squeezed limit, in which one wavenumber is much smaller than the other two. As a first step, we demonstrate that the halo squeezed-limit bispectrum computed from the second-order standard perturbation theory agrees precisely with the responses of *linear* halo power spectrum to large-scale density and potential fluctuations. Since the halo power spectrum responses to arbitrarily small scales can straightforwardly be obtained by separate universe simulations, the response approach provides a novel and powerful technique for modeling the nonlinear halo squeezed-limit bispectrum.

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I. INTRODUCTION

The squeezed-limit bispectrum that quantifies the correlation between two small-scale modes and one large-scale mode captures the impact of the large-scale density environment on the small-scale power spectrum. This coupling between large- and small-scale modes is generated by nonlinear gravitational evolution, and possibly by the inflationary physics that produces local non-Gaussianity in the primordial curvature perturbation (see Ref. [1] for a recent review). Measurement of the squeezed-limit bispectrum can thus be used to test our understanding of gravity and the physics of inflation.

Traditionally the bispectrum is computed with the perturbation theory (see Ref. [2] for a review on standard perturbation theory (hereafter SPT) and Ref. [3] for a review on effective field theory), in which the matter or halo density perturbations are expanded in series of the linear Gaussian perturbation, and the n-point function can be calculated with the Wick's theorem. A novel and powerful approach to computing the squeezed-limit bispectrum is to consider how the small-scale power spectrum *responds to* the large-scale environment [4, 5]. We shall refer it as the "response" approach.

In this paper, we demonstrate that the halo squeezed-limit bispectrum with local primordial non-Gaussianity can be derived from the responses of halo power spectrum to the large-scale density and potential perturbations. Specifically, we take the SPT framework and consider that halo number density traces the underlying density and potential fluctuations, ignoring the large-scale tidal field (see Ref. [6] for a recent review). For simplicity, we assume that the large-scale halo biases are *local in Eulerian space*, but our derivation can be generalized to local Lagrangian bias model with the transformation from Lagrangian to Eulerian space (see e.g. Refs. [7, 8]). Ref. [9] has done a related work to measure how the halo power spectrum responds to a long-wavelength matter fluctuation in N-body simulations, but we shall make a clearer connection between the halo squeezed-limit bispectrum and the halo power spectrum response as well as extend to the cosmology with local primordial non-Gaussianity.

The rest of the paper is organized as follows. In Sec. II we use the SPT to expand the halo number density fluctuation in series of the underlying density and potential perturbations to the second order, and show the leading-order halo squeezed-limit bispectrum. In Sec. III we compute the responses of the linear halo power spectrum to large-scale density and potential perturbations and connect this result to the halo squeezed-limit bispectrum. We discuss the result and future applications in Sec. IV. In App. A we demonstrate the detailed derivation of the halo squeezed-limit bispectrum using the second-order SPT.

II. SECOND-ORDER STANDARD PERTURBATION THEORY

In the peak-background split picture, the long-wavelength perturbations change the local overdensity threshold for halo formation [10]. Therefore, in the large-scale limit, the halo number density is modulated by the long-wavelength perturbations and can be expanded to the second order as [7, 8, 11]

$$n_h(\mathbf{r}|\delta_l, \phi_l) = \bar{n}_h(\mathbf{r}) + \frac{\partial \bar{n}_h}{\partial \delta_l}\delta_l(\mathbf{r}) + \frac{\partial \bar{n}_h}{\partial \phi_l}\phi_l(\mathbf{r}) + \frac{2}{2!}\frac{\partial^2 \bar{n}_h}{\partial \delta_l \partial \phi_l}\delta_l(\mathbf{r})\phi_l(\mathbf{r}) + \frac{1}{2!}\frac{\partial^2 \bar{n}_h}{\partial \delta_l^2}\delta_l^2(\mathbf{r}) + \frac{1}{2!}\frac{\partial^2 \bar{n}_h}{\partial \phi_l^2}\phi_l^2(\mathbf{r}), \quad (1)$$

where $\bar{n}_h$ is the mean halo number density, and δ_l and ϕ_l are linear Gaussian density and potential fluctuations, respectively. For simplicity, in Eq. (1) we neglect $\langle \delta_l^2(\mathbf{r}) \rangle$, $\langle \delta_l(\mathbf{r})\phi_l(\mathbf{r}) \rangle$, and $\langle \phi_l^2(\mathbf{r}) \rangle$, which assure $\langle n_h(\mathbf{r}|\delta_l, \phi_l) \rangle = \bar{n}_h(\mathbf{r})$, as they only contribute to the $k = 0$ mode in Fourier space. We consider that ϕ_l is the primordial potential in the matter-dominated epoch, hence it is related to δ_l by $\delta_l(\mathbf{k}, a) = M(k, a)\phi_l(\mathbf{k})$, with the Poisson operator $M(k, a) = \frac{2}{3}\frac{D(a)}{H_0^2 \Omega_m}k^2 T(k)$. Here a is the scale factor, D is the linear growth normalized to a in the matter-dominated epoch, H_0 is the present-day Hubble, Ω_m is the present-day fractional energy density of matter, and $T(k)$ is the matter transfer function [24]. In the following, we shall omit the scale factor argument for simplicity. Defining the halo bias as

$$b_{ij} = \frac{1}{\bar{n}_h}\frac{\partial^{i+j}\bar{n}_h}{\partial \delta_l^i \partial \phi_l^j}, \quad (2)$$

we can write the halo number density fluctuation as

$$\delta_h(\mathbf{r}) = \frac{n_h(\mathbf{r}|\delta_l, \phi_l)}{\bar{n}_h} - 1 = b_{10}\delta_l(\mathbf{r}) + b_{01}\phi_l(\mathbf{r}) + b_{11}\delta_l(\mathbf{r})\phi_l(\mathbf{r}) + \frac{b_{20}}{2}\delta_l^2(\mathbf{r}) + \frac{b_{02}}{2}\phi_l^2(\mathbf{r}). \quad (3)$$

Eq. (2) indicates that the biases are the responses of the halo mass function to the large-scale density or potential fluctuations. Note that for simplicity we consider both δ_l and ϕ_l are in Eulerian space, hence the biases are the Eulerian biases. Our derivation can be generalized to the Lagrangian bias model by expanding δ_h in Lagrangian space as Eq. (3) and transforming δ_h to Eulerian space using the conservation of the number of halos (see e.g. Refs. [7, 8]).

In general, δ_h traces the underlying *nonlinear* matter density fluctuation δ_m and potential fluctuation Φ, namely

$$\delta_h(\mathbf{r}) = b_{10}\delta_m(\mathbf{r}) + b_{01}\Phi(\mathbf{r}) + b_{11}\delta_m(\mathbf{r})\Phi(\mathbf{r}) + \frac{b_{20}}{2}\delta_m^2(\mathbf{r}) + \frac{b_{02}}{2}\Phi^2(\mathbf{r}). \quad (4)$$

In the presence of the local primordial non-Gaussianity, the potential perturbation is $\Phi(\mathbf{r}) = \phi_l(\mathbf{r}) + f_{\mathrm{NL}}\phi_l^2(\mathbf{r})$ [12], where f_{NL} quantifies the amount of local non-Gaussianity, with $f_{\mathrm{NL}} = 0$ being Gaussian. Using the Poisson operator, we can compute the matter density up to the second order in Fourier space as

$$\delta_m(\mathbf{k}) = \delta_l(\mathbf{k}) + f_{\mathrm{NL}}M(k)\int \frac{d^3q}{(2\pi)^3}\, \phi_l(\mathbf{q})\phi_l(\mathbf{k}-\mathbf{q}) + \int \frac{d^3q}{(2\pi)^3}\, \delta_l(\mathbf{q})\delta_l(\mathbf{k}-\mathbf{q})F_2(\mathbf{q}, \mathbf{k}-\mathbf{q}), \quad (5)$$

where the last term of Eq. (5) is due to the nonlinear gravitational evolution with the kernel F_2 computed from SPT. Combining Eqs. (4)–(5), we have the halo number density fluctuation up to the second order in δ_l and ϕ_l as

$$\begin{aligned}
\delta_h(\mathbf{k}) &= b_{10}\left[\delta_l(\mathbf{k}) + \int \frac{d^3q}{(2\pi)^3}\, \delta_l(\mathbf{q})\delta_l(\mathbf{k}-\mathbf{q})F_2(\mathbf{q}, \mathbf{k}-\mathbf{q}) + f_{\mathrm{NL}}M(k)\int \frac{d^3q}{(2\pi)^3}\, \phi_l(\mathbf{q})\phi_l(\mathbf{k}-\mathbf{q})\right] \\
&\quad + b_{01}\left[\phi_l(\mathbf{k}) + f_{\mathrm{NL}}\int \frac{d^3q}{(2\pi)^3}\, \phi_l(\mathbf{q})\phi_l(\mathbf{k}-\mathbf{q})\right] + b_{11}\int \frac{d^3q}{(2\pi)^3}\, \delta_l(\mathbf{q})\phi_l(\mathbf{k}-\mathbf{q}) \\
&\quad + \frac{b_{20}}{2}\int \frac{d^3q}{(2\pi)^3}\, \delta_l(\mathbf{q})\delta_l(\mathbf{k}-\mathbf{q}) + \frac{b_{02}}{2}\int \frac{d^3q}{(2\pi)^3}\, \phi_l(\mathbf{q})\phi_l(\mathbf{k}-\mathbf{q}). \quad (6)
\end{aligned}$$

The halo power spectrum and bispectrum are defined as

$$\langle \delta_h(\mathbf{k})\delta_h(\mathbf{k'})\rangle = (2\pi)^3 \delta_D(\mathbf{k}+\mathbf{k'})P_{hh}(k)\,, \quad \langle \delta_h(\mathbf{k}_1)\delta_h(\mathbf{k}_2)\delta_h(\mathbf{k}_3)\rangle = (2\pi)^3 \delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_{hhh}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\,, \quad (7)$$

where δ_D is the Dirac delta function. Inserting Eq. (6) into Eq. (7) as well as using the Wick's theorem, we obtain the leading order halo power spectrum as (up to $\delta^n\phi^m$ with $n+m=2$)

$$P_{hh}(k) = b_{10}^2 P_{\delta\delta}(k) + b_{01}^2 P_{\phi\phi}(k) + 2b_{10}b_{01}P_{\delta\phi}(k)\,, \tag{8}$$

where $P_{\delta\delta}$, $P_{\phi\phi}$, and $P_{\delta\phi}$ are linear density-density, potential-potential, and density-potential power spectra, respectively. For the halo squeezed-limit bispectrum, we consider the configuration $k_1 \approx k_2 = k \gg k_3 = k_L$. The detailed derivation is given in App. A, and at the leading order we have (up to $\delta^n\phi^m$ with $n+m=4$)

$$B_{hhh}^{\text{sq}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = b_{10}f_\delta(k)P_{\delta\delta}(k_L) + b_{01}f_\phi(k)P_{\phi\phi}(k_L) + [b_{01}f_\delta(k) + b_{10}f_\phi(k)]\,P_{\delta\phi}(k_L)\,, \tag{9}$$

where

$$
\begin{aligned}
f_\delta(k) &= 2b_{10}b_{20}P_{\delta\delta}(k) + b_{10}^2\left[\frac{47}{21} - \frac{1}{3}\frac{d\ln P_{\delta\delta}}{d\ln k}\right]P_{\delta\delta}(k) + 2b_{10}b_{11}P_{\delta\phi}(k) \\
&\quad + 2b_{01}b_{20}P_{\delta\phi}(k) + b_{10}b_{01}\left[\frac{47}{21} - \frac{1}{3}\frac{d\ln P_{\delta\delta}}{d\ln k}\right]P_{\delta\phi}(k) + 2b_{01}b_{11}P_{\phi\phi}(k)\,,
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
f_\phi(k) &= 2b_{10}b_{11}P_{\delta\delta}(k) + 2b_{01}b_{11}P_{\delta\phi}(k) + 2b_{10}b_{02}P_{\delta\phi}(k) + 2b_{01}b_{02}P_{\phi\phi}(k) \\
&\quad + 4f_{\text{NL}}b_{10}^2 P_{\delta\delta}(k) + 4f_{\text{NL}}b_{01}^2 P_{\phi\phi}(k) + 8f_{\text{NL}}b_{10}b_{01}P_{\delta\phi}(k)\,.
\end{aligned}
\tag{11}
$$

Eqs (9)–(11) are the primary results in this section, i.e. the halo squeezed-limit bispectrum with primordial non-Gaussianity derived from the second-order SPT. We shall show in Sec. III that the same result can be obtained by considering how *linear* halo power spectrum responds to large-scale density and potential fluctuations.

III. LINEAR HALO POWER SPECTRUM RESPONSE

Let us now turn to the response approach. In the presence of δ_l and ϕ_l, at the leading order the halo power spectrum is modulated as

$$P_{hh}(k|\delta_l,\phi_l) = P_{hh}(k)|_{\delta_l,\phi_l=0} + \frac{\partial P_{hh}(k)}{\partial \delta_l}\bigg|_{\delta_l,\phi_l=0}\delta_l + \frac{\partial P_{hh}(k)}{\partial \phi_l}\bigg|_{\delta_l,\phi_l=0}\phi_l\,, \tag{12}$$

where $P_{hh}(k)|_{\delta_l,\phi_l=0}$ is given by Eq. (8), and $\partial P_{hh}(k)/\partial \delta_l$ and $\partial P_{hh}(k)/\partial \phi_l$ are the responses of the halo power spectrum to large-scale density and potential perturbations, respectively. Since the squeezed-limit bispectrum is essentially the coupling between the small-scale power spectrum and its large-scale environment, we compute the correlation between $P_{hh}(k|\delta_l,\phi_l)$ and the large-scale halo density fluctuation $(b_{10}\delta_l + b_{01}\phi_l)$ (as the long-wavelength mode k_3 in the squeezed-limit bispectrum calculation) and obtain

$$\langle P_{hh}(k|\delta_l,\phi_l)(b_{10}\delta_l + b_{01}\phi_l)\rangle = b_{10}\frac{\partial P_{hh}(k)}{\partial \delta_l}\langle\delta_l^2\rangle + b_{01}\frac{\partial P_{hh}(k)}{\partial \phi_l}\langle\phi_l^2\rangle + \left[b_{01}\frac{\partial P_{hh}(k)}{\partial \delta_l} + b_{10}\frac{\partial P_{hh}(k)}{\partial \phi_l}\right]\langle\delta_l\phi_l\rangle\,, \tag{13}$$

where $\langle\delta_l^2\rangle = P_{\delta\delta}(k_L)$, $\langle\phi_l^2\rangle = P_{\phi\phi}(k_L)$, and $\langle\delta_l\phi\rangle = P_{\delta\phi}(k_L)$ are the large-scale linear power spectra. Note that Eq. (13) has the same form as Eq. (9).

There are three ways that the halo power spectrum responds to δ_l and ϕ_l. The first way is through the halo bias, and it can be computed via Eq. (2) as

$$\frac{\partial b_{10}}{\partial \delta_l} = b_{20} - b_{10}^2\,, \quad \frac{\partial b_{01}}{\partial \phi_l} = b_{02} - b_{01}^2\,, \quad \frac{\partial b_{10}}{\partial \phi_l} = \frac{\partial b_{01}}{\partial \delta_l} = b_{11} - b_{10}b_{01}\,. \tag{14}$$

Combining with Eq. (14) with Eq. (8), we have

$$\left.\frac{\partial P_{hh}(k)}{\partial \delta_l}\right|_{\substack{\text{halo} \\ \text{bias}}} = 2(b_{10}b_{20} - b_{10}^3)P_{\delta\delta}(k) + 2(b_{20}b_{01} - 2b_{10}^2 b_{01} + b_{10}b_{11})P_{\delta\phi}(k) + 2(b_{01}b_{11} - b_{10}b_{01}^2)P_{\phi\phi}(k), \quad (15)$$

$$\left.\frac{\partial P_{hh}(k)}{\partial \phi_l}\right|_{\substack{\text{halo} \\ \text{bias}}} = 2(b_{10}b_{11} - b_{10}^2 b_{01})P_{\delta\delta}(k) + 2(b_{01}b_{11} - 2b_{10}b_{01}^2 + b_{10}b_{02})P_{\delta\phi}(k) + 2(b_{01}b_{02} - b_{01}^3)P_{\phi\phi}(k). \quad (16)$$

The second way is through the small-scale linear power spectra. For the response to δ_l, we consider that the effect is due to nonlinear gravitational evolution, i.e. the squeezed-limit matter bispectrum in the absence of local primordial non-Gaussianity, hence the potential power spectra do not respond to δ_l. As a result, at the leading order we have (see e.g. Refs. [4, 5])

$$\frac{\partial P_{\delta\delta}(k)}{\partial \delta_l} = \left[\frac{47}{21} - \frac{1}{3}\frac{d\ln P_{\delta\delta}}{d\ln k}\right]P_{\delta\delta}(k), \quad \frac{\partial P_{\delta\phi}(k)}{\partial \delta_l} = \frac{1}{2}\left[\frac{47}{21} - \frac{1}{3}\frac{d\ln P_{\delta\delta}}{d\ln k}\right]P_{\delta\phi}(k), \quad \frac{\partial P_{\phi\phi}(k)}{\partial \delta_l} = 0. \quad (17)$$

For the response to ϕ_l, we consider that the effect is due to the initial conditions, i.e. the primordial non-Gaussianity, hence all linear power spectra respond identically. Following Ref. [10], the effect of ϕ_l can be regarded as a change of σ_8 locally, and at the leading order we have

$$\frac{\partial P_{xy}(k)}{\partial \phi_l} = \frac{\partial P_{xy}(k)}{\partial \sigma_8}\frac{\partial \sigma_8}{\partial \phi_l} = 4f_{\text{NL}}P_{xy}(k), \quad (18)$$

where $(x, y) \in (\delta, \phi)$. Combining Eqs. (17)–(18) with Eq. (8), we have

$$\left.\frac{\partial P_{hh}(k)}{\partial \delta_l}\right|_{\substack{\text{linear} \\ \text{power} \\ \text{spectra}}} = b_{10}^2\left[\frac{47}{21} - \frac{1}{3}\frac{d\ln P_{\delta\delta}}{d\ln k}\right]P_{\delta\delta}(k) + b_{10}b_{01}\left[\frac{47}{21} - \frac{1}{3}\frac{d\ln P_{\delta\delta}}{d\ln k}\right]P_{\delta\phi}(k), \quad (19)$$

$$\left.\frac{\partial P_{hh}(k)}{\partial \phi_l}\right|_{\substack{\text{linear} \\ \text{power} \\ \text{spectra}}} = 4f_{\text{NL}}P_{hh}(k). \quad (20)$$

The final way is through the reference halo number density. As the halo power spectrum is computed referencing to the mean halo number density, in the presence of long-wavelength fluctuations the mean halo number density measured by the *global* observer is a factor of $(1 + b_{10}\delta_l + b_{01}\phi_l)$ with respect to that measured by the *local* observer, and the halo power spectrum would be rescaled by a factor of $(1 + b_{10}\delta_l + b_{01}\phi_l)^2$. At the leading order, the halo power spectrum responds as

$$\left.\frac{\partial P_{hh}(k)}{\partial \delta_l}\right|_{\substack{\text{reference} \\ \text{density}}} = 2b_{10}P_{hh}(k), \quad \left.\frac{\partial P_{hh}(k)}{\partial \phi_l}\right|_{\substack{\text{reference} \\ \text{density}}} = 2b_{01}P_{hh}(k). \quad (21)$$

Combining the three effects, the leading-order responses of halo power spectrum to δ_l and ϕ_l are

$$\frac{\partial P_{hh}(k)}{\partial \delta_l} = 2b_{10}b_{20}P_{\delta\delta}(k) + b_{10}^2\left[\frac{47}{21} - \frac{1}{3}\frac{d\ln P_{\delta\delta}}{d\ln k}\right]P_{\delta\delta}(k) + 2b_{10}b_{11}P_{\delta\phi}(k)$$

$$+ 2b_{01}b_{20}P_{\delta\phi}(k) + b_{10}b_{01}\left[\frac{47}{21} - \frac{1}{3}\frac{d\ln P_{\delta\delta}}{d\ln k}\right]P_{\delta\phi}(k) + 2b_{01}b_{11}P_{\phi\phi}(k), \quad (22)$$

$$\frac{\partial P_{hh}(k)}{\partial \phi_l} = 2b_{10}b_{11}P_{\delta\delta}(k) + 2b_{01}b_{11}P_{\delta\phi}(k) + 2b_{10}b_{02}P_{\delta\phi}(k) + 2b_{01}b_{02}P_{\phi\phi}(k)$$

$$+ 4f_{\text{NL}}b_{10}^2 P_{\delta\delta}(k) + 4f_{\text{NL}}b_{01}^2 P_{\phi\phi}(k) + 8f_{\text{NL}}b_{10}b_{01}P_{\delta\phi}(k). \quad (23)$$

Eqs (22)–(23) are the main results of this section. We find that they agree precisely with Eqs. (10)–(11), demonstrating that the halo squeezed-limit bispectrum can indeed be derived from considering how halo power spectrum responds to the long-wavelength fluctuations.

IV. DISCUSSION

Using the SPT framework and the local Eulerian bias model, we show the consistency between the halo squeezed-limit bispectrum and the responses of the halo power spectrum to large-scale density and potential fluctuations. Interestingly, for the perturbation theory one needs the second-order computation to obtain the bispectrum, but for the response approach we only have to consider how the *linear* power spectrum responds to large-scale fluctuations. Thus, it not only simplifies the computation but also provides a novel way of understanding the physics of the squeezed-limit bispectrum.

In this paper, we have demonstrated the responses using the perturbative calculation, but the responses can readily be measured from separate universe simulations to nonlinear scales. Specifically, in ΛCDM cosmology the long-wavelength density fluctuation behaves as curvature in the local universe, and one can perform N-body simulations in different density environments to study how the matter power spectrum [4, 13] and the halo mass function [9, 14, 15] are affected. On the other hand, the local primordial non-Gaussianity changes the amplitude of the local power spectrum, hence one can perform simulations with different σ_8's to study the effect on the halo mass function [16, 17].

Combining with the separate universe simulations, the response approach is powerful for exploring the observability as well as modeling the measurement of the squeezed-limit bispectrum, especially in the era of blooming ongoing and upcoming surveys. Here, we discuss two possible applications:

- Construct a new model for the squeezed-limit bispectrum that works better in the nonlinear regime. It has been shown in Ref. [5] that the responses computed using nonlinear matter power spectrum models are in better agreement with the matter squeezed-limit bispectrum measured from simulations at $z \lesssim 1$, compared to the second-order SPT. It can be extended to halos, including the response of the mass function. Note, however, that since galaxies are measured in redshift space with a preferred direction exists, the response of the large-scale tidal field [18–20] has to be taken into account.

- Predict the nonlinear squeezed-limit bispectrum formed by different observables that cannot be computed by the perturbative approach, such as the cross-correlation between the large-scale quasar overdensity and the small-scale Lyman-α forest power spectrum [21]. This is helpful for studying the constraining power on local primordial non-Gaussianity using the squeezed-limit bispectrum of cross-correlation.

Lastly, while we discuss the first-order response of the small-scale power spectrum, one can generalize the calculation to the mth-order response of the n-point function. For example, the mth-order response of the small-scale power spectrum is equivalent to the $(m + 2)$-point function with two small- and m large-scale modes [22], whereas the first-order response of the small-scale n-point function is equivalent to the $(n + 1)$-point function with n small- and one large-scale modes [23]. The response approach thus provides a novel and powerful technique to study the higher-order statistics in the squeezed configurations for the large-scale structure.

Acknowledgments

Appendix A: Halo squeezed-limit bispectrum from the second-order standard perturbation theory

In this appendix we derive explicitly the halo squeezed-limit bispectrum with primordial non-Gaussianity from the second-order SPT. Using Eq. (6) and the Wick's theorem, we obtain the leading-order bispectrum

(up to $\delta^n \phi^m$ with $n + m = 4$) in the squeezed limit such that $k_1 \approx k_2 \gg k_3$ as

$$
\lim_{k_3 \to 0} B_{hhh}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = B_{hhh}^{\mathrm{sq}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)
$$
$$
\begin{aligned}
&= b_{10}^2 b_{20}[P_{\delta\delta}(k_1) + P_{\delta\delta}(k_2)]P_{\delta\delta}(k_3) + 2b_{10}^3[P_{\delta\delta}(k_1)F_2(\mathbf{k}_1, \mathbf{k}_3) + P_{\delta\delta}(k_2)F_2(\mathbf{k}_2, \mathbf{k}_3)]P_{\delta\delta}(k_3) \\
&+ b_{10}^2 b_{11}[P_{\delta\delta}(k_1) + P_{\delta\delta}(k_2)]P_{\delta\phi}(k_3) + b_{10}^2 b_{11}[P_{\delta\phi}(k_1) + P_{\delta\phi}(k_2)]P_{\delta\delta}(k_3) \\
&+ b_{10}b_{01}b_{20}[P_{\delta\phi}(k_1) + P_{\delta\phi}(k_2)]P_{\delta\delta}(k_3) + 2b_{10}^2 b_{01}[P_{\delta\phi}(k_1)F_2(\mathbf{k}_1, \mathbf{k}_3) + P_{\delta\phi}(k_2)F_2(\mathbf{k}_2, \mathbf{k}_3)]P_{\delta\delta}(k_3) \\
&+ b_{10}b_{01}b_{11}[P_{\phi\phi}(k_1) + P_{\phi\phi}(k_2)]P_{\delta\delta}(k_3) + b_{10}b_{01}b_{11}[P_{\delta\phi}(k_1) + P_{\delta\phi}(k_2)]P_{\delta\phi}(k_3) \\
&+ b_{10}b_{01}b_{20}[P_{\delta\delta}(k_1) + P_{\delta\delta}(k_2)]P_{\delta\phi}(k_3) + 2b_{10}^2 b_{01}[P_{\delta\delta}(k_1)F_2(\mathbf{k}_1, \mathbf{k}_3) + P_{\delta\delta}(k_2)F_2(\mathbf{k}_2, \mathbf{k}_3)]P_{\delta\phi}(k_3) \\
&+ b_{10}b_{01}b_{11}[P_{\delta\delta}(k_1) + P_{\delta\delta}(k_2)]P_{\phi\phi}(k_3) + b_{10}b_{01}b_{11}[P_{\delta\phi}(k_1) + P_{\delta\phi}(k_2)]P_{\delta\phi}(k_3) \\
&+ b_{01}^2 b_{20}[P_{\delta\phi}(k_1) + P_{\delta\phi}(k_2)]P_{\delta\phi}(k_3) + 2b_{10}b_{01}^2[P_{\delta\phi}(k_1)F_2(\mathbf{k}_1, \mathbf{k}_3) + P_{\delta\phi}(k_2)F_2(\mathbf{k}_2, \mathbf{k}_3)]P_{\delta\phi}(k_3) \\
&+ b_{01}^2 b_{11}[P_{\delta\phi}(k_1) + P_{\delta\phi}(k_2)]P_{\phi\phi}(k_3) + b_{01}^2 b_{11}[P_{\phi\phi}(k_1) + P_{\phi\phi}(k_2)]P_{\delta\phi}(k_3) \\
&+ b_{10}^2 b_{02}[P_{\delta\phi}(k_1) + P_{\delta\phi}(k_2)]P_{\delta\phi}(k_3) + 2f_{\mathrm{NL}}b_{10}^2 b_{01}[P_{\delta\phi}(k_1) + P_{\delta\phi}(k_2)]P_{\delta\phi}(k_3) \\
&+ 2f_{\mathrm{NL}}b_{10}^3[M(k_2)P_{\delta\phi}(k_1) + M(k_1)P_{\delta\phi}(k_2)]P_{\delta\phi}(k_3) + b_{10}b_{01}b_{02}[P_{\phi\phi}(k_1) + P_{\phi\phi}(k_2)]P_{\delta\phi}(k_3) \\
&+ 2f_{\mathrm{NL}}b_{10}b_{01}^2[P_{\phi\phi}(k_1) + P_{\phi\phi}(k_2)]P_{\delta\phi}(k_3) + 2f_{\mathrm{NL}}b_{10}^2 b_{01}[M(k_2)P_{\phi\phi}(k_1) + M(k_1)P_{\phi\phi}(k_2)]P_{\delta\phi}(k_3) \\
&+ b_{10}b_{01}b_{02}[P_{\delta\phi}(k_1) + P_{\delta\phi}(k_2)]P_{\phi\phi}(k_3) + 2f_{\mathrm{NL}}b_{10}b_{01}^2[P_{\delta\phi}(k_1) + P_{\delta\phi}(k_2)]P_{\phi\phi}(k_3) \\
&+ 2f_{\mathrm{NL}}b_{10}^2 b_{01}[M(k_2)P_{\delta\phi}(k_1) + M(k_1)P_{\delta\phi}(k_2)]P_{\phi\phi}(k_3) + b_{01}^2 b_{02}[P_{\phi\phi}(k_1) + P_{\phi\phi}(k_2)]P_{\phi\phi}(k_3) \\
&+ 2f_{\mathrm{NL}}b_{01}^3[P_{\phi\phi}(k_1) + P_{\phi\phi}(k_2)]P_{\phi\phi}(k_3) + 2f_{\mathrm{NL}}b_{10}b_{01}^2[M(k_2)P_{\phi\phi}(k_1) + M(k_1)P_{\phi\phi}(k_2)]P_{\phi\phi}(k_3) \,, \qquad \text{(A1)}
\end{aligned}
$$

where we assume that k_3 is the long-wavelength mode and only contributes to the linear-order perturbation, i.e. $\delta_h(\mathbf{k}_3) = b_{10}\delta_l(\mathbf{k}_3) + b_{01}\phi_l(\mathbf{k}_3)$. In the exact squeezed limit where $k_1 = k_2 = k$, the products of the linear power spectra and the Poisson operator are

$$
P_{\delta\delta}(k) = M(k_1)P_{\delta\phi}(k_2) = M(k_2)P_{\delta\phi}(k_1) \,, \quad P_{\delta\phi}(k) = M(k_1)P_{\phi\phi}(k_2) = M(k_2)P_{\phi\phi}(k_1) \,. \qquad \text{(A2)}
$$

Furthermore, taking the squeezed limit such that $k_3 = k_L \to 0$ as well as angle-averaging the large-scale mode $\mathbf{k}_3$, the combination of F_2 and the small-scale power spectra at the leading order is given by (see e.g. Ref. [5])

$$
2[P_{xy}(k_1)F_2(\mathbf{k}_1, \mathbf{k}_3) + P_{xy}(k_2)F_2(\mathbf{k}_2, \mathbf{k}_3)] = \left[\frac{47}{21} - \frac{1}{3}\frac{d\ln P_{\delta\delta}}{d\ln k} \right] P_{xy}(k) \,, \qquad \text{(A3)}
$$

where $(x,y) \in (\delta, \phi)$. Combining the above equations, we can simplify the halo squeezed-limit bispectrum with primordial non-Gaussianity as

$$
\begin{aligned}
& B_{hhh}^{\mathrm{sq}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \\
& = \left\{ 2b_{10}^2 b_{20} P_{\delta\delta}(k) + b_{10}^3 \left[\frac{47}{21} - \frac{1}{3} \frac{d \ln P_{\delta\delta}}{d \ln k} \right] P_{\delta\delta}(k) + 2b_{10}^2 b_{11} P_{\delta\phi}(k) \right. \\
& \quad \left. + 2b_{10}b_{01}b_{20} P_{\delta\phi}(k) + b_{10}^2 b_{01} \left[\frac{47}{21} - \frac{1}{3} \frac{d \ln P_{\delta\delta}}{d \ln k} \right] P_{\delta\phi}(k) + 2b_{10}b_{01}b_{11} P_{\phi\phi}(k) \right\} P_{\delta\delta}(k_L) \\
& \quad + \left\{ 2b_{10}b_{01}b_{20} P_{\delta\delta}(k) + b_{10}^2 b_{01} \left[\frac{47}{21} - \frac{1}{3} \frac{d \ln P_{\delta\delta}}{d \ln k} \right] P_{\delta\delta}(k) + 2b_{10}b_{01}b_{11} P_{\delta\phi}(k) + 2b_{01}^2 b_{20} P_{\delta\phi}(k) \right. \\
& \quad + b_{10}b_{01}^2 \left[\frac{47}{21} - \frac{1}{3} \frac{d \ln P_{\delta\delta}}{d \ln k} \right] P_{\delta\phi}(k) + 2b_{01}^2 b_{11} P_{\phi\phi}(k) + 2b_{10}^2 b_{11} P_{\delta\delta}(k) + 2b_{10}b_{01}b_{11} P_{\delta\phi}(k) + 2b_{10}^2 b_{02} P_{\delta\phi}(k) \\
& \quad \left. + 2b_{10}b_{01}b_{02} P_{\phi\phi}(k) + 4f_{\mathrm{NL}} b_{10}^3 P_{\delta\delta}(k) + 4f_{\mathrm{NL}} b_{10}b_{01}^2 P_{\phi\phi}(k) + 8f_{\mathrm{NL}} b_{10}^2 b_{01} P_{\delta\phi}(k) \right\} P_{\delta\phi}(k_L) \\
& \quad + \left\{ 2b_{10}b_{01}b_{11} P_{\delta\delta}(k) + 2b_{01}^2 b_{11} P_{\delta\phi}(k) + 2b_{10}b_{01}b_{02} P_{\delta\phi}(k) + 2b_{01}^2 b_{02} P_{\phi\phi}(k) \right. \\
& \quad \left. + 4f_{\mathrm{NL}} b_{10}^2 b_{01} P_{\delta\delta}(k) + 4f_{\mathrm{NL}} b_{01}^3 P_{\phi\phi}(k) + 8f_{\mathrm{NL}} b_{10}b_{01}^2 P_{\delta\phi}(k) \right\} P_{\phi\phi}(k_L) \,.
\end{aligned}
\tag{A4}
$$

In order to better compare with the linear halo power spectrum response, it is useful to rewrite Eq. (A4) as

$$
B_{hhh}^{\mathrm{sq}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = b_{10} f_\delta(k) P_{\delta\delta}(k_L) + b_{01} f_\phi(k) P_{\phi\phi}(k_L) + \left[b_{01} f_\delta(k) + b_{10} f_\phi(k) \right] P_{\delta\phi}(k_L) \,,
\tag{A5}
$$

where

$$
\begin{aligned}
f_\delta(k) & = 2b_{10}b_{20} P_{\delta\delta}(k) + b_{10}^2 \left[\frac{47}{21} - \frac{1}{3} \frac{d \ln P_{\delta\delta}}{d \ln k} \right] P_{\delta\delta}(k) + 2b_{10}b_{11} P_{\delta\phi}(k) \\
& \quad + 2b_{01}b_{20} P_{\delta\phi}(k) + b_{10}b_{01} \left[\frac{47}{21} - \frac{1}{3} \frac{d \ln P_{\delta\delta}}{d \ln k} \right] P_{\delta\phi}(k) + 2b_{01}b_{11} P_{\phi\phi}(k) \,,
\end{aligned}
\tag{A6}
$$

$$
\begin{aligned}
f_\phi(k) & = 2b_{10}b_{11} P_{\delta\delta}(k) + 2b_{01}b_{11} P_{\delta\phi}(k) + 2b_{10}b_{02} P_{\delta\phi}(k) + 2b_{01}b_{02} P_{\phi\phi}(k) \\
& \quad + 4f_{\mathrm{NL}} b_{10}^2 P_{\delta\delta}(k) + 4f_{\mathrm{NL}} b_{01}^2 P_{\phi\phi}(k) + 8f_{\mathrm{NL}} b_{10}b_{01} P_{\delta\phi}(k) \,.
\end{aligned}
\tag{A7}
$$

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