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Action functional of Cardassian universe

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It is known that Cardassian universe is successful in describing the accelerated expasnsion of the universe but its dynamical equations is hard to get from the action principle. In this paper, we establish the connection between Cardassian universe and $f(T, \mathcal{T})$ gravity where T is the torsion scalar and $\mathcal{T}$ is the trace of the matter energy-momentum tensor. For dust matter, we find the modified Friedmann equations from $f(T, \mathcal{T})$ gravity can correspond to those of Cardassian models and thus a possible origin of Cardassian universe is given. We obtain the original Cardassian model, the modified polytropic Cardassian model and the exponential Cardassian model from the Lagrangians of $f(T, \mathcal{T})$ theory. Furthermore, we give generalized Cardassian models from $f(T, \mathcal{T})$ theory by adding an additional term to the Lagrangians of $f(T, \mathcal{T})$ theory that give the three Cardassian models. Using the observation data of type Ia supernovae, cosmic microwave background radiation and baryon acoustic oscillations, we get the fitting results of the cosmological parameters and give constraints of model parameters for all these models.

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I. INTRODUCTION

Cardassian universe [1–4] has been known to describe the accelerating expansion of the universe with remarkable agreement with observations, whereas it lacks a solid theoretical foundation up to now. In Cardassian models, the Friedmann equation is modified by the introduction of an additional nonlinear term of energy density while without the introduction of cosmological constant or any dynamical dark energy component. In these models, the universe can be flat and yet consist of only matter and radiation, and still be compatible with observations. Matter can be sufficient to provide a flat geometry. The possible origin for Cardassian models is from the consideration of braneworld scenarios, where our observable universe is a three dimensional membrane embedded in extra dimensions[5]. The modified Friedmann equation may result from the existence of extra dimensions. But it is difficult to find a simple higher dimensional theory, i. e., a higher dimensional momentum tensor that produces the Cardassian cosmology[6]. Inspired by the study on correspondence between thermodynamical behavior and gravitational equations, two of us studied the thermodynamic origin of the Cardassian universe[7]. However, it is still hard to get the dynamical equations of this model from the action principle.

To explain the accelerated expansion of the universe, besides adding Cardassian term or unknown fields such as quintessence [8, 9] and phantom [10, 11], there is another kind of theories known as modified gravity, which uses alternative gravity theory instead of Einstein theory, such as $f(R)$ theory [12, 13], MOND cosmology [14], Poincaré gauge theory [15–17], and de Sitter gauge theory [18]. On the other hand, Einstein constructed the "Teleparallel Equivalence of General Relativity" (TEGR) which is equivalent to General Relativity (GR) from the Einstein-Hilbert action[19–23]. In TEGR, the curvatureless Weitzenböck connection takes the place of torsionless Levi-Civita one, and the vierbein is used as the fundamental field instead of the metric. In the Lagrangian of TEGR, the torsion scalar T by contractions of the torsion tensor takes the place of curvature scalar R. The simplest approach in TEGR to modify gravity is $f(T)$ theory [24, 25], whose important advantage is that the field equations are second order but not fourth order as in $f(R)$ theory. Recently, we established two concrete $f(T)$ models which do not change the successful aspects of the ΛCDM scenario under the error band of fitting values as describing the evolution history of the universe including the radiation-dominated era, the matter-dominated era and the present accelerating expansion [26]. We also considered the spherical collapse and virialization in $f(T)$ gravities[27]. Furthermore, extensions of $f(T, \mathcal{T})$ theory[28] where $\mathcal{T}$ is the trace of the matter energy-momentum tensor $\mathcal{T}_{\mu\nu}$ were constructed, whose cosmological implications are rich and varied.

Recently, it is shown[29] that modified gravity models may lead to a Cardassian-like expansion. In this paper, we try to find the relation between Cardassian models and $f(T, \mathcal{T})$ theory. Under the re-considered scheme of $f(T, \mathcal{T})$ theory, we obtain the original Cardassian model, the modified polytropic Cardassian model and the exponential Cardassian model through the action principle and thus give a possible origin of Cardassian universe. Furthermore, we give the generalized Cardassian models by adding an additional term to the Lagrangians of $f(T, \mathcal{T})$ theory that give rise to the three Cardassian models. Using the observation data of type Ia supernovae(SNeIa), cosmic microwave background radiation(CMB) and baryon acoustic oscillations(BAO), we

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get the fitting results of the cosmological parameters and give constraints of model parameters for all these models.

The paper is organized as follows: in section II, with the discussion of the self-consistent form of the Lagrangian of barotropic perfect fluid, we give a new derivation of $f(T,\mathcal{T})$ theory. In section III, the actions of the three Cardassian models from $f(T,\mathcal{T})$ theory are given explicitly and generalized Cardassian models from $f(T,\mathcal{T})$ theory are further given. We also examine the observational constraints of each model in this section. Finally, section IV is devoted to the conclusion and discussion. We use the signature convention (+,-,-,-) in this paper.

II. $f(T,\mathcal{T})$ THEORY WITH BAROTROPIC PERFECT FLUID

A. The Lagrangian of barotropic perfect fluid

There exist two types of Lagrangian $\mathcal{L}_m$ for the perfect fluid in modified gravity theories, so we have to define one or the other of these two. Harko has pointed out that $\mathcal{L}_m = \epsilon(\rho)$ is a more reasonable choice [38] in modified gravity theories, where $\epsilon(\rho)$ is the total energy density of the fluid and ρ is the rest mass density.

In the work of Brown[30], it is shown that the on-shell perfect fluid Lagrangian in GR can be $\mathcal{L}_m = \rho$ or $\mathcal{L}_m = -p$ where ρ is the rest mass density and p is the pressure. Both Lagrangians lead to the same perfect fluid stress-energy tensor concordant with the laws of thermodynamics and hence the same equations of motion. In the past years, some authors adopted some specific form of $\mathcal{L}_m = -p$ from the work of Brown to their alternative theories of gravity [29, 31–36]. However, according to Refs. [37, 38], we have to reconsider how to take the form of $\mathcal{L}_m$ for the perfect fluid in modified gravity theories including $f(T,\mathcal{T})$ theory.

The usual form of the stress-tensor of a barotropic perfect fluid is

$$\mathcal{T}^{\mu\nu} = -[\epsilon(\rho) + p(\rho)]u^\mu u^\nu + p(\rho)g^{\mu\nu} \quad (1)$$

where $\epsilon(\rho)$ and $p(\rho)$ are the total energy density and the pressure of the fluid, respectively, which both depend on the rest mass density ρ. On the other hand, if the Lagrangian of a barotropic perfect fluid $\mathcal{L}_m$ does not depend on the derivatives of the metric, the usual definition of the stress-energy tensor $\mathcal{T}^{\mu\nu}$

$$\mathcal{T}^{\mu\nu} = -\mathcal{L}_m g_{\mu\nu} + 2\frac{\partial \mathcal{L}_m}{\partial g^{\mu\nu}} \quad (2)$$

where $\mathcal{L}_m$ can be assumed to depend on ρ only. Considering the conservation of the matter current $\nabla_\sigma(\rho u^\sigma) = 0$, one can prove that[38, 39]

$$\delta\rho = \frac{1}{2}\rho(g_{\mu\nu} - u_\mu y_\nu)\delta g^{\mu\nu} \quad (3)$$

where the four-velocity of the fluid u^α satisfies the conditions $u^\alpha u_\alpha = 1$. Substituting these results into Eq.(2), one can obtain[38, 40]

$$\mathcal{T}^{\mu\nu} = -\rho\frac{d\mathcal{L}_m}{d\rho}u^\mu u^\nu - \left(\mathcal{L}_m - \rho\frac{d\mathcal{L}_m}{d\rho}\right)g^{\mu\nu}. \quad (4)$$

From a comparison of Eqs.(1) and (4), we have

$$\mathcal{L}_m = \epsilon(\rho) = \rho[c^2 + \int p(\rho)/\rho^2 d\rho]. \quad (5)$$

and

$$\frac{d\epsilon(\rho)}{d\rho} = \frac{\epsilon(\rho) + p(\rho)}{\rho}, \quad (6)$$

where c is the speed of light and the unit $c = 1$ is taken hereinafter. In other words, $\mathcal{L}_m = \epsilon(\rho)$ is a direct and reasonable generalization from $\mathcal{L}_m = \rho$ in GR to $f(T,\mathcal{T})$ theory because Brown's argument becomes invalid in modified gravity theories. When compared with it, $\mathcal{L}_m = -p$ is only a direct employment from GR.

Furthermore, we can verify the conservation of the total energy . Actually, one can easily obtain the divergence of the energy density current

$$\nabla_\sigma(\epsilon u^\sigma) = \left(1 + \int \frac{p}{\rho^2}d\rho + \frac{p}{\rho}\right)\nabla_\sigma(\rho u^\sigma) - p\nabla_\sigma u^\sigma. \quad (7)$$

Under the conservation of matter current $\nabla_\sigma(\rho u^\sigma) = 0$, Eq.(7) is the conservation of the total energy. For example, under Friedmann-Walker-Robertson (FRW) metric it becomes

$$\dot\epsilon + 3H(\epsilon + p) = 0, \quad (8)$$

which is the usual form of energy conservation in cosmology.

B. The field equations in $f(T,\mathcal{T})$ Theory

We can find a set of smooth basis vector fields $\hat{e}_{(\mu)}$ in different patches of the manifold $\mathcal{M}$ and make sure things are well-behaved on the overlaps as usual, where Greek indices run over the coordinate of spacetime. The set of vectors $\mathbf{e}_A$ comprising an orthonormal basis is known as tetrad or vierbein, where Latin indices run over the tangent space T_p at each point p in $\mathcal{M}$. A natural basis of T_p is given by $\hat{e}_{(A)} = \partial/\partial x^A$. Any vector can be expressed as linear combinations of basis vector, so we have

$$\hat{e}_{(A)} = e_A{}^\mu \hat{e}_{(\mu)} \quad (9)$$

where the components $e_A{}^\mu$ form a 4×4 invertible matrix. We will also refer to $e_A{}^\mu$ as the vierbein in accordance with usual practice of blurring the distinction between objects and their components. The vectors $\hat{e}_{(\mu)}$ in terms of $\hat{e}_{(A)}$ are

$$\hat{e}_{(\mu)} = e^A{}_\mu \hat{e}_{(A)} \tag{10}$$

where the inverse vierbeins $e^A{}_\mu$ satisfy

$$e^A{}_\mu e_B{}^\mu = \delta^A_B, e_A{}^\mu e^A{}_\nu = \delta^\mu_\nu. \tag{11}$$

Therefore, the metric is obtained from $e^A{}_\mu$

$$g_{\mu\nu} = \eta_{AB} e^A{}_\mu e^B{}_\nu, \tag{12}$$

or equivalently

$$\eta_{AB} = g_{\mu\nu} e_A{}^\mu e_B{}^\nu, \tag{13}$$

and the root of the metric determinant is given by $|e| = \sqrt{-g} = \det(e^A{}_\mu)$.

In TEGR, one uses the standard Weitzenböck's connection defined as

$$\Gamma^\alpha_{\mu\nu} = e_A{}^\alpha \partial_\nu e^A{}_\mu = -e^A{}_\mu \partial_\nu e_A{}^\alpha. \tag{14}$$

And the covariant derivative D_μ satisfies the equation

$$\mathrm{D}_\mu e^A{}_\nu = \partial_\mu e^A{}_\nu - \Gamma^\alpha_{\nu\mu} e^A{}_\alpha = 0. \tag{15}$$

Then the components of the torsion and contorsion tensors are given by

$$\begin{aligned} T^\alpha{}_{\mu\nu} &= \Gamma^\alpha_{\nu\mu} - \Gamma^\alpha_{\mu\nu} = e_A{}^\alpha(\partial_\mu e^A{}_\nu - \partial_\nu e^A{}_\mu), && (16)\\ K^{\mu\nu}{}_\alpha &= -\frac{1}{2}(T^{\mu\nu}{}_\alpha - T^{\nu\mu}{}_\alpha - T_\alpha{}^{\mu\nu}). && (17) \end{aligned}$$

By introducing another tensor

$$S_\alpha{}^{\mu\nu} = \frac{1}{2}(K^{\mu\nu}{}_\alpha + \delta^\mu_\alpha T^{\beta\nu}{}_\beta - \delta^\nu_\alpha T^{\beta\mu}{}_\beta), \tag{18}$$

we can define the torsion scalar as

$$T \equiv T^\alpha{}_{\mu\nu} S_\alpha{}^{\mu\nu}. \tag{19}$$

The action for $f(T, \mathcal{T})$ gravity takes the following form [28]

$$S = \frac{1}{16\pi G}\int e f(T, \mathcal{T}) d^4x + \int e\mathcal{L}_m d^4x, \tag{20}$$

where $f(T, \mathcal{T})$ is an arbitrary function of the torsion scalar T and the trace $\mathcal{T}$ of the matter tress-energy tensor. On the variation with respect to the vierbein that leads to the field equations, a question that should be noted is how to deal with the variation of the trace of the energy-momentum tensor $\delta\mathcal{T}$. This question has been met in theories with $\mathcal{T}$ included in the action, including $f(R, \mathcal{T})$ theory [29, 31] and $f(T, \mathcal{T})$ theory [28]. With the discussion in last subsection, we can reexamine this question now.

From Eqs. (3) and (6), the variation of ϵ is

$$\delta\epsilon = -\frac{1}{2}(\epsilon + p)(g^{\alpha\beta} - u^\alpha u^\beta)\delta g_{\alpha\beta}. \tag{21}$$

Using (1), (6) and (21), one can express the variation of $\mathcal{T}$ as

$$\begin{aligned} \delta\mathcal{T} &= \delta(3p - \epsilon) \\ &= (3\frac{dp}{d\rho}\frac{\rho}{\epsilon + p} - 1)\delta\epsilon \\ &= (1 - 3\frac{dp}{d\rho}\frac{\rho}{\epsilon + p})(\mathcal{T}^\alpha_\beta + \epsilon\delta^\alpha_\beta)e^\beta_A \delta e^A_\alpha. && (22) \end{aligned}$$

The field equations then read as

$$\begin{aligned} & f e_A{}^\alpha + \frac{4}{e} f_T \partial_\beta(e S_\sigma{}^{\alpha\beta} e_A{}^\sigma) + 4 S_\sigma{}^{\alpha\beta} e_A{}^\sigma \partial_\beta f_T \\ &+ 4 f_T S_\rho{}^{\alpha\sigma} T^\rho{}_{\sigma\beta} e_A{}^\beta + f_\mathcal{T}(1 - 3\frac{dp}{d\rho}\frac{\rho}{\epsilon + p})\epsilon e_A{}^\alpha \\ &= \left(f_\mathcal{T}(3\frac{dp}{d\rho}\frac{\rho}{\epsilon + p} - 1) + 16\pi G\right)\mathcal{T}^\alpha{}_\beta e_A{}^\beta && (23) \end{aligned}$$

where f_T and $f_\mathcal{T}$ denote derivatives with respect to torsion scalar T and the trace of $T^{\mu\nu}$, respectively.

Contrast to $f(T, \mathcal{T})$ theory in previous papers[28, 41–43], this is the new derivation of $f(T, \mathcal{T})$ theory with $\delta\mathcal{T}$ re-considered since we have taken $\mathcal{L}_m = \epsilon(\rho)$ but not $\mathcal{L}_m = -p$. The crucial difference lies in the different choice of the matter Lagrangian $\mathcal{L}_m$. The derivation of the field equations in the references mentioned above depends on the assumption that $\mathcal{L}_m = -p$. And the same assumption is used in works on $f(R, \mathcal{T})$ gravity (see [31]). However, from the discussion in Sec. II.A and also in Refs. [38, 40], $\mathcal{L}_m = \epsilon(\rho)$ would be a more reasonable choice. This is what leads to the difference between the field equations (23) we got and the ones in the literature.

Since $f(T)$ theories are known to violate local Lorentz invariance[44, 45], particular choices of tetrad are important to get viable models in $f(T)$ cosmology, as has been noticed in Ref. [46]. For a flat FRW metric in Cartesian coordinates,

$$ds^2 = dt^2 - a(t)^2(dx^i)^2 \tag{24}$$

where $a(t)$ is the scale factor, the diagonal tetrad $e^A{}_\mu = \mathrm{diag}(1, a, a, a)$ is a good choice to get viable models[46]. The torsion scalar $T = -6H^2$, where $H = \dot{a}/a$ is the Hubble parameter. Then the equations of motion (23) give rise to the modified Friedmann equations

$$f_T H^2 = -\frac{4}{3}\pi G\epsilon - \frac{1}{12}f \tag{25}$$

and

$$4\dot{H} f_T = \left[f_\mathcal{T}\left(3\frac{\partial p}{\partial\epsilon} - 1\right) + 16\pi G\right](\epsilon + p) - 3H\dot{f}_T, \tag{26}$$

which are consequently different from those in previous references for $f(T, \mathcal{T})$ theory. It is easy to confirm the energy conservation (8) from Eqs. (25) and (26).

III. CARDASSIAN UNIVERSE FROM $f(T,\mathcal{T})$ THEORY

A. The action of Cardassian models from $f(T,\mathcal{T})$ theory

In Ref. [26], we studied the cosmology of gravity with the Lagrangian in the forms of $\mathcal{L} \propto -T + \alpha\sqrt{-T} + f(T,\mathcal{L}_m)$ and $\mathcal{L} \propto -T + \beta T^{-1} + f(T,\mathcal{L}_m)$. In the first form, the square root term is easy to be proved as null so α is actually a free parameter, and hence the correction of this term will not affect the local gravity tests. Similar to Ref. [26], here we choose

$$f(T,\mathcal{T}) = -T + \alpha\sqrt{-T} + g(\mathcal{T}). \tag{27}$$

For dust matter, the pressure $p = 0$, then from (5) we have $\epsilon(\rho) = \rho$, and Eq. (25) reduces to

$$H^2 = \frac{8\pi G}{3}\rho + \frac{1}{6}g(\mathcal{T}). \tag{28}$$

where $\mathcal{T} = -\rho$ for dust matter, and $\rho \propto a^{-3}$ from Eq.(8). It is obvious that Eq. (28) is the very equation for Cardassian models and it is easy to find the forms of $f(T,\mathcal{T})$ corresponding to specific Cardassian models. Here, we examine three Cardassian models. The units $8\pi G = 1$ is used hereinafter. For the original Cardassian model (OC)[1]

$$H^2 = \frac{\rho}{3}\left[1 + (\frac{\rho}{\rho_c})^{n-1}\right] \tag{29}$$

where ρ_c is the critical energy density at which the two terms of Eq.(29) are equal, we have

$$g(\mathcal{T}) = 2\rho(\frac{\rho}{\rho_c})^{n-1} = \frac{2}{\rho_c^{n-1}}\left(-\mathcal{T}\right)^n; \tag{30}$$

For the modified polytropic Cardassian model (MPC)[6]

$$H^2 = \frac{\rho}{3}\left[1 + (\frac{\rho}{\rho_c})^{q(n-1)}\right]^{1/q}, \tag{31}$$

we have

$$\begin{aligned} g(\mathcal{T}) =& 2\rho\Big[\left[1 + (\frac{\rho}{\rho_c})^{q(n-1)}\right]^{1/q} - 1\Big] \\ =& 2\mathcal{T}\Big[1 - \left[1 + (\frac{\mathcal{T}}{\rho_c})^{q(n-1)}\right]^{1/q}\Big]; \end{aligned} \tag{32}$$

And for the exponential Cardassian model (EC)[47]

$$H^2 = \frac{\rho}{3}\exp\left[(\frac{\rho}{\rho_c})^{-n}\right], \tag{33}$$

we have

$$\begin{aligned} g(\mathcal{T}) =& 2\rho\Big[\exp\left[(\frac{\rho}{\rho_c})^{-n}\right] - 1\Big] \\ =& 2\mathcal{T}\Big[1 - \exp\left[(\frac{-\mathcal{T}}{\rho_c})^{-n}\right]\Big]. \end{aligned} \tag{34}$$

Therefore, we claim that we find the possible origin of Cardassian models from $f(T,\mathcal{T})$ theory.

B. $f(T,\mathcal{T})$-generalized Cardassian models

Alternatively, inspired by the Lagrangian with term βT^{-1} considered in Ref. [26], if we replace the $\alpha\sqrt{-T}$ term in Eq.(27) with

$$-\frac{3\lambda^2 H_0^4}{T}, \tag{35}$$

we can obtain the $f(T,\mathcal{T})$-generalized Cardassian models. For generalized OC (Model I), the modified FRW equation reads

$$E^2 - \frac{\lambda^2}{4}E^{-2} = \Omega_0(1+z)^3 + \Omega_x(1+z)^{3n}. \tag{36}$$

Here $E(z) = \frac{H(z)}{H_0}$, H_0 is the Hubble parameter, $\Omega_0 \equiv \frac{\rho_0}{3H_0^2} = \Omega_{m0} + \Omega_{b0}$ where Ω_{m0} and Ω_{b0} correspond to dark matter and baryon respectively, and

$$\Omega_x = 1 - \frac{\lambda^2}{4} - \Omega_0. \tag{37}$$

For generalized MPC (Model II), the modified FRW equation reads

$$\begin{aligned} & E^2 - \frac{\lambda^2}{4}E^{-2} \\ =& \left\{\Omega_0^q(1+z)^{3q} + \Big[(\Omega_x + \Omega_0)^q - \Omega_0^q\Big](1+z)^{3qn}\right\}^{1/q} \end{aligned} \tag{38}$$

And for generalized EC (Model III), the modified FRW equation reads

$$\begin{aligned} & E^2 - \frac{\lambda^2}{4}E^{-2} \\ =& \Omega_0(1+z)^3\exp\left[(1+z)^{-3n}\ln\left(\frac{\Omega_x + \Omega_0}{\Omega_0}\right)\right]. \end{aligned} \tag{39}$$

In all the cases, the modified FRW equations can be expressed unifiably as

$$E^2 = \frac{1}{2}\left[\phi(z) + \sqrt{\phi^2(z) + \lambda^2}\right] \tag{40}$$

where $\phi(z)$ is the right hand side of Eqs.(36), (38) or (39).

C. Observational Constraints

In this subsection, using the observational data of SNeIa, CMB and BAO, we give constraints and the best fit parameters of each model. For SNeIa data, we use the joint light-curve analysis" (JLA) sample, which contains 740 spectroscopically confirmed type Ia supernovae with high quality light curves. The distance estimator in this analysis assumes hat supernovae with identical color, shape and galactic environment have on average the same intrinsic luminosity for all redshifts. This hypothesis is quantified by a linear model, yielding a standardized distance modulus[48, 49]

$$\mu_{\rm obs} = m_{\rm B} - \left(M_{\rm B} - A \cdot s + B \cdot C + P \cdot \Delta_M \right) , \qquad (41)$$

where $m_{\rm B}$ is the observed peak magnitude in rest-frame B band, $M_{\rm B}, s, C$ are the absolute magnitude, stretch and color measures, which are specific to the light-curve fitter employed, and $P(M_* > 10^{10} M_\odot)$ is the probability that the supernova occurred in a high-stellar-mass host galaxy. The the stretch, color, and host-mass coefficients (A, B, Δ_M, respectively) are nuisance parameters that should be constrained along with other cosmological parameters.

The CMB temperature power spectrum is sensitive to the matter density, and also it measures precisely the angular diameter distance θ_* at the last-scattering surface. We use the Planck measurement of the CMB temperature fluctuations and the WMAP measurement of the large-scale fluctuations of the CMB polarization. This CMB data are often denoted by "Planck + WP". The geometrical constraints inferred from this data set are the present value of baryon density $\Omega_{b0}h^2$, dark matter $\Omega_{m0}h^2$ and θ_* [26], where h is given by $H_0 = 100h$ km s^{-1} Mpc^{-1}.

The BAO measurement provides a standard ruler to probe the angular dimeter distance versus redshift relation by performing a spherical average of their scale measurement, see, Ref. [50]. We use the measurement of the BAO scale from Ref. [51–53].

In Table.I, we present the best-fit parameters by using the data of CMB+BAO+JLA, and also quote their 1σ bounds from the approximate Fisher Information Matrix. We also examine the constraints on parameters from 1σ to 3σ confidence level for each model and Fig. 1-3 are the illustrations of the constraints on Ω_{m0} and n for Models I, II and III, respectively.

Parameters	Cosmological Models						
	OC	Model I	MPC	Model II	EC	Model III	ΛCDM
Ω_{m0}	$0.255^{+0.009}_{-0.010}$	$0.255^{+0.010}_{-0.010}$	$0.256^{+0.011}_{-0.009}$	$0.256^{+0.011}_{-0.011}$	$0.254^{+0.010}_{-0.010}$	$0.251^{+0.010}_{-0.009}$	$0.257^{+0.009}_{-0.009}$
n	$-0.022^{+0.052}_{-0.054}$	$-0.014^{+0.062}_{-0.055}$	$0.166^{+0.088}_{-0.098}$	$0.377^{+0.102}_{-0.123}$	$0.720^{+0.039}_{-0.035}$	$0.639^{+0.073}_{-0.079}$	–
q	–	–	$1.387^{+0.257}_{-0.222}$	$1.768^{+0.449}_{-0.397}$	–	–	–
λ	–	$0.283^{+0.242}_{-0.246}$	–	$0.915^{+0.278}_{-0.345}$	–	$1.237^{+0.151}_{-0.208}$	–
H_0	$68.46^{+1.232}_{-1.197}$	$68.46^{+1.213}_{-1.239}$	$68.55^{+1.227}_{-1.318}$	$68.55^{+1.379}_{-1.285}$	$68.02^{+1.320}_{-1.241}$	$68.77^{+1.299}_{-1.389}$	$67.98^{+0.736}_{-0.737}$
$\Omega_{b0}h^2$	0.0221 ± 0.0003	0.0221 ± 0.0003	0.0220 ± 0.0003	0.0220 ± 0.0003	0.0222 ± 0.0003	0.0221 ± 0.0003	0.0221 ± 0.0002
A	$0.141^{+0.007}_{-0.006}$	$0.141^{+0.007}_{-0.006}$	$0.140^{+0.006}_{-0.007}$	$0.141^{+0.007}_{-0.007}$	$0.142^{+0.006}_{-0.007}$	$0.142^{0.007}_{-0.006}$	$0.141^{+0.007}_{-0.006}$
B	$3.103^{+0.083}_{-0.079}$	$3.103^{+0.088}_{-0.085}$	$3.101^{+0.082}_{-0.087}$	$3.101^{+0.078}_{-0.085}$	$3.112^{+0.079}_{-0.079}$	$3.112^{+0.086}_{-0.083}$	$3.100^{+0.082}_{-0.086}$
M_B	$-19.10^{+0.031}_{-0.031}$	$-19.10^{+0.031}_{-0.032}$	$-19.09^{+0.032}_{-0.032}$	$-19.09^{+0.038}_{-0.035}$	$-19.14^{+0.031}_{-0.033}$	$-19.11^{+0.035}_{-0.036}$	$-19.11^{+0.026}_{-0.026}$
ΔM	$-0.070^{+0.022}_{-0.025}$	$-0.070^{+0.022}_{-0.024}$	$-0.070^{+0.022}_{-0.021}$	$-0.070^{+0.023}_{-0.022}$	$-0.069^{+0.022}_{-0.022}$	$-0.069^{+0.023}_{-0.021}$	$-0.070^{+0.023}_{0.023}$
$\chi^2_{min}/d.o.f$	683.908/738	683.907/737	683.616/737	683.590/736	688.767/738	685.693/737	684.131/739

TABLE I: Best fitting parameters for all the models.

IV. CONCLUSION AND DISCUSSION

Using the result of the Lagrangian of a barotropic fluid given in Ref.[40], we re-derive $f(T, \mathcal{T})$ gravity, obtaining the modified Friedmann equations. We find the connection between $f(T, \mathcal{T})$ gravity and Cardassin universe. For dust matter, the modified Friedmann equations from $f(T, \mathcal{T})$ theory can correspond to those of Cardassian models and thus a possible origin of Cardassian universe is given. We present the Lagrangians of the original Cardassian model, the modified polytropic Cardassian model and the exponential Cardassian model from $f(T, \mathcal{T})$ theory. Furthermore, we get generalized Cardassian models by adding an additional term to the Lagrangians of $f(T, \mathcal{T})$ theory that give the three Cardassian models. Using the data of CMB+BAO+JLA, we get the fitting results of the cosmological parameters and give constraints of model parameters for all these models.

As one of the candidates for explaining the acceleration of the universe, Cardassian models have advantages in that the universe can be flat and yet consist of only matter and radiation satisfying the conservation laws. But there is not a satisfactory answer in the literature for the origin of the Cardassian models. In our new derivation of $f(T, \mathcal{T})$ theory, the usual energy conservation still holds, which is necessary for Cardassian models. The conclusion that we have given a possible origin of the Cardassian universe from $f(T, \mathcal{T})$ gravity is thus consistent. The connection we have found between the two theories is interesting and will be good in seeking the explanation of the accelerated expansion of the universe.

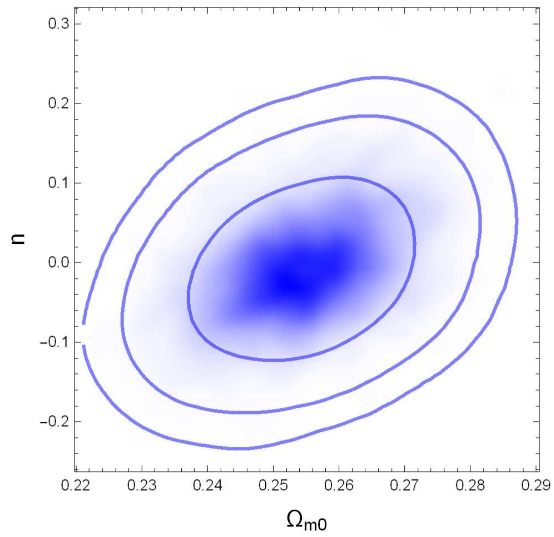

FIG. 1: Constraints on Ω_{m0} and n from 1σ to 3σ confidence level by using JLA SNe Ia + CMB + BAO for model I, while other parameters take their best fitting values.

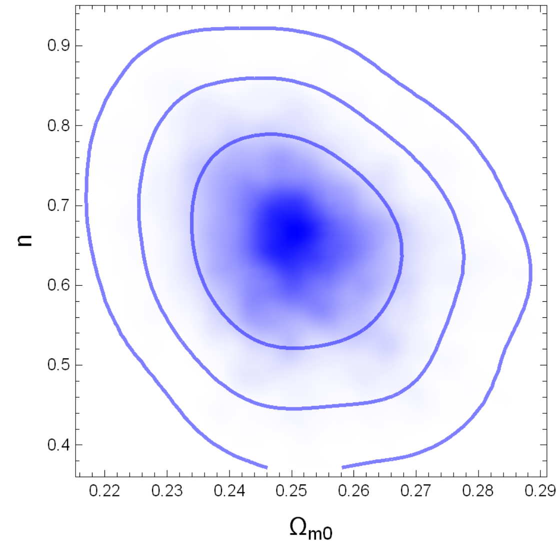

FIG. 3: Constraints on Ω_{m0} and n from 1σ to 3σ confidence level by using JLA SNe Ia + CMB + BAO for model III, while other parameters take their best fitting values.

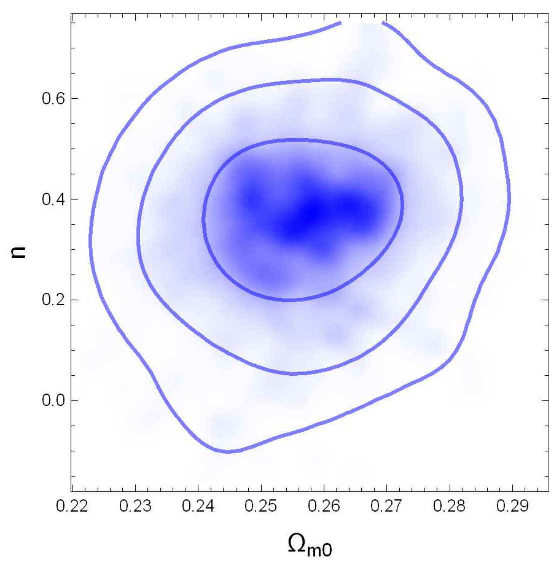

FIG. 2: Constraints on Ω_{m0} and n from 1σ to 3σ confidence level by using JLA SNe Ia + CMB + BAO for model II, while other parameters take their best fitting values.

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