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Constraints on Nonmetricity from Bounds on Lorentz Violation

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Spacetime nonmetricity can be studied experimentally through its couplings to fermions and photons. We use recent high-precision searches for Lorentz violation to deduce first constraints involving the 40 independent nonmetricity components down to levels of order 10^{-43} GeV.

Many theories of gravity, including our most successful theory, General Relativity, associate gravitational phenomena with the geometry of spacetime. In these theories, the notions of distances, angles, and parallelism are essential physical ingredients in specifying the spacetime geometry and the corresponding gravitational degrees of freedom. Mathematically, these ingredients are fixed by introducing a metric and a connection, and the geometry of a general spacetime manifold is then characterized by three tensors, the curvature, the torsion, and the nonmetricity [1]. From this perspective, General Relativity is a comparatively simple and elegant construction based on Riemann geometry with both zero torsion and zero nonmetricity, leaving only curvature to describe gravity.

Numerous alternative theories of gravity make use of more general geometries. One famous example is the Weyl theory of gravitation and electrodynamics [2], which has nonzero curvature and nonmetricity but zero torsion. Another is Einstein-Cartan theory [3], which is based on Riemann-Cartan geometry with dynamical curvature and torsion but zero nonmetricity. Theories of gravity in which all three tensors are nonzero, called metric-affine theories, have also been formulated [4].

An intriguing and vital issue is the extent to which current experimental techniques can constrain the three tensors governing the geometry of our spacetime. While nonzero curvature components in nature are readily associated to known features of gravity, even the existence of torsion and nonmetricity remain open to doubt. The torsion tensor has 24 independent components, most of which have recently been constrained in a model-independent way down to about 10^{-31} GeV using data from laboratory experiments [5, 6]. In constrast, the 40 independent components of the nonmetricity tensor remain unexplored in the laboratory to date.

In this work, we address this surprising lacuna in the literature. We adapt the exceptional sensitivities attained in precision tests of Lorentz symmetry [7] to deduce sharp first constraints for nonmetricity components. The central point is that background nonmetricity in the laboratory can affect a freely falling observer in an orientation-dependent way, while the existence of preferred directions in free fall is the key characteristic of local Lorentz violation [8]. It follows that a background nonmetricity induces effective Lorentz violation in the laboratory, even when the underlying gravitational theory with nonmetricity is locally Lorentz invariant. Precision tests of Lorentz symmetry can thus also serve as high-sensitivity searches for nonmetricity.

Studies of Lorentz symmetry have undergone a substantial revival in recent years following the discovery that minuscule violations of the laws of relativity accessible in the laboratory may arise in theories unifying gravity and quantum physics such as strings [9]. A general and powerful tool to describe phenomena at energies well below the scale of new physics is effective field theory [10]. For Lorentz violation, the general realistic effective field theory is the Standard-Model Extension (SME) [11], which is built by adding all possible coordinate-independent Lorentz-violating terms to the Lorentz-invariant gravitational and matter actions [12]. In the SME, the size and nature of experimental signals from Lorentz-violating operators are determined by coefficients for Lorentz violation, which are therefore appropriate targets for experiments [7]. Here, we identify the correspondence between components of background nonmetricity and certain SME coefficients for Lorentz violation, thereby permitting the extraction of experimental constraints on nonmetricity from existing bounds on Lorentz violation.

To proceed, we postulate that the complete theory of gravity predicts a nonzero nonmetricity in the neighborhood of the Earth, which is thus present as a background in the laboratory. For general couplings to the background nonmetricity, studying the behavior of particles then provides an experimental route to constraining nonmetricity in a model-independent way. The background nonmetricity endows the spacetime with an orientation, thereby inducing effective local Lorentz violation in the particle properties. To extract constraints on nonmetricity, we disregard possible Lorentz-violating contributions from other sources, including any background torsion. Also, we take the primary effects as arising from nonmetricity that is constant in the reference frame of its source, neglecting possible smaller effects involving spacetime derivatives of nonmetricity. In what follows, we adopt the conventions of Ref. [8].

In General Relativity, the spacetime geometry is specified by the Riemann curvature tensor $\widetilde{R}^{\mu}{}_{\nu\alpha\beta}$, which can be constructed by commuting covariant derivatives $\widetilde{D}_{\mu}$ defined using the Levi-Civita connection. In a theory with both curvature and nonmetricity, the geometry is determined by the generalized Riemann tensor $R^{\mu}{}_{\nu\alpha\beta}$ constructed from a generalized covariant derivative D_{μ}, together with the nonmetricity tensor $N_{\mu\alpha\beta} \equiv D_{\mu} g_{\alpha\beta}$ given by the covariant derivative of the metric $g_{\alpha\beta}$. The generalized tensor $R^\mu{}_{\nu\alpha\beta}$ is the sum of $\widetilde{R}^\mu{}_{\nu\alpha\beta}$ and terms involving $N_{\mu\alpha\beta}$. For the laboratory experiments of interest here, gravity and hence $\widetilde{R}^\mu{}_{\nu\alpha\beta}$ are negligible, so we can safely proceed assuming only $N_{\mu\alpha\beta}$ contributes.

The nonmetricity tensor $N_{\mu\alpha\beta}$ can be decomposed in Lorentz-irreducible components as

$$\begin{aligned} N_{\mu\alpha\beta} = \tfrac{1}{18}(&5N_{1\mu}g_{\alpha\beta} - N_{1\alpha}g_{\beta\mu} - N_{1\beta}g_{\mu\alpha} \\ &-2N_{2\mu}g_{\alpha\beta} + 4N_{2\alpha}g_{\beta\mu} + 4N_{2\beta}g_{\mu\alpha}) \\ &+S_{\mu\alpha\beta} + M_{\mu\alpha\beta}, \end{aligned} \tag{1}$$

where

$$\begin{aligned} N_{1\mu} &\equiv g^{\alpha\beta}N_{\mu\alpha\beta}, \quad N_{2\mu} \equiv g^{\alpha\beta}N_{\alpha\mu\beta}, \\ S_{\mu\alpha\beta} &\equiv \tfrac{1}{3}(N_{\mu\alpha\beta} + N_{\alpha\beta\mu} + N_{\beta\mu\alpha}) \\ &\quad -\tfrac{1}{18}(N_{1\mu}g_{\alpha\beta} + N_{1\alpha}g_{\beta\mu} + N_{1\beta}g_{\mu\alpha}) \\ &\quad -\tfrac{1}{9}(N_{2\mu}g_{\alpha\beta} + N_{2\alpha}g_{\beta\mu} + N_{2\beta}g_{\mu\alpha}), \\ M_{\mu\alpha\beta} &\equiv \tfrac{1}{3}(2N_{\mu\alpha\beta} - N_{\alpha\beta\mu} - N_{\beta\mu\alpha}) \\ &\quad -\tfrac{1}{9}(2N_{1\mu}g_{\alpha\beta} - N_{1\alpha}g_{\beta\mu} - N_{1\beta}g_{\alpha\mu}) \\ &\quad +\tfrac{1}{9}(2N_{2\mu}g_{\alpha\beta} - N_{2\alpha}g_{\beta\mu} - N_{2\beta}g_{\alpha\mu}). \end{aligned} \tag{2}$$

Both traces $N_{1\mu}$ and $N_{2\mu}$ contain 4 independent components, while the traceless symmetric piece $S_{\mu\alpha\beta}$ and the traceless mixed-symmetry piece $M_{\mu\alpha\beta}$ each contain 16.

We focus here on experimental signals involving the behavior of a Dirac fermion with arbitrary linear nonmetricity couplings. Neglecting possible couplings other than to nonmetricity and approximating covariant derivatives systematically, the hermitian effective Lagrange density $\mathcal{L}_N$ containing all independent constant-nonmetricity couplings to a Dirac fermion is a series of terms $\mathcal{L}_N^{(d)}$ with operators of increasing mass dimension d,

$$\mathcal{L}_N = \mathcal{L}_0 + \mathcal{L}_N^{(4)} + \mathcal{L}_N^{(5)} + \mathcal{L}_N^{(6)} + \ldots, \tag{3}$$

where $\mathcal{L}_0 = \frac{1}{2}i\overline{\psi}\gamma^\mu \overleftrightarrow{\partial_\mu} \psi - m\overline{\psi}\psi$ and where the other terms are built from fermion bilinears, partial derivatives acting on fermions, the irreducible nonmetricity components, and the Lorentz-group invariants $\eta_{\mu\nu}$ and $\epsilon^{\kappa\lambda\mu\nu}$. Each term is the product of one fermion bilinear and one irreducible piece of the nonmetricity and is required to be hermitian.

The terms with $d = 4$ have no derivatives,

$$\begin{aligned} \mathcal{L}_N^{(4)} = &\zeta_1^{(4)} N_{1\mu}\overline{\psi}\gamma^\mu\psi + \zeta_2^{(4)} N_{1\mu}\overline{\psi}\gamma_5\gamma^\mu\psi \\ &+\zeta_3^{(4)} N_{2\mu}\overline{\psi}\gamma^\mu\psi + \zeta_4^{(4)} N_{2\mu}\overline{\psi}\gamma_5\gamma^\mu\psi. \end{aligned} \tag{4}$$

Analogously, the terms with $d = 5$ have one derivative and take the form

$$\begin{aligned} \mathcal{L}_N^{(5)} = &\tfrac{1}{2}i\zeta_1^{(5)} N_1{}^\mu\overline{\psi}\overleftrightarrow{\partial_\mu}\psi + \tfrac{1}{2}\zeta_2^{(5)} N_1{}^\mu\overline{\psi}\gamma_5\overleftrightarrow{\partial_\mu}\psi \\ &+\tfrac{1}{2}i\zeta_3^{(5)} N_2{}^\mu\overline{\psi}\overleftrightarrow{\partial_\mu}\psi + \tfrac{1}{2}\zeta_4^{(5)} N_2{}^\mu\overline{\psi}\gamma_5\overleftrightarrow{\partial_\mu}\psi \\ &+\tfrac{1}{4}i\zeta_5^{(5)} M_{\mu\nu}{}^\rho\overline{\psi}\sigma^{\mu\nu}\overleftrightarrow{\partial_\rho}\psi \\ &+\tfrac{1}{8}i\zeta_6^{(5)} \epsilon_{\kappa\lambda\mu\nu}M^{\kappa\lambda\rho}\overline{\psi}\sigma^{\mu\nu}\overleftrightarrow{\partial_\rho}\psi \\ &+\tfrac{1}{2}i\zeta_7^{(5)} N_{1\mu}\overline{\psi}\sigma^{\mu\nu}\overleftrightarrow{\partial_\nu}\psi + \tfrac{1}{2}i\zeta_8^{(5)} N_{2\mu}\overline{\psi}\sigma^{\mu\nu}\overleftrightarrow{\partial_\nu}\psi \\ &+\tfrac{1}{4}i\zeta_9^{(5)} \epsilon^{\lambda\mu\nu\rho}N_{1\lambda}\overline{\psi}\sigma_{\mu\nu}\overleftrightarrow{\partial_\rho}\psi \\ &+\tfrac{1}{4}i\zeta_{10}^{(5)} \epsilon^{\lambda\mu\nu\rho}N_{2\lambda}\overline{\psi}\sigma_{\mu\nu}\overleftrightarrow{\partial_\rho}\psi. \end{aligned} \tag{5}$$

To access fermion couplings to the symmetric irreducible piece $S_{\lambda\mu\nu}$ requires considering also operators in $\mathcal{L}_N^{(6)}$, which have two derivatives. Since the other irreducible pieces already appear coupled to operators in $\mathcal{L}_N^{(4)}$ and $\mathcal{L}_N^{(5)}$, we consider here only terms in $\mathcal{L}_N^{(6)}$ involving $S_{\lambda\mu\nu}$,

$$\begin{aligned} \mathcal{L}_N^{(6)} \supset &-\tfrac{1}{4}\zeta_1^{(6)} S_\lambda{}^{\mu\nu}\overline{\psi}\gamma^\lambda\partial_\mu\partial_\nu\psi + \text{h.c.} \\ &-\tfrac{1}{4}\zeta_2^{(6)} S_\lambda{}^{\mu\nu}\overline{\psi}\gamma_5\gamma^\lambda\partial_\mu\partial_\nu\psi + \text{h.c.} \end{aligned} \tag{6}$$

In the above expressions, the coupling constants $\zeta_j^{(d)}$ depend on the details of the theory under consideration. For the special case of Weyl gravity [2], which ties electrodynamics with spacetime geometry, the nonmetricity is determined by the electromagnetic 4-potential A_μ via $N_{\mu\alpha\beta} = A_\mu g_{\alpha\beta}$, with the only nonzero couplings at tree level obeying $4\zeta_1^{(4)} + \zeta_3^{(4)} = 1$ for a minimally coupled unit-charge particle. We remark in passing that this theory is known to be unphysical because it predicts that generic spectral lines cannot exist [13]. Another special case is minimal coupling with covariant derivative defined via the Kosmann lift [14], for which all nonmetricity couplings vanish at tree level. Other choices of minimal and nonminimal couplings are possible, and also radiative corrections generically induce nonminimal couplings, so we proceed here without preconceived notions and retain all couplings for our analysis.

Treating the nonmetricity as a background means that its components behave as scalars under particle Lorentz transformations [8], implying that $\mathcal{L}_N$ describes effective Lorentz violation and that the fermion follows a geodesic in a pseudo-Finsler spacetime [15]. Since the nonmetricity tensor has three indices, all effective couplings of this type are also CPT violating [11]. It follows that the behaviors of particles and antiparticles differ in the presence of background nonmetricity. For each term in $\mathcal{L}_N$, the nonmetricity tensor and accompanying coupling constant together play the role of a coefficient for Lorentz violation in the SME in Minkowski spacetime [8]. Matching each term in $\mathcal{L}_N$ to the corresponding term in the SME yields the correspondences

$$\begin{aligned} b_\mu - mg_\mu^{(A)} &= -(\zeta_2^{(4)} - m\zeta_9^{(5)})N_{1\mu} - (\zeta_4^{(4)} - m\zeta_{10}^{(5)})N_{2\mu}, \\ g_{\mu\nu\alpha}^{(M)} &= -\tfrac{1}{2}\zeta_5^{(5)}(M_{\mu\nu\alpha} - M_{\nu\mu\alpha}) - \tfrac{1}{2}\zeta_6^{(5)}\epsilon_{\mu\nu\rho\sigma}M^{\rho\sigma}{}_\alpha, \\ a_{\mu\alpha\beta}^{(5)(S)} &= -\tfrac{1}{2}\zeta_1^{(6)}S_{\mu\alpha\beta}, \quad b_{\mu\alpha\beta}^{(5)(S)} = -\tfrac{1}{2}\zeta_2^{(6)}S_{\mu\alpha\beta}. \end{aligned} \tag{7}$$

Here, the relevant minimal-SME coefficients governing CPT-odd effects [8] include b_μ and the irreducible components $g^{(A)}_\mu$ and $g^{(M)}_{\mu\nu\alpha}$ of $g_{\mu\nu\alpha}$, while among the nonminimal operators [16] only the totally symmetric and traceless pieces $a^{(5)(S)}_{\mu\alpha\beta}$ and $b^{(5)(S)}_{\mu\alpha\beta}$ of the nonminimal coefficients $a^{(5)}_{\mu\alpha\beta}$ and $b^{(5)}_{\mu\alpha\beta}$ play a role.

Since nonmetricity produces effective Lorentz violation in the laboratory, reporting constaints on nonmetricity components requires also specifying an inertial frame. The nonmetricity can reasonably be taken as approximately uniform throughout the solar system. A suitable frame is then the Sun-centered celestial-equatorial frame [17], which has cartesian coordinates (T, X, Y, Z) with Z axis along the Earth rotation axis and X axis directed towards the vernal equinox 2000. The rotation and revolution of the Earth induces sidereal and annual variations as signals of Lorentz violation [18], and bounds on the SME coefficients in Eq. (7) from numerous experiments have been reported in this frame [7].

The experimental results can be scrutinized independent of fermion flavor because nonmetricity is part of the spacetime geometry. The sharpest constraints on the trace and mixed-symmetry pieces of the nonmetricity are obtained from two experiments with He-Xe dual masers [19, 20]. Using the match (7), the bounds obtained on variations in the maser frequency at the Earth's annual-revolution frequency [19] yield the four conditions

$$
\begin{aligned}
&|\cos\eta[(\zeta^{(4)}_2 - m_n\zeta^{(5)}_9)N_{1T} + (\zeta^{(4)}_4 - m_n\zeta^{(5)}_{10})N_{2T}\\
&\qquad +\tfrac{1}{2}m_n\zeta^{(5)}_5(M_{ZXY} - M_{XYZ}) - \tfrac{3}{4}m_n\zeta^{(5)}_6 M_{TYY}]\\
&\quad -\tfrac{3}{4}m_n\sin\eta[\zeta^{(5)}_5(2M_{XTT} - M_{XYY})\\
&\qquad +2\zeta^{(5)}_6(M_{TYZ} + M_{ZTY})]| < 2.0\times 10^{-27}\ \text{GeV},\\
&\tfrac{3}{4}m_n|\cos\eta[\zeta^{(5)}_5(M_{ZXX} - 2M_{ZTT}) + 2\zeta^{(5)}_6 M_{XTY}]\\
&\qquad -\sin\eta[\zeta^{(5)}_5 M_{YXX} + 2\zeta^{(5)}_6(M_{TZX} + M_{ZTX})]|\\
&\qquad\qquad < 1.6\times 10^{-27}\ \text{GeV},\\
&|(\zeta^{(4)}_2 - m_n\zeta^{(5)}_9)N_{1T} + (\zeta^{(4)}_4 - m_n\zeta^{(5)}_{10})N_{2T}\\
&\quad -\tfrac{1}{2}m_n\zeta^{(5)}_5(M_{XYZ} + 2M_{ZXY}) - \tfrac{3}{4}m_n\zeta^{(5)}_6 M_{TXX}|\\
&\qquad\qquad < 3.8\times 10^{-27}\ \text{GeV},\\
&\tfrac{3}{4}m_n|\zeta^{(5)}_5(M_{ZTT} + M_{ZXX}) - 2\zeta^{(5)}_6(M_{TXY} + M_{XTY})|\\
&\qquad\qquad < 3.6\times 10^{-27}\ \text{GeV},
\end{aligned}
\tag{8}
$$

where m_n is the neutron mass and $\eta \simeq 23.4^\circ$ is the angle between the orbital plane of the Earth and the X-Y plane in the Sun-centered frame, while the bounds obtained on variations in the maser frequency at the Earth's sidereal frequency [20] translate into the two constraints

$$
\begin{aligned}
&|(\zeta^{(4)}_2 - m_n\zeta^{(5)}_9)N_{1X} + (\zeta^{(4)}_4 - m_n\zeta^{(5)}_{10})N_{2X}\\
&\quad -\tfrac{1}{2}m_n\zeta^{(5)}_5(M_{TYZ} + 2M_{ZTY}) + \tfrac{3}{4}m_n\zeta^{(5)}_6 M_{XTT}|\\
&\qquad\qquad < 9.4\times 10^{-34}\ \text{GeV},\\
&|(\zeta^{(4)}_2 - m_n\zeta^{(5)}_9)N_{1Y} + (\zeta^{(4)}_4 - m_n\zeta^{(5)}_{10})N_{2Y}\\
&\quad +\tfrac{1}{2}m_n\zeta^{(5)}_5(M_{TZX} + 2M_{ZTX}) + \tfrac{3}{4}m_n\zeta^{(5)}_6 M_{YTT}|\\
&\qquad\qquad < 1.2\times 10^{-33}\ \text{GeV}.
\end{aligned}
\tag{9}
$$

A complementary constraint comes from bounds on Lorentz violation using a Hg-Cs comagnetometer [21],

$$
\begin{aligned}
&|(\zeta^{(4)}_2 - m_n\zeta^{(5)}_9)N_{1Z} + (\zeta^{(4)}_4 - m_n\zeta^{(5)}_{10})N_{2Z}\\
&\quad +\tfrac{1}{2}m_n\zeta^{(5)}_5(M_{TXY} + 2M_{XTY}) + \tfrac{3}{4}m_n\zeta^{(5)}_6 M_{ZTT}|\\
&\qquad\qquad < 7.0\times 10^{-30}\ \text{GeV}.
\end{aligned}
\tag{10}
$$

Two constraints on the symmetric piece of the nonmetricity can be extracted from bounds on nonminimal SME coefficients [22] obtained via sidereal-variation studies of the hydrogen hyperfine transition [23],

$$
\begin{aligned}
&\sqrt{\frac{\pi}{6}}m_p^2|\zeta^{(6)}_2 S_{TTX}| < 9.0\times 10^{-27}\ \text{GeV},\\
&\sqrt{\frac{\pi}{6}}m_p^2|\zeta^{(6)}_2 S_{TTY}| < 9.0\times 10^{-27}\ \text{GeV},
\end{aligned}
\tag{11}
$$

where m_p is the proton mass. The absence of cosmic-ray Čerenkov radiation [16, 24] provides the tight constraint

$$
|\zeta^{(6)}_1 S_{TTT}| < 1.0\times 10^{-34}\ \text{GeV}^{-1}.
\tag{12}
$$

Finally, bounds on nonminimal SME coefficients [26] extracted using sidereal-variation studies at the muon $g-2$ experiment [25] correspond to the four constraints

$$
\begin{aligned}
&\sqrt{\frac{\pi}{21}}\frac{(\gamma^2-1)}{10\gamma^4 m_\mu}|4\zeta^{(6)}_2 S_{TTX} - 5\zeta^{(6)}_2 S_{XXX} - 5\zeta^{(6)}_2 S_{XYY}|\\
&\qquad\qquad < 4.3\times 10^{-26}\ \text{GeV}^{-2},\\
&\sqrt{\frac{\pi}{21}}\frac{(\gamma^2-1)}{10\gamma^4 m_\mu}|4\zeta^{(6)}_2 S_{TTY} - 5\zeta^{(6)}_2 S_{YYY} - 5\zeta^{(6)}_2 S_{XXY}|\\
&\qquad\qquad < 4.3\times 10^{-26}\ \text{GeV}^{-2},\\
&\sqrt{\frac{\pi}{3}}\frac{2}{3m_\mu}|\zeta^{(6)}_2 S_{TTZ}| < 5.0\times 10^{-26}\ \text{GeV}^{-2},\\
&\sqrt{\frac{\pi}{7}}\frac{(\gamma^2-1)}{15\gamma^4 m_\mu}|2\zeta^{(6)}_2 S_{TTZ} - 5\zeta^{(6)}_2 S_{XXZ} - 5\zeta^{(6)}_2 S_{YYZ}|\\
&\qquad\qquad < 5.0\times 10^{-26}\ \text{GeV}^{-2},
\end{aligned}
\tag{13}
$$

where m_μ is the muon mass and $\gamma \simeq 29.3$ is the muon boost factor.

Some insight about the breadth and quality of the above constraints can be obtained by collating their implications under the assumption that only one nonmetricity component is nonzero at a time. Selecting a canonical set of 16+16 independent components of the mixed and

TABLE I. Laboratory constraints on nonmetricity.

Quantity	Constraint	Quantity	Constraint
$\zeta_2^{(4)} N_{1T}$	10^{-27} GeV	$\zeta_9^{(5)} N_{1T}$	10^{-27}
$\zeta_2^{(4)} N_{1X}$	10^{-33} GeV	$\zeta_9^{(5)} N_{1X}$	10^{-33}
$\zeta_2^{(4)} N_{1Y}$	10^{-33} GeV	$\zeta_9^{(5)} N_{1Y}$	10^{-33}
$\zeta_2^{(4)} N_{1Z}$	10^{-29} GeV	$\zeta_9^{(5)} N_{1Z}$	10^{-29}
$\zeta_4^{(4)} N_{2T}$	10^{-27} GeV	$\zeta_{10}^{(5)} N_{2T}$	10^{-27}
$\zeta_4^{(4)} N_{2X}$	10^{-33} GeV	$\zeta_{10}^{(5)} N_{2X}$	10^{-33}
$\zeta_4^{(4)} N_{2Y}$	10^{-33} GeV	$\zeta_{10}^{(5)} N_{2Y}$	10^{-33}
$\zeta_4^{(4)} N_{2Z}$	10^{-29} GeV	$\zeta_{10}^{(5)} N_{2Z}$	10^{-29}
$\zeta_5^{(5)} M_{TXX}$		$\zeta_6^{(5)} M_{TXX}$	10^{-26}
$\zeta_5^{(5)} M_{TXY}$	10^{-29}	$\zeta_6^{(5)} M_{TXY}$	10^{-27}
$\zeta_5^{(5)} M_{TYY}$		$\zeta_6^{(5)} M_{TYY}$	10^{-27}
$\zeta_5^{(5)} M_{TYZ}$	10^{-33}	$\zeta_6^{(5)} M_{TYZ}$	10^{-27}
$\zeta_5^{(5)} M_{TZX}$	10^{-33}	$\zeta_6^{(5)} M_{TZX}$	10^{-27}
$\zeta_5^{(5)} M_{XTT}$	10^{-27}	$\zeta_6^{(5)} M_{XTT}$	10^{-33}
$\zeta_5^{(5)} M_{XTY}$	10^{-29}	$\zeta_6^{(5)} M_{XTY}$	10^{-27}
$\zeta_5^{(5)} M_{XYY}$	10^{-27}	$\zeta_6^{(5)} M_{XYY}$	
$\zeta_5^{(5)} M_{XYZ}$	10^{-26}	$\zeta_6^{(5)} M_{XYZ}$	
$\zeta_5^{(5)} M_{YTT}$		$\zeta_6^{(5)} M_{YTT}$	10^{-33}
$\zeta_5^{(5)} M_{YXX}$	10^{-26}	$\zeta_6^{(5)} M_{YXX}$	
$\zeta_5^{(5)} M_{ZTT}$	10^{-26}	$\zeta_6^{(5)} M_{ZTT}$	10^{-29}
$\zeta_5^{(5)} M_{ZTX}$	10^{-33}	$\zeta_6^{(5)} M_{ZTX}$	10^{-27}
$\zeta_5^{(5)} M_{ZTY}$	10^{-33}	$\zeta_6^{(5)} M_{ZTY}$	10^{-27}
$\zeta_5^{(5)} M_{ZXX}$	10^{-26}	$\zeta_6^{(5)} M_{ZXX}$	
$\zeta_5^{(5)} M_{ZXY}$	10^{-27}	$\zeta_6^{(5)} M_{ZXY}$	
$\zeta_1^{(6)} S_{TTT}$	10^{-34} GeV^{-1}	$\zeta_2^{(6)} S_{TTT}$	
$\zeta_1^{(6)} S_{TTX}$		$\zeta_2^{(6)} S_{TTX}$	10^{-26} GeV^{-1}
$\zeta_1^{(6)} S_{TTY}$		$\zeta_2^{(6)} S_{TTY}$	10^{-26} GeV^{-1}
$\zeta_1^{(6)} S_{TTZ}$		$\zeta_2^{(6)} S_{TTZ}$	10^{-26} GeV^{-1}
$\zeta_1^{(6)} S_{XXX}$		$\zeta_2^{(6)} S_{XXX}$	10^{-23} GeV^{-1}
$\zeta_1^{(6)} S_{XXY}$		$\zeta_2^{(6)} S_{XXY}$	10^{-23} GeV^{-1}
$\zeta_1^{(6)} S_{XXZ}$		$\zeta_2^{(6)} S_{XXZ}$	10^{-23} GeV^{-1}
$\zeta_1^{(6)} S_{XYY}$		$\zeta_2^{(6)} S_{XYY}$	10^{-23} GeV^{-1}
$\zeta_1^{(6)} S_{YYY}$		$\zeta_2^{(6)} S_{YYY}$	10^{-23} GeV^{-1}
$\zeta_1^{(6)} S_{YYZ}$		$\zeta_2^{(6)} S_{YYZ}$	10^{-23} GeV^{-1}

symmetric pieces of the nonmetricity, we find the results displayed in Table I, where the listed 2σ constraints are understood to hold on the modulus of each quantity. This reveals that the laboratory experiments discussed here yield first sensitivities to 34 of the 40 independent nonmetricity components, with only S_{TXX}, S_{TXY}, S_{TXZ}, S_{TYY}, S_{TYZ}, and S_{XYZ} absent. On the surface of the Earth, a nonmetricity modulus of about 10^{-27} GeV in the modified Poisson equation would compete with conventional gravity, so Table I reveals that experiments already restrict realistic models to comparatively tiny nonmetricity values.

The constraints in Table I are derived assuming uniform cartesian nonmetricity components in the vicinity of the solar system. However, many of these constraints also apply in other scenarios. For example, if the nonmetricity is taken to be sourced by the Sun and so has approximately azimuthal symmetry around the vector normal to the ecliptic plane and passing through the Sun, then the anisotropic nonmetricity components appear roughly unchanged in any laboratory throughout the year. In this scenario, the constraints (8) no longer apply as they are derived from studies of annual variations, but the others remain in force. If instead the Earth is taken as the nonmetricity source, then detecting direction-dependent nonmetricity effects requires rotation of the apparatus in the laboratory frame, so only the constraints (10) and (12) hold. In all special scenarios, other bounds on Lorentz violation obtained in suitable experiments [7] could instead be used to place nonmetricity constraints, albeit at somewhat lower sensitivities than those reported in Table I.

The above analysis considers a single flavor of Dirac fermion. Extending $\mathcal{L}_N$ to include multiple fermion species would generate nonmetricity couplings relevant to experiments searching for Lorentz violation with meson or neutrino oscillations, but the resulting constraints on nonmetricity are weaker than the best sensitivities shown in Table I. Laboratory experiments with various boson species, including photons, also lack the necessary sensitivity to Lorentz violation to achieve competitive constraints on nonmetricity. The results in Table I are thus the sharpest currently attainable in the laboratory.

Astrophysical observations can provide additional nonmetricity constraints. With the comparatively strong assumption that background nonmetricity is uniform on cosmological scales in space and time, and recalling that it violates CPT, then photons [27] and gravitons [28] experience nonmetricity-induced birefringence when propagating over cosmological distances. Existing bounds on cosmological birefringence from searches for Lorentz violation [7] can thus also be used to constrain nonmetricity. The limits on CPT-violating birefringence of gravitons are comparatively weak, so we focus here on photons.

To proceed, we construct the hermitian Lagrange density $\mathcal{L}_N$ containing all effective gauge-invariant CPT-violating contributions to the photon propagator coupled to background nonmetricity. This can again be expanded in the form (3), where now $\mathcal{L}_0 = -F^{\mu\nu}F_{\mu\nu}/4$ with $F_{\mu\nu}$ the electromagnetic field strength and where the leading-order contributions from the irreducible pieces of the nonmetricity involve generalized Chern-Simons terms,

$$
\begin{aligned}
\mathcal{L}_N^{(4)} &= \tfrac{1}{2}\epsilon^{\kappa\lambda\mu\nu}(\zeta_a^{(4)} N_{1\kappa} + \zeta_b^{(4)} N_{2\kappa}) A_\lambda F_{\mu\nu}, \\
\mathcal{L}_N^{(6)} &\supset \tfrac{1}{2}\epsilon^{\kappa\lambda\mu\nu}(\zeta_c^{(6)} M_{\kappa\alpha\beta} + \zeta_d^{(6)} S_{\kappa\alpha\beta}) A_\lambda \partial^\alpha\partial^\beta F_{\mu\nu}. \quad (14)
\end{aligned}
$$

Note that $\mathcal{L}_N^{(d)}$ vanishes for odd d. To extract nonmetricity constraints from bounds on Lorentz violation, we match to the general effective field theory for photon propagation [27], which yields the correspondences

$$(k_{AF}^{(3)})_\kappa = -\zeta_a^{(4)} N_{1\kappa} - \zeta_b^{(4)} N_{2\kappa},$$
$$(k_{AF}^{(5)})^{(S)}_{\kappa\alpha\beta} = -\zeta_d^{(6)} S_{\kappa\alpha\beta}, \quad (k_{AF}^{(5)})^{(M)}_{\kappa\alpha\beta} = -\zeta_c^{(6)} M_{\kappa\alpha\beta} \quad (15)$$

between nonmetricity, the SME coefficients $(k_{AF}^{(3)})_\kappa$, and the traceless symmetric and mixed pieces of $(k_{AF}^{(5)})_{\kappa\alpha\beta}$.

Sharp bounds on all four components of $(k_{AF}^{(3)})_\kappa$ have been obtained from studies of birefringence in the cosmic microwave background. The isotropic component $(k_{AF}^{(3)})_T$ has been extensively explored and constrained to below about 10^{-43} GeV [27, 29–31]. The anisotropic components $(k_{AF}^{(3)})_J$, $J = X, Y, Z$, may exhibit a weak signal but can safely be taken as constrained below 10^{-42} GeV [27, 29, 30]. Taking one nonmetricity component at a time as before then yields the eight constraints

$$|\zeta_a^{(4)} N_{1T}| < 10^{-43} \text{ GeV}, \quad |\zeta_b^{(4)} N_{2T}| < 10^{-43} \text{ GeV},$$
$$|\zeta_a^{(4)} N_{1J}| < 10^{-42} \text{ GeV}, \quad |\zeta_b^{(4)} N_{2J}| < 10^{-42} \text{ GeV}. \quad (16)$$

Only the 16 components $(k_{AF}^{(5)})^{(S)}_{\kappa\alpha\beta}$ produce cosmological birefringence [27], and all are bounded by studies of gamma-ray bursts [27, 32–35]. Taking each nonmetricity component in turn then yields the 16 constraints

$$\zeta_d^{(6)} S_{TTT} < 10^{-35} \text{ GeV}^{-1}, \quad \zeta_d^{(6)} S_{\kappa\alpha\beta} < 10^{-34} \text{ GeV}^{-1}, \quad (17)$$

where $\kappa\alpha\beta$ spans the 15 anisotropic components. The astrophysical constraints improve some laboratory ones but involve different couplings and stronger assumptions.

In summary, we have obtained first constraints involving the 40 independent components of nonmetricity by translating bounds on Lorentz violation from laboratory experiments and astrophysical observations. Given the rapid advances in the search for Lorentz and CPT violation, the prospects for future improvements are excellent.

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