Gravitational-wave cosmography with LISA and the Hubble tension

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Gravitational-wave cosmography with LISA and the Hubble tension

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We propose that stellar-mass binary black holes like GW150914 will become a tool to explore the local Universe within $\sim 100\,\mathrm{Mpc}$ in the era of the Laser Interferometer Space Antenna (LISA). High calibration accuracy and annual motion of LISA could enable us to localize up to ≈ 60 binaries more accurately than the error volume of $\approx 100\,\mathrm{Mpc}^3$ without electromagnetic counterparts under moderately optimistic assumptions. This accuracy will give us a fair chance to determine the host object solely by gravitational waves. By combining the luminosity distance extracted from gravitational waves with the cosmological redshift determined from the host, the local value of the Hubble parameter will be determined up to a few % without relying on the empirically-constructed distance ladder. Gravitational-wave cosmography would pave the way for resolution of the disputed Hubble tension, where the local and global measurements disagree in the value of the Hubble parameter at 3.4σ level, which amounts to $\approx 9\%$.

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I. INTRODUCTION

The discovery of GW150914, the first direct detection of gravitational waves by Advanced LIGO, has significant impacts on physics and astronomy [1]. The key properties of the binary black holes detected so far are the large masses and the high merger rate [2]. In addition to the upcoming observations by ground-based detectors, the prospects for observing binary black holes at low frequency by space-borne detectors such as the Laser Interferometer Space Antenna (LISA: see Ref. [3] for LISA pathfinder), called the evolved LISA (eLISA) for the last several years, are attracting considerable attention [4–7].

Around $7\,\mathrm{mHz}$, LISA would be able to detect ~ 100 nearly-monochromatic binary black holes within $\sim 100\,\mathrm{Mpc}$ at high signal-to-noise ratios assuming the sensitivity of eLISA [7]. This is remarkably different from binary neutron stars, which had been widely considered to be the most promising gravitational wave sources. Indeed, the expected detection number ($\propto \mathcal{M}^{10/3}$) and the maximum detectable distance ($\propto \mathcal{M}^{5/3}$) both depend strongly on the chirp mass $\mathcal{M}$ of the nearly monochromatic binaries [7]. Here, the chirp mass of the binary black hole GW150914 is $28M_\odot$, whereas that of typical binary neutron stars is only $\approx 1.2M_\odot$. We should also note that the ground-based detectors would detect no more than $O(1)$ events within $\sim 100\,\mathrm{Mpc}$ in a year unless the merger rate of compact binaries is substantially higher than $100\,\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$.

Monochromatic binaries of massive black holes may serve as a useful tool in astrometry (positional astronomy), which is the basis of astronomy and astrophysics, because LISA will be able to localize them very accurately [7]. Specifically, host objects (galaxies or clusters) of many extragalactic gravitational-wave sources are likely to be determined without observing electromagnetic counterparts that are presumably absent for binary black holes [8]. For this purpose, LISA has two advantages compared to ground-based detectors. First, high calibration accuracy of LISA will allow us to determine the amplitude of gravitational waves and hence the luminosity distance D with negligible systematic errors [9], in contrast to the ground-based detectors [10] (but see Ref. [11] for improvement). Second, LISA's annual motion induces a Doppler shift to the phase and modulation to the amplitude, and thus the sky location can be determined accurately for the long-lived sources [12, 13]. These two advantages are combined to give a small error volume for the stellar-mass binary black holes, which opens the possibility to determine the host object of the binary.

In this paper, we discuss prospects for determining the Hubble parameter in the local Universe by observing monochromatic stellar-mass binary black holes with LISA. Although the Hubble parameter is arguably the most fundamental quantity in cosmology, its value is still greatly debated. In particular, it has recently been claimed that the Hubble parameter determined by the local measurement, $H_0 = 73.24 \pm 1.74\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [14], is larger by 3.4σ than that obtained from the cosmic microwave background, $66.93 \pm 0.62\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [15]. Local underdensity (cosmic void) seems to contribute at some level to raise the local Hubble parameter [16], but this effect does not account for the majority of 3.4σ [17–19]. While this discrepancy, called the Hubble tension, could be reconciled by invoking nonstandard cosmological ingredients such as dark radiation, it is desirable to carefully analyze possible systematics associated with the current measurements.

For this purpose, the gravitational-wave standard siren is a powerful tool to study the local expansion rate of the Universe [20]. By observing binary black holes, we can determine the luminosity distance from simple principles of physics and can also calibrate the empirically-constructed distance ladder at various distance scales. However, it is well known that gravitational waves do not tell us directly the cosmological redshift of binaries. Thus, the host identification is crucial to extract redshift information. Here, binary black holes like GW150914 could now become promising standard sirens to almost exclusively examine the distance scale around 100 Mpc, thanks to the exquisite localizability described above.

Following our previous work [6, 7], the fiducial values of chirp mass, $\mathcal{M}$, and comoving merger rate, R, are taken to be optimistic values of $28M_\odot$ and $100\,\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$, respectively (see Ref. [2] for an update). Recall this chirp mass is larger by a factor of > 20 than that for typical binary neutron stars. We take the fiducial value of the Hubble parameter, $H_0 = h \times 100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, to be $h = 0.7$. Because we focus on the local Universe in this study, we neglect all minor corrections associated with the cosmological redshift.

II. LOCALIZATION OF BINARY BLACK HOLES WITH LISA

We first describe the prospects for localizing stellar-mass binary black holes by LISA based on Ref. [7]. In this study, we focus on the eLISA's N2A5 configuration [21] as the optimistic sensitivity of LISA and set the fiducial observation period T to be 3 yr. We only consider the four-link (two-arm) configuration, and likely restoration of the six-link (three-arm) configuration is highly welcome. The dependence of our results on the noise curve and observation period is briefly described in Sec. III. Because we discuss the localization accuracy using the result of Ref. [13] derived by a quasimonochromatic approximation where nearly monochromatic but slightly chirping binaries are considered, we restrict our attention to binaries that do not merge within T. Namely, we focus on $f < f_{\mathrm{merge}}(T)$ where

$$f_{\mathrm{merge}}(T) = 19.2\,\mathrm{mHz}\left(\frac{\mathcal{M}}{28M_\odot}\right)^{-5/8}\left(\frac{T}{3\,\mathrm{yr}}\right)^{-3/8}. \quad (1)$$

This frequency range contains the majority of detectable binaries, and high localization accuracy is also achieved in this range due to the high signal-to-noise ratio [7]. In this approximation, the signal-to-noise ratio for a binary at frequency f is given by [7]

$$\rho(f) = 21\left(\frac{\mathcal{M}}{28M_\odot}\right)^{5/3}\left(\frac{T}{3\,\mathrm{yr}}\right)^{1/2}\left(\frac{D}{100\,\mathrm{Mpc}}\right)^{-1} \times\left(\frac{S(f)}{2.3\times10^{-41}\,\mathrm{Hz}^{-1}}\right)^{-1/2}\left(\frac{f}{7\,\mathrm{mHz}}\right)^{2/3}, \quad (2)$$

where $S(f)$ is the noise spectral density of LISA/eLISA [21]. We normalize the frequency and noise spectral density (N2A5 configuration) by 7 mHz, because this turns out later to be the frequency that contributes most to the number of accurately localized sources.

The expected errors of the angular position and luminosity distance are given by [13]

$$\Delta\Omega(f) \sim 3.6\times10^{-4}\,\mathrm{str}\left(\frac{\rho}{20}\right)^{-2}\left(\frac{f}{7\,\mathrm{mHz}}\right)^{-2}, \quad (3)$$

$$\frac{\Delta D}{D} \sim 0.1\left(\frac{\rho}{20}\right)^{-1}, \quad (4)$$

and hence the error volume $\Delta V \equiv (4/3)D^2\Delta\Omega\Delta D$ is estimated to be

$$\Delta V(f) \sim 50\,\mathrm{Mpc}^3\left(\frac{D}{100\,\mathrm{Mpc}}\right)^3\left(\frac{\rho}{20}\right)^{-3}\left(\frac{f}{7\,\mathrm{mHz}}\right)^{-2}. \quad (5)$$

It has to be cautioned that this expression is derived in a conservative manner by averaging over the inclination and sky location of the binary. The precise size of the error volume depends on these parameters, and we only discuss the averaged behavior. In addition, this expression is derived by the Fisher analysis, where a high signal-to-noise ratio is assumed. Although our primary targets are binaries with strong signals that can be localized accurately, this needs future refinement.

The peculiar velocity δv of the host object has to be taken into account for the identification in a redshift catalog of galaxies. From the luminosity distance, D, obtained by LISA, we can approximately estimate the redshift of the host by $H_0 D/c$ with c the speed of light, assuming the Hubble expansion with a speculated value of H_0. However, the actual redshift of the host has an additional drift $\delta v/c$ induced by its peculiar velocity. Thus, for the host identification in a redshift catalog, we need to increase the redshift range by $\pm\sigma/c$, where σ is a typical velocity dispersion of galaxies. We can effectively handle this effect by using the total distance error $\sqrt{(\Delta D)^2 + (\sigma/H_0)^2}$ for calculating ΔV instead of the original one, ΔD. In this study, we conservatively take the fiducial value of σ to be $10^3\,\mathrm{km\,s^{-1}}$ [22], and $\sigma/H_0 = 14\,\mathrm{Mpc}(\sigma/10^3\,\mathrm{km\,s^{-1}})(h/0.7)^{-1}$. Alternatively, we may regard this value as a sum of a more typical peculiar velocity and the uncertainty in the value of H_0. This term dominates the distance error for high signal-to-noise ratios, and the error volume is given by

$$\Delta V(f) \sim 70\,\mathrm{Mpc}^3\left(\frac{D}{100\,\mathrm{Mpc}}\right)^2\left(\frac{\rho}{20}\right)^{-2} \times\left(\frac{f}{7\,\mathrm{mHz}}\right)^{-2}\left(\frac{\sigma}{10^3\,\mathrm{km\,s^{-1}}}\right)\left(\frac{h}{0.7}\right)^{-1} \quad (6)$$

in the limit where $\Delta D/D$ is negligible.

For a given binary, the error volume depends on LISA's design as $[S(f)/T]^{3/2}$ for large errors [Eq. (5)] and as $S(f)/T$ for small errors [Eq. (6)] via the signal-to-noise ratio. This indicates that both the high sensitivity and the longterm operation will be helpful for promoting monochromatic binaries to useful standard sirens.

Once the best estimate of H_0 is obtained (e.g., by binary-black-hole standard sirens), the peculiar velocity may be reversely determined for individual galaxies. The peculiar velocity field could also serve as a useful cosmological probe [22].

III. NUMBER OF ACCURATELY LOCALIZED BINARIES

A crucial question for gravitational-wave cosmography is how many standard sirens are localized to sufficient accuracy. The number density of binary black holes in each logarithmic frequency interval is given by [7]

$$\frac{dn(f)}{d\ln f} = 1.2\times 10^{-5}\,\mathrm{Mpc}^{-3}\left(\frac{\mathcal{M}}{28M_\odot}\right)^{-5/3} \times\left(\frac{R}{100\,\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}}\right)\left(\frac{f}{7\,\mathrm{mHz}}\right)^{-8/3}. \quad (7)$$

Combining this with Eqs. (5) or (6), the number of binary black holes localized more accurately than a given error volume of ΔV in each logarithmic frequency interval is deduced to be

$$\frac{dN(<\Delta V; f)}{d\ln f} \sim \begin{cases} 80\,(\Delta V/100\,\mathrm{Mpc}^3)^{1/2}(f/7\,\mathrm{mHz})^{-2/3} & (\Delta D \gg \sigma/H_0) \\ 70\,(\Delta V/100\,\mathrm{Mpc}^3)^{3/4}(f/7\,\mathrm{mHz})^{-1/6}(\sigma/10^3\,\mathrm{km\,s}^{-1})^{-3/4}(h/0.7)^{3/4} & (\Delta D \ll \sigma/H_0) \end{cases}$$
$$\times\left(\frac{\mathcal{M}}{28M_\odot}\right)^{5/6}\left(\frac{T}{3\,\mathrm{yr}}\right)^{3/4}\left(\frac{S(f)}{2.3\times10^{-41}\,\mathrm{Hz}^{-1}}\right)^{-3/4}\left(\frac{R}{100\,\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}}\right). \quad (8)$$

The smaller one of the above two gives a reasonable estimate, and the transition occurs when $\Delta D = \sigma/H_0$ is satisfied.

Important information extracted from the above expressions may be summarized as follows. By quantifying the frequency dependence (see Ref. [7] for a graphical representation), $f \sim 7\,\mathrm{mHz}$ is found to be abundant in accurately localized binaries for typical configurations of eLISA, and $\Delta V \lesssim 100\,\mathrm{Mpc}^3$ is achieved at 100 Mpc around 7 mHz for both type of errors. The number of accurately localized binaries is proportional to $\mathcal{M}^{5/6}$ for both cases. Stated differently, the number will be determined by the weighted average, $\langle\mathcal{M}^{5/6}\rangle$. As for LISA's design, the above expression indicates that the number is proportional to $[T/S(f)]^{3/4}$. In fact, the realistic increase in time will be faster than $T^{3/4}$, because longer observations will improve the detector sensitivity by removing more foreground noise associated with Galactic binary white dwarfs, so that $S(f)$ will be reduced simultaneously [21].

Figure 1 shows the total number of nonmerging binary black holes localized more accurately than ΔV, $N(<\Delta V)$. The estimate derived by the total distance error is presented as a red curve, and we also show asymptotic relations for cases that one of the distance error becomes dominant. This figure shows that $N(<\Delta V)$ integrated over frequency follows the expected relation, $\propto (\Delta V)^{3/4}$ at small ΔV and $\propto (\Delta V)^{1/2}$ at large ΔV, until it levels off at the detection threshold. For comparison, we indicate the volume corresponding to the typical number density of giant galaxies with $\gtrsim 10^9 M_\odot$ and dwarf galaxies with $\gtrsim 10^7 M_\odot$. In this study, we adopt a galaxy stellar mass function of Ref. [23] derived by the GAMA survey in the local Universe for concreteness, and any reasonable function does not modify our discussions.

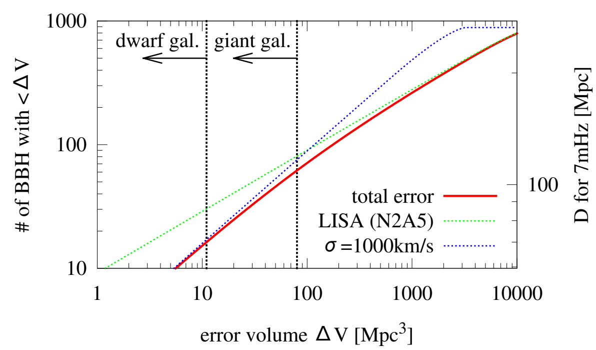

FIG. 1. Cumulative number of nonmerging binary black holes with the error volume smaller than ΔV (the red solid curve). The green and blue dashed curves are asymptotic relations derived using the distance error associate with only the statistical error of LISA and only the peculiar velocity of galaxies, respectively. The blue curve is saturated at ~ 900, which corresponds to the number of binaries with $\rho > 8$ (hypothetical detection threshold). The vertical dashed lines denote the inverse of typical number densities of dwarf (left) and giant (right) galaxies [23]. The right axis shows the luminosity distance corresponding to the number of binaries (left axis) under the assumption that all the nonmerging binaries are uniformly filling the effective volume for 7 mHz.

IV. HOST GALAXY DETERMINATION

The measurement error of the Hubble parameter, $H_0 = cz/D$, is determined by the statistical error of the luminosity distance and contamination of the cosmological redshift due to the peculiar velocity. The cosmic variance due to the coherent peculiar velocity field is nonnegligible in the local measurement concerned here, and its estimation is an entirely different topic from our current problem [17–19]. To the first order in the measurement error, the sum of the distance error and the contamination due to the random peculiar velocity can be approximated by $\sqrt{(\Delta D/D)^2 + [\sigma/(H_0 D)]^2}$. This term behaves as $\propto D^{-1}$ at the small distance and $\propto D$ at the large distance. As the number $N \propto D^3$ of available standard sirens increases, the error reduces approximately as $N^{-1/2} \propto D^{-3/2}$. Thus, the measurement error of the Hubble parameter is conservatively estimated by $(\Delta D/D)/\sqrt{N}$ using Eq. (4) for the farthest standard siren. The problem is how many binary black holes can be utilized as the standard sirens, and the number depends crucially on how many of them can be associated with the unique host.

We first consider the scenario that the majority of binary black holes are associated with giant galaxies with $\gtrsim 10^9 M_\odot$. This seems plausible, because more than 90% of the galaxy stellar mass is contained in giant galaxies [23]. In this scenario, ≈ 60 binaries at $\lesssim 100\,\mathrm{Mpc}$ may be assigned with unique host galaxies. This value is quite promising for determining the local Hubble parameter, because the error in the luminosity distance and the scatter associated with the random peculiar velocity are suppressed as $1/\sqrt{N(<\Delta V)}$ and may become subdominant [24]. Then, the cosmic variance will limit the accuracy to a few % [24], which is better than the extent of the current Hubble tension [14]. The cosmic variance becomes also irrelevant when our aim is to compare the local Hubble parameters obtained by the traditional distance ladder and the gravitational-wave cosmography, sampling an identical volume.

We should also note that most of the globular clusters are associated with galaxies whose number density is $\approx 0.01\,\mathrm{Mpc}^{-3}$ [25], close to the giant galaxies mentioned above. Thus, we can apply our discussions for giant galaxies also to the dynamical formation scenario of binary black holes in globular clusters.

The situation becomes subtle if binary black holes are primarily associated with numerous dwarf galaxies with $\gtrsim 10^7 M_\odot$ [26]. Taking the fact that the completeness of the survey is not very high for dwarfs even in the local Universe, the typical number density should be no lower than $\sim 0.1\,\mathrm{Mpc}^{-3}$. In this case, the number of binary black holes with unique host galaxies is reduced to ≈ 15 located at $\lesssim 60\,\mathrm{Mpc}$ at best. Thus, the determination of the Hubble parameter becomes inaccurate due to the error sources raised above.

Even in this case, LISA will offer fruitful information of the local Universe. Gravitational-wave observations will help to calibrate the distance ladder by providing accurate estimates of the luminosity distance. Furthermore, the number will be sufficient to infer typical host galaxies of stellar-mass binary black holes. Because the host determination of binary black holes is extremely difficult with ground-based detectors [27, 28], investigation of host galaxies can be one of important scientific goals of LISA.

Galaxies in the Local Group may also help to understand the characteristics of typical host galaxies of binary black holes. As pointed out in Ref. [6], Milky-Way equivalent galaxies should typically possess massive binary black holes in the very low frequency range of $\lesssim 0.5\,\mathrm{mHz}$. Even though LISA's sensitivity is low at very low frequency, the signal could be detected with a moderate strength due to the proximity of the Local Group including small galaxies such as the Small Magellanic Cloud. The localization may be easy for such close galaxies.

Although we have restricted our attention to the binaries with unique host candidates for simplicity, binaries with multiple host candidates can also be utilized to extract cosmological information in a statistical manner, e.g., by assigning appropriate likelihood for each candidate in the error volume [29, 30]. This strategy is appealing, because the accessible distance and hence the number of available binaries increase. A large number of dwarf galaxies can be handled in the same manner. A similar strategy also works for the case that the host galaxy exists in a relatively dense structure such as a cluster and thus the error volume of $\approx 100\,\mathrm{Mpc}^3$ or even $\approx 10\,\mathrm{Mpc}^3$ (see vertical lines in Fig. 1) is not small enough to uniquely determine the host galaxy. In this case, we can alternatively take the average redshift of the relevant galaxies, resulting in reduction of the scatter induced by random motions within the structure.

V. COMPARISON WITH OTHER PROPOSALS OF GRAVITATIONAL-WAVE COSMOLOGY

As stated in Sec. I, stellar-mass binary black holes will serve as a unique standard siren to investigate the local Universe around 100 Mpc. Supermassive binary black holes have frequently been discussed as a standard siren for eLISA/LISA (see, e.g., Ref. [31–34], and see also Ref. [35, 36] for an alternative approach). Because targeted redshifts are usually $z \gtrsim 1$ for such sources, they will play an important role to bridge the gap between local and global (i.e., cosmic microwave background) measurements and also to explore the dark-energy equation of state. This strategy, however, heavily relies on accurate localization by electromagnetic counterparts, whose association is not guaranteed (but see also Ref. [30] for statistical redshift determination). Whereas extreme mass ratio inspirals may serve as a probe to the relatively local Hubble parameter of $z \gtrsim 0.1$, the redshift has to be determined by statistical methods even with LISA [29]. In the future, Big Bang Observer (BBO) may become an-other important tool to study cosmology with ultrahigh precision by observing compact binary coalescences from the space [9, 37].

VI. SUMMARY AND FUTURE OUTLOOK

We proposed a possibility of novel gravitational-wave cosmography by stellar-mass binary black holes with a space-borne detector, LISA. High accuracy of the luminosity-distance and angular-position estimation by LISA may allow us to determine host objects. We estimate that the local Hubble parameter will be determined with accuracy of up to a few % via extraction of the cosmological redshift from the hosts. Because the luminosity distance is determined absolutely, gravitational-wave cosmography would give a clue to resolve the disputed Hubble tension [14]. At the same time, LISA will also enable us to investigate host galaxies of binary black holes such as their types [26].

The analysis performed in this study can be improved in various directions. Examples include Bayesian parameter estimation allowing multiple host galaxies, mock simulations using galaxy catalogs, and careful assessment of the finite-number effect and cosmic variance. It would also be interesting to examine the prospect for studying the properties of individual galaxies and clusters of galaxies with binary black holes. We leave these topics for future study.

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comments added At the 11th LISA symposium we were informed that a related work (W. Del Pozzo and A. Sesana) was independently underway.

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