Cosmology with independently varying neutrino temperature and number

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Cosmology with Independently Varying Neutrino Temperature and Number

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We consider Big Bang nucleosynthesis and the cosmic microwave background when both the neutrino temperature and neutrino number are allowed to vary from their standard values. The neutrino temperature is assumed to differ from its standard model value by a fixed factor from Big Bang nucleosynthesis up to the present. In this scenario, the effective number of relativistic degrees of freedom, $N_{\rm eff}^{\rm CMB}$, derived from observations of the cosmic microwave background is not equal to the true number of neutrinos, N_ν. We determine the element abundances predicted by Big Bang nucleosynthesis as a function of the neutrino number and temperature, converting the latter to the equivalent value of $N_{\rm eff}^{\rm CMB}$. We find that a value of $N_{\rm eff}^{\rm CMB} \approx 3$ can be made consistent with $N_\nu = 4$ with a decrease in the neutrino temperature of $\sim 5\%$, while $N_\nu = 5$ is excluded for any value of $N_{\rm eff}^{\rm CMB}$. No observationally-allowed values for $N_{\rm eff}^{\rm CMB}$ and N_ν can solve the lithium problem.

I. INTRODUCTION

Cosmological observations currently provide some of the most informative probes of physics beyond the Standard Model of particle physics. An analysis of the temperature anisotropies from the Cosmic Microwave Background (CMB) and the predictions made by Big Bang Nucleosynthesis (BBN) are two of the most robust methods available to gain an understanding of the physics governing the early Universe.

One of the first such cosmological constraints drawn from such an analysis was the derivation of an upper limit on the number of neutrinos inferred from the abundance of ^{4}He using BBN predictions to be $N_\nu \leq 5$ [1]. More recently, precise measurements of the CMB by the WMAP [2] and PLANCK [3] collaborations placed stringent limits on the effective number of relativistic species $N_{\rm eff}^{\rm CMB}$. The parameter $N_{\rm eff}^{\rm CMB}$ is an estimate of the total energy density contained in relativistic particles at recombination, parametrized in terms of the number of effective two-component neutrinos N_ν. The analysis done by the PLANCK collaboration reveals [3]

$$N_{\rm eff}^{\rm CMB} = 3.15 \pm 0.23, \tag{1}$$

where this value and its best-fit estimate are inferred in a Bayesian treatment by combining the Planck measurements with other cosmological data sources. While $N_{\rm eff}^{\rm CMB}$ includes any particles that are relativistic at recombination, we focus on the case that such "dark radiation" consists entirely of neutrinos and leave more exotic possibilities to future work.

Though not necessarily the case, it is often assumed that CMB measurements of $N_{\rm eff}^{\rm CMB}$ provide a direct probe of the number of neutrinos N_ν. CMB observations are generally only sensitive to the total neutrino energy density at recombination through its effect on the expansion rate. The neutrino energy density at recombination is therefore the direct CMB observable, which does not depend only on N_ν, but also on the neutrino temperature T_ν. The equivalence between N_ν and the value of $N_{\rm eff}^{\rm CMB}$ derived from CMB observations assumes a standard neutrino thermal history, which we refer to henceforth as the "standard model" (SM) temperature of neutrinos $T_{\nu \rm SM}$. The equivalence between N_ν and $N_{\rm eff}^{\rm CMB}$ is broken if nonstandard processes take place after neutrino decoupling, resulting in a temperature deviation from the usual $T_{\nu \rm SM}$.

The possibility that the neutrinos might have a nonstandard temperature has a rich history and has been explored in numerous papers [4–11]. As an example of such a scenario, particles that decay after the neutrinos decouple at a temperature of a few MeV may raise the photon temperature relative to the neutrinos, giving an effective neutrino temperature that is lower than that of the standard cosmological scenario. A similar effect can also occur for MeV-mass dark matter, which can stay in thermal equilibrium long enough to heat the photons relative to the neutrinos.

In this more general case, the statement that $N_{\rm eff}^{\rm CMB} = N_\nu$ no longer holds true and is now generalized to

$$N_{\rm eff}^{\rm CMB} T_{\nu \rm SM}^4 = N_\nu T_\nu^4, \tag{2}$$

where the total neutrino energy density (as long as the neutrinos are relativistic) is proportional to $N_\nu T_\nu^4$. The usual case of $N_{\rm eff}^{\rm CMB} = N_\nu$ is recovered if a standard-model neutrino temperature $T_{\nu SM}$ is assumed, with the more general treatment given by Eq. (2).

This degeneracy between $N_{\rm eff}^{\rm CMB}$ and N_ν can however be disentangled by combining CMB and BBN observables. For example, Nollett and Steigman [10] recently used the combination of BBN abundance predictions and CMB measurements to constrain electromagnetically coupled dark matter particles that raise the photon temperature relative to that of the neutrinos. They also showed in [11] that the opposite effect can be accomplished if a coupling is introduced between the dark matter sector(s) and neutrinos. In this paper, we consider the most general case, in which N_ν and T_ν are treated as free parameters, and then determine the observational constraints obtained from a combination of BBN and the CMB.

Before the neutrinos decouple at the temperature $T_d \approx 2-3$ MeV, the weak rates insure that $T_\nu = T_\gamma$. We make the assumption that the change in T_ν is induced after T_d, but before BBN begins at $T_\gamma \sim 1$ MeV, and that the neutrino temperature subsequently evolves in the standard way as the inverse of the scale factor. This is admittedly a narrow window over which the change is assumed to occur, and it is also true that the onset of BBN is not a sudden process. We discuss these issues in more detail below.

The paper is organized as follows: in section II we discuss how T_ν and N_ν affect BBN and CMB observables, while in section III we explore these effects more precisely using numerical simulations of BBN and its effects on primordial abundance predictions of ^{4}He and deuterium. Combining these results with observational limits on the primordial element abundances, we derive corresponding limits on T_ν and N_ν and then examine the effects on $N_{\rm eff}^{\rm CMB}$. We find that even current observational bounds on ^{4}He and deuterium, when combined with CMB limits, can be consistent with one extra sterile neutrino for a change in T_ν from the standard model value of only $\sim -5\%$, while two additional sterile neutrinos are ruled out. We discuss certain implications of this analysis and conclude in section IV.

II. THE EFFECTS OF N_ν, $N_{\rm eff}^{\rm CMB}$ AND T_ν ON THE CMB AND BBN

Currently, the number of neutrinos N_ν can be probed in two separate eras of cosmic evolution, both producing distinct and independent observables. These two eras are (*i*) *Big Bang nucleosynthesis* (BBN), when light nuclear elements are produced and (*ii*) *Recombination*, when electrically neutral atoms first form allowing photons to free-stream and produce the cosmic microwave background (CMB) radiation. These two eras differ by orders of magnitude in energy, with BBN occurring around ~ 1 MeV and CMB decoupling occurring at the eV scale.

Consider first the standard neutrino thermal history. At sufficiently high temperatures, the weak interactions are sufficiently rapid to keep the neutrinos in equilibrium with the thermal background, so that $T_\nu = T_\gamma$. At a temperature $T_d \approx 2-3$ MeV, the neutrinos decouple from the thermal background. Then T_ν is solely affected by the expansion of the universe, scaling as $T_\nu \propto a^{-1}$, where a is the cosmic scale factor.

When the temperature drops below the mass of the electron, e^+e^- pairs annihilate, heating the photons relative to the neutrinos, and producing a final ratio of

$$T_{\nu\rm SM} = (4/11)^{1/3} T_\gamma. \tag{3}$$

The assumption made in this scenario is that there are no processes that modify either the neutrino temperature or the photon temperature between neutrino decoupling and BBN. We refer to this evolution as the standard model evolution, hence our use of the SM subscript.

Note that the neutrinos are partially heated by the e^+e^- annihilations. Neither the process of neutrino decoupling, nor the annihilations of the e^+e^- occurs sharply at a single temperature, and the resulting overlap between these two processes [9] results in

$$T_{\nu\rm SM} > (4/11)^{1/3} T_\gamma. \tag{4}$$

This effect is usually absorbed into the definition of N_ν rather than T_ν, giving an effective neutrino number of $N_{\rm eff} = 3.046$ [12, 13]. (See also the more recent discussion by de Salas and Pastor [14], which gives a similar value of $N_{\rm eff} = 3.045$). However, this effect is very small compared to the large changes in N_ν and T_ν considered here, so we ignore it in what follows.

Now suppose that this neutrino thermal history is modified by some process similar to those discussed in Refs. [4–11]. Rather than specifying a particular process, we will attempt to keep the discussion as general as possible. Note that the measurable quantity that affects both BBN and the CMB is not the absolute value of T_ν, but the ratio of T_ν to T_γ, since all calculations for BBN and the CMB are scaled off of the background photon temperature. For simplicity, we treat any change in T_ν/T_γ as an effective change in T_ν. However, note that T_ν/T_γ can be altered by changing *either* T_ν or T_γ. We assume here that a physical process occurring after T_d, but before the beginning of BBN, alters T_ν/T_γ by a fixed amount. This can be accounted for by changing the neutrino temperature from its standard model value, given by Eq. (4), to a new value, which we treat as a free parameter in the calculations. Then T_ν evolves in the standard way (inversely proportional to the scale factor), but in such a way that

$$T_\nu / T_{\nu SM} = constant \neq 1, \tag{5}$$

from the beginning of BBN up to the present day.

A second possibility is that T_ν takes its standard model value during BBN, but then changes between BBN and decoupling. For example, entropy release between BBN and decoupling results in an increase in T_γ, so an effective decrease in T_ν at a given value of T_γ. These scenarios are straightforward to analyze: BBN proceeds in the standard way, but the CMB limits are altered by changing the value of $N_{\rm eff}^{\rm CMB}$ relative to N_ν as given by Eq. (6) below. We will not examine such scenarios in detail here.

The model we are examining is admittedly limited in applicability, since the window between T_d and the onset of BBN is narrow. However, there are certainly physical processes which can yield the case we investigate here in appropriate limits. For instance, a decaying particle with a lifetime much shorter than the age of the universe at T_d will heat the photons relative to the neutrinos in the exponential tail of its decay, with negligible effects at later times when BBN begins. An electromagnetically-coupled WIMP with a mass of $1-10$ MeV would annihilate largely after neutrino decoupling and before the onset of BBN, as noted in Ref. [10]. Of course, the most general possible case would allow for $T_\nu/T_{\nu SM}$ to evolve before, during, and after BBN, but such a general treatment is beyond of the scope of this paper.

The CMB measurements are sensitive to the total energy density at the epoch of recombination; the neutrino energy density enters into this calculation only through its total energy density, which is simply proportional to $N_\nu T_\nu^4$. If we allow both N_ν and T_ν to have values that differ from their standard-model values, then the number of neutrinos inferred from CMB measurements will be

$$N_{\rm eff}^{\rm CMB} = N_\nu (T_\nu/T_{\nu SM})^4. \tag{6}$$

We can therefore quantify a possible shift in the neutrino temperature through the ratio of $T_\nu/T_{\nu SM}$.

The processes involved in Big Bang Nucleosynthesis also depend on both N_ν and T_ν, but in a more complex way than the CMB observables do. (For a recent review of BBN, see Ref. [15]). First, the element abundances depend on the expansion rate during BBN given by

$$H^2 = \frac{8\pi G}{3}(\rho_\gamma + \rho_{e^+e^-} + \rho_{\nu\bar{\nu}}), \tag{7}$$

where the three terms that contribute to the total energy density are those of the photons, electron-positron pairs and neutrinos, respectively. The neutrino term includes the contributions of all three standard-model neutrinos as well as any possible nonstandard (e.g., sterile) neutrinos. If the expansion rate was the only place where the neutrino temperature entered into the BBN calculations, then the primordial element abundances would only depend on the total neutrino energy density, just like the CMB observations. That this is not the case is the fundamental reason we can break the degeneracy between $N_{\rm eff}^{\rm CMB}$ and N_ν.

Beyond its role in the expansion rate during BBN, the neutrino temperature also plays a crucial role in the weak interaction rates, which determine the light element abundances. Down to temperatures of ~ 1 MeV, the protons and neutrons are kept in thermal equilibrium via the following weak interaction processes

$$\begin{aligned} n + \nu_e &\leftrightarrow p + e^-, \\ n + e^+ &\leftrightarrow p + \bar{\nu}_e, \\ n &\leftrightarrow p + e^- + \bar{\nu}_e. \end{aligned} \tag{8}$$

The total rates for the conversion of neutrons to protons and protons to neutrons are

$$\begin{aligned} \lambda_{n\to p} = A \int_{m_e}^{\infty} dE_e \frac{E_e |p_e|}{1+\exp[E_e/kT_e]} \\ \times \left\{ \frac{(E_e+Q)^2}{1+\exp[-(E_e+Q)/kT_{\nu_e}]} + \frac{(E_e-Q)^2 \exp(E_e/kT_e)}{1+\exp[(E_e-Q)/kT_{\nu_e}]} \right\}, \end{aligned} \tag{9}$$

and

$$\lambda_{p\to n} = \lambda_{n\to p}(-Q), \tag{10}$$

respectively, where $Q = m_n - m_p$, and the subscripts e and ν_e denote the quantities associated with the electron and the electron-neutrino, respectively. The constant A is determined from the requirement that $\lambda_{n\to p}(T, T_{\nu_e} \to 0) = 1/\tau_n$ (the neutron decay rate).

The important point is that these weak rates are sensitive to the neutrino temperature T_ν, where we assume that neutrino mixing gives all three neutrinos the same temperature. (The actual effect on the various element abundances is discussed in detail in the next section). This dependance on T_ν allows a combination of BBN abundance predictions and CMB observations to yield complementary limits on T_ν and N_ν when these quantities are varied independently.

The two observables we discuss here (BBN and CMB) are not the only means by which the degeneracy between N_ν and T_ν can be broken. A third possibility involves large-scale structure constraints on the neutrino mass. Since these observations are based on an epoch at which the neutrinos have become nonrelativistic, they are actually sensitive to the quantity $T_\nu^3 \Sigma_\nu m_\nu$, where the sum is over all three types of neutrinos, again assumed to have a single common temperature T_ν. A discussion of these limits and their relation to CMB and BBN limits is beyond the scope of this paper, but see the analysis in Ref. [7].

III. NUMERICAL RESULTS, OBSERVATIONAL BOUNDS, AND COMBINED BBN/CMB CONSTRAINTS

In order to investigate the interplay between a varying neutrino number N_ν and neutrino temperature T_ν on light element abundances from BBN, we solve the rate equations and cosmological evolution numerically and present our results in this section. We used the computer code *AlterBBN* [16] originally written by A. Arbey, and later modified by K. P. Hickerson [17]. Our version, modified from Hickerson's version 1.6, along with a supplementary explanation of our analysis is available at [18].

In our simulations, we allow N_ν and T_ν to vary separately. We take T_ν to be the same for all three standard-model neutrinos and any additional sterile neutrinos, which will be the case as long as there is sufficient mixing between all of the neutrino sectors. This corresponds, for instance, to the most interesting cases of mixing with sterile neutrinos to provide a possible explanation for the tension in short-baseline neutrino oscillation experiments (see, e.g., Ref. [7] and references therein).

In our simulations we take a baryon to photon ratio of $\eta = 6.19 \times 10^{-10}$ [19] and a neutron lifetime of $\tau_n = 880.3$ seconds [19], and derive the primordial element abundances as a function of two parameters: N_ν, and a nonstandard neutrino temperature T_ν. We parametrize the shift of the neutrino temperature relative to the standard-model neutrino temperature as the ratio $T_\nu/T_{\nu \mathrm{SM}}$. In order to keep the results as general as possible, we do not assume a particular model or mechanism for the nonstandard value of T_ν; instead, as noted in the previous section, we assume that the neutrino temperature differs by a constant ratio from the standard model neutrino temperature throughout BBN, and that this same ratio is maintained up to the present. Aside from this fixed choice of $T_\nu/T_{\nu SM} \neq 1$, we assume that the neutrino temperature obeys the standard evolution throughout and after BBN, i.e., it decreases as the inverse of the scale factor. One may consider other deviations from our scenario for T_ν; however, we leave such possibilities for future work and focus on the scenario listed above.

Since we assume three standard neutrinos plus an undefined additional contribution to N_ν, it is reasonable to take $N_\nu \geq 3$ to be a physical lower bound. Note, however, that there are brane-world scenarios which achieve a negative change in the relativistic energy density [20], so for completeness we allow N_ν to vary in the range $2 \leq N_\nu \leq 5$.

In Fig. 1, we show the primordial ^{4}He mass fraction, Y_p, and the deuterium and ^{7}Li number densities relative to hydrogen, as a function of N_ν and $T_\nu/T_{\nu SM}$. For the standard model value of the neutrino temperature, $T_\nu/T_{\nu SM} = 1$, we obtain the familiar result that Y_p increases with N_ν, because the increased expansion rate causes the weak rates to freeze out at a higher temperature, resulting in more neutrons, and nearly all of these neutrons (modulo free neutron decay) end up bound into ^{4}He. The deuterium abundance also increases with N_ν at fixed T_ν, as the increased expansion rate allows less time for the deuterium to fuse into heavier elements. On the other hand, the ^{7}Li abundance decreases with increasing N_ν.

The effect of altering T_ν at fixed N_ν is not as obvious, because an increase in T_ν results in both an increase in the weak rates and an increase in the expansion rate. From Fig. 1, we see that increasing T_ν at fixed N_ν results in a net increase in Y_p, indicating that the effect of increasing the expansion rate (which increases Y_p) dominates the effect of increasing the weak rates (which decreases Y_p). Note, however that these two effects begin to cancel for $T_\nu/T_{\nu SM} < 0.9$, at which point Y_p becomes less sensitive to $T_\nu/T_{\nu SM}$. The behavior of deuterium and ^{7}Li is much more straightforward, since these two nuclides are primarily sensitive to the overall expansion rate. Thus, D/H increases with the increased expansion rate produced by an increased value of $T_\nu/T_{\nu SM}$, while ^{7}Li decreases.

These results are not directly comparable to previous studies, but a subset of our results is in qualitative agreement with those of Nollett and Steigman [10] for electromagnetically-coupled WIMPS. In Ref. [10], WIMP annihilation heats the photons relative to the neutrinos, producing a net decrease in T_ν/T_γ. Our case with $N_\nu = 3$ and $T_\nu/T_{\nu SM} < 1$ corresponds roughly to the effect of a 1-10 MeV WIMP, designated "region II" in Fig. 4 of Ref. [10]. In this portion of the parameter space, Nollett and Steigman find a decrease in the production of deuterium and ^{4}He and an increase in ^{7}Li, in agreement with the behavior we see in Fig. 1 as $T_\nu/T_{\nu SM}$ decreases.

Note that while $T_\nu/T_{\nu SM}$ and N_ν are the two parameters that enter directly into the BBN calculation, they are not the most useful to use in our analysis. Instead, we take $N_{\mathrm{eff}}^{\mathrm{CMB}}$ to be one of our parameters, as this is the effective number of neutrinos measured by the CMB. Eq. (6) then leaves only one free parameter, which we can take to be either the neutrino number or temperature. Since it is the neutrino number which is the physically relevant quantity,

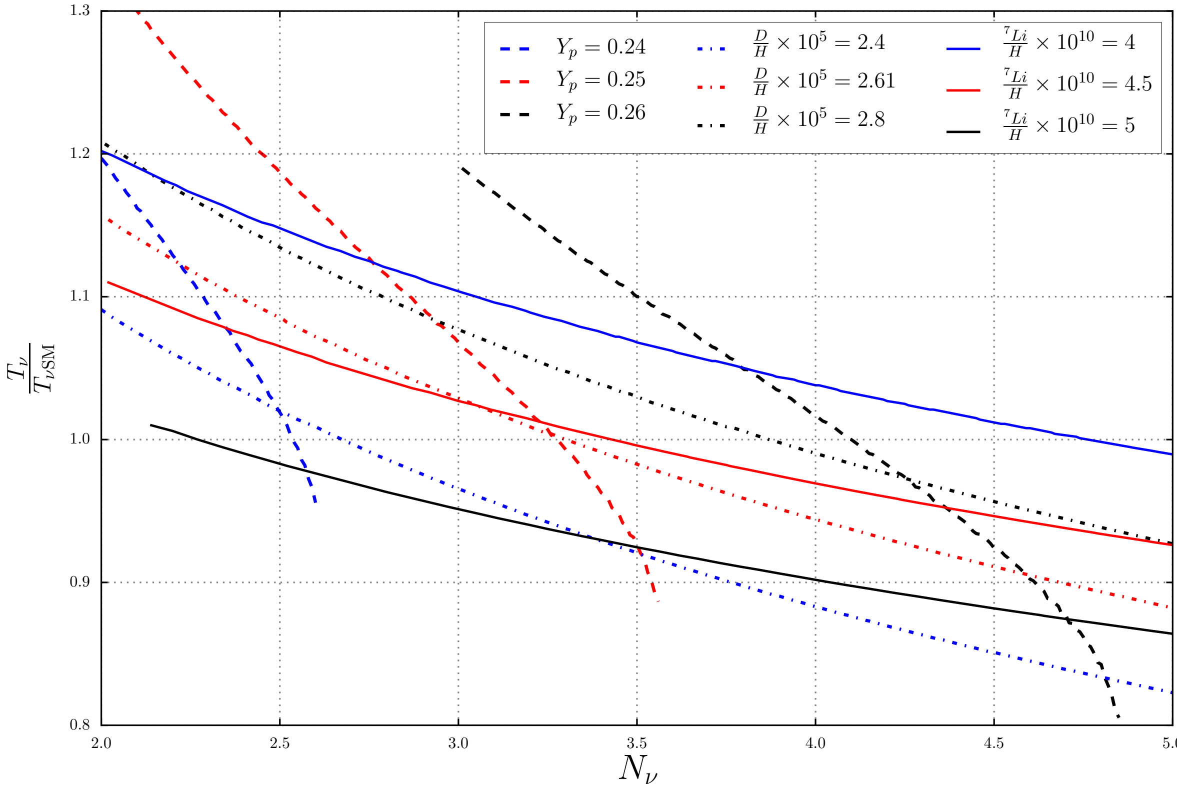

FIG. 1: Predicted primordial abundances of ^{4}He, deuterium, and ^{7}Li in the plane defined by the neutrino number, N_ν, and the ratio of the neutrino temperature to its standard-model value, $T_\nu/T_{\nu SM}$.

we adopt it as our second parameter. For a given set of values of N_ν and $N_{\rm eff}^{\rm CMB}$, the value of $T_\nu/T_{\nu SM}$ can be determined from Eq. (6).

Now consider the observational limits on the primordial ^{4}He mass fraction, Y_p, and the deuterium to hydrogen ratio D/H. For deuterium, the Particle Data group gives [19] $D/H = (2.53 \pm 0.04) \times 10^{-5}$, while a more recent measurement by Cooke et al. gives [21] $D/H = 2.547 \pm 0.033 \times 10^{-5}$. We will therefore take our limit to be

$$D/H = (2.55 \pm 0.03) \times 10^{-5}. \tag{11}$$

Recent estimates of the primordial ^{4}He mass fraction include those of Izotov et al. [22], $Y_p = 0.2551 \pm 0.0022$, and Aver et al. [23], $Y_p = 0.2449 \pm 0.0040$, while the Particle Data Group limit is [19] $Y_p = 0.2465 \pm 0.0097$. Given the discrepancy between these numbers, we will adopt as our limit

$$Y_p = 0.25 \pm 0.01. \tag{12}$$

It is well-known that the current BBN predictions for the primordial ^{7}Li abundance differ significantly from the observationally-inferred values, with the BBN predictions a factor of 3 or more above the observed values. This has been dubbed the "lithium problem" (see, e.g., Ref. [24] for a recent review). Thus, we will not use ^{7}Li to constrain the models examined here; instead, our main interest will be to determine whether these models can ameliorate the lithium problem. To that end, we will adopt a ^{7}Li abundance of [19]

$$Li/H = (1.6 \pm 0.3) \times 10^{-10}. \tag{13}$$

In Fig. 2, we present the predicted element abundances as a function of N_ν, for a variety of $N_{\rm eff}^{\rm CMB}$ values (solid curves). The dashed curve corresponds to the standard temperature case, $T_\nu/T_{\nu\rm SM} = 1$. As is clear in the figure, this curve intersects each $N_{\rm eff}^{\rm CMB}$ curve at the point $N_{\rm eff}^{\rm CMB} = N_\nu$. Curves of constant $N_{\rm eff}^{\rm CMB}$, as defined in Eq. (6),

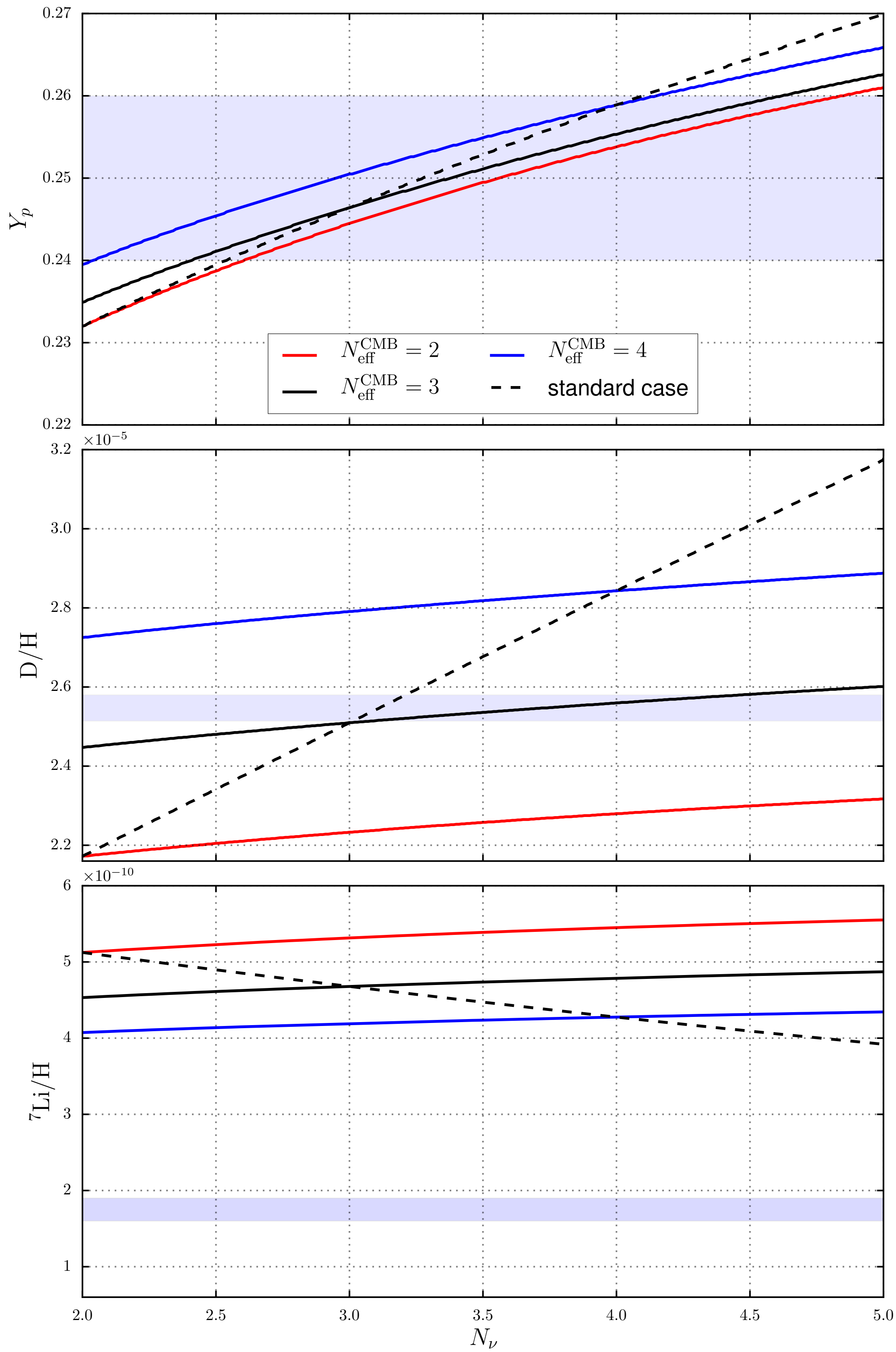

FIG. 2: Predicted primordial abundances of ^{4}He, deuterium, and ^{7}Li as a function of the number of relativistic neutrinos N_ν when the neutrino temperature is allowed to vary from its standard-model value. The neutrino temperature is parametrized in terms of $N_{\rm eff}^{\rm CMB}$ as defined in Eq. (6); solid curves give abundances for the indicated values of $N_{\rm eff}^{\rm CMB}$. Dashed curve gives abundances for the standard-model neutrino temperature. Shaded regions correspond to the observational abundance limits quoted in Eqs. (11) - (13).

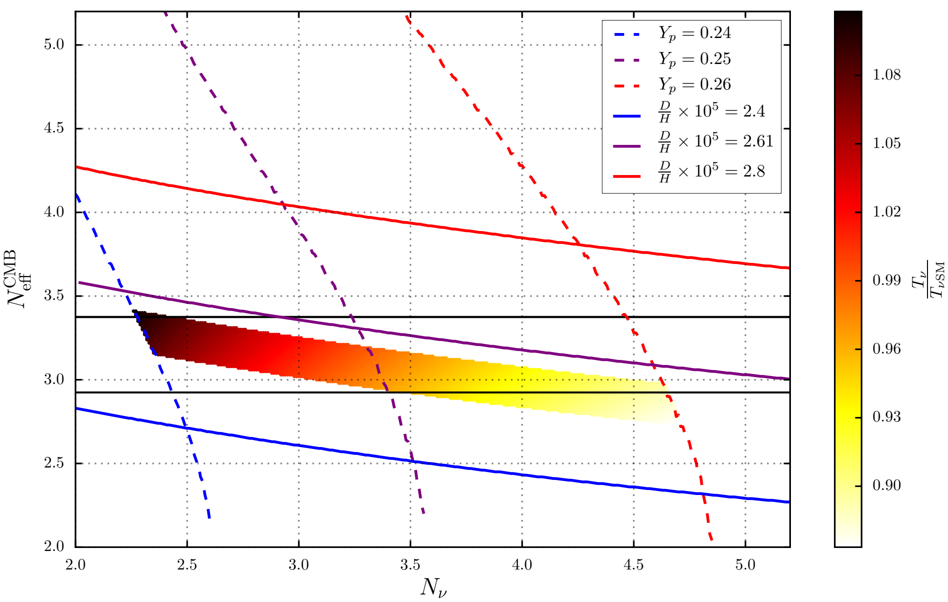

FIG. 3: Allowed region in the N_ν, $N_{\rm eff}^{\rm CMB}$ plane given by combined observational limits on deuterium and ^{4}He, along with CMB bounds on $N_{\rm eff}^{\rm CMB}$. Red-orange-yellow quadrilateral is allowed by the BBN bounds on deuterium and ^{4}He (Eqs. 11 - 12). Black horizontal lines give upper and lower bounds on $N_{\rm eff}^{\rm CMB}$ from CMB observations (Eq. 1). The overlap between these two regions is allowed by both BBN and CMB. Dashed and solid curves correspond to the indicated ^{4}He and deuterium abundances, respectively, while the heat map gives the value of $T_\nu/T_{\nu SM}$ corresponding to a given region of the parameter space allowed by BBN.

correspond to curves of constant neutrino energy density. Thus, tracing the element abundances along each solid curve allows us to see the effect of changing both the neutrino temperature and number in such a way that the neutrino energy density is unchanged. In this case, the only effect on the primordial element abundances comes from the change in the weak rates. Decreasing N_ν at fixed $N_{\rm eff}^{\rm CMB}$ corresponds to increasing T_ν. In the case of ^{4}He, for example, this results in a decrease in Y_p, since the increased neutrino temperature increases the weak rates, allowing them to stay in thermal equilibrium longer and reducing the final neutron abundance.

Now consider the observational bounds from BBN. When we move beyond the standard case for the neutrino temperature (dashed curves in Fig. 2) to allow both N_ν and $N_{\rm eff}^{\rm CMB}$ to vary independently (solid curves), we see the largest effect is a relaxation of the bounds from deuterium. The standard case corresponds to a narrow window near $N_\nu = 3$, while the model we consider here allows N_ν to have any value in the range we have investigated ($2 \le N_\nu \le 5$). This result is easy to understand; for the model discussed here, an increase an N_ν can be compensated by a decrease in T_ν to give an unchanged expansion rate. Since deuterium depends almost entirely on the expansion rate, with only a very weak dependence on the weak rates, we can always find a value of $N_{\rm eff}^{\rm CMB}$ for any given N_ν to give a deuterium abundance in the desired range.

Since Y_p is strongly dependent on both the expansion rate and the weak rates, compensating a change in N_ν with a change in T_ν to leave the expansion rate fixed does not leave the ^{4}He abundance unchanged. Consequently, primordial helium continues to provide an upper bound on N_ν even with the freedom to alter the neutrino temperature. Varying the neutrino temperature relaxes this bound somewhat, but even for $N_{\rm eff}^{\rm CMB}$ as low as 2, we still have the upper bound $N_\nu < 5$.

In Fig. 3 we combine the deuterium and ^{4}He limits to derive overall constraints in the N_ν, $N_{\rm eff}^{\rm CMB}$ plane. This figure shows the complementarity of these two sets of limits, with deuterium giving the upper and lower bounds on $N_{\rm eff}^{\rm CMB}$, and Y_p giving the upper and lower bounds on N_ν. Adding the CMB bound from Eq. (1) tightens the lower bound on $N_{\rm eff}^{\rm CMB}$, but otherwise has very little effect on the excluded region. In particular, even when we include all three sets of limits (deuterium, ^{4}He, and the CMB), the value of N_ν can be as large as 4.5, thus allowing on additional sterile neutrino. On the other hand, a value of $N_\nu = 5$ (two additional sterile neutrinos) is ruled out. Fig. 3 illustrates the additional constraining power of BBN beyond what is available with the CMB alone. When T_ν is allowed to vary freely, a given value of $N_{\rm eff}^{\rm CMB}$ from the CMB no longer provides a constraint on N_ν. But adding the BBN constraint reestablishes the upper bound on N_ν; as we have already noted, this upper bound is derived primarily from limits on ^{4}He, rather than deuterium.

In Fig. 3 we have also superimposed a heat map to illustrate the value of $T_\nu/T_{\nu SM}$ corresponding to a given region of the allowed parameter space. The region for which $N_\nu > 3$ corresponds to a value of T_ν smaller than its standard-model value. Interesting effects are achieved with a very small change in T_ν. For instance, the point corresponding to $N_{\rm eff}^{\rm CMB} = 3$ and $N_\nu = 4$ corresponds to a $\sim 5\%$ decrease in T_ν relative to its standard model value.

We can also use Fig. 3 to derive the combined BBN-CMB *lower* bound on N_ν when the neutrino number and temperature are allowed to vary independently: $N_\nu > 2.3$. This limit is considerably less interesting than our upper bound, as we already have $N_\nu \geq 3$ from the observed neutrinos. However, as noted earlier, this does serve as a constraint on nonstandard models such as those discussed in Ref. [20].

It is interesting to see whether the joint variation of N_ν and T_ν can ameloriate or solve the lithium problem. It is clear from Fig. 2 that although some combinations of N_ν and $N_{\rm eff}^{\rm CMB}$ can reduce the predicted ^{7}Li abundance, this reduction is short of what is needed to close the gap between prediction and observation. Furthermore, the largest reductions in the predicted abundance lie in regions of parameter space that are excluded by the deuterium and ^{4}He observations.

IV. CONCLUSIONS

While observations of the CMB yield very precise limits on cosmological parameters, our results show that Big Bang nucleosynthesis remains an indispensible tool. For models in which the neutrino number and temperature can both vary, the CMB alone cannot produce any limits on N_ν, while a combination of the CMB and BBN yields a very useful bound.

In the models examined here, a value of the neutrino number as determined from the CMB of $N_{\rm eff}^{\rm CMB} \approx 3$ can be consistent with a true neutrino number, N_ν, as large as 4, thus allowing for an additional sterile neutrino. Such a model requires a reduction in the neutrino temperature of approximately 5% relative to the standard model neutrino temperature. However, a value of $N_\nu = 5$ is ruled out for any value of $N_{\rm eff}^{\rm CMB}$.

The obvious direction for future investigation would involve more complex behavior for the evolution of the neutrino temperature, both during and following BBN. Some of these types of behavior have been discussed previously in Refs. [4–11], but these studies do not by any means exhaust all of the interesting possibilities.

Acknowledgments

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