Fundamental implications of intergalactic magnetic field observations

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Fundamental Implications of Intergalactic Magnetic Field Observations

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Helical intergalactic magnetic fields at the $\sim 10^{-14}$ G level on ~ 10 Mpc length scales are indicated by current gamma ray observations. The existence of magnetic fields in cosmic voids and their non-trivial helicity suggest that they must have originated in the early universe and thus have implications for the fundamental interactions. We derive the spectrum of the cosmological magnetic field as implied by observations and MHD evolution, yielding order nano Gauss fields on kiloparsec scales and a "large helicity puzzle" that needs to be resolved by the fundamental interactions. The importance of CP violation and a possible crucial role for chiral effects or axions in the early universe are pointed out.

I. INTRODUCTION

Several independent investigations of gamma rays from blazars indicate the presence of intergalactic magnetic fields [1–7]. Emission of TeV energy gamma rays from blazars and the subsequent electromagnetic cascade in the intergalactic medium is expected to distort the intrinsic blazar spectrum by depleting photons from the TeV range and adding photons in the GeV range. The lack of expected additional photons in the GeV range is explained by invoking an intergalactic magnetic field of strength $\gtrsim 10^{-16}$ GeV. As an intergalactic magnetic field disperses the additional GeV photons, the intergalactic magnetic field hypothesis also predicts a halo of GeV photons around the blazar. An analysis of stacked blazars provides evidence for such a halo and adds support to the derived lower bound on intergalactic magnetic fields [6].

An alternative approach developed in Refs. [8, 9] utilizes the *helical* nature of intergalactic magnetic fields. The reasoning is that intergalactic magnetic fields are measured in cosmic voids, ~ 100 Mpc away from astrophysical sources, and thus were most likely generated in the early universe. (For a review of magnetic fields and some possible astrophysical generation mechanisms see Ref. [10].) Unless the magnetic fields are coherent on very long length scales or are helical at the time of production, they would dissipate and not survive until the present epoch. If the magnetic field generation mechanism was causal, the magnetic fields are not coherent on large length scales and helicity is essential for survival. Furthermore, the observation of helicity can help distinguish between cosmological and astrophysical magnetic fields as a globally preferred sign of the helicity would be indicative of a fundamental production mechanism.

In Refs. [8, 9] it was shown that the helicity of the intergalactic magnetic field leaves a parity odd imprint on the distribution of cascade gamma rays. Thus helicity can be deduced by calculating parity odd correlators of observed gamma ray arrival directions. (Simulations of the process can be found in [11, 12].) Using this technique, it becomes possible to *measure* – not jut bound – the power spectra of intergalactic magnetic fields. Applying this technique on current Fermi-LAT data, Refs. [4, 5] estimate the intergalactic magnetic field to be $\sim 10^{-14}$ G as measured on a length scale ~ 10 Mpc . The statistical significance of these measurements is at $\sim 3.5\sigma$ level in analysis with current data [13]. Further observations, especially using a variety of observational tools, will be able to confirm or refute these findings. For this paper we proceed on the assumption that the accumulating observational evidence is correct.

The existence of helical intergalactic magnetic fields points to an early universe origin and therefore is of interest to particle cosmology. As observational dataset gets larger, it will become possible to measure the magnetic field correlation functions over a range of scales. If the spectrum is flat or red, *i.e.* does not fall off at large length scales, the magnetic field would likely be a product of the big bang or inflation. In this case, the primordial magnetic field may shed light on cosmological initial conditions and it may also have important consequences for the origin of the matter-antimatter asymmetry [14, 15] and other theoretical ideas [16]. If the spectrum is measured to be blue, we expect the magnetic field to have been produced in high energy particle processes, and the helicity of the magnetic field points to an important role for CP violating interactions in the early universe.

For the rest of our discussion, we will assume that the intergalactic magnetic field is stochastic and isotropic, and is generated by a causal mechanism. (If the generation mechanism were acausal, the field may not even be stochastic within our cosmic horizon.) Then the spatial correlation function of the magnetic field is given by [17]

$$\langle B_i(\boldsymbol{x})B_j(\boldsymbol{x}+\boldsymbol{r})\rangle = M_N(r)P_{ij}+M_L(r)\hat{r}_i\hat{r}_j+\epsilon_{ijk}\hat{r}_k M_H(r) \tag{1}$$

where $P_{ij} = \delta_{ij} - \hat{r}_i\hat{r}_j$. $M_N(r)$ and $M_L(r)$ are the "normal" and "longitudinal" power spectra and are related by a differential equation [17]; $M_H(r)$ is the helical power spectrum and is what is measured by the parity odd gamma ray correlators.

We will next determine the full spectrum of the magnetic field in Sec. II, discuss implications and the "large helicity puzzle" in Sec. III, and provide some potential resolutions of the puzzle in Sec. IV. We conclude in Sec. V.

II. THE FULL SPECTRUM

Our first task is to relate the spatial helical correlation function to its counterpart in Fourier space because the magneto-hydrodynamic (MHD) evolution of the magnetic field is carried out in Fourier space while the field correlations are measured in physical space. The Fourier space correlation functions for a stochastic, isotropic magnetic field are written as

$$\begin{aligned}\langle b_i(\boldsymbol{k}) b_j^*(\boldsymbol{k}')\rangle &= \left[\frac{E_M(k)}{4\pi k^2} p_{ij} + i\epsilon_{ijl} k_l \frac{H_M(k)}{8\pi k^2}\right] \\ &\qquad \times (2\pi)^6 \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\end{aligned} \tag{2}$$

where $p_{ij} = \delta_{ij} - \hat{k}_i \hat{k}_j$ and

$$\boldsymbol{b}(\boldsymbol{k}) = \int d^3x \boldsymbol{B}(\boldsymbol{x}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}, \quad \boldsymbol{B}(\boldsymbol{x}) = \int \frac{d^3k}{(2\pi)^3} \boldsymbol{b}(\boldsymbol{k}) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \tag{3}$$

We now use Eq. (3) in (1) to obtain

$$M_H(r) = \frac{1}{2}\int_0^\infty dk\ k H_M(k) \frac{d}{d\rho}\left(\frac{\sin\rho}{\rho}\right) \tag{4}$$

where $\rho = kr$.

Studies of the MHD equations show that a cosmological magnetic field with helicity evolves so that at late times [18–22]

$$E_M(k) = \frac{k}{2}|H_M(k)| = \begin{cases} E_0 (k/k_d)^4, & 0 \le k \le k_d \\ 0, & k_d < k \end{cases} \tag{5}$$

where the first equality is the relation for maximal helicity, the functional dependence k^4 defines the "Batchelor spectrum", and k_d is a dissipation scale that will be discussed below. For $k > k_d$, the spectrum falls off rapidly and so we have set it to zero. Strictly, the Batchelor spectrum only applies for $k < k_I$ where $k_I < k_d$ is the "inertial scale" where the spectrum peaks. For $k_I < k < k_d$, the spectrum falls off as a power law and there is a sharper fall off for $k > k_d$ [67] [23]. For simplicity, we have taken $k_I \approx k_d$, which may also be justified if the magnetic field is generated on very small scales. We shall also assume $H_M(k) \ge 0$ to be concrete. Below we will estimate the power spectrum amplitude, E_0 in Eq. (5).

Gamma ray observations have been used to measure $M_H(r)$. So we use Eq. (5) in (4) to obtain $M_H(r)$

$$M_H(r) = \frac{E_0 k_d}{\rho_d^5}[(\rho_d^3 - 8\rho_d)\sin\rho_d + 4(\rho_d^2 - 2)\cos\rho_d + 8] \tag{6}$$

where $\rho_d \equiv k_d r$. One can check: $M_H(r) \propto r$ as $r \to 0$ and $M_H(r) \to \sin(k_d r)/r^2$ as $r \to \infty$, so $M_H(r)$ is well-behaved for all r.

Any observation will measure a "smeared" $M_H(r)$. For example, gamma ray observations in Refs. [4, 5] measure M_H on a certain distance scale r that is determined from the energies of observed gamma rays. However, for statistical purposes, the observed gamma rays are binned according to their energies – in 10 GeV wide bins in Refs. [4, 5]. This means that observations yield M_H that is smeared over a range, Δr, of r. With present day observations, r is typically on the order of Mpc, and $l_d = 2\pi/k_d$ is typically kpc, so that $\rho_d = k_d r \gg 1$. The precise smearing function depends on the binning procedure and experimental details (*e.g.* energy dependence of time exposure of the experiment), however, with current parameters $l_d \ll \Delta r \lesssim r$.

Let us write $\Delta\rho_d = k_d \Delta r$. Then, from Eq. (6), the smearing procedure will effectively replace the oscillating trigonometric functions by (weighted) averages. For example,

$$\frac{\sin\rho_d}{\rho_d^2} \to \frac{1}{\Delta\rho_d}\int_{\rho_d}^{\rho_d+\Delta\rho_d} d\rho \frac{\sin\rho}{\rho^2} \approx \frac{\mathcal{O}(1)}{\rho_d^2}. \tag{7}$$

Since $\rho_d \gg 1$, the ρ_d^3 term in the square bracket in Eq. (6) will dominate and we can write

$$M_H(r) \approx \frac{E_0 k_d}{\rho_d^2} \tag{8}$$

Therefore a measurement of $M_H(r)$ at $r = r_*$, denoted M_{H*}, will give

$$E_0 = k_d r_*^2 M_{H*} \tag{9}$$

The magnetic field energy and helicity spectra in Eq. (5) become,

$$E_M(k) = \frac{k}{2}|H_M(k)| = k_d r_*^2 |M_{H*}| \left(\frac{k}{k_d}\right)^4 \tag{10}$$

From Eq. (50) of Ref. [8] [68] we have the estimate

$$|M_{H*}| \sim (10^{-14}\ \mathrm{G})^2 \tag{11}$$

and $r_* \sim 10$ Mpc. Subsequent (and ongoing) analyses [5] show rough agreement with these estimates and future observations should be able to pin down the values more accurately. Other analyses [1–3, 6] do not provide measurements of the field strength but they do provide lower bounds if they assume a coherence scale and a spectrum. These lower bounds on the field strength are on the order of 10^{-16} G (see Fig. 12 of Ref. [10]).

The energy density in the magnetic field is

$$\mathcal{E} = \frac{1}{2}\langle \boldsymbol{B}^2\rangle = \int dk\ E_M(k) \sim (10^{-14}\ \mathrm{G})^2 \frac{k_d^2 r_*^2}{5} \tag{12}$$

Similarly the helicity density is given by

$$\begin{aligned} H &= \lim_{V\to\infty}\left\langle \frac{1}{V}\int_V d^3x \boldsymbol{A}\cdot\boldsymbol{B}\right\rangle \\ &= \int dk\ H_M(k) \sim (10^{-14}\ \mathrm{G})^2 \frac{k_d r_*^2}{2} \end{aligned} \tag{13}$$

where $\boldsymbol{B} = \mathrm{curl}(\boldsymbol{A})$.

Next we discuss the dissipation length scale l_d. In Ref. [24], the authors considered a homogeneous magnetic field and calculated the damping rate of small perturbations on this background. The dominant dissipation of the small perturbations is due to the damping of fast magnetosonic modes. Hence this mechanism sets the dissipation scale that then depends on the strength of the background uniform field.

The damping of a *stochastic, helical* magnetic field has been discussed in Ref. [25, 26]. The evolution of the dissipation scale, which roughly coincides with the coherence scale for the Batchelor spectrum, depends on properties of the magnetic field at the time it was generated. The result for the dissipation scale at the present epoch is (see Eqs. (4) and (5) of [27])

$$\begin{aligned} l_{d0} \;&=\; 0.45 \text{ pc}\sqrt{n}\left(\frac{\Omega_{B\mathrm{Rad}g}}{0.083}\right)^{1/2} x^{-2/(n+2)} \\ &\times\left(\frac{T_g}{100 \text{ MeV}}\right)^{-n/(n+2)} \end{aligned} \tag{14}$$

where n is the spectral index for the magnetic field, $\Omega_{B\mathrm{Rad}g}$ is the ratio of the energy density in magnetic fields to that in radiation (in all relativistic species), T_g is the temperature, and all quantities are taken at the time of magnetic field generation (denoted by subscript "g"). Also, $x = 2.3\times10^{-9}$ is a numerical factor. This formula yields

$$l_{d0} \approx 1 \text{ pc} - 1 \text{ kpc} \tag{15}$$

for magnetic field generation at the electroweak epoch ($T_g = 100$ GeV), for $n = 2-5$ – larger n gives smaller l_{d0} – and with $\Omega_{B\mathrm{Rad}g} = 0.083$. The index n is defined in [27] by the relation $\rho_B \propto l^{-n}$ where ρ_B is the energy density in magnetic fields on a length scale l *at the epoch of magnetogenesis*. Translating this into our language with the relation in Eq. (12) we have $n = 5$ for the Batchelor spectrum, and $n = 3$ based on a model of processes that might have occured during a first order phase transition [28].

With $r_* = 10$ Mpc in Eq. (10), we get the full spectrum of the observed cosmological magnetic field to be

$$E_M(k) = \frac{k}{2}|H_M(k)| \approx 10^{-22}\left(\frac{k}{k_d}\right)^4\left(\frac{1 \text{ kpc}}{l_{d0}}\right) \text{G}^2 \text{ Mpc} \tag{16}$$

for $0 < k < k_d$, and $E_M(k) \approx 0$ for $k_d < k$, where $k_d = 2\pi/l_{d0}$ and l_{d0} estimated as in Eq. (15).

Eq. (12) now gives the magnetic field energy density at the present epoch,

$$\mathcal{E}_0 \sim (3\times10^{-10} \text{ G})^2 \left(\frac{1 \text{ kpc}}{l_{d0}}\right)^2 \tag{17}$$

and Eq. (13) gives

$$H_0 \sim 3\times10^{-20} \text{ G}^2 \text{ kpc} \left(\frac{1 \text{ kpc}}{l_{d0}}\right) \tag{18}$$

In natural units ($\hbar = 1 = c$), with the conversions 1 G $= 1.95\times10^{-20}$ GeV^2 $= 5\times10^7$ cm^{-2}, we can also write

$$H_0 \sim 2\times10^{17} \text{ cm}^{-3} \left(\frac{1 \text{ kpc}}{l_{d0}}\right) \tag{19}$$

To summarize this section, Eq. (16) gives the closed form expressions for the magnetic field spectra, and, (17) and (18) (or (19)) give the magnetic energy and helicity densities as indicated by current observations. These estimates use existing observational data together with some theoretical input. As more data accumulates, the method described in Ref. [8] and implemented in Refs. [4, 5], can be used to directly, *i.e.* without additional theoretical input, measure the helical power spectrum.

III. IMPLICATIONS AND THE LARGE HELICITY PUZZLE

To get a feel for these estimates, we compare the energy density in magnetic fields to that in photons,

$$\Omega_{B\gamma0} = \frac{\mathcal{E}_0}{\rho_{\gamma0}} \sim 10^{-8}\left(\frac{1 \text{ kpc}}{l_{d0}}\right)^2. \tag{20}$$

where $\rho_{\gamma0} = 4.6\times10^{-34}$ gms/cm^3 $\approx (4\times10^{-6} \text{ G})^2$ is the energy density in photons at the present epoch.

To proceed further we would like to estimate $\Omega_{B\gamma}$ at earlier times. The full details of the evolution are complicated because of episodes (*e.g.* e^+e^- annihilation), viscosity, finite electrical conductivity, and unknown factors (*e.g.* neutrino masses). However a simple approximate picture emerges from various studies within the context of conventional MHD [20, 25, 29–32]. Most crucially, helicity is found to be conserved, so the helicity density $H \propto a^{-3}$ where $a(t)$ is the cosmic scale factor. The inertial scale, also the scale where most of the magnetic energy is stored, grows as $l_I \propto a\times a^{2/3}$ in the radiation dominated era and as $l_I \propto a$ greater than the temperature at matter-radiation equality $T_{\mathrm{eq}} \approx 1$ eV) as long as the helicity is maximal [20]. So, from the relations in Eqs. (12) and (13), the energy density scaling is $\mathcal{E} \propto a^{-4}\times a^{-2/3}$ in the radiation dominated era and $\mathcal{E} \propto a^{-4}$ in the matter dominated era. With these scalings, and with the cosmic cooling rate $T \propto a^{-1}$ and the temperature at big bang nucleosynthesis (BBN) $T_{\mathrm{BBN}} \sim 0.1$ MeV, we get $\Omega_{B\gamma\mathrm{BBN}} \sim 10^{-5}(1 \text{ kpc}/l_{I0})^2$. Requiring $\Omega_{B\gamma\mathrm{BBN}} \lesssim 1$, this means that the inertial scale today (assumed to be close to the dissipation scale today, l_{d0}) is observationally constrained to be larger than ~ 10 pc.

Spectral distortions of the cosmic microwave background (CMB) also provide a means to probe small scale magnetic fields for cosmological redshift z between 10^3 and 10^6 [33–36]. As of now the bounds from COBE/FIRAS measurements of the CMB spectrum are not competitive with the BBN bound. Proposed experiments, such as PIXIE, can change this situation and be able to detect CMB $\mu-$distortions for $l_{d0} \sim 1$ kpc (see Figs. 2 and 3 of Ref. [36]). Small scale magnetic fields may also leave an imprint on the CMB anisotropies through non-linear effects [37–40].

The estimate in Eq. (17) shows that intergalactic magnetic fields that are indicated by gamma ray observations may be of $\sim 3 \times 10^{-10}$ G strength on 1 kpc scales. During structure formation, the field would get compressed within galaxies by a factor $(\rho_{\rm gal}/\rho_{\rm c})^{1/3}$, where $\rho_{\rm gal} \approx 10^{-24}$ gm/cm^3 is the baryonic density in the galactic disk and $\rho_{\rm c} \approx 10^{-31}$ gm/cm^3 is the cosmic baryon density. If we assume flux freezing during structure formation, the magnetic field strength will increase by $(\rho_{\rm gal}/\rho_{\rm c})^{2/3} \approx 10^5$ and the coherence scale will decrease by $(\rho_{\rm c}/\rho_{\rm gal})^{1/3} \approx 10^{-2}$. With these numbers, and $l_{d0} = 1$ kpc, a galaxy would inherit a magnetic field with strength $\sim 3 \times 10^{-5}$ G and coherence ~ 10 pc. A somewhat larger value of $l_{d0} \sim 10$ kpc would lead to estimates that are closer to observations of the random component of the Milky Way magnetic field, $4-6$ μG on $10-100$ pc [41]. This conclusion is in line with that of Ref. [27] where the authors argue that magnetic fields in galaxy clusters may arise directly from the intergalactic magnetic field. However, these relatively large seed fields during galaxy formation do not obviate the need for a galactic dynamo such as discussed in Ref. [42]. Dissipative and dynamical effects within the galaxy will diminish the magnetic field over time and dynamo action is necessary even to maintain the field (see Sec. 2 of [43]).

We now turn to the helicity of the magnetic field, a quantity that is parity (P) odd and also odd under combined charge and parity (CP) transformations. Hence observed non-zero magnetic helicity indicates a period of CP violation in the early universe, as is also necessary for the generation of the observed cosmic matter-antimatter asymmetry. Thus it is natural to compare the observed magnetic helicity to the cosmic baryon number density, $n_{b0} \approx 10^{-7}$ cm^{-3},

$$\eta_{Bb0} \equiv \frac{H_0}{n_{b0}} \sim 2 \times 10^{24} \left(\frac{1 \text{ kpc}}{l_d}\right) \tag{21}$$

This estimate, also discussed in Ref. [36], raises a challenge for fundamental physics – what processes can generate such a large helicity to baryon number ratio?

The simplest particle physics based scenarios of magnetogenesis are based on the evidence that a baryon number changing process via an electroweak sphaleron [44] also produces magnetic fields with $\sim 10^2$ helicity [45, 46]. Then the magnetic helicity is proportional to the baryon number and we get $\eta_{Bb0} \sim 10^2$ [47, 48]. Even in the unbroken phase of the electroweak model, where the electroweak sphaleron solution does not exist *per se*, we expect gauge field production to occur during changes of Chern-Simons number which is necessary for baryon number violation.

A more realistic view of the production of cosmic matter asymmetry is that baryon number violating processes occur so as to produce both baryons and antibaryons but with a slight excess of baryon production. In terms of magnetic fields this means that both left- and right-handed helical fields are produced but with a slight excess of left-handed helicity that is given by the fundamental CP violation [45]. Within the context of baryogenesis in the standard model, CP violation is extremely weak [49] and the total helicity is tiny compared to the energy density in the magnetic field [45]. The estimates of Ref. [50] show that the energy density in magnetic fields after the phase transition can be comparable to that in other forms of radiation. Thus the energy density in magnetic fields may be much larger than that implied by magnetic helicity alone. However, the problem we are encountering based on observation, is that the initial magnetic *helicity* also needs to be much larger (see Eq. (21)). Is there some dynamics beyond standard MHD that could potentially increase the magnetic helicity and saturate the maximal helicity condition in the early universe?

IV. CHIRAL EFFECTS, AXIONS AND MAGNETIC FIELDS

A simple possibility to resolve the large helicity puzzle is to look for a mechanism that selectively amplifies one handedness of the magnetic field. Then, if we start with a magnetic field, even with zero net helicity, the dynamics will amplify one of the two helicities, increase the magnetic field energy density, and also saturate the helicity at its maximal value. This has been the focus of earlier studies starting with [51], of the "chiral magnetic effect" [52], in which a magnetic field induces an electric current $\boldsymbol{j} \propto \boldsymbol{B}$, which results in the amplification of certain Fourier modes of only one handedness. More importantly for us, however, the chiral magnetic effect also selectively *dissipates* one handedness of the magnetic field (see, for example, [53]). Thus, if baryon number violating interactions (or other dynamics) produce a large but non-helical magnetic field, the chiral magnetic effect can dissipate one of the two helicities, the handedness being determined by the sign of the chiral imbalance, and thus reduce the magnetic field energy by half while saturating the helicity at its maximal value.

More quantitatively, in terms of comoving variables (denoted by a subscript c), the magnetic field evolution takes the form [53]

$$\begin{aligned}\partial_\eta \boldsymbol{B}_c \ &= \ \nabla_c \times (\boldsymbol{v}_c \times \boldsymbol{B}_c) + \gamma_{Dc} \nabla_c^2 \boldsymbol{B}_c \\ &\quad + \gamma_{\omega c} \nabla_c \times (\nabla_c \times \boldsymbol{v}_c) + \gamma_{Bc} \nabla_c \times \boldsymbol{B}_c, \end{aligned} \tag{22}$$

where $\sigma_c \sim 10^2$ is the comoving plasma electrical conductivity in the early universe, $\gamma_{Dc} = 1/\sigma_c$, $\gamma_{\omega c} = e\Delta\mu_c^2/4\pi^2\sigma_c$, $\gamma_{Bc} = e^2\Delta\mu_c/2\pi^2\sigma_c$, $\Delta\mu_c$ is the chiral asymmetry of the medium as given by the difference of the left- and right- chemical potentials, ∇_c is differentiation with respect to the spatial metric δ_{ij} and v_c is the comoving velocity. All quantities have been rescaled so that the unit of length is given by the inverse cosmic temperature [53].

After some manipulation, and ignoring the velocity field for the present analysis, the MHD equations can be written in terms of the right- and left- circularly polarized modes $B_\pm(k)$ as [53]

$$\partial_\eta |B^+|^2 = 2(-\gamma_D k^2 + \gamma_B k)|B^+|^2 \quad (23)$$
$$\partial_\eta |B^-|^2 = 2(-\gamma_D k^2 - \gamma_B k)|B^-|^2 \quad (24)$$

The evolution of the ensemble averaged comoving chemical potential is given by

$$\frac{d\Delta\mu_c}{d\eta} = -c_\Delta \alpha \int \frac{kdk}{2\pi^2}\partial_\eta \left[|B^+|^2 - |B^-|^2\right] - \Gamma_F \Delta\mu_c \ . \quad (25)$$

where Γ_F tells us the strength of the left-right flipping due to fermion-Higgs interaction and c_Δ is a numerical constant.

From Eqs. (23) and (24), the difference of the two helical amplitudes of the magnetic field Fourier modes, $\Delta B \equiv |B^+(k)| - |B^-(k)|$, evolves according to

$$\partial_\eta \Delta B = +\frac{k_p}{\sigma_c}(|B^+| + |B^-|) + \mathcal{O}(\Delta B) \quad (26)$$

where η is the conformal time, σ is the electrical conductivity of the plasma and $k_p = e^2\Delta\mu_c/2\pi^2$. Thus ΔB grows in proportion to the summed amplitudes of the two helicities of the magnetic field and the field tends to become maximally helical on a time scale $\tau \sim \sigma_c/k_p \propto 2\pi^2\sigma_c/(e^2\Delta\mu_c)$. Hence the helicity is most efficiently generated for large chiral imbalance.

A chiral imbalance might arise naturally above the electroweak scale since the weak interactions distinguish between left- and right-handed particles at a fundamental level. At these epochs the relevant magnetic field is the hypermagnetic (not electromagnetic) field. There are also some difficulties with this idea. First the chiral imbalance has to be relatively large for the time scales of helicity dissipation to be shorter than the Hubble expansion time scale. Further, the scenario has to assume magnetic field generation prior to the electroweak epoch. Magneto genesis may well take place during inflationary reheating but this requires a whole new set of interactions in the fundamental action and, depending on the strength of those interactions, the magnetic field might be helical even when first generated, thus obviating the need for any chiral effects.

Scenarios that evolve magnetic fields in a homogeneous chiral medium have received some attention in the literature [54, 55]. The precise dynamics, however, needs further investigation since the analysis outlined above ignores the plasma velocity field. The joint evolution of the magnetic field and the plasma velocity is essential to see effects such as the inverse cascade of helical fields [56]. For an inhomogeneous chiral medium, even the equations necessary to describe dynamics with spatially varying chirality have not yet been established (recent attempts can be found in [57, 58]).

Another possibility is to consider axion-MHD, since a time dependent axion field behaves very much like a chiral asymmetry in the MHD equations. Differences with the chiral asymmetry arise in the dynamical equations for the axion as compared to the anomaly equations for the chiral asymmetry. As discussed in Ref. [16], the axion-MHD equations (in Minkowski space and in Lorentz-Heaviside units) are

$$\ddot{\varphi} - \nabla^2\varphi + m_a^2\varphi - g_{a\gamma}\mathbf{E}\cdot\mathbf{B} = 0 \quad (27)$$
$$\nabla\times\mathbf{B} - g_{a\gamma}\dot{\varphi}\mathbf{B} - g_{a\gamma}\nabla\varphi\times\mathbf{E} - \sigma\mathbf{E} - \sigma\mathbf{v}\times\mathbf{B} - \dot{\mathbf{E}} = 0 \quad (28)$$
$$\dot{\mathbf{B}} + \nabla\times\mathbf{E} = 0 \quad (29)$$

where ϕ is the axion field, $\mathbf{v}$ is the plasma velocity field, $g_{a\gamma}$ is the photon-axion coupling, m_a is the mass of the axion which turns on at the QCD epoch, and σ is the electrical conductivity of the plasma. In the MHD approximation, we ignore the displacement current term ($\dot{\mathbf{E}}$), then solve Eq. (28) for $\mathbf{E}$ and substitute in Eq. (29). The resulting expressions are quite messy but simplify if we only work to leading order in $g_{a\gamma}/\xi$ where ξ is a length (or time) scale associated with the gradient (or scale of time variation) of φ.

It is convenient to define:

$$\mathbf{a} \equiv \frac{g_{a\gamma}}{\sigma}\nabla\varphi \quad (30)$$
$$\mathbf{b} \equiv \left(\frac{1}{\sigma}\nabla\times\mathbf{B} - \mathbf{v}\times\mathbf{B}\right) - \frac{g_{a\gamma}}{\sigma}\dot{\varphi}\mathbf{B} \quad (31)$$
$$\equiv \mathbf{c} - \frac{g_{a\gamma}}{\sigma}\dot{\varphi}\mathbf{B} \quad (32)$$

Then the dynamical MHD equation for $\mathbf{B}$ becomes

$$\dot{\mathbf{B}} = -\nabla\times\mathbf{E} = -\nabla\times\left(\frac{\mathbf{b} + (\mathbf{a}\cdot\mathbf{b})\mathbf{a} - \mathbf{a}\times\mathbf{b}}{1+\mathbf{a}^2}\right) \quad (33)$$

To linear order in $g_{a\gamma}$, this simplifies to

$$\dot{\mathbf{B}} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \frac{1}{4\sigma}\nabla^2\mathbf{B} + \frac{g_{a\gamma}}{\sigma}\nabla\times\left(\dot{\phi}\mathbf{B} + \nabla\phi\times\mathbf{c}\right) \quad (34)$$

The first two terms on the right-hand side are the standard advection and diffusion terms of MHD; the last term is the contribution of the axion field. Assuming the axion field is approximately homogeneous so that gradients are small, only the $\dot{\phi}$ term survives. This term is equivalent to the usual chiral-magnetic effect that also appears in Eq. (22) with the identification

$$\gamma_{Bc} = \frac{e^2\Delta\mu_c}{2\pi^2\sigma_c} \longleftrightarrow \frac{g_{a\gamma}}{\sigma}\dot{\varphi}. \quad (35)$$

The time scale for the growth of helicity is now $\sigma/(g_{a\gamma}\dot{\varphi})$ and is short only if the axion field varies rapidly.

The assumption of approximate homogeneity of the axion field at early times may be valid in certain cosmological scenarios in which there is a period of inflation at low energy scales. However, without a period of inflation, the universe will contain a network of axion strings until the QCD epoch, and it is dubious to assume that inhomogeneities in the axion field are unimportant. The evolution of the magnetic field will then be affected by the network of axion strings as they move through the magnetized plasma. Even if the axion field is initially homogeneous, the electromagnetic source term in Eq. (27), with $\mathbf{E}$ replaced by the expression in Eq. (33), will be inhomogeneous and will induce inhomogeneities in the axion field. We plan to analyze some of these issues in the future.

V. CONCLUSIONS

The main point of this paper is that current observational evidence for intergalactic magnetic fields has profound implications for fundamental interactions. The indicated magnetic fields must have originated in the early universe since they are seen in voids and are helical. With more data, the method of Ref. [8] that uses parity odd correlators of gamma ray arrival directions, has the capability to uncover the full power spectrum of magnetic fields. If we uncover a red spectrum, we would know that magnetic fields were generated by an acausal mechanism. The magnetic fields would then provide valuable information about the earliest moments of the universe. If the spectrum turns out to be blue, the properties of the magnetic field will give us important clues about particle physics beyond the standard model.

The observation of magnetic helicity implies a strong role for fundamental CP violation in the early universe. Since helical magnetic fields are closely connected with baryon number violating processes, the observation of helical magnetic fields can inform us about matter-genesis. But baryogenesis by itself is insufficient to explain the large helicity that is indicated by observations. We have suggested that there may be a role for the chiral-magnetic effect, either due to a primordial chiral asymmetry or due to an axion field, to drive magnetic helicity to its maximal value. Then the standard model must be extended to allow for successful baryogenesis *and* the chiral magnetic effect should play a role in cosmology. This would have implications for particle physics close to the electroweak scale and may perhaps also be testable at the LHC or future accelerator experiments. Future observations (*e.g.* by the Cherenkov Telescope Array [59]) will further clarify the case for intergalactic magnetic fields and allow for more precise measurements of their power spectra.

Before closing it is worthwhile to step back and point out some caveats to the considerations in this paper.

First, the observational situation could be misleading. It is true that the claimed existence of inter-galactic magnetic fields is based on analyses carried out by several independent groups using different methodologies [1–7]. However all these analyses are based on either the blazar spectra or the distribution of diffuse gamma rays. There is also a claim that we don't fully understand the cascade process and that there might be instabilities [60–64], even though direct observations [65] and simulations [66] suggest that the instability is not operative. Nonetheless it would be extremely valuable to have a separate independent tool to observe intergalactic magnetic fields.

Second, only one method actually claims to be able to *measure* the correlation function of the magnetic field [8, 9]. In the present paper, we have combined the measurement of the correlation function at a single spatial separation [4, 5] with theoretical studies of the evolution of magnetic fields to find the full magnetic field spectrum [20, 25, 29–32]. If there are any new ingredients in the evolution of magnetic fields – *e.g.* additional drivers of turbulence, chiral or other effects – the evolution might be different than what we have supposed and this would alter some of our conclusions.

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