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Holographic Heavy-Light Chiral Effective Action

Yizhuang Liu* and Ismail Zahed†
Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794-3800, USA

We propose a variant of the $D4$-$D8$ construction to describe the low energy effective theory of heavy-light mesons, interacting with the lowest lying pseudoscalar and vector mesons. The heavy degrees of freedom are identified with the $D8_L$-$D8_H$ string low energy modes, and are approximated near the world volume of $N_f - 1$ light $D8_L$ branes, by fundamental vector field valued in $U(N_f - 1)$. The effective action follows from the reduction of the bulk Dirac-Born-Infeld (DBI) and Chern-Simons (CS) actions, and is shown to exhibit both chiral and heavy-quark symmetry. The action interpolates continuously between the $U(N_f)$ case with massless mesons, and the $U(N_f - 1)$ case with heavy-light mesons. The heavy-light meson radial spectrum is Regge-like. The one-pion and two-pion couplings to the heavy-light multiplets are evaluated. The partial widths for the charged decays $G \to H + \pi$ are shown to be comparable to the recently reported full widths for both the charm and bottom mesons.

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I. INTRODUCTION

In QCD the light quark sector (u, d, s) is dominated by the spontaneous breaking of chiral symmetry. The heavy quark sector (c, b, t) is characterized by heavy-quark symmetry [5]. The combination of both symmetries led to the conclusion that the heavy-light doublet $(0^-, 1^-) = (D, D^*)$ has a chiral partner $(0^+, 1^+) = (\tilde{D}, \tilde{D}^*)$ that is about one consituent mass heavier [3, 4]. This observation is supported by the BaBar collaboration [1] and the CLEOII collaboration [2].

More recently, the Belle collaboration [6] and the BESIII collaboration [7] have reported the observations of multiquark exotics, with quantum numbers uncommensurate with the excited states of charmonia and bottomia, such as the neutral $X(3872)$ and the charged $Z_c(3900)^\pm$ and $Z_b(10610)^\pm$ to cite afew. These sightings and more have been supported by the DO collaboration at Fermilab [8], and the LHCb collaboration at CERN [9]. They provide a window to new phenomena involving heavy-light multiquark states.

Theoretical arguments have predicted the occurence of some of these exotics as molecular bound states mediated by one-pion exchange much like deuterons or deusons [10–17]. Non-molecular heavy exotics were also discussed using constituent quark models [19], heavy solitonic baryons [20, 21], instantons [22] and QCD sum rules [23]. The molecular mechanism favors the formation of shallow bound states near treshold, while the non-molecular or quarkonium mechanism leads to deeply bound states.

The holographic approach offers a useful framework for discussing both the spontaneous breaking of chiral symmetry and confinement, in the double limit of large N_c and large t′Hooft coupling $\lambda = g^2 N_c$. An example is the $D4$-$D8$ model suggested by Sakai and Sugimoto [24]. In short, the model consists of N_f probe $D8$ and $D\overline{8}$ branes in a background of N_c $D4$ branes. The induced gravity on the probe branes, cause them to fuse in the infrared providing a geometrical mechanism for the spontaneous breaking of chiral symmetry. The DBI action on the probe branes, provides a low-energy effective action for the light pseudoscalars with full global chiral symmetry, where the vectors and axial-vector light mesons are dynamical gauge particles of a hidden chiral symmetry [25].

The purpose of this paper is to address the dual concepts of chiral and heavy-quark symmetries by using the holographic construction. We will show that a variant of the $D4$-$D8$ construct, composed of $(N_f - 1)$ light and one heavy probe branes allows a geometrical set up for the derivation of the leading heavy-light (HL) effective action in conformity with chiral and heavy-quark symmetry. The heavy-light mesons are identified with the string low energy modes, and approximated by bi-fundamental and local vector fields in the vicinity of the light probe branes. Their masses follow from the vev of the moduli span by the dilaton fields in the DBI action. We note that few approaches were proposed for the description of heavy-light mesons using holography without the strictures of chiral symmetry [26, 27].

The organization of the paper is as follows: In section 2 we briefly outline the geometrical set up for the derivation of the heavy-light effective action (HL). We identify the pertinent light and heavy fields and explicit their contributions to the (expanded) DBI and CS actions. In section 3, we detail the analysis of the HL meson spectrum. In section 4, we derive the chiral interactions to the HL mesons and deduce their corresponding axial couplings within and across the HL meson multiplets. We use these couplings to estimate the HL charm and bottom one pion charged decays. Our conclusions are in section 5.

*Electronic address: yizhuang.liu@stonybrook.edu
†Electronic address: ismail.zahed@stonybrook.edu

II. HOLOGRAPHIC EFFECTIVE ACTION

A. D-brane set up

The $D4$-$D8$ construction proposed by Sakai and Sugimoto [24] for the description of the spontaneous breaking of chiral symmetry and the ensuing chiral effective action is by now well-known, and will not be repeated here. Instead, consider the variant with N_f-1 light $D8$-$\bar{D}8$ (L) and one heavy (H) probe branes in the cigar-shaped geometry that spontaneously breaks chiral symmetry. Also and for simplicity, the light probe branes are always assumed in the anti-podal configuration. A schematic description of the set up for $N_f=3$ is shown in Fig. 1. We assume that the L-brane world volume consists of $R^4\times S^1\times S^4$ with $[0-9]$-dimensions. The light 8-branes are embedded in the $[0-3+5-9]$-dimensions and set at the antipodes of S^1 which lies in the 4-dimension. The warped $[5-9]$-space is characterized by a finite size R and a horizon at U_{KK}.

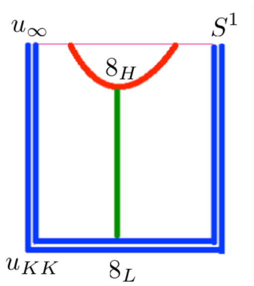

FIG. 1: $N_f-1=2$ antipodal 8_L light branes, and 1 8_H heavy brane shown in the τU plane, with a massive HL-string connecting them. When the latter is massless, the 8_H brane coincides with the 8_L branes, transmuting to N_f $8+\bar{8}$ light branes.

B. DBI and CS actions

The lowest open string modes streched between the H- and L-branes as shown in Fig. 1, when viewed near the L brane world volume, consist of tranverse modes Φ_M and longitudinal modes Ψ, both fundamental with respect to the flavor group $SU(N_f-1)$. At non-zero brane separation, these fields acquire a vev that makes the vector field massive [28]. Strictly speaking these fields are bi-local, but near the L-branes we will approximate them by local vector fields that are described by the standard DBI action. In this respect, our construction is distinct from the approaches developed in [26].

With this in mind and to leading order in the $1/\lambda$ expansion, the effective action on the probe L-branes consists of the non-Abelian DBI (D-brane Born-Infeld) and CS (Chern-Simons) action. After integrating over the S^4, the leading contribution to the DBI action is

$$S_{\rm DBI}\approx-\kappa\int d^4x dz\,{\rm Tr}\left(f(z)\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}+g(z)\mathbf{F}_{\mu z}\mathbf{F}^{\nu z}\right) \quad (1)$$

The warping factors are

$$f(z)=\frac{R^3}{4U_z}\,,\qquad g(z)=\frac{9}{8}\frac{U_z^3}{U_{KK}} \quad (2)$$

with $U_z^3=U_{KK}^3+U_{KK}z^2$, and $\kappa\equiv\tilde{T}(2\pi\alpha')^2$ [24]. The effective fields in the field strengths are (M,N run over (μ,z))

$$\mathbf{F}_{MN}=\left(\begin{array}{cc}F_{MN}-\Phi_{[M}\Phi^\dagger_{N]} & \partial_{[M}\Phi_{N]}+A_{[M}\Phi_{N]}\\ -\partial_{[M}\Phi^\dagger_{N]}-\Phi^\dagger_{[M}A_{N]} & -\Phi^\dagger_{[M}\Phi_{N]}\end{array}\right) \quad (3)$$

Specifically, using (3)) we can recast the trace contribution in (1) in the form $f\mathcal{L}_f+g\mathcal{L}_g$ with

$$\begin{aligned}\mathcal{L}_f&={\rm Tr}(F_{\mu\nu}-a^{\mu\nu})(F^{\mu\nu}-a^{\mu\nu})-2f^\dagger_{\mu\nu}f^{\mu\nu}+b^{\mu\nu}b_{\mu\nu}\\ \mathcal{L}_g&={\rm Tr}(F_{\mu z}-a_{\mu z})(F^{\mu z}-a^{\mu z})-2f^\dagger_{\mu z}f^{\mu z}+b^{\mu z}b_{\mu z}\end{aligned} \quad (4)$$

and

$$\begin{aligned}a_{MN}&=\Phi_{[M}\Phi^\dagger_{N]}\\ f_{MN}&=\partial_{[M}\Phi_{N]}+A_{[M}\Phi_{N]}\\ b_{MN}&=\Phi^\dagger_{[M}\Phi_{N]}\end{aligned} \quad (5)$$

The CS contribution to the effective action is (form notation used)

$$S_{\rm CS}=\frac{N_c}{24\pi^2}\int_{R^{4+1}}{\rm Tr}\left(\mathbf{A}\mathbf{F}^2-\frac{1}{2}\mathbf{A}^3\mathbf{F}+\frac{1}{10}\mathbf{A}^5\right) \quad (6)$$

where the normalization to N_c is fixed by integrating the F_4 RR flux over the S^4. The matrix valued 1-form gauge field is

$$\mathbf{A}=\left(\begin{array}{cc}A & \Phi\\ -\Phi^\dagger & 0\end{array}\right) \quad (7)$$

For N_f coincidental branes, the Φ multiplet is massless. However, their brane world-volume supports an adjoint and traceless scalar Ψ in addition to the adjoint gauge field A_M both of which are hermitean and $N_f \times N_f$ valued, which we have omitted from the DBI action in so far for simplicity. Their leading contribution from the DBI action is of the form (omitting the warping factors)

$$\frac{1}{2}\mathrm{Tr}\,|\nabla_M\Psi|^2 - \frac{1}{4}\mathrm{Tr}\left([\Psi,\Psi]^2\right) \tag{8}$$

with $\nabla_M\Psi = \partial_M\Psi + i[A_M,\Psi]$. The extrema of the potential contribution in (8) or $[[\Psi,\Psi],\Psi]=0$ define a moduli [28]. For N_f-1 light branes separated from one heavy brane, we identify one of the moduli solution with a finite vev v as,

$$\Psi = \begin{pmatrix} -\frac{v}{N_f-1}\mathbf{1}_{N_f-1} & 0 \\ 0 & v \end{pmatrix} \tag{9}$$

Since the upper block diagonal contribution commutes with A_μ, only the Φ multiplet acquires a Higgs-like mass through the first contribution in (8)

$$\frac{1}{2}M^2\mathrm{Tr}\left(\Phi_M^\dagger\Phi_M\right) \equiv \frac{v^2N_f^3}{(N_f-1)^2}\mathrm{Tr}\left(\Phi_M^\dagger\Phi_M\right) \tag{10}$$

The vev is related to the separation between the light and heavy branes [28], which we take it to be the length of of the HL string of mass M, i.e. $v\sim M$. Below, M is also the degenerate (in practice mean) heavy meson mass for the bi-fundamental field Φ which will be identified with the HL meson doublets, such as (D,D^*) or (B,B^*). Throughout, we will refer to the large mass limit also as the heavy quark limit.

III. HL MESON SPECTRUM

To investigate the holographic spectrum of the heavy-light mesons, we first specialize to the case where the simplified gauge potential 1-form (11)

$$\mathbf{A} \rightarrow \begin{pmatrix} 0 & \Phi \\ -\Phi^\dagger & 0 \end{pmatrix} \tag{11}$$

is inserted in the unexpanded DBI action

$$-\kappa\int d^4x dz\sqrt{-g} \\ \times\left(\det\left(g_{MN}+2\pi\alpha' F_{MN}+\nabla_M\Psi^\dagger\nabla_N\Psi\right)\right)^{\frac{1}{2}} \tag{12}$$

As we noted earlier, the warped mass term in (10) follows by using (9) and expanding (12) in leading order

$$-\tilde\nu^2\int d^4xdz(U_z^2)(g^{zz}\Phi_z^\dagger\Phi_z + g^{xx}\Phi^{\dagger\mu}\Phi_\mu) \tag{13}$$

Here we have set $\tilde\nu^2 = \tilde T\nu^2$ with the HL mass parameter $\nu\sim v$, and defined the warpings as

$$U_z^2g^{zz} = \frac{9}{4}\left(\frac{U_z}{R}\right)^{\frac{3}{2}}\frac{U_z^3}{U_{KK}} \equiv \frac{9}{4}a(z)\frac{U_z^3}{U_{KK}} \\ U_z^2g^{xx} = U_z^2\left(\frac{R}{U_z}\right)^{\frac{3}{2}} = \left(\frac{U_z}{R}\right)^{\frac{3}{2}}\frac{R^3}{U_z} \tag{14}$$

The corresponding field-strength 2-form (3) is

$$\mathbf{F}_{MN} \rightarrow \begin{pmatrix} -\Phi_{[M}\Phi_{N]}^\dagger & \partial_{[M}\Phi_{N]} \\ -\partial_{[M}\Phi_{N]}^\dagger & -\Phi_{[M}^\dagger\Phi_{N]} \end{pmatrix} \tag{15}$$

Inserting (15) into (1) and combining it with (13) yield to quadratic order

$$\frac{S_\Phi}{2\kappa} = \\ -\int dzd^4xf(z)(\partial_\mu\Phi_\nu^\dagger - \partial_\nu\Phi_\mu^\dagger)(\partial^\mu\Phi^\nu - \partial^\nu\Phi^\mu) \\ -\int dzd^4xg(z)(\partial_\mu\Phi^\dagger - \partial_z\Phi_\mu^\dagger)(\partial^\mu\Phi - \partial^z\Phi^\mu) \\ -\tilde\nu^2\int dzd^4xa(z)(2f(z)\Phi^{\dagger\mu}\Phi_\mu + g(z)\Phi^\dagger\Phi) \tag{16}$$

A. Mode analysis

To find the HL mass spectrum, we first make use of the general decomposition

$$\Phi_\mu = \epsilon_\mu(z)e^{ip\cdot x}, \qquad \Phi = -i\epsilon(z)e^{ip\cdot x} \tag{17}$$

into the equations of motion following from (16) to obtain

$$2f(p^2\epsilon^\mu - p^\mu\epsilon\cdot p) + \frac{d}{dz}\left(g\left(p^\mu\epsilon - \frac{d\epsilon^\mu}{dz}\right)\right) + 2a\tilde\nu^2f\epsilon^\mu = 0 \\ g\left(\epsilon p^2 - p\cdot\frac{d\epsilon}{dz}\right) + a\tilde\nu^2g\epsilon = 0 \tag{18}$$

We can simplify (18) by redefining

$$\frac{d\tilde\epsilon_\mu}{dz} = \frac{d\epsilon_\mu}{dz} - p_\mu\epsilon \\ \epsilon_\mu = \tilde\epsilon_\mu + p_\mu\int dz\epsilon \tag{19}$$

in terms of which (18) now reads

$$-\frac{d}{dz}\left(g\frac{d\tilde{\epsilon}_\mu}{dz}\right)$$
$$+2f(p^2\tilde{\epsilon}_\mu - p^\mu(p\cdot\tilde{\epsilon})) + 2\tilde{\nu}^2 af\tilde{\epsilon}_\mu + 2a\tilde{\nu}^2 fp_\mu\int dz\epsilon = 0$$
$$-gp\cdot\frac{d\tilde{\epsilon}}{dz} + a\tilde{\nu}^2 g\epsilon = 0 \qquad (20)$$

1. *Transverse modes*

The transverse modes solution to (20) with $\tilde{\epsilon}\cdot p = 0$, yields $\epsilon = 0$ and $\tilde{\epsilon}$ satisfying

$$-\frac{d}{dz}\left(g\frac{d\tilde{\epsilon}_\mu}{dz}\right) + 2f(p^2 + \tilde{\nu}^2 a)\tilde{\epsilon}_\mu = 0 \qquad (21)$$

for $\tilde{\nu}, a \neq 0$. The heavy-light states correspond to the normalizable bulk modes with $p^2 = -m_n^2$. Let $\phi_n(z)$ denote these normalizable modes. They satisfy the warped and massive eigenvalue equation

$$-\frac{d}{dz}\left(g\frac{d\phi_n}{dz}\right) + 2f(-m_n^2 + \tilde{\nu}^2 a)\phi_n = 0 \qquad (22)$$

We note that for coincidental branes with zero vev or $\tilde{\nu} = 0$, (22) reduces to the equation for the pionic zero mode in the massless limit with $m_n = 0$. As a result, the spontaneous breaking of chiral symmetry will be enlarged from $SU(N_f - 1)$ to $SU(N_f)$, in the limit of N_f coincidental branes in agreement with the original analysis in [24]. This point will be emphasized further below in the ensuing chiral effective action.

In terms of the modes (22), the transverse mode decomposition is

$$\Phi_\mu(x,z) = \sum_{n=1}\phi_n(z)B_\mu^n(x)$$
$$\Phi_z(x,z) = 0 \qquad (23)$$

Inserting (23) into the vector contribution in (16) we obtain

$$2\kappa\sum_{n,m}\int dzf\phi_m\phi_n\int d^4x(\partial_{[\mu}B_{\nu]}^m)^\dagger(\partial^{[\mu}B^{n,\nu]})$$
$$+2\kappa\sum_{n,m}\int(2a\tilde{\nu}^2 f\phi_m\phi_n + g\phi_n'\phi_m')dz\int d^4x(B_\mu^m)^\dagger B^{n,\mu} \qquad (24)$$

We now note that the coefficient of the second contribution in (24) can be integrated by parts to satisfy the identity

$$\int dz\,(2a\tilde{\nu}^2 f\phi_n - \frac{d}{dz}(g\frac{d\phi_n}{dz}))\phi_m = 2m_n^2\int dzf\phi_m\phi_n \qquad (25)$$

thanks to (19). This suggests to normalize ϕ_n as

$$4\kappa\int dzf\phi_m\phi_n = \delta_{m,n} \qquad (26)$$

which brings the quadratic HL vector contribution (24) to the canonical form

$$\sum_n\int d^4x\left(\frac{1}{2}\partial_{[\mu}B_{\nu]}^{n\dagger}\partial^{[\mu}B^{n\nu]} + m_n^2 B_\mu^{n\dagger}B^{n\mu}\right) \qquad (27)$$

2. *Longitudinal modes*

The longitudinal modes correspond to $\tilde{\epsilon}\cdot p \neq 0$. For $p^2 = -m^2$ nonzero, these modes are of the form $\tilde{\epsilon}^\mu = p^\mu\epsilon_1$. In this case both ϵ, ϵ_1 are non-zero and satisfy the coupled equations

$$-\frac{d}{dz}\left(g\frac{d\epsilon_1}{dz}\right) + 2\tilde{\nu}^2 af\epsilon_1 + 2a\tilde{\nu}^2 f\int\epsilon = 0$$
$$a\tilde{\nu}^2\epsilon + m^2\frac{d\epsilon_1}{dz} = 0 \qquad (28)$$

from which we have

$$\epsilon = -\frac{d\epsilon_1}{dz} + \frac{d}{dz}\left(\frac{1}{2a\tilde{\nu}^2 f}\frac{d}{dz}\left(g\frac{d\epsilon_1}{dz}\right)\right) \qquad (29)$$

or equivalently

$$m^2\epsilon = a\tilde{\nu}^2\epsilon - \frac{d}{dz}\left(\frac{1}{2af}\frac{d}{dz}(ag\epsilon)\right) \qquad (30)$$

We note that (30) does not lead to the pionic zero mode in bulk for $\tilde{\nu}, m \to 0$, in contrast to (22). This is consistent with the counting of Golstone modes in the limit when all the D-branes are coincidental.

In terms of the original fields Φ_μ and Φ_z, the expansion now reads

$$\Phi_z(x,z) = \sum_n\epsilon_n(z)D_n(x)$$
$$\Phi_\mu(x,z) = \sum_n\frac{-1}{2afm_n^2}\frac{d}{dz}(ag\epsilon_n)\partial_\mu D_n(x) \qquad (31)$$

after using the relation between $\tilde{\epsilon}$ and ϵ in (20). Inserting (31) into the pertinent quadratic parts in (16) yields

$$2\kappa\sum_{m,n}\int dz\, ag\frac{\tilde\nu^2\epsilon_m\epsilon_n}{m_n^2}\int d^4x\partial_\mu D_n^\dagger\partial^\mu D^m$$
$$+2\kappa\sum_{m,n}\int dz\,\tilde\nu^2 ag\epsilon_m\epsilon_n\int d^4x D_m^\dagger D_n \qquad (32)$$

which suggests the normalization

$$2\kappa\int dz\, ag\epsilon_m\epsilon_n=\frac{m_n^2}{\tilde\nu^2}\delta_{mn} \qquad (33)$$

as a result, (32) takes the canonical form for the free HL pseudoscalars

$$\sum_n\int d^4x\left(\partial_\mu D_n^\dagger\partial^\mu D_n+m_n^2D_n^\dagger D_n\right) \qquad (34)$$

B. Heavy quark limit

The heavy quark limit will be sought through the rescaling $z=\tilde z/\tilde\nu^\beta$ with $\tilde\nu\to\infty$. With this in mind (22) reads

$$-\tilde\nu^{2\beta}\frac{g}{2f}\frac{d^2\phi_n}{d\tilde z^2}+\tilde\nu^\beta\frac{g'}{2f}\frac{d\phi_n}{d\tilde z}+\tilde\nu^2a\phi_n=m_n^2\phi_n \qquad (35)$$

with the limiting values

$$a\to a_0+\frac{a_0}{2}\left(\frac{\tilde z^2}{U_{KK}^2}\right)\frac{1}{\tilde\nu^{2\beta}}$$
$$\frac{g}{2f},\frac{g'}{2f}\to\frac{g_0}{2f_0},\frac{g_0'}{2f_0}+\mathcal{O}\left(\frac{1}{\tilde\nu^\beta}\right) \qquad (36)$$

and $a_0=a(U_{KK})=(U_{KK}/R)^{\frac{3}{2}}$. Notice that

$$\frac{d}{dz}\left(\frac{1}{2af}\frac{d(ag)}{dz}\right)\approx a_0+\mathcal{O}\left(\frac{1}{\tilde\nu^{2\beta}}\right) \qquad (37)$$

This means that after the rescaling, (30) reduces to (37) as $\tilde\nu\to\infty$. A consistent $\tilde\nu\to\infty$ limit is achieved by setting $2\beta=2-2\beta\to\beta=\frac{1}{2}$ in (35). Matching the leading poweres in $\tilde\nu$ gives

$$-\frac{g_0}{2f_0}\frac{d^2\phi_n}{d\tilde z^2}+\frac{a_0}{2}\frac{\tilde z^2\phi_n}{U_{KK}^2}=\tilde m_n^2\phi_n \qquad (38)$$

with the squared mass

$$m_n^2=\tilde\nu^2a_0+\tilde\nu\tilde m_n^2=M^2+\tilde\nu\tilde m_n^2 \qquad (39)$$

We have identified the heavy quark mass as $M=\tilde\nu\sqrt{a_0}$. More specifically, we can re-write the solutions to (38) as

$$m_n^2 = M^2+\tilde\nu\sqrt{a_0}\left(\frac{g_0}{U_{KK}^2f_0}\right)^{\frac{1}{2}}\left(n+\frac{1}{2}\right)$$
$$= M^2+M\left(\frac{2m_\rho^2}{0.67}\right)^{\frac{1}{2}}\left(n+\frac{1}{2}\right) \qquad (40)$$

using the rho mass $m_\rho=\sqrt{0.67}M_{KK}$ [24]. (40) is Regge-like with an intercept M^2 and a slope of about Mm_ρ. Note that the mass splitting Δm between the odd-parity H-multiplet (say $n=0$) and the even-parity G-multiplet (say $n=1$) is finite in this model, with

$$\Delta m=m_G-m_H\approx\frac{m_\rho}{\sqrt{2}\sqrt{0.67}} \qquad (41)$$

in the large M limit. The splitting is about $\Delta m\approx 665$ MeV, which is larger than the reported mean of 420 MeV for charm and 396 MeV for bottom [2].

In the D3/D7 set up and its variants of heavy-light mesons presented in [26], the heavy-light meson mass spectrum was found to be of the form $m_{HL}=m_H+\mathcal{O}(1/\sqrt{\lambda})$ in the chiral limit. In contrast, our result (40) shows a leading correction of order $\mathcal{O}(\lambda^0)$ when chiral symmetry is spontaneously broken in the holographic construct.

IV. HL CORRELATION FUNCTIONS

The HL correlation functions from the current-current correlator

$$\Pi^{AB}(q)=i\int d^4xe^{iq\cdot x}\left\langle 0|T^*\left(\mathbb{J}^A(x)\mathbb{J}^B(0)\right)|0\right\rangle \qquad (42)$$

with the even and odd parity multiplet assigments

$$(\mathbb{J}^P,\mathbb{J}_\mu^V)=(\bar qi\gamma_5Q,\bar q\gamma_\mu Q) \qquad (\mathbb{J}^S,\mathbb{J}_\mu^A)=(\bar qQ,\bar qi\gamma_\mu\gamma_5Q) \qquad (43)$$

In the holographic approach, (42) can be obtained from the boundary effective action $\mathbf{S}_B$, by inserting sources in the UV and integrating out the bulk fields using the equations of motion [29–31].

A. Vector and Axial-Vector polarizations

We now define the bulk HL vector and scalar source fields in momentum space as

$$\Phi_\mu(p,z)\to\frac{\mathcal{V}(p,z)}{\mathcal{V}(p,z_\Lambda)}\mathbb{S}_\mu(p)$$
$$\Phi_z(p,z)\to 0 \qquad (44)$$

with $z_\Lambda/U_{KK} \gg 1$ setting the UV cutoff. The boundary-to-bulk propagator $\mathcal{V}(p,z)$ satisfies the off-shell version of (22),

$$-\frac{d}{dz}\left(g\frac{d\mathcal{V}}{dz}\right)+2f(p^2+\tilde{\nu}^2 a)\mathcal{V}=0 \tag{45}$$

subject to the axial and vector boundary conditions

$$\begin{aligned}\mathcal{V}(p,0) = &\quad 0, \qquad \textit{axial}\\ \partial_z\mathcal{V}(p,0) = &\quad 0, \qquad \textit{vector}\end{aligned} \tag{46}$$

To construct $\mathbf{S}_B$, we first make some general observations regarding the transverse eigenmode equation (45-46). In general, the equation admits two independent solutions. The first is f_1 which is square integrable in the UV limit, and as $p^2 \to -m_n^2$, f_1 aproaches the normalized eigenmodes ϕ_n given in (22). The second is f_2 which is another independent solution that is not square integrable in the UV,. We normalize it using the Wronskian normalized Wronskian

$$4\kappa g\left(f_2\frac{d}{dz}f_1 - f_1\frac{d}{dz}f_2\right) = -1 \tag{47}$$

Also, we note that near the UV boundary with $z\to\infty$, (45) simplifies to

$$-\frac{d}{dz}\left(z^2\frac{d\phi_n}{dz}\right)+\mathbf{c}(p)\,z^{\frac{1}{3}}\phi_n \approx 0 \tag{48}$$

with $\mathbf{c}(p)$ a p-dependent function. The two independent solutions to (48) take the asymptotic forms in terms of the modified Bessel functions

$$\begin{aligned} f_1(p,z) &\approx \frac{K_3(6\sqrt{\mathbf{c}(p)}z^{\frac{1}{6}})}{\sqrt{z}} \to \frac{e^{-6\sqrt{\mathbf{c}(p)}z^{\frac{1}{6}}}}{z^{\frac{7}{12}}} \equiv f_{1,asy}(p,z)\\ f_2(p,z) &\approx \frac{I_3(6\sqrt{\mathbf{c}(p)}z^{\frac{1}{6}})}{\sqrt{z}} \to \frac{e^{6\sqrt{\mathbf{c}(p)}z^{\frac{1}{6}}}}{z^{\frac{7}{12}}} \equiv f_{2,asy}(p,z)\end{aligned} \tag{49}$$

The general solutions to (45) satisfying the boundary conditions (46) are

$$\begin{aligned}\mathcal{V}(p,z) = &\quad f_2(p,0)f_1(p,z) - f_1(p,0)f_2(p,z),\ \textit{axial}\\ \mathcal{V}(p,z) = &\quad f_2'(p,0)f_1(p,z) - f_1'(p,0)f_2(p,z),\ \textit{vector}\end{aligned} \tag{50}$$

We now insert (44) using (50) into the massive quadratic action for Φ. The result is the boundary action ($z_\Lambda\to\infty$)

$$\mathbf{S}_B = -\int\frac{d^4q}{(2\pi)^4}\mathbb{S}_\mu^\dagger(q)\left(2\kappa g(z_\Lambda)\frac{\partial_z\mathcal{V}(p,z_\Lambda)}{\mathcal{V}(p,z_\Lambda)}\right)\mathbb{S}^\mu(q) \tag{51}$$

from which we read the axial polarization function at the boundary

$$\Pi_A(q^2) = 4\kappa g(z_\Lambda)\frac{f_2(q,0)f_1'(q,z_\Lambda) - f_1(p,0)f_2'(q,z_\Lambda)}{f_2(q,0)f_1(q,z_\Lambda) - f_1(q,0)f_2(q,z_\Lambda)} \tag{52}$$

In the vicinity of the poles, (52) is dominated by

$$\Pi_A(q^2) \to -\frac{f_2(q,0)}{f_1(q,0)}4\kappa g(z_\Lambda)\frac{f_1'(q,z_\Lambda)}{f_2(q,z_\Lambda)} \tag{53}$$

To simplify (53), we now note that near the poles the identity (47) simplifies

$$4\kappa g\frac{f_1'}{f_2} = -\frac{1}{f_2^2} \to -\frac{1}{f_{2,asy}^2} \tag{54}$$

where the last relation holds at the UV boundary $z = z_\Lambda$. (54) when used in (53) reduces the axial polarization function to

$$\Pi_A(p^2) = \frac{f_2(p,0)}{f_1(p,0)}f_{2,asy}^{-2}(p,z_\Lambda) \tag{55}$$

A similar reasoning with the vector sources gives the vector polarization function

$$\Pi_V(q^2) = \frac{f_2'(q,0)}{f_1'(q,0)}f_{2,asy}^{-2}(q,z_\Lambda) \tag{56}$$

The poles in the axial-vector correlator (55) are given by $f_1(m_n,0) = 0$, while those of the vector correlator (56) are given by $f_1'(m_n,0) = 0$, in agreement with the mass spectrum in (22). The residues are sensitive to the UV cutoff z_Λ. They are further discussed in the Appendix.

B. Scalar and Pseudo-Scalar polarizations

Similarly, for the scalar and pseudo-scalar mesons, we refer to the square integrable solutions by $\tilde{f}_1$ and to the non-square integrable function by $\tilde{f}_2$, and require the normalization through the Wronskian

$$\frac{\kappa\tilde{\nu}^2}{p^2}\frac{\tilde{g}}{\tilde{f}}\left(\tilde{f}_1\frac{d\tilde{f}_2}{dz} - \tilde{f}_2\frac{d\tilde{f}_1}{dz}\right) = -1 \tag{57}$$

in comparison to the vector normalization in (47). A repeat of the arguments for the vector polarizations lead to the pseudo-scalar and scalar polarizations

$$\begin{aligned}\tilde{\Pi}_S(q^2) &= \frac{\tilde{f}_2(q,0)}{\tilde{f}_1(q,0)}\tilde{f}_{2,ayy}^{-2}(p,z_\Lambda)\\ \tilde{\Pi}_P(q^2) &= \frac{\tilde{f}_2'(q,0)}{\tilde{f}_1'(q,0)}\tilde{f}_{2,ayy}^{-2}(q,z_\Lambda)\end{aligned} \tag{58}$$

respectively. It is readily checked that in the heavy quark limit $\Pi = \tilde{\Pi}$, and the $\mathbb{JJ}$ correlators exhibit heavy-quark symmetry. This degeneracy follows from the rigid $O(4)$ symmetry of the vector fields in 5-dimensions.

V. HL CHIRAL INTERACTIONS

A. Chiral symmetry

To identify properly the nature of the chiral transformation on the holographic field decomposition, we will recall in this section how this identification is made in the constituent quark model. For that consider the HL sources $H_\pm$ in the effective action for bare constituent quarks

$$-\bar{\psi}_L H_+ Q_R - \bar{\psi}_R H_- Q_L + \text{c.c} \tag{59}$$

with $H_\pm$ HL sources for the $(0^\pm, 1^\pm)$ multiplet with $\pm$ chiralities. Here $\psi_{L,R}$ refer to the bare light quarks and $Q_{L,R}$ to the heavy quarks. Let U be a generic pion field. Under rigid chiral symmetry, the pion field transforms linearly as $U \to LUR^\dagger$, and the fundamental quarks transform as $\psi_L \to L\psi_L, \psi_R \to R\psi_R$, so that

$$(H_+, H_-) \to (LH_+, RH_-) \tag{60}$$

Rigid chiral symmetry is better enforced through the decomposition $U = \xi_L \xi_R^\dagger$, with both $\xi_L \to L\xi_L h^\dagger(x)$ and $\xi_R \to R\xi_R h^\dagger(x)$ transforming non-linearly. Consider now the dressed constituent quarks $\chi_{L,R} = \xi_{L,R}^\dagger \psi_{L,R}$. It follows, that the corresponding dressed HL effective fields with odd parity are

$$H = (\gamma_\mu D^\mu + i\gamma_5 D) \tag{61}$$

with the identification

$$\begin{aligned} D^\mu = &\quad \xi_L^\dagger H_+^\mu + \xi_R^\dagger H_-^\mu \\ D = &\quad \xi_L^\dagger H_+ + \xi_R^\dagger H_-^\mu \end{aligned} \tag{62}$$

Under chiral symmetry (61) transforms as $H \to h(x)H$. Similarly, for the $(0^+, 1^+)$ multiplet we can define the dressed HL effective fields with even parity as

$$G = (D_0 + \gamma_\mu \gamma_5 D_1^\mu) \tag{63}$$

which transforms as $G \to h(x)G$ under chiral symmetry We now seek to enforce these symmetries on the bulk fields in holography.

B. Holographic identification

In the axial gauge $A_M(x, z \to \infty) \to 0$, the residual gauge transformation g satisfies $\partial_M g \to 0$ at infinity. Following the arguments in [24], we identify the rigid L, R chiral transformations as $L = g(z \to +\infty) = g_+$ and $R = g(z \to -\infty) = g_-$. In the axial gauge (bare gauge), the pion field $\pi(x)$ is identified as

$$U(x) = e^{\frac{i}{f_\pi}\pi(x)} = Pe^{-\int_{-\infty}^{+\infty} A_z(x,z')dz'} \tag{64}$$

As expected, under rigid chiral transformations $U \to LUR^\dagger$. A useful gauge choice is the one where the pion field is identified with the zero mode in the holographic direction (dressed gauge)

$$A_z(x,z) = -\frac{i}{f_\pi}\pi(x)\psi_0'(z), \qquad A_\mu(x,z) = 0 \tag{65}$$

The non-normalizable pion zero-mode running along the $(N_f - 1)$ coincidental $D8_L$ branes is [24]

$$\psi_0(z) = \frac{1}{\pi}\arctan\left(\frac{z}{U_{KK}}\right) \tag{66}$$

This dressed gauge is reached by noting that the pion identification in (64) allows us to slice the holonomy along the z-direction through

$$\xi_\pm(x,z) = Pe^{-\int_0^{\pm z} A_z(x,z')dz'} \tag{67}$$

and identify

$$\begin{aligned} \xi_L(x) = &\quad \xi_+(x, z \to \infty) \\ \xi_R^{-1}(x) = &\quad \xi_-(x, z \to \infty) \end{aligned} \tag{68}$$

The bare HL meson fields transform as fundamental fields under rigid chiral transformations

$$\Phi(x, z \to \pm\infty) \to g_\pm \Phi(x, z \to \pm\infty) \tag{69}$$

This means that the bare $\Phi(z \to \pm\infty)$ are the analogue of the bare $H_\pm$. In particular for charm, the low lying $(\tilde{\phi}_0, \phi_0)$ modes contribute to the (D, D^*) meson multiplet H, and the first excited $(\tilde{\phi}_1, \phi_1)$ modes contribute to the (D_0, D_1) meson multiplet G. The dressed HL meson fields are readily identifiedas

$$\begin{aligned} \Phi_\mu(x,z) = &\quad \xi_+(x,z)\phi_0(z)D_\mu(x) \\ \Phi_z(x,z) = &\quad \xi_+(x,z)\tilde{\phi}_0(z)D(x) \end{aligned} \tag{70}$$

C. Quadratic HL holographic action

The holographic effective action with all quadratic terms in the HL fields can now be constructed without recourse to the heavy quark limit. For that, we follow the construction in [24] and supplement the A_M field with external flavor sources $A_{L,R}$ by defining

$$A_\mu = V_\mu + 2\psi_0 A_\mu + \sum_n \mathbb{A}_{\mu,n}\psi_n \tag{71}$$

with A_z still given by (65). Here, the external sources are defined as $A_{L,R} = (V \pm A)$. For odd values of n we identify $\mathbb{A}_{\mu,n} = v_{\mu,n}$ with the light 1^{--} flavor vector excitations, and for even values of n we identify $\mathbb{A}_{\mu,n} = a_{\mu,n}$ with the light 1^{++} flavor axial excitations. Using the additional definition

$$\mathbf{A}_\mu(x,z) = e^{-\frac{i}{f_\pi}\psi_0\pi}(\partial_\mu + A_\mu)e^{\frac{i}{f_\pi}\psi_0\pi} \tag{72}$$

we have the identities

$$\begin{aligned}
e^{-\frac{i}{f_\pi}\psi_0\pi}f_{\mu,\nu} &= \sum_n \phi_n(\partial_{[\mu}D_{n,\nu]} + \mathbf{A}_{[\mu}D_{n,\nu]})\\
e^{-\frac{i}{f_\pi\psi_0}\pi}f_{\mu,z} &= \sum_n \left(\tilde\phi_n(\partial_\mu D_n + \mathbf{A}_\mu D_n) - \phi_n' D_{\mu,n}\right)
\end{aligned} \tag{73}$$

The DBI contributions which are quadratic in D, D^* and linear in $\mathbf{A}$ follows by inserting (71-73). The resulting action is of the form $S_2 = \kappa f\mathbf{S}_1 + \kappa g\mathbf{S}_2$ with

$$\begin{aligned}
\mathbf{S}_1 = &\\
&+4\sum_{m,n}\phi_m\phi_n D_m^{\mu\dagger}(\partial_\mu\mathbf{A}_\nu - \partial_\nu\mathbf{A}_\mu + [\mathbf{A}_\mu,\mathbf{A}_\nu])D_n^\nu\\
&-4\sum_{m,n}\phi_m\phi_n D_m^{\dagger\mu}(\mathbf{A}_\nu\mathbf{A}_\mu - h_{\mu\nu}\mathbf{A^2})D_n^\nu\\
&-2\sum_{mn}\phi_m\phi_n\partial_{[\mu}D^\dagger_{n,\nu]}\mathbf{A}^{[\mu}D_m^{\nu]}\\
&-2\sum_{m,n}\phi_m\phi_n D_m^{[\mu\dagger}\mathbf{A}^{\nu]}\partial_{[\mu}D_{n,\nu]}
\end{aligned} \tag{74}$$

$$\begin{aligned}
\mathbf{S}_2 = &\\
&-2\sum_{m,n}\tilde\phi_m\tilde\phi_n(\partial_\mu D_m^\dagger\mathbf{A}^\mu D_n - D_n^\dagger\mathbf{A}^\mu\partial_\mu D_m)\\
&+2\sum_{m,n}\tilde\phi_m\phi_n'(D_n^{\mu\dagger}\mathbf{A}_\mu D_m - D_m^\dagger\mathbf{A}_\mu D_n^\mu)\\
&+2\sum_{m,n}\tilde\phi_m\phi_n(D_m^\dagger\partial_z\mathbf{A}_\mu D_n^\mu - D_n^{\mu\dagger}\partial_z\mathbf{A}_\mu D_m)\\
&+2\sum_{m,n}\tilde\phi_m\tilde\phi_n D_m^\dagger\mathbf{A^2}D_n
\end{aligned} \tag{75}$$

where we have omitted the traces and the integrations for notational simplicity. Here we have defined $\mathbf{A^2} = \mathbf{A}^\mu\mathbf{A}_\mu$. The expansion of $\mathbf{A}_\mu$ in terms of the pion and vector mesons in leading orders, read

$$\begin{aligned}
\mathbf{A}_\mu = &\frac{i}{f_\pi}\psi_0\partial_\mu\pi + \frac{1}{8f_\pi^2}[\pi,\partial_\mu\pi] + ...\\
&+V_\mu - \frac{i}{f_\pi}\psi_0[\pi,V_\mu] + 2A_\mu\psi_0 - \frac{2i}{f_\pi}\psi_0^2[\pi,A_\mu] + ...\\
&+\sum_n\psi_{2n-1}v_{n,\mu} + \sum_n\psi_{2n}a_{n,\mu} + ...
\end{aligned} \tag{76}$$

The unexpanded CS contribution involving the Φ fields receives several contributions from (6). They are

$$S_{CS} = S_{\Phi^2,A} + S_{\Phi^2,A^2} + S_{\Phi^2,A^3} + S_{\Phi^4,A} + S_{\Phi^4} \tag{77}$$

with each of the contributions given in form-notations as follows

$$\begin{aligned}
S_{\Phi^2,A} = &-\frac{N_c}{24\pi^2}(d\Phi^\dagger A d\Phi + d\Phi^\dagger dA\Phi + \Phi^\dagger dAd\Phi)\\
S_{\Phi^2,A^2} = &-\frac{N_c}{16\pi^2}(d\Phi^\dagger A^2\Phi + \Phi^\dagger A^2 d\Phi)\\
&-\frac{N_c}{16\pi^2}\Phi^\dagger(AdA + dAA)\Phi\\
S_{\Phi^2,A^3} = &-\frac{5N_c}{48\pi^2}\Phi^\dagger A^3\Phi\\
S_{\Phi^4,A} = &+\frac{N_c}{8\pi^2}\Phi^\dagger\Phi\Phi^\dagger A\Phi\\
S_{\Phi^4} = &+\frac{N_c}{16\pi^2}\Phi^\dagger\Phi(\Phi^\dagger d\Phi + d\Phi^\dagger\Phi)
\end{aligned} \tag{78}$$

The Φ field is defined explicitly in (70) and the A field is defined in (71). We have omitted the flavor trace and the integration which is 5-dimensional here. The latter will reduce to 4-dimensions after inserting (70) and integrating over the HL meson holographic wavefunctions.

We note that when only the pion field is retained in (71) (no vector mesons), (77-78) simplifies as

$$S_{\Phi^2,A^2} + S_{\Phi^2,A^3} \to 0 \tag{79}$$

Also, the first quadratic contribution $S_{\Phi^2,A}$ does not vanish and will be discussed in details below. In addition, the quartic contributions in (78) do not vanish and contribute

$$\begin{aligned}
&S_{\Phi^4,A} + S_{\Phi^4} \to\\
&\frac{iN_c}{8\pi^2}D^\dagger DD^\dagger\mathbf{A}D + \frac{iN_c}{16\pi^2}D^\dagger D(D^\dagger dD + dD^\dagger D)
\end{aligned} \tag{80}$$

Finally, the DBI quadratic holographic action (74) together with the CS parts (78) are the most general pion and vector meson interactions with HL mesons with finite masses. The HL mesons are characterized by a pionic-like zero mode as noted in (22) in the massless limit. So (74) and (78) interpolate continuously between massless and massive HL mesons with exact heavy quark symmetry asymptotically as we further detail below.

D. One-pion interaction

Now we consider the sourceless case with $A_{L,R} = 0$ and all $a_{n,\mu}, v_{n,\mu} = 0$. In this case, in leading order in the pion field, the bare heavy meson fields in (70) reads

$$\Phi_\mu \approx \left(1+\frac{i}{f_\pi}\psi_0\pi\right)\phi_1 D_\mu$$
$$\Phi_z \approx \left(1+\frac{i}{f_\pi}\psi_0\pi\right)\tilde\phi_1 D$$
$$A_z = -\frac{i}{f_\pi}\pi\psi_0'(z),\ \ A_\mu = 0 \tag{81}$$

in terms of which the contributions (4-5) are

$$f_{\mu\nu} \approx \left(1+\frac{i}{f_\pi}\psi_0\pi\right)\phi_1\partial_{[\mu}D_{\nu]} + \frac{i}{f_\pi}\psi_0\phi_1\partial_{[\mu}\pi D_{\nu]}$$
$$f^0_{\mu z} \approx \left(1+\frac{i}{f_\pi}\psi_0\pi\right)\tilde\phi_1\partial_\mu D - \frac{i}{f_\pi}\pi\psi_0\tilde\phi_1' D_\mu + \frac{i}{f_\pi}\psi_0\tilde\phi_1\partial_\mu\pi D \tag{82}$$

and

$$a_{\mu\nu} \approx \left(1+\frac{i}{f_\pi}\psi_0\pi\right)D_{[\mu}D^\dagger_{\nu]}\left(1-\frac{i}{f_\pi}\psi_0\pi\right)\phi_1^2$$
$$a_{\mu z} \approx \left(1+\frac{i}{f_\pi}\psi_0\pi\right)D_{[\mu}D^\dagger_{z]}\left(1-\frac{i}{f_\pi}\psi_0\pi\right)\phi_1\tilde\phi_1 \tag{83}$$

with

$$F_{\mu\nu} = \mathcal{O}(\pi^2), \qquad F_{z\mu} \approx \psi_0'\frac{i}{f_\pi}\partial_\mu\pi \tag{84}$$

In developing the gauge and heavy meson fields in (70), we have omitted the contributions from the tower of vector and axial fields, and the contribution of the excited heavy mesons for simplicity. They will be recalled below. Note that in the dressed gauge, the leading pion contribution is the current algebra result

$$\frac{f_\pi^2}{4}\mathrm{Tr}(U^{-1}\partial_\mu U)^2 \tag{85}$$

with $f_\pi^2/M_{KK}^2 = 4\kappa/\pi$.

1. $(0^-, 1^-)$ *multiplet*

The leading contribution to the interaction between the heavy-light mesons to the pions follows only from the first contribution in the CS term in (6) in the form

$$S_{\rm CS} \to -\frac{N_c}{16\pi^2}\int dz d^4x \mathrm{Tr}(d\Phi^\dagger dA\Phi + \Phi^\dagger dA d\Phi) \tag{86}$$

Using the identification (61), we can re-write (86) as

$$S^\pi_{\rm CS} = -\frac{N_c}{32\pi^2 f_\pi}\int dz\psi_0'(z)\phi_0^2 \times\int d^4x\,\mathbf{Tr}\left(\gamma\cdot\partial\bar H\gamma_5\gamma\cdot\partial\pi H + \text{c.c.}\right) \tag{87}$$

with $\bar H = \gamma^0 H^\dagger\gamma^0$. We note that the bolded trace in (87) is now both over flavor and spin. To obtain the non-relativistic reduction of (87), we first decompose the positive frequency part of $H \to \mathbb{H}_+ + \mathbb{H}_-$ as

$$\mathbb{H}_\pm = \frac{e^{-iMx_0}}{\sqrt{2M}}(\gamma_\mu D_\mu + i\gamma_5 D)\frac{1\pm\gamma_0}{2} \tag{88}$$

which gives

$$S^\pi_{\rm CS} = S^+_{\rm CS} + S^-_{\rm CS}$$
$$S^\pm_{\rm CS} = \frac{iN_c}{32\pi^2 f_\pi}\int dz\psi_0'(z)\phi_0^2 \times\int d^4x\,\mathrm{Tr}\partial_i\pi(\pm(D_iD^\dagger - DD_i^\dagger) + \epsilon^{ijk}D_kD_j^\dagger) \tag{89}$$

Keeping only the contribution from $\mathbb{H}_+$, which transforms homogeneously as the $(\frac12,\frac12)$ representation under heavy-light spin transformations, is equivalent to deforming the CS contribution (87) to

$$S^\pi_{\rm CS} \to S^+_{\rm CS} = -\frac{N_c}{32\pi^2 f_\pi}\int dz\psi_0'(z)\phi_0^2 \times\int d^4x\,\mathbf{Tr}\left((\gamma\cdot\partial + iM)\bar H\gamma_5\gamma\cdot\partial\pi H + \text{c.c.}\right) \tag{90}$$

Amusingly, this deformation can be viewed as a fermion loop with a massive instead of a massless quark, with $\gamma\cdot\partial \to \gamma\cdot\partial + iM$ acting as a massive projector. Therefore, keeping positive energy naturally selects the $\mathbb{H}_+$ contribution. The modified term is actually a normal term which requires a metric and may come from a missing piece of the low-energy effective field theory action of our underlying brane configuration with large transverse separation. Assuming this, the z-integration can be performed exactly to give

$$\int dz\psi_0'(z)\phi_0^2 = \frac{1}{\pi U_{KK}}\frac{1}{4f_0\kappa} = \frac{1}{\pi\tilde T(2\pi\alpha')^2R^3} = \frac{108\pi^3 l_s^2}{M_{KK}N_c}\times\frac{2M_{KK}}{\pi\lambda l_s^2} = \frac{216\pi^2}{N_c\lambda} \tag{91}$$

which allows for (90) to take the standard leading one-pion interaction form

$$S^{+}_{\rm CS} = \frac{g_H}{f_\pi}\int d^4x\,{\rm Tr}\partial_i\pi(D_iD^\dagger - DD_i^\dagger + \epsilon^{ijk}D_kD_j^\dagger) \quad (92)$$

with the pseudo-vector pion axial coupling g_H to the $(0^-, 1^-)$ multiplet given by

$$g_H \equiv \frac{27}{4\lambda} \quad (93)$$

In [24] a fit to the low lying meson spectrum led to $\lambda \sim 8.7$ which implies that $g_H \sim 0.78$ in our holographic model. The holographic result is close to the reported value of $g_H \sim 0.65$, as measured through the charged pion decay $D^* \to D\pi$ [2].

2. $(0^+, 1^+)$ multiplet

Using the analogue of the non-relativistic reduction (88) for the G-multiplet with

$$G \to \mathbb{G}_+ = \frac{e^{-iMx_0}}{\sqrt{2M}}(D_0 + \gamma_\mu\gamma_5 D_1^\mu)\frac{1+\gamma_0}{2} \quad (94)$$

a similar reasoning shows that the pseudo-vector pion axial coupling g_G to the $(0^+, 1^+)$ multiplet follows similarly with the mode substitution $\phi_0 \to \phi_1$ in (87-91). As a result, we have $g_G = g_H$ in our holographic construct.

The intra-multiplet one-pion interaction is now seen to follow from the DBI contribution only,

$$\begin{aligned} S^\pi_{HG} = & \\ + \frac{4\kappa}{f_\pi}&\int f\phi_0\phi_1\psi_0(z)dz \int d^4x{\rm Tr}\partial_0\pi(D_iD^\dagger_{1i} + D_{1i}D_i^\dagger) \\ + \frac{2\kappa}{f_\pi}&\int g\tilde\phi_0\tilde\phi_1\psi_0(z)dz \int d^4x{\rm Tr}\partial_0\pi(DD_0^\dagger + D_0D^\dagger) \end{aligned} \quad (95)$$

We note the following identity

$$2\int dz\, f\psi_0\phi_0\phi_1 = \int dz\, g\psi_0\tilde\phi_0\tilde\phi_1 \quad (96)$$

which allows to rewrite (95) in standard form

$$\frac{g_{HG}}{f\pi}\mathbf{Tr}\left(\gamma_5\bar{G}Hv^\mu A_\mu\right) + {\rm c.c} \quad (97)$$

The pseudo-vector axial cross pion coupling is

$$g_{HG} = 4\kappa\int dz\, f\psi_0\phi_0\phi_1 = \frac{2^{\frac14}}{2\pi}\left(\frac{M_{KK}}{M}\right)^{\frac12} = 0.23\left(\frac{m_\rho}{M}\right)^{\frac12} \quad (98)$$

the last relation follows from the substitution of the rho mass $m_\rho = \sqrt{0.67}M_{KK}$ [24]. The axial coupling is seen to vanish in the heavy quark limit $M = \tilde\nu\sqrt{a_0} \to \infty$.

We observe that in the present holographic model all one-pion couplings to the HL mesons are pseudo-vectors. In particular, the Goldberger-Treiman combination $g_{GH}\Delta m/f_\pi$ does not support a pseudo-scalar coupling as initially noted in the chiral symmetric constructs without confinement in [3, 4]. Using the empirical value of $m_\rho \sim 770$ MeV and the charm mass with $m_C = 1275$ MeV we find that $g_{HG,C} \sim 0.18$, while for bottom with $m_B \sim 4180$ MeV we find $g_{HG,B} \sim 0.10$.

3. One-pion radiative widths

The strong intra-multiplets decay $G \to H + \pi$ follow from (97). Both the chargeless and charged pion decay of charmed and bottom mesons with final momentum p_π read

$$\begin{aligned} \Gamma(G \to H + \pi^0) &= \frac{1}{4\pi}\left(\frac{g_{HG,C}}{f_\pi}\right)^2(m_G - m_H)^2|p_{\pi^0}| \\ \Gamma(G \to H + \pi^\pm) &= \frac{2}{4\pi}\left(\frac{g_{HG,C}}{f_\pi}\right)^2(m_G - m_H)^2|p_{\pi^\pm}| \end{aligned} \quad (99)$$

Our holographic result for the HL charm meson gives

$$\begin{aligned} &\Gamma(D_1^0(2420) \to D^{*+}(2010)\pi^-) = \\ &\frac{1}{2\pi}\left(\frac{0.18}{93}\right)^2(411)^2(354) = 36\,{\rm MeV} \end{aligned} \quad (100)$$

wich is comparable to the measured full width of $= (27.4 \pm 2.5)$ MeV at $p_{\pi^-} = 354$ MeV [32]. The partial width for the charged decay of the HL bottom meson is

$$\begin{aligned} &\Gamma(B_1^0(5721) \to B^{*+}(5325)\pi^-) = \\ &\frac{1}{2\pi}\left(\frac{0.1}{93}\right)^2(400)^2(362) = 11\,{\rm MeV} \end{aligned} \quad (101)$$

which is to be compared to a full width of 23 ± 5 MeV at $p_{\pi^-} = 362$ MeV [32]. Also, we have the partial decay width

$$\begin{aligned} &\Gamma(B_1^+(5721) \to B^{*0}(5325)\pi^+) = \\ &\frac{1}{2\pi}\left(\frac{0.1}{93}\right)^2(402)^2(409) = 12\,{\rm MeV} \end{aligned} \quad (102)$$

which is to be compared to the measured full width of 49^{+12}_{-16} MeV at $p_{\pi^+} = 409$ MeV [32].

E. Two pion interaction

The two pion interaction terms can be derived using similar arguments. All terms quadratic in D, D^* and linear in $\mathbf{A}$ following from the DBI action (74) follow from the two contributions

$$
\begin{aligned}
&-4i\sum_{m,n} f\phi_m\phi_n \,\mathrm{Tr}(\mathbf{A}_0 D^\mu_m D^\dagger_{\mu n}) \\
&-2i\sum_{m,n} g\tilde{\phi}_m\tilde{\phi}_n \,\mathrm{Tr}(\mathbf{A}_0 D_m D^\dagger_n)
\end{aligned} \tag{103}
$$

Inserting (76) in (103) and performing the z-integrations give

$$
-\frac{i}{8f_\pi^2}\sum_n \mathrm{Tr}\left([\pi,\partial_0\pi](D_{n,\mu}D_n^{\mu\dagger} + D_n D_n^\dagger)\right) \tag{104}
$$

which is of order M^0. For instance, (104) describes the parity conserving two pion radiative decays within the same multiplets $(0^\pm, 1^\pm)$, i.e. $H \to H + 2\pi$ and $G \to G + 2\pi$.

In addition, there are also parity non-conserving two pion interactions from the Chern-Simons term, leading to radiative decays of the type $G \to H + 2\pi$. To leading order in the heavy-quark limit, we have

$$
\begin{aligned}
&\frac{1}{8f_\pi^2 M}\sum_{m,n}\int dz\,\kappa g\tilde{\phi}_m\phi_n' \int d^4x \\
&\times \mathrm{Tr}\left([\pi,\partial_i\pi](D_m D^\dagger_{ni} - D_{n,i}D^\dagger_m + \epsilon^{ijk}D_{mk}D^\dagger_{nj})\right)
\end{aligned} \tag{105}
$$

with

$$
\begin{aligned}
&\int dz\,\kappa g\tilde{\phi}_0\phi_1' = \left(\frac{g_0}{f_0}\right)^{1/2}\int dz\,\kappa f\phi_0\phi_1' = \\
&\frac{\sqrt{2}}{4}\left(\frac{g_0}{f_0}\right)^{\frac14}\left(\frac{M}{U_{KK}}\right)^{\frac12} = \frac{2^{\frac34}}{4}\,(M_{KK}M)^{\frac12}
\end{aligned} \tag{106}
$$

(106) yields the two-pion cross coupling G_{GH}/f_π^2 in (105) as

$$
G_{GH} = \frac{2^{\frac34}}{32}\left(\frac{M_{KK}}{M}\right)^{\frac12} = 0.06\left(\frac{m_\rho}{M}\right)^{\frac12} \tag{107}
$$

which is relatively small. The couplings G_{GH} in (107) and g_{GH} in (98) are both of order $1/\sqrt{M}$ and vanish in the heavy quark limit.

VI. CONCLUSIONS

We have presented a top-down holographic approach to the HL mesons interacting with the lightest pseudoscalar mesons. The geometrical set up consists of $N_f - 1$ light $D8$-$\overline{D8}$ probe branes plus one heavy brane in the background of N_c $D4$ branes. We have identified the HL degrees of freedom with the string low energy degrees of freedom near the world volume of the light branes. They are represented by bi-fundamental vector fields that are approximately local in the vicinity of the light branes.

We have shown how the holographic effective action emerges from the bulk DBI and Chern-simons actions, and explicitly verified that it enjoys both chiral and heavy quark symmetry in the limit of a heavy quark mass, modulo a suitable deformation of the CS contribution. The HL holographic effective action reduces continuously to the chiral effective action for $SU(N_f)$ when the heavy quark mass is removed in the limit of coincidental N_f light branes. This construction can be made more realistic through the use of improved holographic QCD [33].

The HL effective action allows for a description of the HL meson internal structure in the strong coupling $\lambda = g^2 N_c$ limit. In particular, the squared mass spectrum is shown to be Regge-like with fixed intercept M^2 and a slope of about Mm_ρ. In leading order, the splitting between the even and odd parity multiplets is fixed by the rho mass m_ρ. However, it is found to be larger than the reported empirical splitting for charm and bottom.

We have made explicit use of the HL effective action to extract the pertinent axial charges for the low lying HL multiplets $H = (0^-, 1^-)$ and $G = (0^+, 1^+)$ both of which are degenerate in the heavy quark limit. Holography shows that the axial couplings are equal with $g_H = g_G = 27/4\lambda$, and close to the experimentally reported value of $g_H = 0.65$, for $\lambda \sim 8.7$ which is the value selected in [24]. The inter-multiplet coupling is fixed by the ratio of the rho to heavy quark mass as $g_{GH} = 0.23\,(m_\rho/M)^{\frac12}$. Estimates of the one-pion partial decay widths are overall in qualitative agreement with the data.

The shortcomings of the holographic limit are rooted in the double limit of large N_c and strong 't Hooft coupling $\lambda = g^2 N_c$. The corrections are notoriously hard to calculate. Also, our near-coincidental brane approximation for the Chern-Simons contribution shows the need for an additional mass deformation in the heavy mass limit, which combines to form a massive projector, that requires a better first principle undertanding. Overall, the leading order results we have established are in fair agreement with data. The holographic HL chiral effective action provides valuable results for the few pion decays without and with vector mesons that could be compared with the upcoming experiments involving especially HL bottom mesons. It also provides a framework for discussing electromagnetic decays, as well as single and double heavy baryons. Some of these issues will be addressed next.

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VIII. APPENDIX: HL DECAY CONSTANTS

The axial and vector polarization functions (55-56) exhibit poles, with the squared axial decay constants as residues

$$f^2_{A_n,V_n} = -\lim_{q^2\to -m_n^2} \left((q^2+m_n^2)\ \Pi_{A,V}(q^2) \right) \tag{108}$$

In the heavy mass limit, the residues at the poles of the scalar and pseudoscalar polarization functions (55-56) are related by heavy quark symmetry which holds in our case. In the standard definitions, the pseudoscalar constant is $m_P f_P \equiv f_A$ and vector decay constant is $m_{P^*} f_{P^*} \equiv f_V$, with

$$\begin{aligned} \langle 0|\bar{q} i\gamma_\mu\gamma_5 Q|P(p)\rangle =& \quad f_P p_\mu \\ \langle 0|\bar{q}\gamma_\mu Q|P^*(p)\rangle =& \quad f_{P^*}\epsilon_\mu \end{aligned} \tag{109}$$

for $P = D, \bar{B}, ...$ and $P^* = D^*, \bar{B}^*, ...$ respectively. (109) are measurable through the weak leptonic decays $D \to \bar{l}\nu_l$ and $\bar{B} \to l\bar{\nu}_l$.

1. Generalities

To explicit the decay constants as residues at the poles of polarization functions derived above, we first reorganize the equation for the bulk-to-boundary propagator $\mathcal{V}(p,z)$ in the heavy quark limit using (38),

$$-\frac{d^2\mathcal{V}}{\omega_0 d\tilde{z}^2} + (\sqrt{\omega_0}\tilde{z})^2\mathcal{V} = -\frac{2(p^2+M^2)f_0}{\omega_0\tilde{\nu}g_0}\mathcal{V} \tag{110}$$

with $\omega_0^2 = f_0 a_0/g_0 U_{KK}^2$. We can set

$$\alpha \equiv -\frac{p^2+M^2}{\omega_0\tilde{\nu}}\frac{f_0}{g_0} - \frac{1}{2} \to \frac{\tilde{m}_n^2}{\omega_0}\frac{f_0}{g_0} - \frac{1}{2} \tag{111}$$

where the last identity holds for $p^2 = -m_n^2$. Using the parametrization $x = \sqrt{2\omega_0}\tilde{z} = \sqrt{2\omega_0\tilde{\nu}}z$, we can re-write (110) in the compact harmonic form

$$\frac{d^2\mathcal{V}}{dx^2} + \left(\alpha + \frac{1}{2} - \frac{1}{4}x^2\right)\mathcal{V} = 0 \tag{112}$$

The normalized square integrable solutions to (112) are parabolic cylinder functions $D_\alpha(x)$ (solutions to the harmonic problem)

$$f_1(\alpha, x) = \frac{(2\omega_0\tilde{\nu})^{\frac{1}{4}}\sqrt{2}}{(4f_0\kappa\Gamma(\alpha+1)\sqrt{2\pi})^{\frac{1}{2}}} D_\alpha(x) \tag{113}$$

In the heavy quark limit with $\tilde{\nu} \to 0$, the spectrum in (112) is harmonic for $p^2 = -m_n^2$, and identical to the harmonic spectrum in (39-40).

2. Axial decay constants: f_{2k+1}

For the odd harmonic states with $\alpha = 2k+1$, we have

$$\left(\frac{df_1}{dz}\right)(\alpha, 0) = \frac{(2\omega_0\tilde{\nu})^{\frac{3}{4}}\sqrt{2}(2k+1)!!}{(4f_0\kappa(2k+1)!\sqrt{2\pi})^{\frac{1}{2}}}(-1)^k \tag{114}$$

which gives through the Wronskian

$$f_2(\alpha, 0) = \frac{1}{2\kappa g_0}\frac{(4f_0\kappa(2k+1)!\sqrt{2\pi})^{\frac{1}{2}}}{\sqrt{2}(2k+1)!!(2\omega_0\tilde{\nu})^{\frac{3}{4}}}(-1)^{k+1} \tag{115}$$

Also, in the vicinity of $\alpha = 2k+1$ we note that,

$$\alpha - 2k - 1 = -\frac{p^2+m_{2k+1}^2}{\omega_0\tilde{\nu}}\frac{f_0}{g_0} \tag{116}$$

as well as

$$\begin{aligned} f_1(\alpha, 0) =& \quad \frac{(2\omega_0\tilde{\nu})^{\frac{1}{4}}\sqrt{2}(2k)!!}{(4f_0\kappa(2k+1)!\sqrt{2\pi})^{\frac{1}{2}}} \\ & \times\frac{\sqrt{2\pi}}{2}(-1)^{k+1} \times (\alpha - 2k - 1) \end{aligned} \tag{117}$$

Combining the above results, we finally find that

$$\begin{aligned} & f^2_{2,asy}(2k+1, z_\Lambda)f^2_{2k+1} = \\ & \frac{1}{2\kappa g_0}\frac{4f_0\kappa(2k+1)!\sqrt{2\pi}}{(2k+1)!(2\omega_0\tilde{\nu})\sqrt{2\pi}}\frac{g_0(\omega_0\tilde{\nu})}{f_0} = 1 \end{aligned} \tag{118}$$

3. Vector decay constants: f_{2k}

A rerun of the previous steps for the even harmonic states with $\alpha = 2k$ gives

$$f_1(\alpha, 0) = (2\omega_0\tilde{\nu})^{\frac{1}{4}}\frac{\sqrt{2}(2k-1)!!}{(4f_0\kappa(2k)!\sqrt{2\pi})^{\frac{1}{2}}}(-1)^k \tag{119}$$

which combined with the Wronskian gives

$$\left(\frac{df_2}{dz}\right)(\alpha,0) = \frac{1}{2\kappa g_0}\frac{(4f_0\kappa(2k)!\sqrt{2\pi})^{\frac{1}{2}}}{\sqrt{2}(2k-1)!!(2\omega_0\tilde{\nu})^{\frac{1}{4}}}(-1)^k \quad (120)$$

Also near $\alpha = 2k$ we have

$$\frac{d}{dz}f_1(\alpha,0) = (2\omega_0\tilde{\nu})^{\frac{3}{4}}\frac{\sqrt{2}(2k)!!(-1)^k\sqrt{2\pi}}{(4f_0\kappa(2k)!\sqrt{2\pi})^{\frac{1}{2}}}(\alpha-2k-1) \quad (121)$$

and therefore

$$f^2_{2,asy}(2k,z_\Lambda)f^2_{2k} = 1 \quad (122)$$

4. Estimate

The dependence on the cutoff z_Λ reflects on the dependence on the heavy quark mass which is M in the large mass limit. In our D-brane set up in section II we have made the assumption that the stringy HL modes are approximated by local bi-fundamental vector fields in the world-volume of the light branes. This approximation precludes us from a rigorous evaluation of the dependence of z_Λ. Qualitatively, we may estimate $z_\Lambda(M)$ by identifying M with the mass of a straight Nambu-Goto string pending from the heavy 8_H-brane to the light 8_L-branes as shown in Fig. 1. In the Einstein frame and for large z_Λ, we have

$$\begin{aligned} M \approx & \frac{1}{2\pi l_s^2}\int_0^{z_\Lambda} dz\,(-g_{tt}g_{zz})^{\frac{1}{2}} \\ \approx & \frac{1}{2\pi l_s^2}\int_0^{z_\Lambda} dz\left(\frac{4U_{KK}}{9U_z}\right)^{\frac{1}{2}} \approx \frac{1}{\pi l_s^{\frac{4}{3}}}\left(\frac{2}{9}\lambda M_{KK} z_\Lambda^2\right)^{\frac{1}{3}} \end{aligned} \quad (123)$$

which shows that $z_\Lambda \approx M^{\frac{3}{2}}$ in our estimate. Using (123) and the explicit form of f_2 where the overall constant is fixed by the Wronskian, we can make an estimate of the decay constants in (118) and (122). The vector (n-even) and axial-vector (n-odd) decay constants are

$$f_n(M) = \frac{f(n)}{\sqrt{n!}}\frac{2^{n-\frac{33}{32}}}{\pi^{\frac{37}{16}}3^{\frac{25}{18}}}\frac{\mathbb{C}_M^{\frac{n}{2}+\frac{11}{16}}}{e^{\mathbb{C}_M}}\sqrt{N_c}\,\lambda^{\frac{17}{16}}M_{KK}^2 \quad (124)$$

Here $\mathbb{C}_M$ is the dimensionless combination

$$\mathbb{C}_M = \frac{1}{\sqrt{2}}\left(\frac{9\pi}{\lambda}\right)^3\left(\frac{M}{2M_{KK}}\right)^4 \quad (125)$$

with $M_{KK} = m_\rho/\sqrt{0.67} = 1.22\,m_\rho = 941$ MeV. The constant $f(n)$ is related to the expansion of the parabolic cylinder functions. Note that (124) is of order $\sqrt{N_c}$.

The holographic result (124-125) holds in the heavy quark limit. In particular, we note that the ratio of the pseudo-scalar D to B meson decay constants following from (124-125) using the canonical definition $f_{Q_n} = f_n/m_n$ and to leading order in $1/\lambda$, is

$$\frac{f_{B_n}}{f_{D_n}} = \left(\frac{m_B}{m_D}\right)^{2n+\frac{7}{4}}\left(1+\mathcal{O}\left(\frac{1}{\lambda^3}\right)\right) \quad (126)$$

For $n = 0$, (126) is to be compared to $f_B/f_D = (m_D/m_B)^{\frac{1}{2}}$ from general arguments [5]. While the two ratios reduce to 1 in the heavy mass limit, they differ sharply at finite masses owing to our crude estimate in (123), and more generally our use of local bi-fundamental fields to describe non-local string modes. This last concern can be altogether bypassed in the bottom-up approach [34].

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