Λ_{c} semileptonic decays in a quark model

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Λ_c Semileptonic Decays in a Quark Model

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Hadronic form factors for semileptonic decay of the Λ_c are calculated in a nonrelativistic quark model. The full quark model wave functions are employed to numerically calculate the form factors to all relevant orders in ($1/m_c$, $1/m_s$). The form factors obtained satisfy relationships expected from the heavy quark effective theory (HQET). The differential decay rates and branching fractions are calculated for transitions to the ground state and a number of excited states of Λ. The branching fraction of the semileptonic decay width to the total width of Λ_c has been calculated and compared with other theoretical estimates and experimental results. The branching fractions for $\Lambda_c \to \Lambda^* l^+ \nu_l \to \Sigma\pi l^+ \nu_l$ and $\Lambda_c \to \Lambda^* l^+ \nu_l \to N\bar{K} l^+ \nu_l$ are also calculated. Apart from decays to the ground state $\Lambda(1115)$, it is found that decays through the $\Lambda(1405)$ provide a significant portion of the branching fraction $\Lambda_c \to X_s l \nu_l$. A new estimate for $f = B(\Lambda_c^+ \to \Lambda l^+ \nu_l)/B(\Lambda_c^+ \to X_s l^+ \nu_l)$ is obtained.

I. INTRODUCTION AND MOTIVATION

Semileptonic decays of hadrons are the main sources for precise knowledge on Cabibo-Kobayashi-Maskawa (CKM) matrix elements [1]. The form factors that parametrize the non-perturbative QCD effects in these transitions play a crucial role in the extraction of CKM matrix elements and the precision depends on how well the form factors are calculated.

A great deal of work has been done on semileptonic decay processes to calculate and improve the modeling of the form factors. For example, monopole type form factors were used to study semileptonic decay of heavy mesons by Wirbel, Stech and Bauer [2]. Isgur, Scora, Grinstein and Wise caculated the semileptonic B and D meson decays in a non-relativistic quark model [3]. Lattice QCD calculations of semileptonic decay form factors have been done in ref [4]. These are a very few out of a huge number of articles. More work has been done on semileptonic meson decays than baryon decays. Pervin, Roberts and Capstick worked on semileptonic baryon decays of Λ_Q [5] and Ω_Q [6]) in a constituent quark model. Some baryon decays have also been addressed in QCD sum rules [7], perturbative lattice QCD [8] and a number of other approaches [9].

The description of the weak decays of heavy hadrons are somewhat simplified because of the so-called heavy quark symmetry. This was first pointed out by Isgur and Wise [10]. Hadrons containing one heavy quark Q (with $m_Q >> \Lambda_{\rm QCD}$) possess this symmetry, which has been formalized into the heavy quark effective theory (HQET). In HQET the properties of the hadrons are governed by the light degrees of freedom and are independent of the heavy quark degrees of freedom. For semileptonic decays of heavy hadrons, HQET reduces the number of independent form factors needed to describe the decays.

In this paper, we examine the semileptonic decays of the Λ_c^+ to a number of Λs, including the ground state. Because it is the lightest charmed baryon, Λ_c^+ plays an important role in understanding charm and bottom baryons. The lowest-lying bottom baryon is most often detected through its weak decay to Λ_c^+. In addition, the study of all of the Λ_c^+-type and Σ_c-type baryons are directly linked to the understanding of the ground state of Λ_c^+, as these baryons eventually decay into a Λ_c^+.

Among the branching fractions of the Λ_c, $\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+)$ is used to normalize most of its other branching fractions. The Particle Data Group (PDG), in their previous version [11] reported that there was no model independent measurement of $\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+)$. Two model-dependent measurements were reported, with two different results obtained from different assumptions. The model that calculated branching fractions $\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+)$ from semileptonic decays, estimated that

$$\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+) = RfF\frac{B(D \to Xl^+\nu_l)}{1+|\frac{V_{cd}}{V_{cs}}|^2}\tau(\Lambda_c^+), \quad (1)$$

where,

$$R = B(\Lambda_c^+ \to pK^-\pi^+)/B(\Lambda_c^+ \to \Lambda l^+\nu_l),$$

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$$f = B(\Lambda_c^+ \to \Lambda l^+\nu_l)/B(\Lambda_c^+ \to X_s l^+\nu_l),$$
$$F = B(\Lambda_c^+ \to X_s l^+\nu_l)/B(D \to X_s l^+\nu_l).$$

They estimated $B(\Lambda_c^+ \to pK^-\pi^+) = (7.3 \pm 1.4)\%$ with the theoretical estimate of $f = F = 1.0$ with significant uncertainties.

However, in their most recent release, PDG [12] reports a model independent measurement of $\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+)$. A. Zupanc *et al.* (Belle Collaboration) [13] measured it to be $6.84^{+0.32}_{-0.40}\%$, while M. Ablikim *et al.* (BESIII Collaboration) [14] measured it to be $5.84 \pm 0.27 \pm 0.23\%$. The PDG fit is $6.35 \pm 0.33\%$ that leads to a new estimate of

$$f = B(\Lambda_c^+ \to \Lambda l^+\nu_l)/B(\Lambda_c^+ \to X_s l^+\nu_l) = 0.87^{+0.13}_{-0.17},$$

with the assumption of $F = 1.0$. Pervin, Roberts and Capstick (PRCI) [5] estimated the value of f to be 0.85 ± 0.04. Mott and Roberts [15] later estimated the rare decay branching fractions of the Λ_b using two different methods. Their results indicated that the results were sensitive to the precision with which the form factors were estimated, and this further implied that f could be even smaller than 0.85. The semileptonic branching fraction, $\mathcal{B}(\Lambda_c^+ \to \Lambda l^+\nu_l)$ is reported to be $2.8 \pm 0.5\%$ with the assumption that the Λ_c^+ decays only to the ground state $\Lambda(1115)$. No semileptonic decays to excited Λ have been reported. This provides the motivation for our work.

There have been a number of theoretical articles on the semileptonic decay of Λ_c^+ in recent years. Gutsche *et al.* used a covariant quark model to estimate the branching fraction for $\Lambda_c \to \Lambda l^+\nu_l$ [16]. Liu *et al.* used QCD light cone sum rules to examine this decay [17], while Ikeno and Oset have examined the semileptonic decay to the $\Lambda(1405)$, treating that state as a dynamically generated molecular state [18].

In the work presented herein, we work in the framework of a constituent quark model. Such models have been quite successful in explaining the main features of hadron phenomenology. In computing the form factors for $\Lambda_c \to \Lambda^*$, we have deployed two approximations. In the first approximation, single component wave functions are used to compute the analytic form factors for $\Lambda_c \to \Lambda^*$ transitions. As in PRCI [5] a variational diagonalization of a quark model Hamiltonian was used to extract the single component wave functions and the quark operators were reduced to their non-relativistic Pauli form. In the second method we keep the full relativistic form of the quark spinors and use the full quark model wave functions. We believe that this second method provides more reliable numerical values of the form factors as it uses fewer approximations.

We calculate the decay widths and branching fractions for decays to ground state and a number of excited $\Lambda^{(*)}$. We also study the decay widths and branching fractions of two other decay channels, namely $\Lambda_c^+ \to \Sigma\pi l^+\nu_l$ and $\Lambda_c^+ \to N\bar{K}l^+\nu_l$, via a set of Λ resonances.

The rest of this paper is organized as follows: in section II, we discuss the hadronic matrix elements and decay rates. Section III presents a concise overview of HQET and the relationships predicted by HQET among the form factors for the transitions we study. In section IV we describe the model we employ to calculate the form factors. Section V is devoted to discussing the numerical results such as form factors, decay rates and branching fractions. Section VI presents our conclusions and outlook. A number of details of the calculation are shown in the appendices.

II. MATRIX ELEMENTS AND DECAY WIDTHS

A. Semileptonic decay ($\Lambda_c^+ \to \Lambda l^+\nu_l$)

1. *Matrix Elements*

Fig.1 depicts the semileptonic decay $\Lambda_c^+ \to \Lambda^{(*)}l^+\nu_l$. We work in the rest frame of the parent Λ_c. The transition matrix element for the decay is

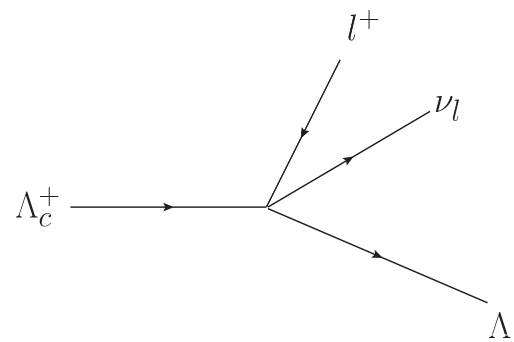
$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cs} L^\mu \langle \Lambda(p', s')|J_\mu|\Lambda_c(p, s)\rangle, \tag{2}$$

where V_{cs} is the CKM matrix element, $L^\mu = \bar{u}_{\nu_l}\gamma^\mu(1-\gamma_5)v_l$ is the lepton current and $J_\mu = \bar{s}\gamma_\mu(1-\gamma_5)c$ is the hadronic current. The momenta of the Λ_c, Λ, l, ν_l are labeled as p, p', p_l and p_{ν_l}, respectively. The hadronic matrix element is defined as

$$H_\mu = \langle \Lambda^{(*)}|J_\mu|\Lambda_c\rangle. \tag{3}$$

The hadronic matrix elements are parametrized in terms of a number of form factors. For transitions from the ground state Λ_c ($J^P = \frac{1}{2}^+$) to the ground state Λ ($J^P = \frac{1}{2}^+$), the matrix elements for the vector (V_μ) and axial-vector (A_μ)

FIG. 1: Semileptonic decay $\Lambda_c^+ \to \Lambda^{(*)} l^+ \nu_l$



currents are, respectively,

$$\langle \Lambda(p', s')|V_\mu|\Lambda_c(p, s)\rangle = \bar{u}(p', s')\left(\gamma_\mu F_1 + \frac{p_\mu}{m_{\Lambda_c}} F_2 + \frac{p'_\mu}{m_\Lambda} F_3\right) u(p, s), \tag{4}$$

$$\langle \Lambda(p', s')|A_\mu|\Lambda_c(p, s)\rangle = \bar{u}(p', s')\left(\gamma_\mu G_1 + \frac{p_\mu}{m_{\Lambda_c}} G_2 + \frac{p'_\mu}{m_\Lambda} G_3\right) \gamma_5 u(p, s), \tag{5}$$

where the F_i's and G_i's are the form factors and $s(s')$ is the spin of $\Lambda_c(\Lambda)$. The matrix elements for transitions to a daughter baryon with $J^P = \frac{3}{2}^-$ are

$$\langle \Lambda(p', s')|V_\mu|\Lambda_c(p, s)\rangle = \bar{u}^\alpha(p', s')\left[\frac{p_\alpha}{m_{\Lambda_c}}\left(\gamma_\mu F_1 + \frac{p_\mu}{m_{\Lambda_c}} F_2 + \frac{p'_\mu}{m_\Lambda} F_3\right) + g_{\alpha\mu} F_4\right] u(p, s), \tag{6}$$

$$\langle \Lambda(p', s')|A_\mu|\Lambda_c(p, s)\rangle = \bar{u}^\alpha(p', s')\left[\frac{p_\alpha}{m_{\Lambda_c}}\left(\gamma_\mu G_1 + \frac{p_\mu}{m_{\Lambda_c}} G_2 + \frac{p'_\mu}{m_\Lambda} G_3\right) + g_{\alpha\mu} G_4\right] \gamma_5 u(p, s). \tag{7}$$

The Rarita-Schwinger spinor $\bar{u}^\alpha$ satisfies the conditions

$$p'_\alpha \bar{u}^\alpha(p', s') = 0, \quad \bar{u}^\alpha(p', s')\gamma_\alpha = 0, \quad \bar{u}^\alpha(p', s')\not{p}' = m_{\Lambda^{3/2}} \bar{u}^\alpha(p', s'). \tag{8}$$

The corresponding matrix elements for transitions to a daughter baryon with $J^p = \frac{5}{2}^+$ are

$$\langle \Lambda(p', s')|V_\mu|\Lambda_c(p, s)\rangle = \bar{u}^{\alpha\beta}(p', s')\frac{p_\alpha}{m_{\Lambda_c}}\left[\frac{p_\beta}{m_{\Lambda_c}}\left(\gamma_\mu F_1 + \frac{p_\mu}{m_{\Lambda_c}} F_2 + \frac{p'_\mu}{m_\Lambda} F_3\right) + g_{\beta\mu} F_4\right] u(p, s).$$
$$\langle \Lambda_c(p', s')|A_\mu|\Lambda_c(p, s)\rangle = \bar{u}^{\alpha\beta}(p', s')\frac{p_\alpha}{m_{\Lambda_c}}\left[\frac{p_\beta}{m_{\Lambda_c}}\left(\gamma_\mu G_1 + \frac{p_\mu}{m_{\Lambda_c}} G_2 + \frac{p'_\mu}{m_\Lambda} G_3\right) + g_{\beta\mu} G_4\right] \gamma_5 u(p, s).$$

The spinor $\bar{u}^{\alpha\beta}$ satisfies the conditions

$$p'_\alpha \bar{u}^{\alpha\beta}(p', s') = p'_\beta \bar{u}^{\alpha\beta}(p', s') = 0, \quad \bar{u}^{\alpha\beta}(p', s')\gamma_\alpha = \bar{u}^{\alpha\beta}(p', s')\gamma_\beta = 0,$$
$$\bar{u}^{\alpha\beta}(p', s')\not{p}' = m_{\Lambda^{5/2}} \bar{u}^{\alpha\beta}(p', s'), \quad \bar{u}^{\alpha\beta}(p', s') g_{\alpha\beta} = 0.$$

Here we have shown the hadronic transition matrix elements for the decays to daughter baryons with natural parity. For decays to states with unnatural parity, the matrix elements are constructed by switching γ_5 from the equations defining the G_i to the equations defining the F_i.

2. *Decay Width*

The differential decay rate for the transition $\Lambda_c \to \Lambda^{(*)} l^+ \nu_l$ is

$$d\Gamma = \frac{1}{2m_{\Lambda_c}} \overline{|\mathcal{M}|^2} \frac{d^3p_l d^3p_{\nu_l} d^3p'}{2E_l 2E_{\nu_l} 2E'} \frac{(2\pi)^4 \delta^4(p - p' - p_l - p_{\nu_l})}{(2\pi)^3(2\pi)^3(2\pi)^3}, \tag{9}$$

where

$$\overline{|\mathcal{M}|^2} = \frac{G_F^2}{2}|V_{cs}|^2\frac{1}{2}\sum_{\text{spins}} H_\mu^\dagger H_\nu L^{\mu\dagger} L^\nu,$$
$$= \frac{G_F^2}{4}|V_{cs}|^2 H_{\mu\nu}L^{\mu\nu}. \tag{10}$$

$\overline{|\mathcal{M}|^2}$ is the squared amplitude averaged over the initial spins (the factor of $\frac{1}{2}$) and summed over the final spins. The most general Lorentz form of the hadronic tensor can be written as

$$H_{\mu\nu} = \alpha g_{\mu\nu} + \beta_{PP}P_\mu P_\nu + \beta_{PL}P_\mu L_\nu + \beta_{LP}L_\mu P_\nu + \beta_{LL}L_\mu L_\nu + i\gamma\epsilon_{\mu\nu\rho\sigma}P_\rho L_\sigma, \tag{11}$$

where we have defined $P = p'$ and $L = p - p'$. The lepton tensor is

$$L^{\mu\nu} = 8[p_l^\mu p_{\nu_l}^\nu + p_{\nu_l}^\mu p_l^\nu - g^{\mu\nu}(p_l.p_{\nu_l}) + i\epsilon^{\mu\nu\alpha\beta}p_{l\alpha}p_{\nu_l\beta}]. \tag{12}$$

Integrating over the lepton momenta allows us to write the lepton tensor as

$$\int \frac{d^3p_l d^3p_{\nu_l}}{(2\pi)^3(2\pi)^3 2E_l 2E_{\nu_l}} L^{\mu\nu} = \int d\Omega_l (Ag^{\mu\nu} + A'L^\mu L^\nu), \tag{13}$$

where $q^2 = (p - p')^2$ and

$$A = -\frac{(q^2 - m_l^2)^2(2q^2 + m_l^2)}{384\pi^6 q^4}, \quad A' = \frac{(q^2 - m_l^2)^2(q^2 + 2m_l^2)}{192\pi^6 q^6}. \tag{14}$$

The complete expression for the differential decay rate becomes

$$\frac{d\Gamma}{dq^2} = \frac{|V_{cs}|^2}{192}\frac{G_F^2}{\pi^3 m_{\Lambda_c}^3}\lambda^{1/2}(m_{\Lambda_c}^2, m_\Lambda^2, q^2)\frac{(q^2 - m_l^2)^2}{4q^4}\Bigg(-6\alpha q^2 + \beta_{PP}\Big[2q^2\big((P\cdot L) - m_\Lambda^2\big) + m_l^2\big(4(P\cdot L) - m_\Lambda^2\big)\Big] \tag{15}$$
$$+ \Big[\beta_{LP}(P\cdot L) + \beta_{PL}(P\cdot L) + \beta_{LL}q^2\Big]3m_l^2\Bigg),$$

where $P\cdot L = \frac{1}{2}(m_{\Lambda_c}^2 - m_\Lambda^2 - q^2)$ and $\lambda^{1/2}(x,y,z) = (x^2 + y^2 + z^2 - 2xy - 2yz - 2zx)^{1/2}$. When contracted with the lepton tensor, all of the βs (except β_{PP}) are proportional to powers of the lepton mass m_l and thus give small contributions to the decay rate. The complete form of β_{PP} is given in appendix E 1.

B. $\Lambda_c \to \Lambda^* l^+\nu_l \to \Sigma\pi l^+\nu_l / N\bar{K}l^+\nu_l$

We include six $\Lambda^{(*)}$ in this calculation. We denote these as Λ_i, $i = 1\ldots6$. In this notation, $\Lambda_1 = \Lambda(1115)\ 1/2^+$; $\Lambda_2 = \Lambda(1600)\ 1/2^+$; $\Lambda_3 = \Lambda(1405)\ 1/2^-$; $\Lambda_4 = \Lambda(1520)\ 3/2^-$; $\Lambda_5 = \Lambda(1890)\ 3/2^+$; $\Lambda_6 = \Lambda(1820)\ 5/2^+$. With the exception of Λ_1, these excited Λ_i are not stable particles and will decay strongly to $\Sigma\pi$ or $N\bar{K}$. Thus we study the four-body decays, $\Lambda_c \to \Lambda_i l^+\nu_l \to \Sigma\pi l^+\nu_l$ and $\Lambda_c \to \Lambda_i l^+\nu_l \to N\bar{K}l^+\nu_l$ as shown in Fig. 2(a, b). There are other contributions to each of these four-body final states, two of which are shown in Fig. 2 (c, d). However, in each case, the intermediate resonance is very heavy and very far from the mass shell. Thus, we expect these contributions to be small.

1. *Kinematics*

Fig 3 shows the kinematic diagram for the four-particle decay $\Lambda_c \to \Sigma\pi l^+\nu_l$. We define

$$P \equiv p_\Sigma + p_\pi, \quad Q \equiv p_\Sigma - p_\pi, \quad L \equiv p_l + p_\nu, \tag{16}$$

so that $p_{\Lambda_c} = P + L$. In the rest frame of the Λ_c, the back-to back momenta $\vec{P}$ and $\vec{L}$ define a common z-axis. In the rest frame of the daughter hadrons, θ_h^* is the polar angle between the pion momentum and $\vec{P}$. Similarly, in the rest frame of the lepton pair, θ_l^* is the polar angle between the lepton momentum and $\vec{L}$. ϕ^* is then the angle between the lepton and hadron planes.

FIG. 2: (a) shows the semileptonic decay $\Lambda_c^+ \to \Lambda^* l^+ \nu_l$ followed by the strong decay $\Lambda \to \Sigma\pi$; (b) shows the semileptonic decay $\Lambda_c^+ \to \Lambda^* l^+ \nu_l$ followed by the strong decay $\Lambda \to N\bar{K}$; (c) shows the strong decay $\Lambda_c^+ \to \Sigma_c^* \pi$ followed by the semileptonic decay $\Sigma_c^* \to \Sigma l^+ \nu_l$; (d) shows the strong decay $\Lambda_c^+ \to D^* N$ followed by the semileptonic decay $D^* \to \bar{K} l^+ \nu_l$.

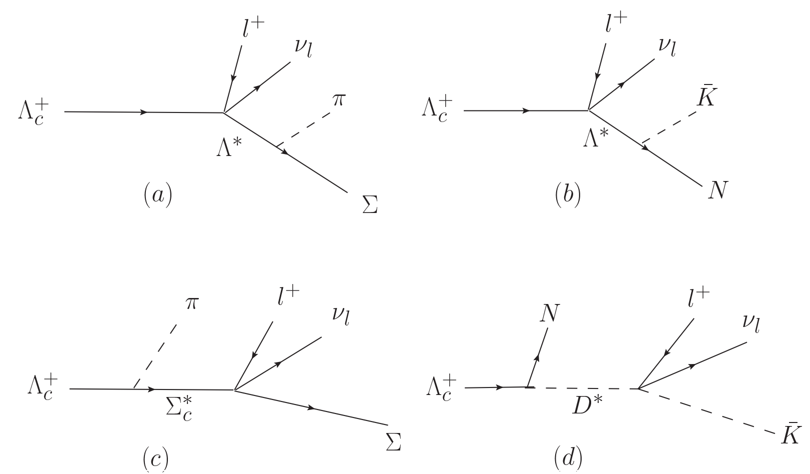
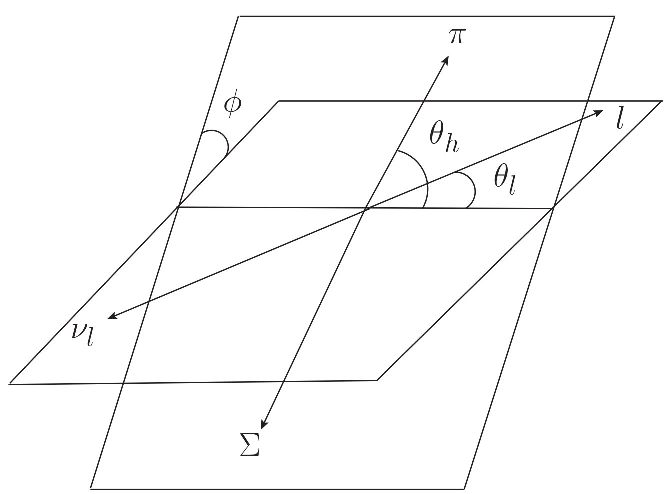


FIG. 3: Kinematics for the process $\Lambda_c \to \Sigma\pi l\nu$. The lepton momenta define the lepton plane, while the momenta of the hadrons define the hadron plane.



In the overall rest frame of Λ_c, the momenta P and L are

$$P = \Big(\frac{1}{2m_{\Lambda_c}}(m_{\Lambda_c}^2 + S_{\Sigma\pi} - q^2),\ 0,\ 0,\ \frac{1}{2m_{\Lambda_c}}\lambda^{1/2}(m_{\Lambda_c}^2, S_{\Sigma\pi}, q^2)\Big),$$
$$L = \Big(\frac{1}{2m_{\Lambda_c}}(m_{\Lambda_c}^2 - S_{\Sigma\pi} + q^2),\ 0,\ 0,\ -\frac{1}{2m_{\Lambda_c}}\lambda^{1/2}(m_{\Lambda_c}^2, S_{\Sigma\pi}, q^2)\Big).$$

In the rest frame of the daughter hadrons, the momenta p_Σ and p_π are

$$p_\pi = \Big(\frac{1}{2\sqrt{S_{\Sigma\pi}}}(S_{\Sigma\pi} - m_\Sigma^2 + m_\pi^2),\ \frac{1}{2\sqrt{S_{\Sigma\pi}}}\lambda^{1/2}(S_{\Sigma\pi}, m_\Sigma^2, m_\pi^2)\sin\theta_h^*,\ 0,\ \frac{1}{2\sqrt{S_{\Sigma\pi}}}\lambda^{1/2}(S_{\Sigma\pi}, m_\Sigma^2, m_\pi^2)\cos\theta_h^*\Big),$$
$$p_\Sigma = \Big(\frac{1}{2\sqrt{S_{\Sigma\pi}}}(S_{\Sigma\pi} + m_\Sigma^2 - m_\pi^2),\ -\frac{1}{2\sqrt{S_{\Sigma\pi}}}\lambda^{1/2}(S_{\Sigma\pi}, m_\Sigma^2, m_\pi^2)\sin\theta_h^*,\ 0,\ -\frac{1}{2\sqrt{S_{\Sigma\pi}}}\lambda^{1/2}(S_{\Sigma\pi}, m_\Sigma^2, m_\pi^2)\cos\theta_h^*\Big).$$

In the rest frame of the lepton pair, the lepton momenta are

$$p_l = \Big(\frac{1}{2\sqrt{q^2}}(q^2 + m_l^2),\ \frac{1}{2\sqrt{q^2}}(q^2 - m_l^2)\sin\theta_l^*,\ 0,\ \frac{1}{2\sqrt{q^2}}(q^2 - m_l^2)\cos\theta_l^*\Big),$$

$$p_\nu = \Big(\frac{1}{2\sqrt{q^2}}(q^2 - m_l^2),\ -\frac{1}{2\sqrt{q^2}}(q^2 - m_l^2)\sin\theta_l^*,\ 0,\ -\frac{1}{2\sqrt{q^2}}(q^2 - m_l^2)\cos\theta_l^*\Big).$$

2. Matrix Elements

The hadron matrix elements for the decays $\Lambda_c \to \Lambda_i l\nu_l \to BMl\nu_l$, where B is a baryon with $J^P = 1/2^+$ and M is a pseudoscalar meson, can be written as

$$\langle (B(p_B)M(p_M))_i | J_\mu | \Lambda_c(p_{\Lambda_c})\rangle = \bar{u}(p_B)\Upsilon^s R(P)\mathcal{J}_\mu^i u(P+L). \tag{17}$$

In this expression, Υ^s represents the strong decay vertex, p_B and p_M are the momenta of the daughter baryon B and meson M, respectively, $R(P)$ is the propagator with momentum P. J_μ is the weak current leading to the weak decay, while $\mathcal{J}_\mu^i$ is the matrix element for the semileptonic decay $\Lambda_c \to \Lambda_i$, written in terms of the form factors of section II A 1. In this notation, the momenta of eqn. 16 are more generally written as

$$P \equiv p_B + p_M, \quad Q \equiv p_B - p_M, \quad L \equiv p_l + p_\nu. \tag{18}$$

When the intermediate baryon has $J^P = 1/2^+$, the hadron matrix elements are

$$\begin{aligned}\langle B(p_B)M(p_M)|V_\mu^i|\Lambda_c(P+L)\rangle &= g_{\Lambda_i BM}\bar{u}(p_B)\gamma_5(\not{P} + M_\Gamma^i)\nabla^i\left[\gamma_\mu F_1^i + \frac{(P+L)_\mu}{m_{\Lambda_c}}F_2^i + \frac{P_\mu}{m_{\Lambda_i}}F_3^i\right]u(P+L),\\ \langle B(p_B)M(p_M)|A_\mu^i|\Lambda_c(P+L)\rangle &= g_{\Lambda_i BM}\bar{u}(p_B)\gamma_5(\not{P} + M_\Gamma^i)\nabla^i\left[\gamma_\mu G_1^i + \frac{(P+L)_\mu}{m_{\Lambda_c}}G_2^i + \frac{P_\mu}{m_{\Lambda_i}}G_3^i\right]\gamma_5 u(P+L),\end{aligned} \tag{19}$$

where $\nabla^i = 1/(P^2 - M_\Gamma^{i\,2})$ and $M_\Gamma^i = m_{\Lambda_i} - i\Gamma_i/2$, with m_{Λ_i} and Γ_i the mass and total decay width of the Λ_i, respectively. $g_{\Lambda_i BM}$ is the strong coupling constant for the decay $\Lambda_i \to BM$.

For an intermediate state with $J^P = 3/2^-$, the hadron matrix elements are

$$\begin{aligned}\langle B(p_B)M(p_M)|V_\mu^i|\Lambda_c(P+L)\rangle =& g_{\Lambda_i BM}\bar{u}(p_B)\gamma_5\frac{p_{M_\alpha}}{m_M}R^{\alpha\beta}(P)\nabla^i\Bigg[\frac{(P+L)_\beta}{m_{\Lambda_c}}\Bigg(\gamma_\mu F_1^i + \frac{(P+L)_\mu}{m_{\Lambda_c}}F_2^i\\ &+ \frac{P_\mu}{m_{\Lambda_i}}F_3^i\Bigg) + g_{\beta\mu}F_4^i\Bigg]u(P+L),\\ \langle B(p_B)M(p_M)|A_\mu^i|\Lambda_c(P+L)\rangle =& g_{\Lambda_i BM}\bar{u}(p_B)\gamma_5\frac{p_{M_\alpha}}{m_M}R^{\alpha\beta}(P)\nabla^i\Bigg[\frac{(P+L)_\beta}{m_{\Lambda_c}}\Bigg(\gamma_\mu G_1^i + \frac{(P+L)_\mu}{m_{\Lambda_c}}G_2^i\\ &+ \frac{p_\mu}{m_{\Lambda_i}}G_3^i\Bigg) + g_{\beta\mu}G_4^i\Bigg]\gamma_5 u(P+L),\end{aligned} \tag{20}$$

where $R^{\alpha\beta}(P)$ is the Rarita-Schwinger tensor for a massive spin 3/2 propagator, which takes the form

$$R^{\alpha\beta}(P) = -(\not{P} + M_\Gamma^i)\left[g^{\alpha\beta} - \frac{1}{3}\gamma^\alpha\gamma^\beta - \frac{2P^\alpha P^\beta}{3m_{\Lambda_i}^2} - \frac{\gamma^\alpha P^\beta - \gamma^\beta P^\alpha}{3m_{\Lambda_i}}\right].$$

For an intermediate state with $J^P = 5/2^+$, the hadronic matrix elements are

$$\begin{aligned}\langle B(p_B)M(p_M)|V_\mu^i|\Lambda_c(p_{\Lambda_c})\rangle =& g_{\Lambda_i BM}\bar{u}(p_B)\gamma_5\frac{p_M^{\alpha'}p_M^{\beta'}}{m_M^2}R^{\alpha\beta}_{\alpha'\beta'}(P)\nabla^i\frac{(P+L)_\alpha}{m_{\Lambda_c}}\Bigg[\frac{(P+L)_\beta}{m_{\Lambda_c}}\Bigg(\gamma_\mu F_1^i + \frac{(P+L)_\mu}{m_{\Lambda_c}}F_2^i\\ &+ \frac{P_\mu}{m_{\Lambda_i}}F_3^i\Bigg) + g_{\beta\mu}F_4^i\Bigg]u(p),\\ \langle B(p_B)M(p_M)|A_\mu^i|\Lambda_c(P+L)\rangle =& g_{\Lambda_i BM}\bar{u}(p_B)\gamma_5\frac{p_M^{\alpha'}p_M^{\beta'}}{m_M^2}R^{\alpha\beta}_{\alpha'\beta'}(P)\nabla^i\frac{(P+L)_\alpha}{m_{\Lambda_c}}\Bigg[\frac{(P+L)_\beta}{m_{\Lambda_c}}\Bigg(\gamma_\mu G_1^i + \frac{(P+L)_\mu}{m_{\Lambda_c}}G_2^i\\ &+ \frac{P_\mu}{m_{\Lambda_i}}G_3^i\Bigg) + g_{\beta\mu}G_4^i\Bigg]\gamma_5 u(P+L),\end{aligned} \tag{21}$$

where $R^{\alpha\beta}_{\alpha'\beta'}(P)$ is the Rarita-Schwinger propagator tensor for a massive particle with total angular momentum 5/2 [19].

We need to cast the matrix elements from the previous three equations into a more general form that makes it easier to organize the calculation. The most general form of the contribution of the ith state to the matrix element for the four-body decay $\Lambda_c^+ \to \Lambda_i l^+ \nu_l \to BMl^+\nu_l$ can be written

$$M_\nu^i = \bar{u}(p_B)\left(\sum_{j=1}^{16} c_j^i \mathcal{O}_j\right) u(P+L),$$

where the Lorentz-Dirac operators $\mathcal{O}_i$ are

$$\mathcal{O}_1 = \gamma_\nu,\quad \mathcal{O}_2 = \not{P}\gamma_\nu,\quad \mathcal{O}_3 = P_\nu,\quad \mathcal{O}_4 = \not{P}P_\nu,\quad \mathcal{O}_5 = L_\nu,\quad \mathcal{O}_6 = \not{P}L_\nu,\quad \mathcal{O}_7 = Q_\nu,\quad \mathcal{O}_8 = \not{P}Q_\nu,$$
$$\mathcal{O}_9 = \gamma_\nu\gamma_5,\quad \mathcal{O}_{10} = \not{P}\gamma_\nu\gamma_5,\quad \mathcal{O}_{11} = P_\nu\gamma_5,\quad \mathcal{O}_{12} = \not{P}P_\nu\gamma_5,\quad \mathcal{O}_{13} = L_\nu\gamma_5,\quad \mathcal{O}_{14} = \not{P}L_\nu\gamma_5,\quad \mathcal{O}_{15} = Q_\nu\gamma_5,\quad \mathcal{O}_{16} = \not{P}Q_\nu\gamma_5.$$

Because of the forms of the propagators in eqns. 19 - 21, there are no terms containing $\not{L}$ or $\not{Q}$ among the $\mathcal{O}_i$. For the cases of eqns. 20 and 21, the meson momentum p_M can be replaced by $P - p_B$, and any factors of $\not{p}_B$ can be commuted leftward until they are adjacent to the spinor $\bar{u}(p_B)$. The Dirac equation can then be used to write this as the scalar m_B.

The c_j^i can be written

$$c_j^i = \frac{g_{\Lambda_i BM}}{P^2 - M_\Gamma^{i\,2}} \sum_k (C_{jk}^{i,F} F_k + C_{jk}^{i,G} G_k). \tag{22}$$

where k runs from 1 to 3 for spin $\frac{1}{2}$ states and from 1 to 4 for states with higher spin.

In the above, we have shown the forms for the Λ_i states with natural parity. For the states with unnatural parity, the weak and strong vertices each acquire an extra multiplicative factor of γ_5.

3. *Decay Width*

The differential decay rate for the decay $\Lambda_c^+ \to BMl^+\nu_l$ is,

$$d\Gamma = \frac{1}{2m_{\Lambda_c}} \frac{G_F^2}{2} |V_{cs}|^2 \frac{d^3p_B d^3p_M d^3p_l d^3p_{\nu_l}}{2E_B 2E_M 2E_l 2E_{\nu_l}} \delta^4(p_{\Lambda_c} - p_B - p_M - p_l - p_{\nu_l}) H_{\mu\nu} L^{\mu\nu}, \tag{23}$$

The hadron tensor that arises from each intermediate state i can be written as,

$$\begin{aligned} H_{\mu\nu}^i = \sum_{\text{spins}} M_\mu^{i\dagger} M_\nu^i &= \alpha^i g_{\mu\nu} + \beta_{PP}^i P_\mu P_\nu + \beta_{PQ}^i P_\mu Q_\nu + \beta_{QP}^i Q_\mu P_\nu + \beta_{QQ}^i Q_\mu Q_\nu \\ &+ \beta_{QL}^i Q_\mu L_\nu + \beta_{LQ}^i L_\mu Q_\nu + \beta_{LL}^i L_\mu L_\nu + \beta_{PL}^i P_\mu L_\nu + \beta_{PL}^i P_\mu L_\nu \\ &+ i\gamma_a^i \epsilon^{\mu\nu\rho\delta} P_\rho Q_\delta + i\gamma_b^i \epsilon^{\mu\nu\rho\delta} L_\rho P_\delta + i\gamma_c^i \epsilon^{\mu\nu\rho\delta} L_\rho Q_\delta + i\gamma_d^i \epsilon^{\sigma\mu\rho\delta} L_\sigma P_\rho Q_\delta P_\nu + i\gamma_e^i \epsilon^{\sigma\mu\rho\delta} L_\sigma P_\rho Q_\delta Q_\nu \\ &+ i\gamma_f^i \epsilon^{\sigma\mu\rho\delta} L_\sigma P_\rho Q_\delta L_\nu + i\gamma_g^i \epsilon^{\sigma\nu\rho\delta} L_\sigma P_\rho Q_\delta P_\mu + i\gamma_h^i \epsilon^{\sigma\nu\rho\delta} L_\sigma P_\rho Q_\delta Q_\mu + i\gamma_k^i \epsilon^{\sigma\nu\rho\delta} L_\sigma P_\rho Q_\delta L_\mu. \end{aligned}$$

In this expression,

$$\alpha^i = \sum_{j,k=1}^{16} a_{jk}^i c_j^{i\dagger} c_k^i, \tag{24}$$

with similar forms for all of the other coefficients. The terms in γ_i do not contribute to the decay rates that we consider, due to the symmetry of the lepton tensor.

For the process $\Lambda_c \to BMl\nu_l$ we examine the contribution from each Λ_i individually, as well as the coherent contribution of all the Λ_i. For the coherent sum, we write

$$M_\nu = \bar{u}(p_B)\left(\sum_{j=1}^{16} \mathcal{C}_j \mathcal{O}_j\right) u(p_{\Lambda_c}) = \bar{u}(p_B) \sum_{i=1}^{6} \left(\sum_{j=1}^{16} c_j^i \mathcal{O}_j\right) u(p_{\Lambda_c}), \tag{25}$$

which ultimately leads to

$$\mathcal{C}_j = \sum_{i=1}^{6} c_j^i. \tag{26}$$

Integrating over the lepton momenta, and making use of eqn. (13) leads to

$$\begin{aligned}
\frac{d\Gamma}{dS_{BM}dq^2d\theta_h d\theta_l d\phi} =& \frac{|V_{cs}|^2}{2}\frac{4\pi G_F^2}{128 m_{\Lambda_c}^3 S_{BM}}\sin\theta_h \lambda^{1/2}(m_{\Lambda_c}^2, S_{BM}^2, q^2)\lambda^{1/2}(S_{BM}^2, m_B^2, m_M^2)\Big(\alpha\Big[4A + A'q^2\Big] \\
&+ \beta_{PP}\Big[AS_{BM} + A'(P\cdot L)^2\Big] + \Big[\beta_{PQ} + \beta_{QP}\Big]\Big[A(P\cdot Q) + A'(P\cdot L)(Q\cdot L)\Big] \\
&+ \beta_{QQ}\Big[A(Q\cdot Q) + A'(Q\cdot L)^2\Big] \\
&+ \Big[(\beta_{LP} + \beta_{PL})(P\cdot L) + (\beta_{LQ} + \beta_{QL})(Q\cdot L) + \beta_{LL}q^2\Big]\Big[A + A'q^2\Big]\Big),
\end{aligned} \tag{27}$$

where

$$\begin{aligned}
P\cdot P &= p_{\Lambda^*}^2 \equiv S_{BM}, \\
P\cdot Q &= m_B^2 - m_M^2, \\
P\cdot L &= (m_{\Lambda_c}^2 - S_{BM} - q^2)/2, \\
L\cdot L &= q^2, \\
Q\cdot Q &= 2m_B^2 + 2m_M^2 - S_{BM}, \\
Q\cdot L &= \frac{1}{2S_{BM}}\Big[(m_B^2 - m_M^2)(m_{\Lambda_c}^2 - S_{BM} - q^2) + \cos\theta_h \lambda^{1/2}(m_{\Lambda_c}^2, S_{BM}, q^2)\lambda^{1/2}(S_{BM}, m_B^2, m_M^2)\Big].
\end{aligned}$$

III. HEAVY QUARK EFFECTIVE THEORY

The heavy quark effective theory (HQET) has been a very useful tool in the study of the electroweak decays of hadrons containing one heavy quark. In this effective theory, the matrix elements are expanded in increasing orders of $1/m_Q$, where m_Q is the mass of the heavy quark. This expansion has facilitated the extraction of CKM matrix elements with decreasing model dependence.

Hadrons containing a single charm or beauty quark are considered to be heavy hadrons as the mass $m_Q >> \Lambda_{\rm QCD}$. For such hadrons, HQET reduces the number of independent form factors required to describe the transitions mediated by electroweak transitions that change a heavy quark of one flavor into a heavy quark of different flavor. At leading order in the $1/m_Q$ expansion, such heavy to heavy transitions require a single form factor, the so-called Isgur-Wise function. This is the case independent of the total angular momentum of the daughter hadron (we assume that the parent hadron is a ground-state hadron), integer (meson) or half-integer (baryon). For transitions between a ground-state heavy hadron and a light one, HQET is not as powerful. However, for transitions between a heavy baryon (ground state) and a light one, HQET indicates that a pair of form factors is all that is needed to describe the transition, independent of the angular momentum of the daughter baryon.

The semileptonic decays $\Lambda_c \to \Lambda^*$ fall into this second category, and are therefore described by two independent form factors. We may represent one of these light baryons of angular momentum J by a generalized Rarita-Schwinger field $u^{\mu_1\cdots\mu_n}(p)$ where $n = J - 1/2$. This field is symmetric under exchange of any pair of its Lorentz indices, and satisfies the conditions

$$\begin{aligned}
\not{p}u^{\mu_1\cdots\mu_n}(p) &= m_\Lambda u^{\mu_1\cdots\mu_n}(p), \gamma_{\mu_1}u^{\mu_1\cdots\mu_n}(p) = 0, \\
p_{\mu_1}u^{\mu_1\cdots\mu_n}(p) &= 0, u_\mu^{\mu\cdots\mu_n}(p) = 0.
\end{aligned}$$

The matrix element we are interested in is

$$\langle\Lambda^*(p')|\bar{s}\Gamma c|\Lambda_c^+(p)\rangle = \bar{u}^{\mu_1\cdots\mu_n}M_{\mu_1\ldots\mu_n}\Gamma u(p), \tag{28}$$

where $\Gamma = \gamma^\mu$ or $\gamma^\mu\gamma_5$ defines vector or axial vector current and $M_{\mu_1\mu_2\ldots\mu_n}$ is a tensor. The most general tensor can be constructed as

$$M_{\mu_1\mu_2\ldots\mu_n} = v_{\mu_1}\ldots v_{\mu_n}A_n, \tag{29}$$

where A_n is the most general Lorentz scalar that can be constructed. This takes the form

$$A_n = \xi_1^{(n)} + \not{v}\xi_2^{(n)}, \tag{30}$$

where $v = p/m_{\Lambda_c}$ is the velocity of the parent baryon. For the transitions to daughter baryons with unnatural parity, $M_{\mu_1\mu_2\ldots\mu_n}$ must be a pseudo-tensor. This is easily constructed by including a factor of γ_5, so that

$$M_{\mu_1\mu_2\ldots\mu_n} = v_{\mu_1}\ldots v_{\mu_n}\left(\zeta_1^{(n)} + \not{v}\zeta_2^{(n)}\right)\gamma_5. \tag{31}$$

A. Form Factors

The matrix elements can be written in terms of six general form factors for spin $\frac{1}{2}^{\pm}$, or eight general form factors for spin $\frac{3}{2}^{\pm}$ and $\frac{5}{2}^{+}$, as shown in Section II A 1. Comparing the predictions of HQET with the most general form of the matrix elements leads to a number of relations among the general form factors F_i/G_i and the HQET form factors ξ_i/ζ_i.

For spin $\frac{1}{2}^{+}$, these relationships are

$$F_1 = \xi_1^{(0)} - \xi_2^{(0)}, G_1 = \xi_1^{(0)} + \xi_2^{(0)}, F_2 = G_2 = 2\xi_2^{(0)}, F_3 = G_3 = 0. \tag{32}$$

For spin $\frac{1}{2}^{-}$, they are

$$F_1 = -(\zeta_1^{(0)} + \zeta_2^{(0)}), G_1 = -(\zeta_1^{(0)} - \zeta_2^{(0)}), F_2 = G_2 = -2\zeta_2^{(0)}, F_3 = G_3 = 0. \tag{33}$$

For spin $\frac{3}{2}^{-}$, they are

$$F_1 = \xi_1^{(1)} - \xi_2^{(1)}, G_1 = \xi_1^{(1)} + \xi_2^{(1)}, F_2 = G_2 = 2\xi_2^{(1)}, F_3 = G_3 = 0, F_4 = G_4 = 0. \tag{34}$$

For spin $\frac{3}{2}^{+}$, the relationships are

$$F_1 = -(\zeta_1^{(1)} + \zeta_2^{(1)}), G_1 = -(\zeta_1^{(1)} - \zeta_2^{(1)}), F_2 = G_2 = -2\zeta_2^{(1)}, F_3 = G_3 = 0, F_4 = G_4 = 0. \tag{35}$$

For spin $\frac{5}{2}^{+}$, they are

$$F_1 = \xi_1^{(2)} - \xi_2^{(2)}, G_1 = \xi_1^{(2)} + \xi_2^{(2)}), F_2 = G_2 = 2\xi_2^{(2)}, F_3 = G_3 = 0, F_4 = G_4 = 0. \tag{36}$$

B. Decay Width

At leading order in HQET, the differential decay rates take simple forms for all the excited states we discuss. This general form is

$$\frac{d\Gamma}{dq^2} = \Phi^J X\left[\left(A_1 + A_2\frac{m_l^2}{q^2}\right)\xi_1^2 + \frac{m_\Lambda}{m_{\Lambda_c}}\left(B_1 + B_2\frac{m_l^2}{q^2}\right)\xi_1\xi_2 + \frac{1}{m_{\Lambda_c}^2}\left(C_1 + C_2\frac{m_l^2}{q^2}\right)\xi_2^2\right], \tag{37}$$

where Φ^J is a dimensionless quantity that depends on the angular momentum of the daughter baryon. X, A_i, B_i, C_i are, respectively,

$$X = \frac{4\pi^3}{m_{\Lambda_c}^3}\frac{G_F^2}{2}|v_{cs}|^2\lambda^{1/2}(m_{\Lambda_c}^2, m_\Lambda^2, q^2),$$
$$A_1 = \left[m_{\Lambda_c}^4 + m_{\Lambda_c}^2\left(q^2 - 2m_\Lambda^2\right) + m_\Lambda^4 + m_\Lambda^2 q^2 - 2q^4\right],$$
$$A_2 = \left[2m_{\Lambda_c}^4 - m_{\Lambda_c}^2\left(4m_\Lambda^2 + q^2\right) + 2m_\Lambda^4 - m_\Lambda^2 q^2 - q^4\right],$$

$$B_1 = 2\Big[m_{\Lambda_c}^4 - 2m_{\Lambda_c}^2\Big(m_\Lambda^2 - 2q^2\Big) + \Big(m_\Lambda^2 - q^2\Big)^2\Big],$$
$$B_2 = 4\Big[m_{\Lambda_c}^4 + m_{\Lambda_c}^2\Big(q^2 - 2m_\Lambda^2\Big) + \Big(m_\Lambda^2 - q^2\Big)^2\Big],$$
$$C_1 = \Big[m_{\Lambda_c}^4\Big(m_\Lambda^2 + 2q^2\Big) - m_{\Lambda_c}^2\Big(2m_\Lambda^4 - 3m_\Lambda^2 q^2 + q^4\Big) + \Big(m_\Lambda^2 - q^2\Big)^3\Big],$$
$$C_2 = \Big[m_{\Lambda_c}^4\Big(2m_\Lambda^2 + q^2\Big) + m_{\Lambda_c}^2\Big(-4m_\Lambda^4 + 3m_\Lambda^2 q^2 + q^4\Big) + 2\Big(m_\Lambda^2 - q^2\Big)^3\Big].$$

The decay width for states with total spin J does not depend on parity. The Φ^J for states with angular momentum J are

$$\Phi^{1/2} = 4,$$
$$\Phi^{3/2} = \frac{2}{3m_{\Lambda_c}^2 m_\Lambda^2}\lambda\big(m_{\Lambda_c}^2, m_\Lambda^2, q^2\big),$$
$$\Phi^{5/2} = \frac{1}{10m_{\Lambda_c}^4 m_\Lambda^4}\lambda^2\big(m_{\Lambda_c}^2, m_\Lambda^2, q^2\big).$$

IV. THE MODEL

A. Wave Function Components

In our model, a baryon state has the form

$$|\Lambda_Q(\vec{p}, s)\rangle = 3^{-3/2}\int d^3p_\rho d^3p_\lambda C^A \Psi_{\Lambda_Q}^S |q_1(\vec{p}_1, s_1)q_2(\vec{p}_2, s_2)q_3(\vec{p}_3, s_3)\rangle, \tag{38}$$

where Λ_Q is a flavored baryon (Λ_c^+ or Λ) having a flavored quark (c or s) Q, which may or may not be considered heavy. $q_i(\vec{p}_i, s_i)$ is the creation operator for quark q_i with momentum $\vec{p}_i$ and spin s_i. $|q_1(\vec{p}_1, s_1)q_2(\vec{p}_2, s_2)q_3(\vec{p}_3, s_3)\rangle$ is the three quark state with quarks q_i having momenta and spins $(\vec{p}_i, s_i)$. $\vec{p}_\rho = \frac{1}{\sqrt{2}}(\vec{p}_1 - \vec{p}_2)$ and $\vec{p}_\lambda = \frac{1}{\sqrt{6}}(\vec{p}_1 + \vec{p}_2 - 2\vec{p}_3)$ are the Jacobi momenta. C^A is the antisymmetric color wave function and $\Psi_{\Lambda_Q}^S = \phi_{\Lambda_Q}\psi_{\Lambda_Q}\chi_{\Lambda_Q}$ is a symmetric combination of flavor, momentum and spin wave functions. For Λ_Q the flavor wave function is

$$\phi_{\Lambda_Q} = \frac{1}{\sqrt{2}}(ud - du)Q. \tag{39}$$

This is antisymmetric under the exchange of the first two quarks, so the spin-space wave function must also be antisymmetric under such exchange.

The total spin of a system of three spin-$\frac{1}{2}$ particles can be either $\frac{3}{2}$ or $\frac{1}{2}$. The maximally stretched spin states are

$$\chi_{3/2}^S(+3/2) = |\uparrow\uparrow\uparrow\rangle,$$
$$\chi_{1/2}^\rho(+1/2) = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\uparrow\rangle - \downarrow\uparrow\uparrow\rangle),$$
$$\chi_{1/2}^\lambda(+1/2) = -\frac{1}{\sqrt{6}}(|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle),$$

where the superscript S indicates that the state is totally symmetric under the exchange of any pair of quarks, while ρ, λ denote the mixed-symmetric states that are antisymmetric and symmetric under the exchange of first two spins, respectively.

The momentum-space wave function ψ_{Λ_Q} can be constructed from the Clebsch-Gordan sum of the product of wave functions of the two jacobi momenta p_ρ, p_λ with total angular momentum $\vec{L} = \vec{l}_\rho + \vec{l}_\lambda$,

$$\psi_{LM_L n_\rho l_\rho n_\lambda l_\lambda}(p_\lambda, p_\rho) = \sum_m C_{l_\rho m, l_\lambda M_L - m}^{LM_L}\psi_{n_\rho l_\rho m}(p_\rho)\psi_{n_\lambda l_\lambda M_L - m}(p_\lambda). \tag{40}$$

This wave function is then coupled to the spin wave function χ_{Λ_Q} to give a spin-momentum wave function of total spin J and parity $(-)^{(l_\rho+l_\lambda)}$,

$$\Psi_{JM} = \sum_{M_L} C^{JM}_{LM_L,SM-M_L}\psi_{LM_Ln_\rho l_\rho n_\lambda l_\lambda}\chi_S(M-M_L). \tag{41}$$

The full wave function is then constructed as

$$\Psi_{\Lambda_Q,J^P,M} = \phi_{\Lambda_Q}\sum_i \eta_i \Psi^i_{JM} \tag{42}$$

The η_i are the coefficients determined by diagonalizing the Hamiltonian in the basis of the states Ψ^i_{JM} [5]. In this model the expansion is restricted to $N \le 2$, where $N = 2(n_\rho + n_\lambda) + l_\rho + l_\lambda$.

In the notation introduced above, the wave functions for states with $J^P = \frac{1}{2}^+$ are written as

$$\begin{aligned}\Psi_{\Lambda_Q,\frac{1}{2}^+M} = \phi_{\Lambda_Q}\Bigg(&\Big[\eta_1\psi_{000000}(\vec{p}_\rho,\vec{p}_\lambda)+\eta_2\psi_{001000}(\vec{p}_\rho,\vec{p}_\lambda)+\eta_3\psi_{000010}(\vec{p}_\rho,\vec{p}_\lambda)\Big]\chi^\rho_{\frac{1}{2}}(M)\\ &+\eta_4\psi_{000101}(\vec{p}_\rho,\vec{p}_\lambda)\chi^\lambda_{\frac{1}{2}}(M)+\eta_5\Big[\psi_{1M_L0101}(\vec{p}_\rho,\vec{p}_\lambda)\chi^\lambda_{\frac{1}{2}}(M-M_L)\Big]_{1/2,M}\\ &+\Big[\eta_6\psi_{1M_L0101}(\vec{p}_\rho,\vec{p}_\lambda)\chi^S_{\frac{3}{2}}(M-M_L)\Big]_{1/2,M}+\eta_7\Big[\psi_{2M_L0101}(\vec{p}_\rho,\vec{p}_\lambda)\chi^S_{\frac{3}{2}}(M-M_L)\Big]_{1/2,M}\Bigg),\end{aligned} \tag{43}$$

where we have used $[\psi_{LM_Ln_\rho l_\rho n_\lambda l_\lambda}(\vec{p}_\rho,\vec{p}_\lambda)\chi_S(M-M_L)]_{J,M}$ as a shorthand notation for the Clebsch-Gordan sum $\sum_{M_L} C^{JM}_{LM_L,SM-M_L}\psi_{LM_Ln_\rho l_\rho n_\lambda l_\lambda}(\vec{p}_\rho,\vec{p}_\lambda)\chi_S(M-M_L)$.

In our analytic calculation of the form factors we have used the following single component representation of the Λ states with different J^P:

$$\begin{aligned}\Psi_{1/2^+,M} &= \phi_\Lambda\Big[\psi_{000000}(\vec{p}_\rho,\vec{p}_\lambda)\chi^\rho_{1/2}(M-M_L)\Big]_{1/2,M};\\ \Psi_{1/2^+_1,M} &= \phi_\Lambda\Big[\psi_{000010}(\vec{p}_\rho,\vec{p}_\lambda)\chi^\rho_{1/2}(M-M_L)\Big]_{1/2,M};\\ \Psi_{1/2^-,M} &= \phi_\Lambda\Big[\psi_{1M_L0001}(\vec{p}_\rho,\vec{p}_\lambda)\chi^\rho_{1/2}(M-M_L)\Big]_{1/2,M};\\ \Psi_{3/2^-,M} &= \phi_\Lambda\Big[\psi_{1M_L0001}(\vec{p}_\rho,\vec{p}_\lambda)\chi^\rho_{1/2}(M-M_L)\Big]_{3/2,M};\\ \Psi_{3/2^+,M} &= \phi_\Lambda\Big[\psi_{2M_L0002}(\vec{p}_\rho,\vec{p}_\lambda)\chi^\rho_{1/2}(M-M_L)\Big]_{3/2,M};\\ \Psi_{5/2^+,M} &= \phi_\Lambda\Big[\psi_{2M_L0002}(\vec{p}_\rho,\vec{p}_\lambda)\chi^\rho_{1/2}(M-M_L)\Big]_{5/2,M}.\end{aligned} \tag{44}$$

For details of the construction of the wave functions, see appendix D.

ψ_{nlm} is expanded in the harmonic oscillator basis, whose wave functions in momentum space are,

$$\psi_{nlm}(\vec{p}) = \left[\frac{2n!}{(n+l+\frac{1}{2})!}\right]^{\frac{1}{2}}(i)^l(-1)^n\frac{1}{\alpha^{l+\frac{3}{2}}}e^{-\frac{p^2}{(2\alpha^2)}}L_n^{l+\frac{1}{2}}(p^2/\alpha^2)\mathcal{Y}_{lm}(\vec{p}), \tag{45}$$

where, $L_n^\beta(x)$ are the generalized Laguerre polynomials with $p = |\vec{p}|$ and $\mathcal{Y}_{lm}(\vec{p})$ are the solid harmonics.

B. Extraction of Form Factors

The hadron matrix elements for any arbitrary current $\bar{s}\Gamma c$ take the form

$$\langle\Lambda(p_\Lambda,s')|\bar{s}\Gamma c|\Lambda_c(0,s)\rangle = \int d^3p'_\lambda d^3p'_\rho d^3p_\lambda d^3p_\rho C^{A*}C^A\Psi^*_\Lambda(s')$$

$$\times \langle q_1' q_2' s | \bar{s}\Gamma c | q_1 q_2 c\rangle \Psi_{\Lambda_c}(s), \tag{46}$$

where $\langle q_1' q_2' s|\bar{s}\Gamma c|q_1 q_2 c\rangle = \langle q_1' q_2'|q_1 q_2\rangle\langle s|\bar{s}\Gamma c|c\rangle$. In our spectator approximation $\langle q_1' q_2'|q_1 q_2\rangle$ gives delta functions in spin, momentum and flavor.

The analytic expressions for the form factors shown in appendix C are obtained using the single-component wave functions of eq. 44. We also calculate the form factors numerically using the full multi-component wave functions extracted from the diagonalization of the Hamiltonian. For this, we adapted the semi-analytic approach used by Mott and Roberts [15] in their calculation of the rare dileptonic decay of Λ_b. In this method some of the calculation is done analytically, leaving a couple of integrations to be done numerically.

In the rest frame of the parent Λ_c, we write the initial quark momenta in terms of the Jacobi momenta as

$$\vec{p}_1 = \frac{1}{\sqrt{2}}\vec{p}_\rho + \frac{1}{\sqrt{6}}\vec{p}_\lambda, \quad \vec{p}_2 = -\frac{1}{\sqrt{2}}\vec{p}_\rho + \frac{1}{\sqrt{6}}\vec{p}_\lambda, \quad \vec{p}_3 \equiv \vec{p} = -\sqrt{\frac{2}{3}}\vec{p}_\lambda.$$

We use the spectator approximation in which the first two quarks are unaffected by the transition. This allows us to integrate over the Jacobi momenta separately and write the matrix element as

$$\langle\Lambda|\bar{s}\Gamma c|\Lambda_c\rangle = \sum_{b',b} h_{b'}^{\Lambda^*} h_b^{\Lambda_c}\delta_{s_1's_1}\delta_{s_2's_2}(-1)^{l_{\lambda'}+l_\lambda} \times B_{n_\rho l_\rho m_\rho}^{n_{\rho'}l_{\rho'}m_{\rho'}}(\alpha_\rho,\alpha_{\rho'}) D_{\Gamma;n_\lambda l_\lambda m_\lambda s_q}^{n_{\lambda'}l_{\lambda'}m_{\lambda'}s_{q'}}(\alpha_\lambda,\alpha_{\lambda'}), \tag{47}$$

where the coefficients $h_{b(b')}$ are the products of the normalization of the baryon states, the expansion coefficients η_i, and the various Clebsch-Gordan coefficients that appear in the parent (daughter) baryon wave function. The indices $b(b')$ contain all the relevant quantum numbers being summed over for the parent (daughter) baryon state. $B_{n_\rho l_\rho m_\rho}^{n_{\rho'}l_{\rho'}m_{\rho'}}$ is the spectator overlap,

$$B_{n_\rho l_\rho m_\rho}^{n_{\rho'}l_{\rho'}m_{\rho'}}(\alpha_\rho,\alpha_{\rho'}) = \int d^3p_\rho d^3p_{\rho'}\psi^*_{n_{\rho'}l_{\rho'}m_{\rho'}}(\alpha_{\rho'};\vec{p}_{\rho'})\psi_{n_\rho l_\rho m_\rho}(\alpha_\rho;\vec{p}_\rho)\delta(p_\rho - p_{\rho'}). \tag{48}$$

This integral can be done analytically and is given in appendix B.

The interaction overlap $D_{\Gamma;n_\lambda l_\lambda m_\lambda s_q}^{n_{\lambda'}l_{\lambda'}m_{\lambda'}s_{q'}}$ is

$$D_{\Gamma;n_\lambda l_\lambda m_\lambda s_q}^{n_{\lambda'}l_{\lambda'}m_{\lambda'}s_{q'}}(\beta,\beta') = \int d^3p\psi^*_{n_{\lambda'}l_{\lambda'}m_{\lambda'}}(\beta';\vec{p}')\,\langle s(\vec{p}+\vec{p}_\Lambda,s_{q'})|\bar{s}\Gamma c|c(\vec{p},s_q)\rangle\psi_{n_\lambda l_\lambda m_\lambda}(\beta;\vec{p}),$$

where $\beta^{(\prime)} = \sqrt{2/3}\alpha_\lambda^{(\prime)}$ is the reduced length parameter for the parent (daughter) baryon, $p' = (2m_\sigma/\tilde{m}_\Lambda)\vec{p}_\Lambda + \vec{p}$, $\tilde{m}_\Lambda = m_s + 2m_\sigma$, and m_s and m_σ are the masses of the strange quark and each light quark, respectively. In terms of the generalized Laguerre polynomials,

$$D_{\Gamma;n_\lambda l_\lambda m_\lambda s_q}^{n_{\lambda'}l_{\lambda'}m_{\lambda'}s_{q'}}(\beta,\beta') = \int d^3p\,\exp\left(-\frac{p'^2}{2\beta'^2} - \frac{p^2}{2\beta^2}\right)\mathcal{L}_{n_{\lambda'}}^{l_{\lambda'}+\frac{1}{2}*}\left(\frac{p'^2}{\beta'^2}\right)\mathcal{Y}^*_{l_{\lambda'}m_{\lambda'}}(\vec{p}')$$
$$\times\langle s(\vec{p}_\Lambda+\vec{p},s_{q'})|\bar{s}\Gamma c|c(\vec{p},s_q)\rangle\mathcal{L}_{n_\lambda}^{l_\lambda+\frac{1}{2}}\left(\frac{p^2}{\beta^2}\right)\mathcal{Y}_{l_\lambda m_\lambda}(\vec{p}). \tag{49}$$

The angular dependence in the exponential is eliminated by using the substitutions

$$\vec{p} = \vec{k} + a\vec{p}_\Lambda, \quad \vec{p}' = \vec{k} + a'\vec{p}_\Lambda, \tag{50}$$

where

$$a = -\frac{m_\sigma\alpha_\lambda^2}{\tilde{m}_\Lambda\alpha_{\lambda\lambda'}^2}, \quad a' = \frac{m_\sigma\alpha_{\lambda'}^2}{\tilde{m}_\Lambda\alpha_{\lambda\lambda'}^2}. \tag{51}$$

and $\alpha_{\lambda\lambda'} = \sqrt{(\alpha_\lambda^2+\alpha_{\lambda'}^2)/2}$. The interaction overlap then takes the form

$$D_{\Gamma;n_\lambda l_\lambda m_\lambda s_q}^{n_{\lambda'}l_{\lambda'}m_{\lambda'}s_{q'}}(\beta,\beta') = \exp\left(\frac{-3m_\sigma^2}{2m_\Lambda^2}\frac{p_\Lambda^2}{\alpha_{\lambda\lambda'}^2}\right)\int d^3k e^{-\alpha^2k^2}\mathcal{L}_{n_{\lambda'}}^{l_{\lambda'}+\frac{1}{2}*}\left(\frac{p'^2}{\beta'^2}\right)\mathcal{Y}^*_{l_{\lambda'}m_{\lambda'}}(\vec{p}')$$
$$\times\langle s(\vec{p}_\Lambda+\vec{p},s_{q'})|\bar{s}\Gamma c|c(\vec{p},s_q)\rangle\mathcal{L}_{n_\lambda}^{l_\lambda+\frac{1}{2}}\left(\frac{p^2}{\beta^2}\right)\mathcal{Y}_{l_\lambda m_\lambda}(\vec{p}), \tag{52}$$

where $\alpha^2 = \frac{\alpha_\lambda^2+\alpha_{\lambda'}^2}{2\alpha_\lambda^2\alpha_{\lambda'}^2}$. The Laguerre polynomials and the solid harmonics are functions of k and p_Λ. The details of the semi-analytic calculations are given in appendix A.

V. NUMERICAL RESULTS

A. Form Factors

The form factors in this work are calculated using the parameters for the quark model wave functions taken from [20]. The quark masses relevant for this calculation are shown in Tables I, while the wave function size parameters are shown in table II. The calculated form factors are parametrized to have the simple form

$$F = (a_0 + a_2 q^2 + a_4 q^4) \exp\left(\frac{-3m_\sigma^2}{2m_\Lambda^2}\frac{p_\Lambda^2}{\alpha_{\lambda\lambda'}^2}\right), \tag{53}$$

where q^2 is the momentum transfer $(p_{\Lambda_c} - p_\Lambda)^2$. p_Λ is calculated in the rest frame of the parent Λ_c, and takes the form

$$p_\Lambda = \frac{1}{2m_{\Lambda_c}}\lambda^{1/2}(m_{\Lambda_c}^2, m_\Lambda^2, q^2). \tag{54}$$

The parameters for the form factors we obtain are given in table III.

TABLE I: Quark masses used in [20].

m_q GeV	m_s GeV	m_c GeV
0.2848	0.5553	1.8182

TABLE II: Baryon masses and wave function size parameters, α_λ and α_ρ obtained from [20]. All values are in GeV.

State, J^P	Mass (GeV)		Size parameters (GeV)	
	Experiment	Model	α_λ	α_ρ
$\Lambda_c(2286)\frac{1}{2}^+$	2.29	2.27	0.424	0.393
$\Lambda(1115)\frac{1}{2}^+$	1.12	1.10	0.387	0.372
$\Lambda(1600)\frac{1}{2}^+_1$	1.60	1.71	0.387	0.372
$\Lambda(1405)\frac{1}{2}^-$	1.41	1.48	0.333	0.320
$\Lambda(1520)\frac{3}{2}^-$	1.52	1.53	0.333	0.308
$\Lambda(1890)\frac{3}{2}^+$	1.89	1.81	0.325	0.303
$\Lambda(1820)\frac{5}{2}^+$	1.82	1.81	0.325	0.303

Figure 4 shows the form factors for the transitions to the ground state and the excited states that we consider. In the language of HQET, the form factors (F_1, G_1) associated with leading order in the $1/m_c$ expansion are dominant, while all of the others are smaller. With the exception of transitions to the $\Lambda(1600)1/2^+$, all of the form factors have their largest absolute values at their respective non-recoil points.

B. Comparison with HQET

In Section III A, we obtained expressions for the general transition form factors in terms of the leading order HQET form factors. Those expressions can be inverted to write the HQET form factors in terms of the general ones. Since the pair of leading order HQET form factors are valid for both the vector and axial-vector hadronic matrix elements, we can extract them from both sets of general form factors. The expressions for ξ_1 and ξ_2 are shown in table IV, and the curves are shown in Fig. 5.

The leading order HQET expectation is that the extraction of ξ_1 and ξ_2 should be independent of whether they are extracted from axial or vector form factors. However, the curves we obtain indicate that there is some sensitivity to which set of form factors is used. This sensitivity can be attributed to the fact that our form factors include effects

TABLE III: Coefficients in parametrization of the form factors, from eqn. 53.

Transition	$a_n(\text{GeV}^{-n})$	F_1	F_2	F_3	F_4	G_1	G_2	G_3	G_4
$\Lambda_c \to \Lambda(1115)$	a_0	1.382	-0.235	-0.146	–	0.868	-0.440	0.203	–
	a_2	-0.073	0.022	-0.003	–	0.013	-0.116	-0.009	–
	a_4	0.000	0.006	-0.001	–	0.004	0.003	0.000	–
$\Lambda_c \to \Lambda(1600)$	a_0	0.172	0.036	-0.015	–	0.144	-0.002	0.021	–
	a_2	-0.257	0.121	0.020	–	-0.102	0.160	-0.040	–
	a_4	0.025	-0.008	-0.001	–	0.005	-0.026	0.004	–
$\Lambda_c \to \Lambda(1405)$	a_0	0.300	-0.797	0.162	–	0.881	-0.516	0.027	–
	a_2	-0.126	0.028	-0.010	–	-0.058	-0.066	0.025	–
	a_4	0.008	-0.003	-0.000	–	0.002	0.009	-0.001	–
$\Lambda_c \to \Lambda(1520)$	a_0	1.496	-0.530	-0.172	0.094	0.613	-0.810	0.351	-0.170
	a_2	-0.080	0.019	-0.005	0.001	0.005	0.122	-0.010	0.008
	a_4	0.002	0.003	-0.001	0.000	0.002	-0.001	0.000	0.000
$\Lambda_c \to \Lambda(1890)$	a_0	0.251	-0.358	0.165	-0.090	0.625	-0.257	0.016	0.040
	a_2	-0.079	-0.107	-0.006	0.004	-0.030	-0.041	0.021	-0.011
	a_4	0.005	0.950	0.000	-0.000	0.001	0.006	-0.001	0.001
$\Lambda_c \to \Lambda(1820)$	a_0	1.148	-0.441	-0.177	0.139	0.322	-0.677	0.381	-0.325
	a_2	-0.059	0.008	-0.005	0.001	0.002	0.089	-0.008	0.013
	a_4	0.002	0.001	-0.001	0.000	0.000	-0.002	0.000	0.000

TABLE IV: The leading order HQET form factors in terms of the general form factors. The second and third columns show the expressions in terms of the vector form factors, while the fifth and sixth columns show them in terms of the axial-vector form factors. The fourth and seventh columns show the ratios ξ_2/ξ_1 (ζ_2/ζ_1) calculated at the non-recoil point.

State, J^P	Vector			Axial Vector		
	ξ_1 (ζ_1)	ξ_2 (ζ_2)	ξ_2/ξ_1 (ζ_2/ζ_1)	ξ_1 (ζ_1)	ξ_2 (ζ_2)	ξ_2/ξ_1 (ζ_2/ζ_1)
$\Lambda(1115)$, $\frac{1}{2}^+$	$F_1 + F_2/2$	$F_2/2$	-0.093	$G_1 - G_2/2$	$G_2/2$	-0.202
$\Lambda(1600)$, $\frac{1}{2}^+$			0.095			-0.007
$\Lambda(1405)$, $\frac{1}{2}^-$	$-F_1 + F_2/2$	$-F_2/2$	-0.571	$-G_1 - G_2/2$	$-G_2/2$	-0.414
$\Lambda(1520)$, $\frac{3}{2}^-$	$F_1 + F_2/2$	$F_2/2$	-0.215	$G_1 - G_2/2$	$-G_2/2$	-0.398
$\Lambda(1890)$, $\frac{3}{2}^+$	$-F_1 + F_2/2$	$-F_2/2$	-0.416	$-G_1 - G_2/2$	$-G_2/2$	-0.259
$\Lambda(1820)$, $\frac{5}{2}^+$	$F_1 + F_2/2$	$F_2/2$	-0.238	$G_1 - G_2/2$	$G_2/2$	-0.512

that arise in all orders of $1/m_c$, while the relationships between the ξ_i (ζ_i) and the F_i and G_i are obtained at leading order. Higher order terms in the $1/m_c$ expansion will modify the expressions shown in eqns. 32 - 36, and hence the inverted relationships.

C. Decay Widths

1. $\Lambda_c^+ \to \Lambda^{(*)} l^+ \nu_l$

The differential decay rates, $d\Gamma/dq^2$ (in $\text{s}^{-1}\text{GeV}^{-2}$), for the semileptonic decays $\Lambda_c^+ \to \Lambda^{(*)} l \nu_l$ are shown in figure 6. Fig 6(a) shows the decay rates for the transition to the elastic channel (the ground state) as well as to the excited states that we consider. The elastic channel is dominant but the decay rates for the decays to $1/2^-$ and $3/2^-$ are significant. Fig 6(b) shows an enlarged version of the decay rates to the excited states. This figure shows that the rates for decays to the radially excited $1/2^+$, the $3/2^+$ and the $5/2^+$ states are small compared to the rates for the $1/2^-$ and $3/2^-$ states.

The integrated total decay widths that we obtain for $\Lambda_c^+ \to \Lambda^{(*)} l^+ \nu_l$ are shown in table V. Also shown are the results presented in PRCI. The calculated total decay widths to the elastic channel are $1.92 \times 10^{11} s^{-1}$ for $l = e$, and $1.86 \times 10^{11} s^{-1}$ for $l = \mu$. The branching fractions calculated are $\frac{\Gamma(\Lambda_c^+ \to \Lambda l^+ \nu_l)}{\Gamma_{\Lambda_c}} = 3.84\%$ for the electron channel,

FIG. 4: Form factors plotted as functions of q^2 for transitions to (a) $\Lambda(1115)\frac{1}{2}^+$; (b) $\Lambda(1600)\frac{1}{2}^+$; (c) $\Lambda(1405)\frac{1}{2}^-$; (d) $\Lambda(1520)\frac{3}{2}^-$; (e) $\Lambda(1890)\frac{3}{2}^+$; (f) $\Lambda(1820)\frac{5}{2}^+$.

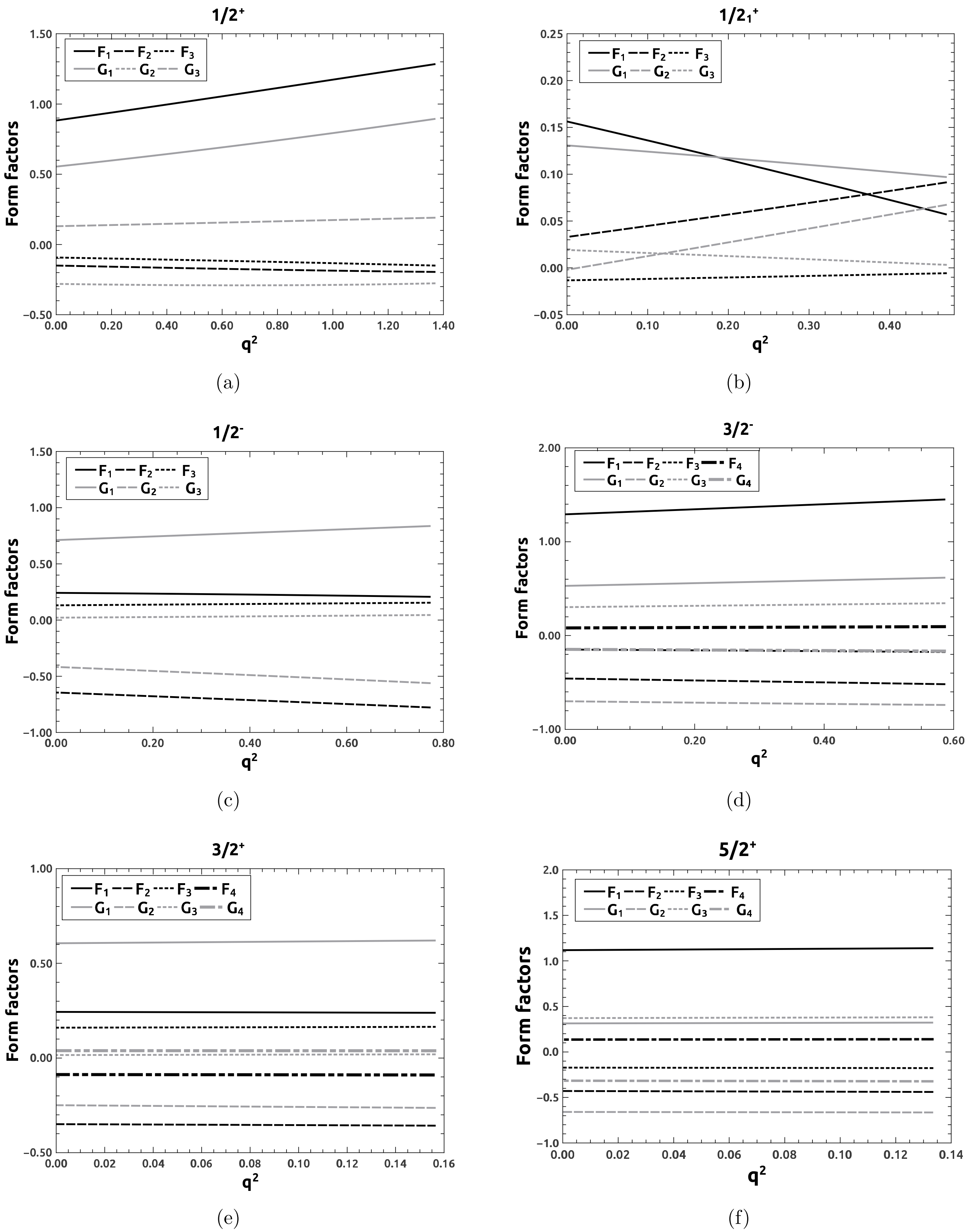
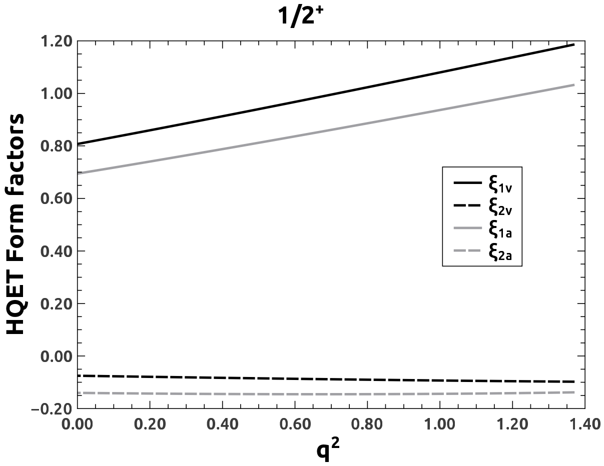
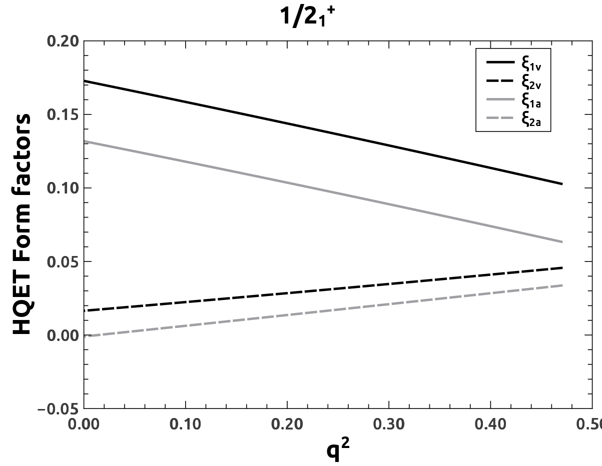


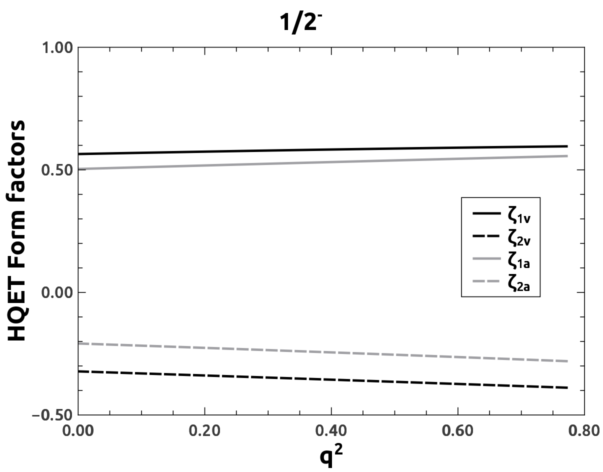
FIG. 5: Vector and axial vector form factors obtained using HQET, for the states that we treat in this work. The graphs shown are for (a) $\Lambda(1115)1/2^+$; (b) $\Lambda(1600)1/2^+$; (c) $\Lambda(1405)1/2^-$; (d) $\Lambda(1520)3/2^-$; (e) $\Lambda(1890)3/2^+$; (f) $\Lambda(1820)5/2^+$.



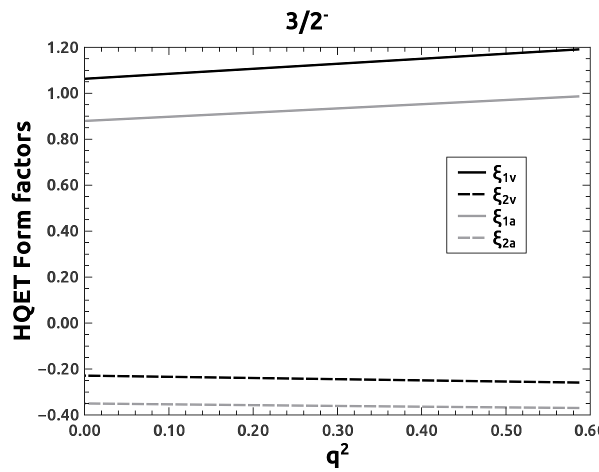
(a)



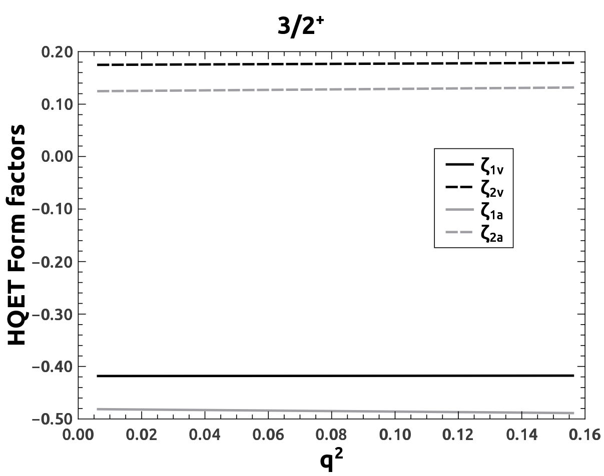
(b)



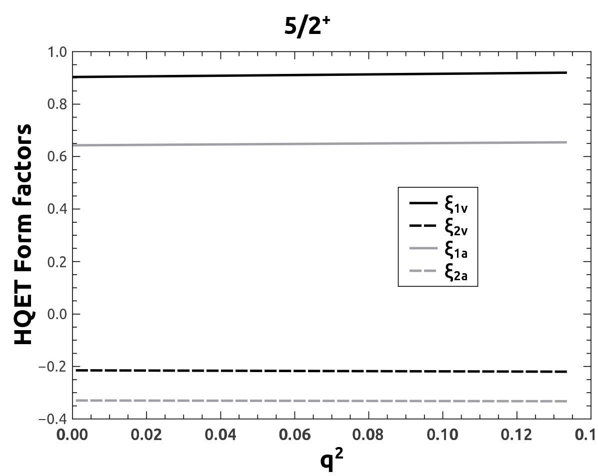
(c)



(d)

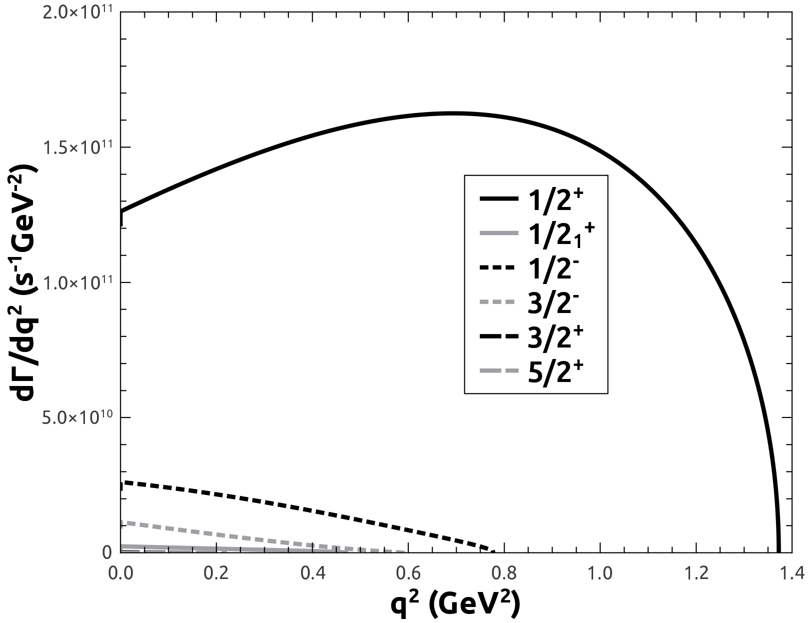


(e)

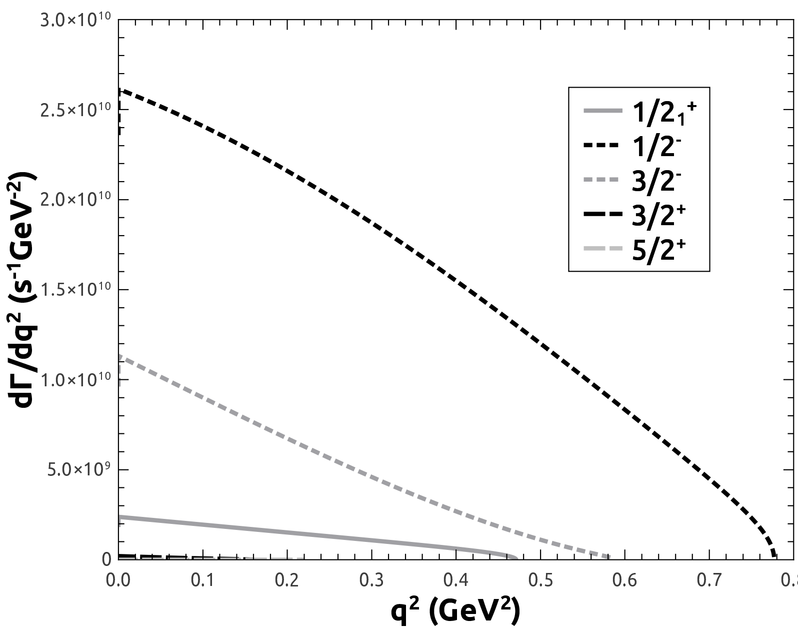


(f)

FIG. 6: Differential decay rates $d\Gamma/dq^2$ (in units of $\mathrm{s}^{-1}\mathrm{GeV}^{-2}$) for the semileptonic decays $\Lambda_c^+ \to \Lambda^{(*)}l^+\nu_l$. (a) shows the decay rate for all states considered, while (b) shows the decay rates for the excited states only.



(a)



(b)

and 3.72% for the muon channel. Γ_{Λ_c} is the total decay width of the Λ_c. Table VI compares our results with other theoretical estimates [16–18] and the experimental results from the Belle [13] and BESIII [21, 22] collaborations. Our results are in very good agreement with the most recent experimental result from BESIII.

From table V, it is evident that the elastic channel dominates the semileptonic decay rate of the Λ_c but does not saturate it. We find that the branching fraction to the $\Lambda^*(1405)$ state with $J^P = 1/2^-$ is 6% of the total semileptonic decay, while the branching fraction to $\Lambda^*(1520)$ is 1% of the total. Decays to the other states we consider are significantly smaller.

TABLE V: Integrated total decay widths for $\Lambda_c^+ \to \Lambda^{(*)} l^+ \nu_l$ in units of $10^{11} s^{-1}$, for the states we consider in this work. Also shown are the results obtained in PRCI. The last row shows the branching fraction of the elastic decay channel, where Γ_{Total} is the total semileptonic decay width assuming the decays shown in the table saturate the semileptonic decays.

Spin	Mass (GeV)	Model estimates		
		This Work		PRCI [5]
		$\Lambda_c^+ \to \Lambda^* e^+ \nu_e$	$\Lambda_c^+ \to \Lambda^* \mu^+ \nu_\mu$	
$\frac{1}{2}^+$	1.115	1.92	1.86	2.10
$\frac{1}{2_1}^+$	1.600	0.63×10^{-2}	0.55×10^{-2}	2.00×10^{-2}
$\frac{1}{2}^-$	1.405	0.12	0.11	0.19
$\frac{3}{2}^-$	1.519	2.97×10^{-2}	2.6×10^{-2}	5.00×10^{-2}
$\frac{3}{2}^+$	1.890	1.58×10^{-4}	1.01×10^{-4}	−−
$\frac{5}{2}^+$	1.820	0.66×10^{-4}	0.42×10^{-4}	−−
Total		2.08	2.00	2.36
$\Gamma_{\Lambda_c^+ \to \Lambda l^+ \nu_l}/\Gamma_{\text{total}}$		0.92	0.93	0.89

TABLE VI: Branching fractions of the semileptonic decay $\Lambda_c^+ \to \Lambda(1115) l^+ \nu_l$, compared with other theoretical estimates and experimental results. In the table, CQM refers to the covariant quark model of reference [16], while LCSR refers to the light cone sum rules of reference [17].

Branching fraction	Model estimates (%)				Experimental results(%)	
	This work	PRCI [5]	CQM [16]	LCSR [17]	Belle [13]	BESIII [21, 22]
$\Gamma_{\Lambda_c^+ \to \Lambda e^+ \nu_e}/\Gamma_{\Lambda_c^+}$	3.84	4.2	2.78	3.05 ± 0.27	2.9 ± 0.5	$3.63 \pm 0.38 \pm 0.20$
$\Gamma_{\Lambda_c^+ \to \Lambda \mu^+ \nu_\mu}/\Gamma_{\Lambda_c^+}$	3.72		2.69	1.96 ± 0.32	2.7 ± 0.6	$3.49 \pm 0.46 \pm 0.27$

2. $\Lambda_c^+ \to \Sigma\pi l^+ \nu_l$ and $\Lambda_c^+ \to N\bar{K} l^+ \nu_l$

Table VII lists the states that we have included in this study, their total and partial widths in the $\Sigma\pi$ and $N\bar{K}$ channels, and the corresponding strong coupling constants, ($g_{\Lambda\Sigma\pi}$, $g_{\Lambda N\bar{K}}$). The $\Lambda(1405)$ lies just below the $N\bar{K}$ threshold, so its coupling to this channel must be estimated by other means. We use the value estimated by Schat, Scoccola and Gobbi [23], but also explore the effects on the decay rate of allowing departures from their value.

Figures 7 and 8 show the differential decay rates $d\Gamma/dq^2$ and $d\Gamma/dS_{\Sigma\pi}$, respectively, for the decays $\Lambda_c^+ \to \Lambda^{(*)} l^+ \nu_l \to \Sigma\pi l^+ \nu_l$. The dominant contribution to this total decay width is through the $\Lambda(1405)$ resonance. Transitions through the $\Lambda(1115)$ and $\Lambda(1520)$ also provide a significant contribution to the total decay rate for $\Lambda_c^+ \to \Sigma\pi l^+ \nu_l$. The contributions from the transitions through the $\Lambda(1600)$, $\Lambda(1890)$ and $\Lambda(1820)$ are small.

The differential decay rates $d\Gamma/dq^2$ and $d\Gamma/dS_{N\bar{K}}$ for the decay $\Lambda_c^+ \to \Lambda^{(*)} l^+ \nu_l \to N\bar{K} l^+ \nu_l$ are shown in figs 9 and 10, respectively. In the total decay width, the transition through the $\Lambda(1520)$ is dominant, and the transition through the sub-threshold $\Lambda(1405)$ is still large. The contributions to the total rate, from transitions through the $\Lambda(1600)$, $\Lambda(1890)$ and $\Lambda(1820)$, are small.

The integrated total decay widths are shown in table VIII. In this calculation, we assume $\Lambda_c^+ \to \Lambda^* \to \Sigma\pi$ and $\Lambda_c^+ \to \Lambda^* \to N\bar{K}$ are the dominant decay modes, and that these two decay modes are saturated by contributions from the states we consider. We also assume that other semileptonic decay modes of the Λ_c are suppressed. The total decay width $\Gamma_{\Lambda_c^+ \to \Lambda^{(*)} l^+ \nu_l \to \Sigma\pi l^+ \nu_l}$ and $\Gamma_{\Lambda_c^+ \to \Lambda^{(*)} l^+ \nu_l \to N\bar{K} l^+ \nu_l}$ are calculated to be $\sim 0.18 \times 10^{11} s^{-1}$, and $\sim 0.1 \times 10^{11} s^{-1}$ respectively. We calculate the semileptonic branching fraction $\frac{\Gamma_{\Lambda_c^+ \to \Lambda^{(*)} l^+ \nu_l \to \Sigma\pi l^+ \nu_l}}{\Gamma_{\Lambda_c}}$ to be 0.08, while the branching fraction $\frac{\Gamma_{\Lambda_c^+ \to \Lambda^{(*)} l^+ \nu_l \to N\bar{K} l^+ \nu_l}}{\Gamma_{\Lambda_c}}$ is 0.04. Our calculation contradicts the assumption of the CLEO collaboration [24] that the elastic channel saturates the semileptonic decay of Λ_c. In our model, we find the branching fractions for the multi-particle final states are 12% of the total semileptonic decay. This suggests that the semileptonic decay of Λ_c^+ is

TABLE VII: Parameters of the excited Λ states that we use in our study. Shown are their total decay widths, and their partial decay widths and the strong couplings for the decays $\Lambda^* \to \Sigma\pi$ and $\Lambda^* \to N\bar{K}$.

Spin of $\Lambda^{(*)}$	Mass(GeV)	Total width (MeV)	Partial width (MeV)		Strong coupling constant	
			$\Gamma_{\Lambda^*\to\Sigma\pi}$	$\Gamma_{\Lambda^*\to N\bar{K}}$	$g_{\Lambda\Sigma\pi}$	$g_{\Lambda N\bar{K}}$
$\frac{1}{2}^+$	1.115	-	-	-	15.73	14.03
$\frac{1}{2}^+_1$	1.600	150	52.5	33.8	8.21	5.76
$\frac{1}{2}^-$	1.405	50.5	50.5	-	1.57	1.90
$\frac{3}{2}^-$	1.519	15.7	6.6	7.1	3.64	15.38
$\frac{3}{2}^+$	1.890	100	6.5	27.5	0.14	1.05
$\frac{5}{2}^+$	1.820	80.0	8.8	48.0	0.40	8.45

TABLE VIII: Integrated decay widths for $\Lambda_c^+ \to \Lambda^{(*)}l^+\nu_l \to \Sigma\pi l^+\nu_l$ and $\Lambda_c^+ \to \Lambda^{(*)}l^+\nu_l \to N\bar{K}l^+\nu_l$ in units of $10^{11}s^{-1}$, for individual $\Lambda^{(*)}$ states for both $l^+ = e^+$ and $l^+ = \mu^+$. The last row shows the coherent totals for the four-body decays $\Lambda_c^+ \to \Sigma\pi l\nu_l$ and $\Lambda_c^+ \to NKl\nu_l$.

Spin of $\Lambda^{(*)}$	Mass(GeV)	$\Gamma_{\Lambda_c\to\Lambda^*e^+\nu_e\to\Sigma\pi e^+\nu_e}$	$\Gamma_{\Lambda_c\to\Lambda^*\mu^+\nu_\mu\to\Sigma\pi\mu^+\nu_\mu}$	$\Gamma_{\Lambda_c\to\Lambda^*e^+\nu_e\to N\bar{K}e^+\nu_e}$	$\Gamma_{\Lambda_c\to\Lambda^*\mu^+\nu_\mu\to N\bar{K}\mu^+\nu_\mu}$
$\frac{1}{2}^+$	1.115	4.18×10^{-2}	3.81×10^{-2}	1.86×10^{-2}	1.65×10^{-2}
$\frac{1}{2}^+_1$	1.600	1.66×10^{-3}	1.44×10^{-4}	9.79×10^{-4}	8.42×10^{-4}
$\frac{1}{2}^-$	1.405	9.54×10^{-2}	8.78×10^{-2}	1.82×10^{-2}	1.65×10^{-2}
$\frac{3}{2}^-$	1.519	2.21×10^{-2}	1.98×10^{-2}	2.54×10^{-2}	2.25×10^{-2}
$\frac{3}{2}^+$	1.890	2.11×10^{-5}	1.66×10^{-5}	1.12×10^{-4}	9.00×10^{-5}
$\frac{5}{2}^+$	1.820	2.23×10^{-5}	1.74×10^{-5}	1.81×10^{-4}	1.43×10^{-4}
Total		18.31×10^{-2}	16.59×10^{-2}	9.33×10^{-2}	8.23×10^{-2}

not saturated to decay to elastic channel and further investigation is needed to see evidence of the channels we have discussed here.

We have treated the $\Lambda(1405)$ as a three-quark state that is the lightest excitation of the Λ, with $J^P = 1/2^-$. In fig. 7, this state contributes a clear resonant structure at $\sqrt{S_{\Sigma\pi}} \approx 1405$. This would suggest that examination of the decay channel $\Lambda_c^+ \to \Sigma\pi l^+\nu_l$ would provide confirmation of this state as a three-quark state. If no evidence is found for this resonance then it may well be that this state is not a simple three-quark state.

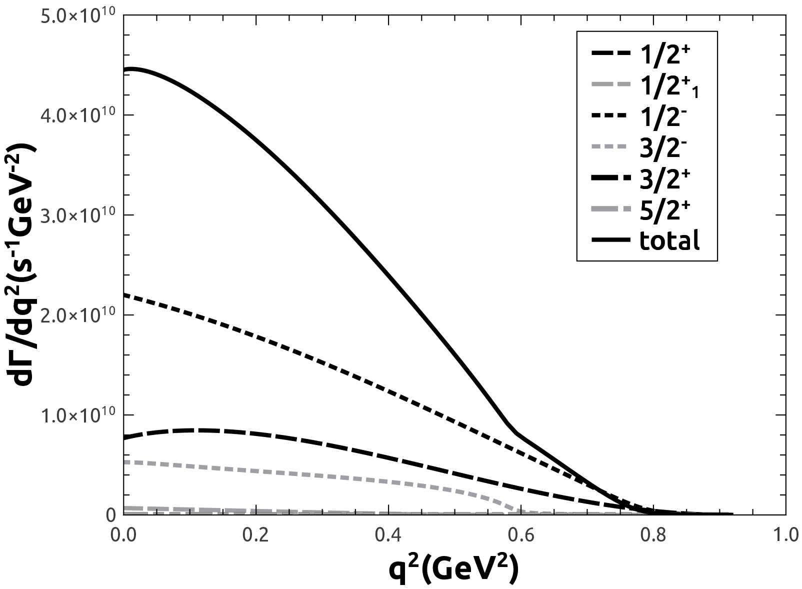
There are a number of other conjectures regarding the structure of the $\Lambda(1405)$. It has been suggested that it could be a dynamically generated molecular state of $\bar{K}N$ and $\Sigma\pi$ [18, 25–28], or a multi-quark state [29]. Recently, Roca and Oset [30] explained it as a molecular state of $\bar{K}N$. Hall *et al.* [31] drew the same conclusion based on a lattice simulation. Ikeno and Oset [18] have estimated the semileptonic decay rate of the Λ_c to this state, assuming that it is a dynamically-generated molecular state. They obtained a value of 2×10^{-5} for the branching fraction $\mathcal{B}(\Lambda_c^+ \to \Lambda(1405)l^+\nu_l) = 2\times10^{-5}$. For $\Lambda_c^+ \to \Lambda(1405)l^+\nu_l \to \Sigma\pi l^+\nu_l$ our branching fraction/ratio is $\sim 2.0\times10^{-3}$, while for $\Lambda_c^+ \to \Lambda(1405)l^+\nu_l \to N\bar{K}l^+\nu_l$ it is $\sim 0.4\times10^{-3}$. Our values are therefore about 20 times larger than the prediction by Ikeno and Oset.

VI. CONCLUSIONS AND OUTLOOK

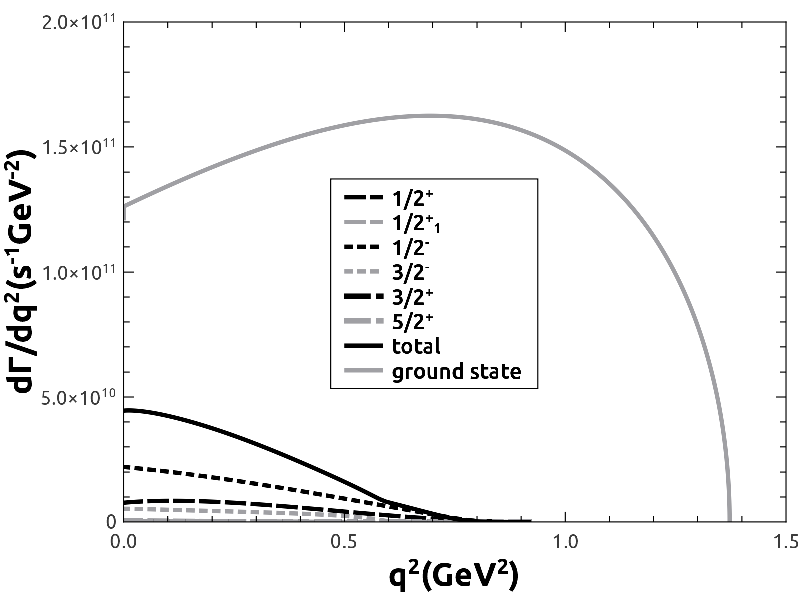
In this work, semileptonic decays of the Λ_c^+ have been studied using a constituent quark model to calculate the required form factors. These form factors for the $\Lambda_c \to \Lambda^{(*)}$ transitions have been obtained both analytically and numerically, using the harmonic oscillator basis to describe the baryon wave functions. The form factors obtained in this model are compared with the HQET expectations at leading order, and are seen to be largely consistent with those expectations. The decay rates of Λ_c^+ to the ground state and a number of excited Λ states have been evaluated.

The original motivation for this work was that there was no model independent calculation for $\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+)$ reported in the previous edition of PDG [32]. PDG estimated $B(\Lambda_c^+ \to pK^-\pi^+) = RfF\frac{B(D\to Xl^+\nu_l)}{1+|\frac{V_{cd}}{V_{cs}}|^2}\tau(\Lambda_c^+)$, based on the measurements by the ARGUS [33] and CLEO [34] Collaborations, using the semileptonic decays of the Λ_c. They assumed that $f = \frac{B(\Lambda_c^+\to\Lambda l^+\nu_l)}{B(\Lambda_c^+\to X_s l^+\nu_l)} = 1.0$ and $F = \frac{B(\Lambda_c^+\to X_s l^+\nu_l)}{B(D\to X_s l^+\nu_l)} = 1.0$. The latest edition reports a model independent measurement that makes the old estimate obsolete. A. Zupanc *et al.* (Belle Collaboration) [13] and M. Ablikim (BESIII Collaboration) [14] measured $B(\Lambda_c^+ \to pK^-\pi^+)$ to be $6.84^{+0.32}_{-0.40}$ and $5.84\pm0.27\pm0.23\%$ respectively. PDG reports their fit for $B(\Lambda_c^+ \to pK^-\pi^+)$ to be $6.35\pm0.33\%$. This result lets us estimate the branching fraction

FIG. 7: (a) The differential decay rates $d\Gamma/dq^2$ for the four-body decay $\Lambda_c^+ \to \Lambda^{(*)} l^+ \nu_l \to \Sigma\pi l^+ \nu_l$. (b) Comparison of the decay rates of the four-body semileptonic decay to the elastic channel $\Lambda_c^+ \to \Lambda l^+ \nu_l$.



(a)



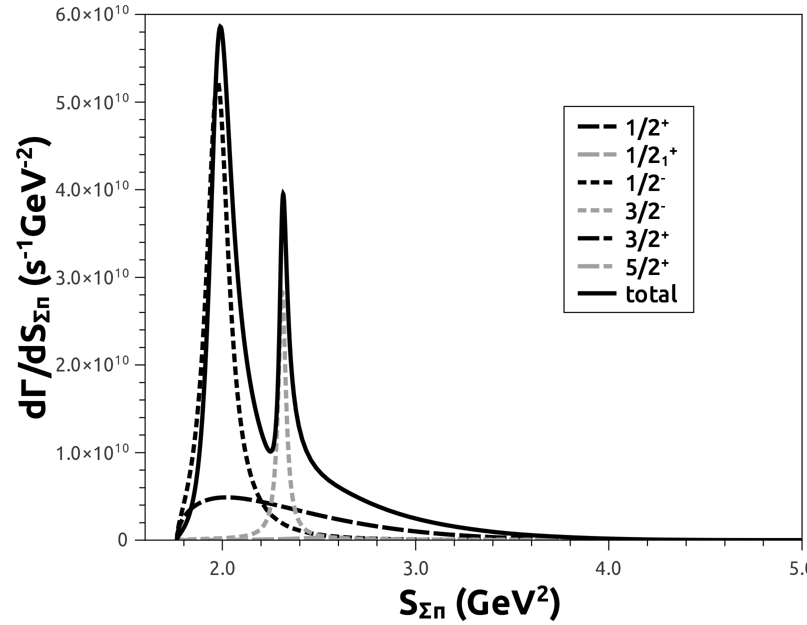
(b)

$f = 0.87^{+0.13}_{-0.17}$, still assuming that $F = 1.0$.

We have calculated branching fractions of the semileptonic decays and they are in a good agreement with the calculations done by Pervin *et al.* in PRCI [5]. The branching fraction of the decay to the elastic channel has been calculated to be 3.84% (for $l = e$) and 3.72% (for $l = \mu$). Our prediction is in agreement with the recent results from BESIII [21, 22] that measured it to be $3.63 \pm 0.38 \pm 0.20\%$ (for $l = e$) and $3.49 \pm 0.46 \pm 0.27$ (for $l = \mu$).

We have used the form factors obtained to examine the semileptonic decays to two four-particle final states, namely $\Sigma\pi l^+\nu_l$ and $NK l^+\nu_l$. We find that the branching fraction for these two channels totals 12% of the inclusive

FIG. 8: The differential decay rates $d\Gamma/dS_{\Sigma\pi}$ for decays via the states we consider, along with the coherent total. The black solid curve shows the coherent total decay rate for the $\Sigma\pi l\nu$ final state.



semileptonic decay $\Lambda_c \to X_s l^+ \nu_l$. We estimate $f = 0.88$, in disagreement with the CLEO [35] assumption that the decay to the ground state $\Lambda(1115)$ saturates the semileptonic decays of the Λ_c.

The two lowest-lying Λ resonances, the $\Lambda(1405)1/2^-$ and $\Lambda(1520)3/2^-$ are seen to be important in both the rate and the shape of the spectrum. The $\Lambda(1405)$ produces a sharp resonant structure in the $S_{\Sigma\pi}$ spectrum, suggesting that this state may be detectable in the $\Lambda_c^+ \to \Sigma\pi l^+ \nu_l$ transition. The $\Lambda(1520)$ also generates sharp resonant structures in both the $\Sigma\pi$ and $N\bar{K}$ decay spectra. This state may therefore also be detectable in these channels. This can have some impact on baryon phenomenology, as it would confirm these states as orbital excitations of the ground states Λ. The broader resonances that were included in the study are less likely to be identifiable in the decay spectra.

In this calculation we have assumed that the states we include saturate the resonant decays of the Λ_c. The available phase space limits the number of excited states that can contribute significantly to the semileptonic decay rate. There is ample phase space to produce some of the lighter excitations, such as the $\Lambda(1670)1/2^-$ and $\Lambda(1690)3/2^-$, but the very small wave function overlap with that of the Λ_c means that the form factors are tiny, so that the decays are very effectively suppressed.

The work presented in this manuscript can be extended in a number of directions. The form factors calculated here may be used to study any of the polarization observables that can arise in these semileptonic decays. With a suitable parametrization of the factorization assumption, they can also be used to examine a number of nonleptonic decays of the Λ_c. The form factors were evaluated using the harmonic oscillator basis, and this leads to form factors that have exponential dependence on q^2. One possible extension of the project would be to use a different basis, such as the sturmian basis, to extract the form factors. This basis leads to form factors with multipole dependence on q^2, closer to popular expectations. The semianalytic method we have developed for use with the harmonic oscillator basis can easily be adapted for the sturmian basis. The semileptonic decays of the Λ_b to both charmed and charmless final states may also be re-examined.

ACKNOWLEDGEMENTS

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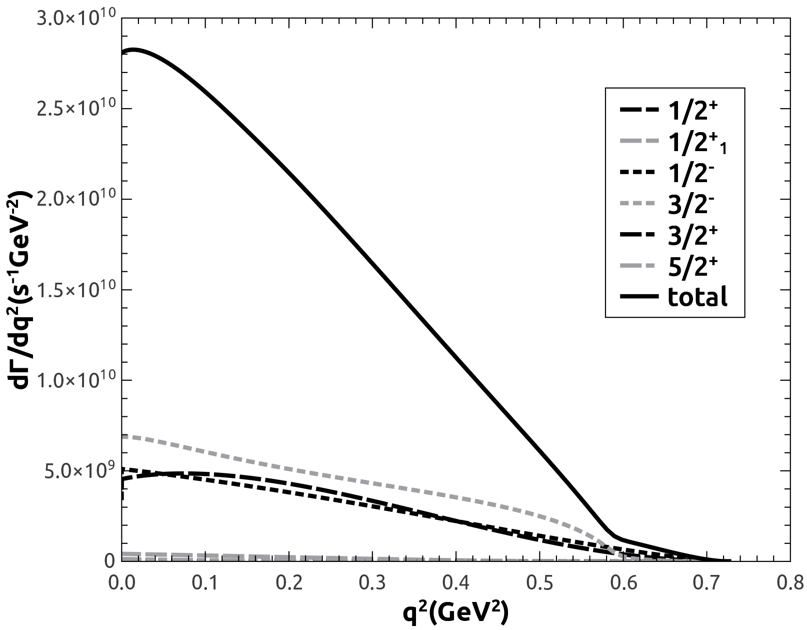
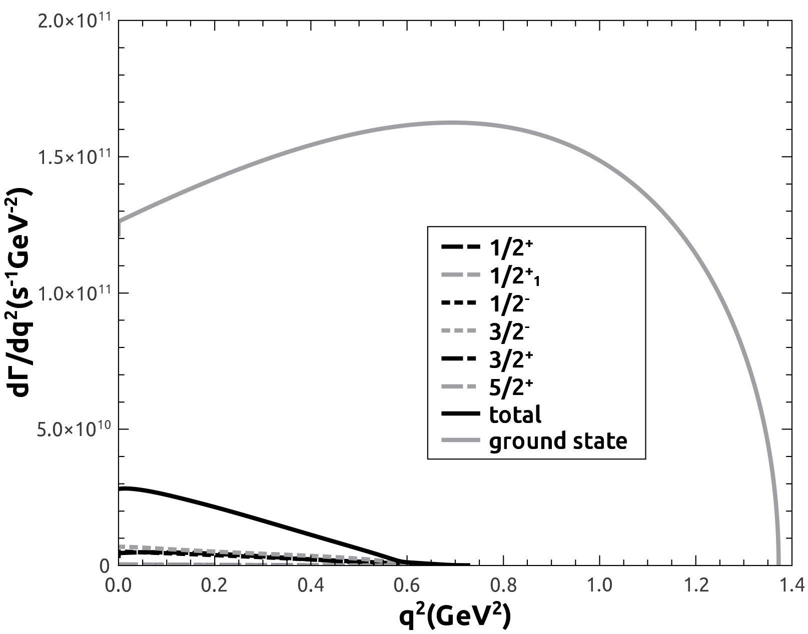
FIG. 9: (a) The differential decay rates $d\Gamma/dq^2$ for the decay $\Lambda_c^+ \to \Lambda^{(*)} l^+ \nu_l \to N\bar{K} l^+ \nu_l$. (b) Comparison the decay rates to the excited states with the semileptonic decay to the elastic channel $\Lambda_c^+ \to \Lambda(1115) l^+ \nu_l$.



(a)



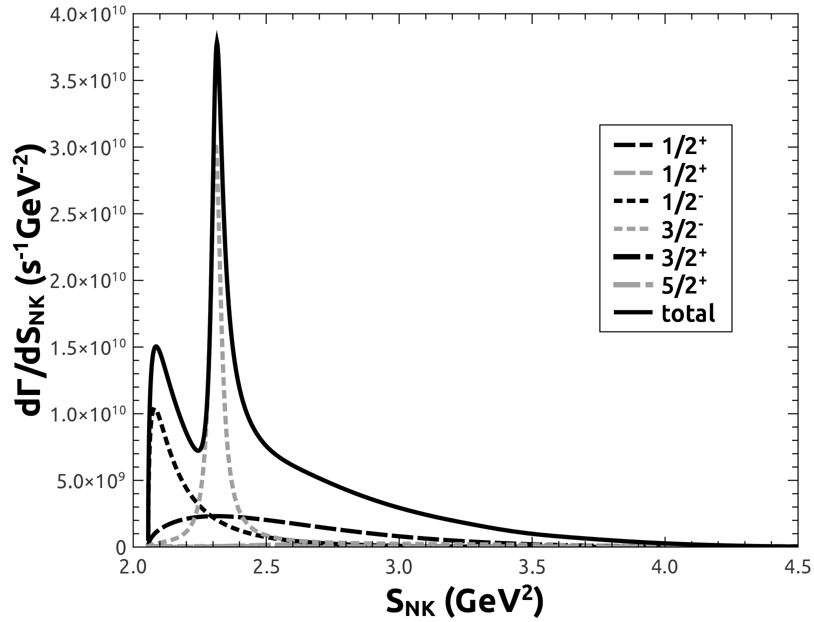
(b)

Appendix A: Semi Analytic Treatment of Hadronic Matrix Elements

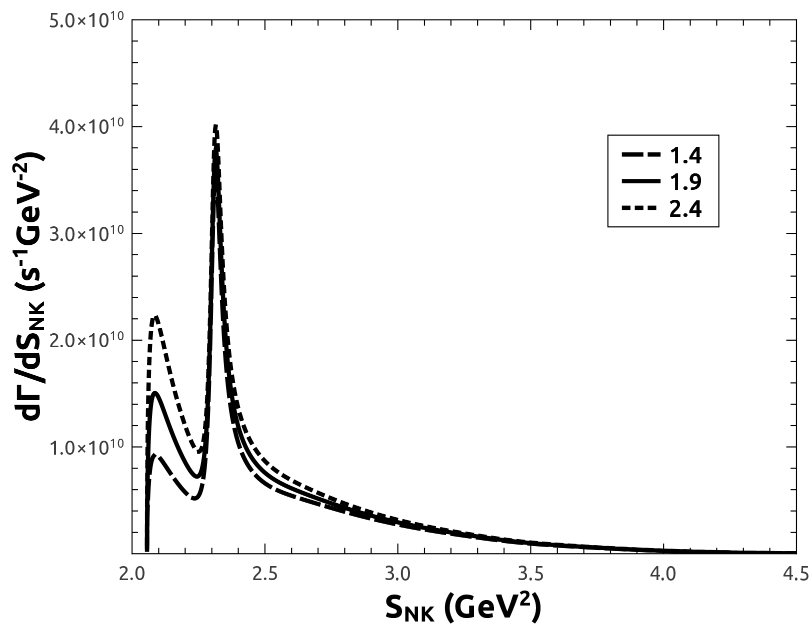
The hadronic matrix element can be written in the form,

$$\langle \Lambda | \bar{s} \Gamma c | \Lambda_c \rangle = \sum_{b',b} h_{b'}^{\Lambda^*} h_b^{\Lambda_c} \delta_{s_1' s_1} \delta_{s_2' s_2} (-1)^{l_{\lambda'} + l_\lambda} \times B_{n_\rho l_\rho m_\rho}^{n_{\rho'} l_{\rho'} m_{\rho'}}(\alpha_\rho, \alpha_{\rho'}) D_{\Gamma; n_\lambda l_\lambda m_\lambda s_q}^{n_{\lambda'} l_{\lambda'} m_{\lambda'} s_{q'}}(\alpha_\lambda, \alpha_{\lambda'}), \tag{A1}$$

FIG. 10: (a) The differential decay rate through each of the Λ^* states considered in this calculation along with the coherent total. The black solid curve shows the differential decay rate for $\Lambda_c \rightarrow NKl\nu_l$. (b) The differential decay rate $d\Gamma/dS_{N\bar{K}}$ for different values of $g_{\Lambda N\bar{K}}$ for $\Lambda(1405)$ state.



(a)



(b)

where the coefficients $h^A_{b(b')}$ are the products of the normalization of the baryon state A, the expansion coefficients η_i, and the various Clebsch-Gordan coefficients that appear in the parent (daughter) baryon wave function. The indices $b(b')$ contain all the relevant quantum numbers being summed over for the parent (daughter) baryon state. $B^{n_{\rho'}l_{\rho'}m_{\rho'}}_{n_\rho l_\rho m_\rho}$

is the spectator overlap,

$$\begin{aligned}
B^{n_{\rho'}l_{\rho'}m_{\rho'}}_{n_\rho l_\rho m_\rho}(\alpha_\rho,\alpha_{\rho'}) =& \int d^3p_\rho d^3p_{\rho'}\psi^*_{n_{\rho'}l_{\rho'}m_{\rho'}}(\alpha_{\rho'};\vec{p}_{\rho'})\psi_{n_\rho l_\rho m_\rho}(\alpha_\rho;\vec{p}_\rho)\delta(p_\rho - p_{\rho'})\\
=&\delta_{l_\rho l_{\rho'}}\delta_{m_\rho m_{\rho'}}N^*_{n_{\rho'}l_{\rho'}}(\alpha_{\rho'})N_{n_\rho l_\rho}(\alpha_\rho)\int d^3p_\rho\ \exp(-p_\rho^2/2\alpha_\rho^2)\ \exp(-p_\rho^2/2\alpha_{\rho'}^2)\\
&\times\mathcal{Y}^*_{l_{\rho'}m_{\rho'}}(\vec{p}_\rho)\mathcal{Y}_{l_\rho m_\rho}(\vec{p}_\rho)\mathcal{L}^{l_{\rho'}+1/2*}_{n_{\rho'}}(p_\rho^2/\alpha_{\rho'}^2)\mathcal{L}^{l_\rho+1/2}_{n_\rho}(p_\rho^2/\alpha_\rho^2)\\
=&N^*_{n_{\rho'}l_\rho}(\alpha_{\rho'})N_{n_\rho l_\rho}(\alpha_\rho)\int d^3p_\rho\ \exp(-b^2p_\rho^2)\\
&\times\mathcal{Y}^*_{l_\rho m_\rho}(\vec{p}_\rho)\mathcal{Y}_{l_\rho m_\rho}(\vec{p}_\rho)\mathcal{L}^{l_\rho+1/2*}_{n_{\rho'}}(p_\rho^2/\alpha_{\rho'}^2)\mathcal{L}^{l_\rho+1/2}_{n_\rho}(p_\rho^2/\alpha_\rho^2),
\end{aligned}$$

where $b^2=\frac{\alpha_\rho^2+\alpha_{\rho'}^2}{2\alpha_\rho^2\alpha_{\rho'}^2}$. The results for the overlap integrals that appear in this calcuation are shown in appendix B.

The interaction overlap $D^{n_{\lambda'}l_{\lambda'}m_{\lambda'}s_{q'}}_{\Gamma;n_\lambda l_\lambda m_\lambda s_q}$ is

$$\begin{aligned}
D^{n_{\lambda'}l_{\lambda'}m_{\lambda'}s_{q'}}_{\Gamma;n_\lambda l_\lambda m_\lambda s_q}(\beta,\beta') =& \exp\left(\frac{-3m_\sigma^2}{2m_\Lambda^2}\frac{p_\Lambda^2}{\alpha_{\lambda\lambda'}^2}\right)\int d^3k e^{-\alpha^2k^2}\mathcal{L}^{l_{\lambda'}+\frac{1}{2}*}\left(\frac{{p'}^2}{\beta'^2}\right)\mathcal{Y}^*_{l_{\lambda'}m_{\lambda'}}(\vec{p}')\\
&\times\langle s(\vec{p}_\Lambda+\vec{p},s_{q'})|\bar{s}\Gamma c|c(\vec{p},s_q)\rangle\mathcal{L}^{l_\lambda+\frac{1}{2}}\left(\frac{p^2}{\beta^2}\right)\mathcal{Y}_{l_\lambda m_\lambda}(\vec{p}),
\end{aligned}\tag{A2}$$

where, $\beta^{(\prime)}=\sqrt{2/3}\alpha_\lambda^{(\prime)}$, $\alpha^2=\frac{\alpha_\lambda^2+\alpha_{\lambda'}^2}{2\alpha_\lambda^2\alpha_{\lambda'}^2}$ and $\alpha_{\lambda\lambda'}=\sqrt{(\alpha_\lambda^2+\alpha_{\lambda'}^2)/2}$.

Using the changes in variable $\vec{p}=\vec{k}+a\vec{p}_\Lambda$, and $\vec{p}'=\vec{k}+a'\vec{p}_\Lambda$, where $a=-\frac{m_\sigma}{\tilde{m}_\Lambda}\frac{\alpha_\lambda^2}{2\alpha_{\lambda\lambda'}^2}$ and $a'=\frac{m_\sigma}{\tilde{m}_\Lambda}\frac{\alpha_{\lambda'}^2}{2\alpha_{\lambda\lambda'}^2}$, the solid harmonics take the form $\mathcal{Y}_{lm}(a\vec{p}_1+b\vec{p}_2)$, and can be decomposed using the addition theorem as,

$$\mathcal{Y}_{lm}(b_1\vec{p}_1+b_2\vec{p}_2)=\sum_{\lambda=0}^{l}\sum_{\mu_\lambda=-\lambda}^{\lambda}B^{\lambda\mu_\lambda}_{lm}\mathcal{Y}_{\lambda\mu_\lambda}(\vec{p}_1)\mathcal{Y}_{l-\lambda m-\mu_\lambda}(\vec{p}_2),$$

where

$$B^{\lambda\mu_\lambda}_{lm}=\sqrt{\frac{4\pi(2l+1)!}{(2\lambda+1)!(2l-2\lambda+1)!}}b_1^\lambda b_2^{l-\lambda}C^{lm}_{\lambda\mu_\lambda l-\lambda m-\mu_\lambda}.$$

Equation A2 then takes the form

$$D^{n_{\lambda'}l_{\lambda'}m_{\lambda'}s_{q'}}_{\Gamma;n_\lambda l_\lambda m_\lambda s_q}(\beta,\beta')=\exp\left(\frac{-3m_\sigma^2}{2m_\Lambda^2}\frac{p_\Lambda^2}{\alpha_{\lambda\lambda'}^2}\right)\sum_{\lambda'\mu_{\lambda'}}\sum_{\lambda\mu_\lambda}B^{\lambda'\mu_{\lambda'}*}_{l_{\lambda'}m_{\lambda'}}B^{\lambda\mu_\lambda}_{l_\lambda m_\lambda}\mathcal{I}^{n_{\lambda'}l_{\lambda'}s_{q'}:\lambda'\mu_{\lambda'}}_{\Gamma;n_\lambda l_\lambda s_q:\lambda\mu_\lambda}(\alpha^2),$$

where

$$\begin{aligned}
\mathcal{I}^{n_{\lambda'}l_{\lambda'}s_{q'}:\lambda'\mu_{\lambda'}}_{\Gamma;n_\lambda l_\lambda s_q:\lambda\mu_\lambda}(\alpha^2)=&\int d^3k e^{-\alpha^2k^2}\mathcal{L}^{l_{\lambda'}+\frac{1}{2}*}_{n_{\lambda'}}(\frac{{p'}^2}{\beta'^2})\mathcal{Y}^*_{\lambda'\mu_{\lambda'}}(\vec{k})\mathcal{Y}^*_{l-\lambda'm-\mu_{\lambda'}}(\vec{p}_\Lambda)\\
&\times\langle s(\vec{p}_\Lambda+\vec{p},s_{q'})|\bar{s}\Gamma c|c(\vec{p},s_q)\rangle\mathcal{L}^{l_\lambda+\frac{1}{2}}_{n_\lambda}(\frac{p^2}{\beta^2})\mathcal{Y}_{\lambda\mu_\lambda}(\vec{k})\mathcal{Y}_{l-\lambda m-\mu_\lambda}(\vec{p}_\Lambda).
\end{aligned}\tag{A3}$$

The quark current $\langle s(\vec{p}_\Lambda+\vec{p},s_{q'})|\bar{s}\Gamma c|c(\vec{p},s_q)\rangle$ can be written in its most general form as

$$\langle s(\vec{p}_\Lambda+\vec{p},s_{q'})|\bar{s}\Gamma c|c(\vec{p},s_q)\rangle=\sum_{l=0}^{2}\sum_{m_l=-l}^{l}\xi^{s_{q'}s_q}_{\Gamma;lm_l}(\vec{k},\vec{p}_\Lambda)\mathcal{Y}_{lm_l}(\vec{k}),\tag{A4}$$

where $\xi^{s_{q'}s_q}_{\Gamma;lm_l}(\vec{k},\vec{p}_\Lambda)$ can be expanded in Legendre polynomials as

$$\xi^{s_{q'}s_q}_{\Gamma;lm_l}(\vec{k},\vec{p}_\Lambda)=\sum_{l_0=0}^{\infty}\zeta^{s_{q'}s_q}_{\Gamma;lm_ll_0}(k,p_\Lambda)P_{l_0}(x). \tag{A5}$$

Here, $x=\hat{k}\cdot\hat{p}_\Lambda$. The coefficients $\zeta^{s_{q'}s_q}_{\Gamma;lm_ll'}(k,\vec{p}_\Lambda)$ are obtained as

$$\zeta^{s_{q'}s_q}_{\Gamma;lm_ll_0}(k,p_\Lambda)=\left(\frac{2l_0+1}{2}\right)\int_{-1}^{1}dx\xi^{s_{q'}s_q}_{\Gamma;lm_l}(\vec{k},\vec{p}_\Lambda)P_{l_0}(x), \tag{A6}$$

and the integral on the right hand side is evaluated numerically. In practice, the sum in eq. A5 includes a finite number of terms, determined by the values of l_λ, n_λ, $l_{\lambda'}$, $n_{\lambda'}$ and the maximum value of l in eq. A4. The Legendre polynomial $P_{l_0}(x)$ can be written as

$$P_{l_0}(x)=\left(\frac{4\pi}{2l_0+1}\right)\sum_{m_0=-l_0}^{l_0}Y^*_{l_0m_0}(\hat{p}_\Lambda)Y_{l_0m_0}(\hat{k}),$$

Thus, $\xi^{s_{q'}s_q}_{\Gamma;lm_l}(\vec{k},\vec{p}_\Lambda)$ takes the form

$$\xi^{s_{q'}s_q}_{\Gamma;lm_l}(\vec{k},\vec{p}_\Lambda)=2\pi\sum_{l_0=0}^{\infty}\sum_{m_0=-l_0}^{l_0}Y^*_{l_0m_0}(\hat{p}_\Lambda)Y_{l_0m_0}(\hat{k})\left[\int_{-1}^{1}dx\xi^{s_{q'}s_q}_{\Gamma;lm_l}(\vec{k},\vec{p}_\Lambda)P_{l_0}(x)\right]. \tag{A7}$$

The product of the Laguerre polynomials $\mathcal{L}^{l_{\lambda'}+\frac{1}{2}*}_{n_{\lambda'}}(p'^2/\beta'^2)\times\mathcal{L}^{l_\lambda+\frac{1}{2}}_{n_\lambda}(p^2/\beta^2)$ can be written as

$$\mathcal{L}^{l_{\lambda'}+\frac{1}{2}*}_{n_{\lambda'}}\left(\frac{p'^2}{\beta'^2}\right)\mathcal{L}^{l_\lambda+\frac{1}{2}}_{n_\lambda}\left(\frac{p^2}{\beta^2}\right)=\sum_{\delta\rho\sigma}D_{\delta\rho\sigma}k^{2\delta}p^{2\rho}_\Lambda(\vec{k}\cdot\vec{p}_\Lambda)^\sigma, \tag{A8}$$

where $(\vec{k}\cdot\vec{p}_\Lambda)$ can be expanded as

$$\begin{aligned}(\vec{k}\cdot\vec{p}_\Lambda)^\sigma&=\sum_{l_\sigma=0}^{\sigma}k^\sigma p^\sigma_\Lambda c'^\sigma_{l_\sigma}P_{l_\sigma}(\hat{k}\cdot\hat{p}_\Lambda)\\&=\sum_{l_\sigma=0}^{\sigma}k^\sigma p^\sigma_\Lambda C'^\sigma_{l_\sigma}\frac{4\pi}{2l_\sigma+1}Y_{l_\sigma m_\sigma}(\hat{k})Y^*_{l_\sigma m_\sigma}(\hat{p}_\Lambda),\end{aligned}$$

where $P_{l_\sigma}(\hat{k}\cdot\hat{p}_\Lambda)$ is the Legendre polynomial and $C'^\sigma_{l_\sigma}$ is defined as

$$\begin{aligned}C'^{2r}_{2s}&=(2r)!\frac{2^{2s}(4s+1)(r+s)!}{(2r+2s+1)!(r-s)!},C'^{2r}_{2s+1}=0,\\C'^{2r+1}_{2s+1}&=(2r+1)!\frac{2^{2s+1}(4s+3)(r+s+1)!}{(2r+2s+3)!(r-s)!},C'^{2r+1}_{2s}=0.\end{aligned}$$

Eqn. A3 therefore becomes

$$\begin{aligned}\mathcal{I}^{n_{\lambda'}l_{\lambda'}s_{q'}:\lambda'\mu_{\lambda'}}_{\Gamma;n_\lambda l_\lambda s_q:\lambda\mu_\lambda}(\alpha^2)=&\sum_{l=0}^{2}\sum_{m_l=-l}^{l}\sum_{l_0=0}^{\infty}\sum_{m_0=-l_0}^{l_0}\sum_{\delta\rho\sigma}\sum_{l_\sigma=0}^{\sigma}\int d^3ke^{-\alpha^2k^2}\xi^{s_{q'}s_q}_{\Gamma;lm_l}(\vec{k},\vec{p}_\Lambda)\\&\times D_{\delta\rho\sigma}k^{2\delta}p^{2\rho}_\Lambda k^\sigma p^\sigma_\Lambda C'^\sigma_{l_\sigma}\frac{4\pi}{2l_\sigma+1}\mathcal{Y}^*_{l-\lambda'm-\mu_{\lambda'}}(\vec{p}_\Lambda)Y^*_{l_0m_0}(\hat{p}_\Lambda)Y^*_{l_\sigma m_\sigma}(\hat{p}_\Lambda)\mathcal{Y}_{l-\lambda m-\mu_\lambda}(\vec{p}_\Lambda)\\&\times Y_{l_0m_0}(\hat{k})Y_{l_\sigma m_\sigma}(\hat{k})\mathcal{Y}^*_{\lambda'\mu_{\lambda'}}(\vec{k})\mathcal{Y}_{lm_l}(\vec{k})\mathcal{Y}_{\lambda\mu_\lambda}(\vec{k}).\end{aligned} \tag{A9}$$

The angular integrations in eq. A9 are evaluated using the properties of the spherical and solid harmonics. The remaining integral is done numerically.

Appendix B: Spectator Overlap

The spectator overlaps for the set of quantum numbers $(n_\rho l_\rho m_\rho, n_{\rho'} l_{\rho'} m_{\rho'})$ used in our calculation are listed in this appendix. We define $\alpha_{\rho\rho'} = \sqrt{\alpha_\rho^2 + \alpha_{\rho'}^2/2}$.

$$
\begin{aligned}
B_{000}^{000} &= \left[\frac{\alpha_\rho \alpha_{\rho'}}{\alpha_{\rho\rho'}^2}\right]^{3/2}, \quad B_{010}^{010} = \left[\frac{\alpha_\rho \alpha_{\rho'}}{\alpha_{\rho\rho'}^2}\right]^{5/2}, \\
B_{000}^{100} &= -B_{100}^{000} = \sqrt{\frac{3}{2}}\left[\frac{\alpha_\rho \alpha_{\rho'}}{\alpha_{\rho\rho'}^2}\right]^{3/2} \frac{\alpha_{\rho'}^2 - \alpha_\rho^2}{\alpha_{\rho'}^2 + \alpha_\rho^2}, \\
B_{100}^{100} &= \left[\frac{\alpha_\rho \alpha_{\rho'}}{\alpha_{\rho\rho'}^2}\right]^{3/2} \left[\frac{5}{2}\left(\frac{\alpha_\rho \alpha_{\rho'}}{\alpha_{\rho\rho'}^2}\right)^2 - \frac{3}{2}\right], \\
B_{010}^{110} &= -B_{110}^{010} = \sqrt{\frac{5}{2}}\left[\frac{\alpha_\rho \alpha_{\rho'}}{\alpha_{\rho\rho'}^2}\right]^{5/2} \frac{\alpha_{\rho'}^2 - \alpha_\rho^2}{\alpha_{\rho'}^2 + \alpha_\rho^2}, \\
B_{110}^{110} &= \left[\frac{\alpha_\rho \alpha_{\rho'}}{\alpha_{\rho\rho'}^2}\right]^{5/2} \left[\frac{7}{2}\left(\frac{\alpha_\rho \alpha_{\rho'}}{\alpha_{\rho\rho'}^2}\right)^2 - \frac{5}{2}\right].
\end{aligned}
$$

Appendix C: Analytic Expressions For The Form Factors

The analytical expressions for the form factors for transition to Λ^* states with the J^P are shown. We obtained these form factors using the single component wave-functions in the harmonic oscillator basis.

1. $1/2^+$

$$
\begin{aligned}
F_1 &= I_H\left[1 + \frac{m_\sigma}{\alpha_{\lambda\lambda'}^2}\left(\frac{\alpha_{\lambda'}^2}{m_s} - \frac{\alpha_\lambda^2}{m_c}\right)\right], \\
F_2 &= -I_H\left[\frac{m_\sigma}{m_s}\frac{\alpha_{\lambda'}^2}{\alpha_{\lambda\lambda'}^2} - \frac{\alpha_\lambda^2\alpha_{\lambda'}^2}{4m_c m_s \alpha_{\lambda\lambda'}^2}\right], \\
F_3 &= -I_H\frac{m_\sigma}{m_c}\frac{\alpha_\lambda^2}{\alpha_{\lambda'}^2}, \\
G_1 &= I_H\left[1 - \frac{\alpha_\lambda^2 {\alpha_{\lambda'}}^2}{12\alpha_{\lambda\lambda'}^2 m_c m_s}\right], \\
G_2 &= -I_H\left[\frac{m_\sigma}{m_s}\frac{\alpha_{\lambda'}^2}{\alpha_{\lambda\lambda'}^2} + \frac{\alpha_\lambda^2\alpha_{\lambda'}^2}{12 m_c m_s \alpha_{\lambda\lambda'}^2}\left(1 + \frac{12 m_\sigma^2}{\alpha_{\lambda\lambda'}^2}\right)\right], \\
G_3 &= I_H\left[\frac{m_\sigma}{m_c}\frac{\alpha_{\lambda'}^2}{\alpha_{\lambda\lambda'}^2} + \frac{m_\sigma^2\alpha_\lambda^2\alpha_{\lambda'}^2}{m_c m_s \alpha_{\lambda\lambda'}^4}\right],
\end{aligned}
$$

where

$$
I_H = \left(\frac{\alpha_\lambda \alpha_{\lambda'}}{\alpha_{\lambda\lambda'}^2}\right)^{3/2} \exp\left(-\frac{3m_\sigma^2}{2m_\Lambda^2}\frac{p_\Lambda^2}{\alpha_{\lambda\lambda'}^2}\right).
$$

2. $1/2_1^+$

$$F_1 = I_H \frac{1}{2\alpha_{\lambda\lambda'}^2}\left[(\alpha_\lambda^2 - \alpha_{\lambda'}^2) - \frac{m_\sigma}{3\alpha_{\lambda\lambda'}^2}\left(\frac{\alpha_{\lambda'}^2}{m_s}(7\alpha_\lambda^2 - 3\alpha_{\lambda'}^2) + \frac{\alpha_\lambda^2}{m_c}(7\alpha_{\lambda'}^2 - 3\alpha_\lambda^2)\right)\right],$$
$$F_2 = -I_H \frac{\alpha_{\lambda'}^2}{6m_s\alpha_{\lambda\lambda'}^4}(7\alpha_\lambda^2 - 3\alpha_{\lambda'}^2)\left[m_\sigma - \frac{\alpha_\lambda^2}{4m_c}\right],$$
$$F_3 = I_H \frac{\alpha_\lambda^2 m_\sigma}{6m_c\alpha_{\lambda\lambda'}^4}(7\alpha_{\lambda'}^2 - 3\alpha_\lambda^2),$$
$$G_1 = I_H\left[\frac{(\alpha_\lambda^2 - \alpha_{\lambda'}^2)}{2\alpha_{\lambda\lambda'}^2} - \frac{\alpha_\lambda^2\alpha_{\lambda'}^2}{72\alpha_{\lambda\lambda'}^4 m_c m_s}(7\alpha_\lambda^2 - 3\alpha_{\lambda'}^2)\right],$$
$$G_2 = -I_H \frac{\alpha_{\lambda'}^2}{6m_s\alpha_{\lambda\lambda'}^4}\left[(7\alpha_\lambda^2 - 3\alpha_{\lambda'}^2)\left(m_\sigma + \frac{\alpha_\lambda^2}{6m_c}\right) + \frac{7m_\sigma^2\alpha_\lambda^2}{m_c\alpha_{\lambda\lambda'}^2}(\alpha_\lambda^2 - \alpha_{\lambda'}^2)\right],$$
$$G_3 = -I_H \frac{\alpha_\lambda^2 m_\sigma}{6m_c\alpha_{\lambda\lambda'}^4}\left[(7\alpha_{\lambda'}^2 - 3\alpha_\lambda^2) + \frac{7m_\sigma\alpha_{\lambda'}^2}{m_c\alpha_{\lambda\lambda'}^2}(\alpha_\lambda^2 - \alpha_{\lambda'}^2)\right],$$

where

$$I_H = \sqrt{\frac{3}{2}}\left(\frac{\alpha_\lambda\alpha_{\lambda'}}{\alpha_{\lambda\lambda'}^2}\right)^{3/2}\exp\left(-\frac{3m_\sigma^2}{2m_\Lambda^2}\frac{p_\Lambda^2}{\alpha_{\lambda\lambda'}^2}\right).$$

3. $1/2^-$

$$F_1 = I_H \frac{\alpha_\lambda}{6}\left[\frac{3}{m_s} - \frac{1}{m_c}\right],$$
$$F_2 = -I_H\left[\frac{2m_\sigma}{\alpha_\lambda} - \frac{\alpha_\lambda}{2m_s} + \frac{2m_\sigma^2\alpha_\lambda}{m_c\alpha_{\lambda\lambda'}^2} - \frac{m_\sigma\alpha_\lambda}{6m_c m_s\alpha_{\lambda\lambda'}^2}(3\alpha_\lambda^2 - 2\alpha_{\lambda'}^2)\right],$$
$$F_3 = I_H \frac{2m_\sigma^2\alpha_\lambda}{m_c\alpha_{\lambda\lambda'}^2},$$
$$G_1 = I_H\left[\frac{2m_\sigma}{\alpha_\lambda} - \frac{\alpha_\lambda}{6m_c} + \frac{m_\sigma\alpha_\lambda}{6m_c m_s\alpha_{\lambda\lambda'}^2}(3\alpha_\lambda^2 - 2\alpha_{\lambda'}^2)\right],$$
$$G_2 = I_H\left[-\frac{2m_\sigma}{\alpha_\lambda} + \frac{\alpha_\lambda}{2m_s} + \frac{\alpha_\lambda}{3m_c}\right],$$
$$G_3 = I_H \frac{\alpha_\lambda}{3m_c}\left[1 - \frac{m_\sigma}{2m_s\alpha_{\lambda\lambda'}^2}(3\alpha_\lambda^2 - 2\alpha_{\lambda'}^2)\right],$$

where

$$I_H = \left(\frac{\alpha_\lambda\alpha_{\lambda'}}{\alpha_{\lambda\lambda'}^2}\right)^{5/2}\exp\left(-\frac{3m_\sigma^2}{2m_\Lambda^2}\frac{p_\Lambda^2}{\alpha_{\lambda\lambda'}^2}\right).$$

4. $3/2^-$

$$F_1 = I_H \frac{3m_\sigma}{\alpha_\lambda}\left[1 + \frac{m_\sigma}{\alpha_{\lambda\lambda'}^2}\left(\frac{\alpha_{\lambda'}^2}{m_s} + \frac{\alpha_\lambda^2}{m_c}\right)\right],$$

$$F_2 = -I_H \left[\frac{3m_\sigma^2}{m_s}\frac{\alpha_\lambda^2}{\alpha_{\lambda\lambda'}^2\alpha_\lambda} - \frac{5\alpha_\lambda\alpha_{\lambda'}^2 m_\sigma}{4\alpha_{\lambda\lambda'}^2 m_c m_s}\right],$$
$$F_3 = -I_H \left[\frac{3m_\sigma^2\alpha_\lambda}{m_c\alpha_{\lambda\lambda'}^2} + \frac{\alpha_\lambda}{2m_c}\right],$$
$$F_4 = I_H \frac{\alpha_\lambda}{m_c},$$
$$G_1 = I_H \left[\frac{3m_\sigma}{\alpha_\lambda} - \frac{\alpha_\lambda}{2m_c}\left(1 + \frac{3m_\sigma\alpha_{\lambda'}^2}{2m_s\alpha_{\lambda\lambda'}^2}\right)\right],$$
$$G_2 = -I_H \left[\frac{3m_\sigma^2}{m_s}\frac{\alpha_{\lambda'}^2}{\alpha_\lambda\alpha_{\lambda\lambda'}^2} + \frac{m_\sigma\alpha_\lambda\alpha_{\lambda'}^2}{4m_c m_s\alpha_{\lambda\lambda'}^4}(\alpha_{\lambda\lambda'}^2 + 12m_\sigma^2)\right],$$
$$G_3 = I_H \frac{\alpha_\lambda}{m_c\alpha_{\lambda\lambda'}}\left[\frac{\alpha_{\lambda\lambda'}^2}{2} + 3m_\sigma^2 + \frac{\alpha_{\lambda'}^2 m_\sigma}{m_s\alpha_{\lambda\lambda'}^2}(\alpha_{\lambda\lambda'}^2 + 6m_\sigma^2)\right],$$
$$G_4 = -I_H \left[\frac{\alpha_\lambda}{m_c} + \frac{m_\sigma}{m_c m_s}\frac{\alpha_\lambda\alpha_{\lambda'}^2}{\alpha_{\lambda\lambda'}^2}\right],$$

where

$$I_H = -\frac{1}{\sqrt{3}}\left(\frac{\alpha_\lambda\alpha_{\lambda'}}{\alpha_{\lambda\lambda'}^2}\right)^{5/2}\exp\left(-\frac{3m_\sigma^2}{2m_\Lambda^2}\frac{p_\Lambda^2}{\alpha_{\lambda\lambda'}^2}\right).$$

5. $3/2^+$

$$F_1 = -I_H \frac{m_\sigma}{2}\left[\frac{5}{m_s} - \frac{3}{m_c}\right],$$
$$F_2 = I_H \frac{m_\sigma}{\alpha_\lambda}\left[\frac{6m_\sigma}{\alpha_\lambda} - \frac{5\alpha_\lambda}{2m_s} + \frac{6m_\sigma^2\alpha_\lambda}{m_c\alpha_{\lambda\lambda'}^2} - \frac{m_\sigma\alpha_\lambda}{2m_c m_s\alpha_{\lambda\lambda'}^2}(\alpha_\lambda^2 - 2\alpha_{\lambda'}^2)\right],$$
$$F_3 = I_H \frac{2m_\sigma^2\alpha_\lambda}{m_c\alpha_{\lambda\lambda'}^2},$$
$$F_3 = -I_H \frac{m_\sigma}{m_c}\left[1 + \frac{6m_\sigma^2}{\alpha_{\lambda\lambda'}^2}\right],$$
$$F_4 = I_H \frac{2m_\sigma}{m_c},$$
$$G_1 = -I_H \left[\frac{6m_\sigma^2}{\alpha_\lambda^2} - \frac{m_\sigma}{2m_c} + \frac{m_\sigma^2}{6\alpha_{\lambda\lambda'}^2 m_c m_s}\left(11\alpha_\lambda^2 - 6\alpha_{\lambda'}^2\right)\right],$$
$$G_2 = I_H \left[\frac{6m_\sigma^2}{\alpha_\lambda^2} - \frac{5m_\sigma}{2m_s} - \frac{2m_\sigma}{m_c} + \frac{5\alpha_\lambda^2}{12m_c m_s} - \frac{2m_\sigma^2\alpha_\lambda^2}{3\alpha_{\lambda\lambda'}^2 m_c m_s}\right],$$
$$G_3 = -I_H \left[\frac{m_\sigma}{2m_c} - \frac{5\alpha_\lambda^2}{24m_c m_s} - \frac{m_\sigma^2}{4m_c m_s\alpha_{\lambda\lambda'}^2}(5\alpha_\lambda^2 - 2\alpha_{\lambda'}^2)\right],$$
$$G_4 = -I_H \frac{5\alpha_\lambda^2}{6m_c m_s},$$

where

$$I_H = \frac{1}{\sqrt{5}}\left(\frac{\alpha_\lambda\alpha_{\lambda'}}{\alpha_{\lambda\lambda'}^2}\right)^{7/2}\exp\left(-\frac{3m_\sigma^2}{2m_\Lambda^2}\frac{p_\Lambda^2}{\alpha_{\lambda\lambda'}^2}\right).$$

6. $5/2^+$

$$F_1 = I_H \frac{3m_\sigma^2}{\alpha_\lambda^2}\left[1+\frac{m_\sigma}{\alpha_{\lambda\lambda'}^2}\left(\frac{\alpha_{\lambda'}^2}{m_s}+\frac{\alpha_\lambda^2}{m_c}\right)\right],$$
$$F_2 = -I_H \frac{m_\sigma^2}{m_s\alpha_{\lambda\lambda'}^2}\left[\frac{3m_\sigma\alpha_{\lambda'}^2}{\alpha_\lambda^2}-\frac{1}{4m_c}(8\alpha_\lambda^2+7\alpha_{\lambda'}^2)\right],$$
$$F_3 = -I_H \frac{m_\sigma}{m_c}\left[1+\frac{3m_\sigma^2}{\alpha_{\lambda\lambda'}^2}\right],$$
$$F_4 = I_H \frac{2m_\sigma}{m_c},$$
$$G_1 = I_H \left[\frac{3m_\sigma^2}{\alpha_\lambda^2}-\frac{m_\sigma}{m_c}-\frac{m_\sigma^2}{12m_sm_c\alpha_{\lambda\lambda'}^2}(8\alpha_\lambda^2+15\alpha_{\lambda'}^2)\right],$$
$$G_2 = -I_H \frac{m_\sigma^2}{m_s\alpha_{\lambda\lambda'}^2}\left[\frac{3m_\sigma\alpha_{\lambda'}^2}{\alpha_\lambda^2}+\frac{1}{12m_c}(8\alpha_\lambda^2+3\alpha_{\lambda'}^2)+\frac{3m_\sigma^2\alpha_{\lambda'}^2}{m_c\alpha_{\lambda\lambda'}^2}\right],$$
$$G_3 = I_H \frac{m_\sigma}{m_c}\left[1+\frac{3m_\sigma^2}{\alpha_{\lambda\lambda'}^2}+\frac{m_\sigma\alpha_{\lambda'}^2}{m_s\alpha_{\lambda\lambda'}^2}(1+\frac{6m_\sigma^2}{\alpha_{\lambda\lambda'}^2})\right],$$
$$G_4 = -I_H \frac{2m_\sigma}{m_c}\left[1+\frac{m_\sigma}{m_s}\frac{\alpha_{\lambda'}^2}{\alpha_{\lambda\lambda'}^2}\right],$$

where

$$I_H = \frac{1}{\sqrt{2}}\left(\frac{\alpha_\lambda\alpha_{\lambda'}}{\alpha_{\lambda\lambda'}^2}\right)^{7/2}\exp\left(-\frac{3m_\sigma^2}{2m_\Lambda^2}\frac{p_\Lambda^2}{\alpha_{\lambda\lambda'}^2}\right).$$

Appendix D: Wave Functions

The baryon wave functions are expanded in the harmonic oscillator basis. For Λ states with spin-parity $J^P=\frac{1}{2}^+$, the wave function expansion is

$$\begin{aligned}\Psi_{\Lambda_Q,\frac{1}{2}^+M} =&\phi_{\Lambda_Q}\Bigg(\left[\eta_1\psi_{000000}(\vec{p}_\rho,\vec{p}_\lambda)+\eta_2\psi_{001000}(\vec{p}_\rho,\vec{p}_\lambda)+\eta_3\psi_{000010}(\vec{p}_\rho,\vec{p}_\lambda)\right]\chi^\rho_{\frac{1}{2}}(M)\\ &+\eta_4\psi_{000101}(\vec{p}_\rho,\vec{p}_\lambda)\chi^\lambda_{\frac{1}{2}}(M)+\eta_5\left[\psi_{1M_L0101}(\vec{p}_\rho,\vec{p}_\lambda)\chi^\lambda_{\frac{1}{2}}(M-M_L)\right]_{1/2,M}\\ &+\eta_6\left[\psi_{1M_L0101}(\vec{p}_\rho,\vec{p}_\lambda)\chi^S_{\frac{3}{2}}(M-M_L)\right]_{1/2,M}+\eta_7\left[\psi_{2M_L0101}(\vec{p}_\rho,\vec{p}_\lambda)\chi^S_{\frac{3}{2}}(M-M_L)\right]_{1/2,M}\Bigg).\end{aligned} \tag{D1}$$

where the η_i's are the expansion coefficients and $[\psi_{LM_Ln_\rho m_\rho n_\lambda m_\lambda}\chi_S(M-M_L)]_{J,M}$ is a short-hand notation for the Clebsch-Gordan sum $\sum_{M_L} C^{JM}_{LM_L,SM-M_L}$.

For $J^P=\frac{1}{2}^-$ and $J^P=\frac{3}{2}^-$, the wave function expansion is

$$\begin{aligned}\Psi_{\Lambda_Q,J^-M} =&\phi_{\Lambda_Q}\Bigg(\eta_1\left[\psi_{1M_L0001}(\vec{p}_\rho,\vec{p}_\lambda)\chi^\rho_{\frac{1}{2}}(M-M_L)\right]_{J,M}+\eta_2\left[\psi_{1M_L0100}(\vec{p}_\rho,\vec{p}_\lambda)\chi^\lambda_{\frac{1}{2}}(M-M_L)\right]_{J,M}\\ &+\eta_3\left[\psi_{1M_L0100}(\vec{p}_\rho,\vec{p}_\lambda)\chi^S_{\frac{3}{2}}(M-M_L)\right]_{J,M}\Bigg).\end{aligned} \tag{D2}$$

For $J^P=\frac{3}{2}^+$, the wave function is

$$\Psi_{\Lambda_Q,\frac{3}{2}^+M} =\phi_{\Lambda_Q}\Bigg(\eta_1\left[\psi_{2M_L0002}(\vec{p}_\rho,\vec{p}_\lambda)\chi^\rho_{\frac{1}{2}}(M-M_L)\right]_{3/2,M}+\eta_2\left[\psi_{2M_L0200}(\vec{p}_\rho,\vec{p}_\lambda)\chi^\rho_{\frac{1}{2}}(M-M_L)\right]_{3/2,M}$$

$$+ \eta_3 \left[\psi_{1M_L0101}(\vec{p}_\rho, \vec{p}_\lambda) \chi^\lambda_{\frac{1}{2}}(M - M_L) \right]_{3/2,M} + \eta_4 \left[\psi_{2M_L0101}(\vec{p}_\rho, \vec{p}_\lambda) \chi^\lambda_{\frac{1}{2}}(M - M_L) \right]_{3/2,M}$$
$$+ \eta_5 \psi_{000101}(\vec{p}_\rho, \vec{p}_\lambda) \chi^S_{\frac{3}{2}}(M) + \eta_6 \left[\psi_{1M_L0101}(\vec{p}_\rho, \vec{p}_\lambda) \chi^S_{\frac{3}{2}}(M - M_L) \right]_{3/2,M}$$
$$+ \eta_7 \left[\psi_{2M_L0101}(\vec{p}_\rho, \vec{p}_\lambda) \chi^S_{\frac{3}{2}}(M - M_L) \right]_{3/2,M} \Bigg). \quad \text{(D3)}$$

For $J^P = \frac{5}{2}^+$, the wave function is

$$\Psi_{\Lambda_Q, \frac{5}{2}^+ M} = \phi_{\Lambda_Q} \Bigg(\eta_1 \left[\psi_{2M_L0200}(\vec{p}_\rho, \vec{p}_\lambda) \chi^\rho_{\frac{1}{2}}(M - M_L) \right]_{5/2,M} + \eta_2 \left[\psi_{2M_L0002}(\vec{p}_\rho, \vec{p}_\lambda) \chi^\rho_{\frac{1}{2}}(M - M_L) \right]_{5/2,M}$$
$$+ \eta_3 \left[\psi_{2M_L0101}(\vec{p}_\rho, \vec{p}_\lambda) \chi^\lambda_{\frac{1}{2}}(M - M_L) \right]_{5/2,M} + \eta_4 \psi_{1M_L0101}(\vec{p}_\rho, \vec{p}_\lambda) \chi^S_{\frac{3}{2}}(M)$$
$$+ \eta_5 \left[\psi_{2M_L0101}(\vec{p}_\rho, \vec{p}_\lambda) \chi^S_{\frac{3}{2}}(M - M_L) \right]_{5/2,M} \Bigg). \quad \text{(D4)}$$

No other states are expected to have significant overlap with the decaying ground state Λ_c in the spectator approximation.

Appendix E: Hadron Tensors

1. Hadron Tensor in $\Lambda_c^+ \to \Lambda^* l^+ \nu_l$ transitions

a. $1/2^+$

$$\alpha = 2 \left[Y F_1^2 + X G_1^2 \right]. \quad \text{(E1)}$$

$$\beta_{++} = \sum_{i=1,j=1}^{i=3,j=3} (A_{ij} F_i F_j + A'_{ij} G_i G_j), \quad \text{(E2)}$$

where

$$A_{11} = 2,, \qquad A'_{11} = 2,$$
$$A_{22} = \frac{1}{2m^2_{\Lambda_c}} X, \qquad A'_{22} = \frac{1}{2m^2_{\Lambda_c}} Y,$$
$$A_{33} = \frac{1}{2m^2_{\Lambda}} X, \qquad A'_{33} = \frac{1}{2m^2_{\Lambda}} Y,$$
$$A_{12} = \frac{2}{m_{\Lambda_c}} (m_{\Lambda_c} + m_\Lambda), \qquad A'_{12} = \frac{-2}{m_{\Lambda_c}} (m_{\Lambda_c} - m_\Lambda),$$
$$A_{23} = \frac{1}{m_\Lambda m_{\Lambda_c}} X, \qquad A'_{23} = \frac{1}{m_\Lambda m_{\Lambda_c}} Y,$$
$$A_{31} = \frac{2}{m_\Lambda} (m_{\Lambda_c} + m_\Lambda), \qquad A'_{31} = \frac{-2}{m_\Lambda} (m_{\Lambda_c} - m_\Lambda).$$

$$\gamma(1/2^+) = 4 F_1 G_1$$

where $X \equiv [(m_{\Lambda_c} + m_\Lambda)^2 - q^2]$ and $Y \equiv [(m_{\Lambda_c} - m_\Lambda)^2 - q^2]$.

b. $1/2^-$

$$\alpha = 2\left[XF_1^2 + YG_1^2\right], \tag{E3}$$

$$\beta_{++} = \sum_{i=1,j=1}^{i=3,j=3} (A_{ij}F_iF_j + A'_{ij}G_iG_j), \tag{E4}$$

where

$$\begin{aligned}
A_{11} &= 2, & A'_{11} &= 2,\\
A_{22} &= \frac{1}{2m_{\Lambda_c}^2}Y, & A'_{22} &= \frac{1}{2m_{\Lambda_c}^2}X,\\
A_{33} &= \frac{1}{2m_{\Lambda}^2}Y, & A'_{33} &= \frac{1}{2m_{\Lambda}^2}X,\\
A_{12} &= \frac{-2}{m_{\Lambda_c}}(m_{\Lambda_c} - m_\Lambda), & A'_{12} &= \frac{2}{m_{\Lambda_c}}(m_{\Lambda_c} + m_\Lambda),\\
A_{23} &= \frac{1}{m_\Lambda m_{\Lambda_c}}Y, & A'_{23} &= \frac{1}{m_\Lambda m_{\Lambda_c}}X,\\
A_{31} &= \frac{-2}{m_\Lambda}(m_{\Lambda_c} - m_\Lambda), & A'_{31} &= \frac{2}{m_\Lambda}(m_{\Lambda_c} + m_\Lambda),
\end{aligned}$$

$$\gamma(1/2^+) = 4F_1G_1. \tag{E5}$$

c. $3/2^-$

$$\alpha = \sum_{i=1,j=1}^{i=4,j=4} \frac{1}{Z}(B_{ij}F_iF_j + B'_{ij}G_iG_j), \tag{E6}$$

where $Z = 3m_{\Lambda_c}^2 m_\Lambda^2$ and the non vanishing coefficients are

$$\begin{aligned}
&B_{11} = XY^2, \quad B_{14} = B'_{14} = -2m_{\Lambda_c}m_\Lambda XY,\\
&B_{44} = 4m_{\Lambda_c}^2 m_\Lambda^2 X, \quad B'_{11} = X^2Y, \quad B'_{44} = 4m_{\Lambda_c}^2 m_\Lambda^2 Y.
\end{aligned}$$

$$\beta_{++} = \sum_{i=1,j=1}^{i=4,j=4} \frac{1}{Z}(A_{ij}F_iF_j + A'_{ij}G_iG_j), \tag{E7}$$

$$
\begin{aligned}
A_{11} =& XY, \\
A_{12} =& \frac{1}{m_{\Lambda_c}}(m_{\Lambda_c}+m_\Lambda)XY, \\
A_{13} =& \frac{1}{m_\Lambda}(m_{\Lambda_c}+m_\Lambda)XY, \\
A_{14} =& 2m_{\Lambda_c}\big[m_{\Lambda_c}(m_{\Lambda_c}^2-m_\Lambda^2-q^2) \\
& + m_\Lambda(m_{\Lambda_c}^2-m_\Lambda^2)\big], \\
A_{22} =& \frac{1}{4m_{\Lambda_c}^2}X^2Y, \\
A_{23} =& \frac{1}{2m_{\Lambda_c}m_\Lambda}X^2Y, \\
A_{24} =& X(m_{\Lambda_c}^2-m_\Lambda^2-q^2), \\
A_{33} =& \frac{1}{4m_\Lambda^2}X^2Y, \\
A_{34} =& \frac{m_{\Lambda_c}}{m_\Lambda}X(m_{\Lambda_c}^2-m_\Lambda^2-q^2), \\
A_{44} =& m_{\Lambda_c}^2X,
\end{aligned}
\qquad
\begin{aligned}
A'_{11} =& XY, \\
A'_{12} =& \frac{-1}{m_{\Lambda_c}}(m_{\Lambda_c}-m_\Lambda)XY, \\
A'_{13} =& \frac{-1}{m_\Lambda}(m_{\Lambda_c}-m_\Lambda)XY, \\
A'_{14} =& -2m_{\Lambda_c}\big[m_{\Lambda_c}(m_{\Lambda_c}^2-m_\Lambda^2-q^2) \\
& - m_\Lambda(m_{\Lambda_c}^2-m_\Lambda^2)\big], \\
A'_{22} =& \frac{1}{4m_{\Lambda_c}^2}XY^2, \\
A'_{23} =& \frac{1}{2m_{\Lambda_c}m_\Lambda}XY^2, \\
A'_{24} =& Y(m_{\Lambda_c}^2-m_\Lambda^2-q^2), \\
A'_{33} =& \frac{1}{4m_\Lambda^2}XY^2, \\
A'_{34} =& \frac{m_{\Lambda_c}}{m_\Lambda}Y(m_{\Lambda_c}^2-m_\Lambda^2-q^2), \\
A'_{44} =& m_{\Lambda_c}^2Y,
\end{aligned}
$$

$$\gamma = \sum_{i=1,j=1}^{i=4,j=4}\frac{1}{Z}C_{ij}F_iG_j,$$

$$C_{11} = 2XY, \quad C_{14} = -2m_{\Lambda_c}m_\Lambda Y, \quad C_{41} = -2m_{\Lambda_c}m_\Lambda X, \quad C_{44} = -4m_{\Lambda_c}^2m_\Lambda^2.$$

d. $3/2^+$

$$\alpha = \sum_{i=1,j=1}^{i=4,j=4}\frac{1}{Z}(B_{ij}F_iF_j + B'_{ij}G_iG_j), \tag{E8}$$

where $Z = 3m_{\Lambda_c}^2m_\Lambda^2$ and the non vanishing coefficients are

$$
\begin{aligned}
&B_{11} = X^2Y, \quad B_{14} = B'_{14} = -2m_{\Lambda_c}m_\Lambda XY, \\
&B_{44} = 4m_{\Lambda_c}^2m_\Lambda^2Y, \quad B'_{11} = XY^2, \quad B'_{44} = 4m_{\Lambda_c}^2m_\Lambda^2X,
\end{aligned}
$$

$$\beta_{++} = \sum_{i=1,j=1}^{i=4,j=4}\frac{1}{Z}(A_{ij}F_iF_j + A'_{ij}G_iG_j), \tag{E9}$$

$$\begin{aligned}
A_{11} &= XY,\\
A_{12} &= \frac{-1}{m_{\Lambda_c}}(m_{\Lambda_c}-m_\Lambda)XY,\\
A_{13} &= -\frac{1}{m_\Lambda}(m_{\Lambda_c}+m_\Lambda)XY,\\
A_{14} &= -2m_{\Lambda_c}\big[m_{\Lambda_c}(m_{\Lambda_c}^2-m_\Lambda^2-q^2)\\
&\qquad -m_\Lambda(m_{\Lambda_c}^2-m_\Lambda^2)\big],\\
A_{22} &= \frac{1}{4m_{\Lambda_c}^2}XY^2,\\
A_{23} &= \frac{1}{2m_{\Lambda_c}m_\Lambda}XY^2,\\
A_{24} &= Y(m_{\Lambda_c}^2-m_\Lambda^2-q^2),\\
A_{33} &= \frac{1}{4m_\Lambda^2}XY^2,\\
A_{34} &= \frac{m_{\Lambda_c}}{m_\Lambda}Y(m_{\Lambda_c}^2-m_\Lambda^2-q^2),\\
A_{44} &= m_{\Lambda_c}^2Y,
\end{aligned}$$

$$\begin{aligned}
A'_{11} &= XY,\\
A'_{12} &= \frac{1}{m_{\Lambda_c}}(m_{\Lambda_c}+m_\Lambda)XY,\\
A'_{13} &= \frac{1}{m_\Lambda}(m_{\Lambda_c}+m_\Lambda)XY,\\
A'_{14} &= 2m_{\Lambda_c}\big[m_{\Lambda_c}(m_{\Lambda_c}^2-m_\Lambda^2-q^2)\\
&\qquad +m_\Lambda(m_{\Lambda_c}^2-m_\Lambda^2)\big],\\
A'_{22} &= \frac{1}{4m_{\Lambda_c}^2}X^2Y,\\
A'_{23} &= \frac{1}{2m_{\Lambda_c}m_\Lambda}X^2Y,\\
A'_{24} &= X(m_{\Lambda_c}^2-m_\Lambda^2-q^2),\\
A'_{33} &= \frac{1}{4m_\Lambda^2}X^2Y,\\
A'_{34} &= \frac{m_{\Lambda_c}}{m_\Lambda}X(m_{\Lambda_c}^2-m_\Lambda^2-q^2),\\
A'_{44} &= m_{\Lambda_c}^2X,
\end{aligned}$$

$$\gamma = \sum_{i=1,j=1}^{i=4,j=4}\frac{1}{Z}C_{ij}F_iG_j,$$

$$\begin{aligned}
C_{11} &= 2XY,\\
C_{14} &= -2m_{\Lambda_c}m_\Lambda X,\\
C_{41} &= -2m_{\Lambda_c}m_\Lambda Y,\\
C_{44} &= -4m_{\Lambda_c}^2m_\Lambda^2.
\end{aligned}$$

e. $5/2^+$

$$\alpha = \sum_{i=1,j=1}^{i=4,j=4}\frac{1}{Z}(B_{ij}F_iF_j + B'_{ij}G_iG_j), \tag{E10}$$

where $Z' = 20m_{\Lambda_c}^4m_\Lambda^4$ and the non vanishing coefficients are

$$\begin{aligned}
B_{11} &= -X^2Y^3,\\
B_{14} &= B'_{14} = 2m_{\lambda_c}m_\Lambda X^2Y^2,\\
B_{44} &= -3m_{\Lambda_c}^2m_\Lambda^2X^2Y,\\
B'_{11} &= -X^3Y^2,\\
B'_{44} &= -3m_{\Lambda_c}^2m_\Lambda^2XY^2,
\end{aligned}$$

$$\beta_{++} = \sum_{i=1,j=1}^{i=4,j=4}\frac{1}{Z}(A_{ij}F_iF_j + A'_{ij}G_iG_j), \tag{E11}$$

$$
\begin{aligned}
A_{11} =& X^2Y^2, \\
A_{12} =& \frac{1}{m_{\Lambda_c}}(m_{\Lambda_c}+m_\Lambda)X^2Y^2, \\
A_{13} =& \frac{1}{m_\Lambda}(m_{\Lambda_c}+m_\Lambda)X^2Y^2, \\
A_{14} =& 2m_{\Lambda_c}XY\big[m_{\Lambda_c}(m_{\Lambda_c}^2-m_\Lambda^2-q^2) \\
& \qquad - m_\Lambda(m_{\Lambda_c}^2-m_\Lambda^2)\big], \\
A_{22} =& \frac{1}{4m_{\Lambda_c}^2}X^3Y^2, \\
A_{23} =& \frac{1}{2m_{\Lambda_c}m_\Lambda}X^3Y^2, \\
A_{24} =& X^2Y(m_{\Lambda_c}^2-m_\Lambda^2-q^2), \\
A_{33} =& \frac{1}{4m_\Lambda^2}X^3Y^2, \\
A_{34} =& \frac{m_{\Lambda_c}}{m_\Lambda}X^2Y(m_{\Lambda_c}^2-m_\Lambda^2-q^2), \\
A_{44} =& m_{\Lambda_c}^2X\lambda(m_{\Lambda_c}^2,m_\Lambda^2,q^2),
\end{aligned}
\qquad
\begin{aligned}
A'_{11} =& X^2Y^2, \\
A'_{12} =& \frac{-1}{m_{\Lambda_c}}(m_{\Lambda_c}-m_\Lambda)X^2Y^2, \\
A'_{13} =& \frac{-1}{m_\Lambda}(m_{\Lambda_c}-m_\Lambda)X^2Y^2, \\
A'_{14} =& -2m_{\Lambda_c}XY\big[m_{\Lambda_c}(m_{\Lambda_c}^2-m_\Lambda^2-q^2) \\
& \qquad - m_\Lambda(m_{\Lambda_c}^2-m_\Lambda^2)\big], \\
A'_{22} =& \frac{1}{4m_{\Lambda_c}^2}X^2Y^3, \\
A'_{23} =& \frac{1}{2m_{\Lambda_c}m_\Lambda}X^2Y^3, \\
A'_{24} =& XY^2(m_{\Lambda_c}^2-m_\Lambda^2-q^2), \\
A'_{33} =& \frac{1}{4m_\Lambda^2}X^2Y^3, \\
A'_{34} =& \frac{m_{\Lambda_c}}{m_\Lambda}XY^2(m_{\Lambda_c}^2-m_\Lambda^2-q^2), \\
A'_{44} =& m_{\Lambda_c}^2Y\lambda(m_{\Lambda_c}^2,m_\Lambda^2,q^2),
\end{aligned}
$$

where $\lambda(m_{\Lambda_c}^2,m_\Lambda^2,q^2)=(m_{\Lambda_c}^4+m_\Lambda^4+q^4-2m_{\Lambda_c}^2m_\Lambda^2-2m_{\Lambda_c}^2q^2-2m_\Lambda^2q^2)$.

$$
\gamma=\sum_{i=1,j=1}^{i=4,j=4}\frac{1}{Z}C_{ij}F_iG_j,
$$

$$
\begin{aligned}
C_{11} &= 2X^2Y^2, \\
C_{14} &= -2m_{\Lambda_c}m_\Lambda XY^2, \\
C_{41} &= -2m_{\Lambda_c}m_\Lambda X^2Y, \\
C_{44} &= -2m_{\Lambda_c}^2m_\Lambda^2XY.
\end{aligned}
$$

2. Hadron tensor in $\Lambda_c^+\to\Lambda^*l^+\nu_l\to\Sigma\pi l^+\nu_l/NKl^+\nu_l$ transitions

The most general form of the contribution of the ith state to the matrix element for the four-body decay $\Lambda_c^+\to\Lambda_il^+\nu_l\to BMl^+\nu_l$ can be written

$$
M_\nu^i=\bar{u}(p_B)\left(\sum_{j=1}^{16}c_j^i\mathcal{O}_j\right)u(P+L),
$$

where the Lorentz-Dirac operators $\mathcal{O}_i$ are

$$
\begin{aligned}
&\mathcal{O}_1=\gamma_\nu,\;\; \mathcal{O}_2=\not{P}\gamma_\nu,\;\; \mathcal{O}_3=P_\nu,\;\; \mathcal{O}_4=\not{P}P_\nu,\;\; \mathcal{O}_5=L_\nu,\;\; \mathcal{O}_6=\not{P}L_\nu,\;\; \mathcal{O}_7=Q_\nu,\;\; \mathcal{O}_8=\not{P}Q_\nu,\\
&\mathcal{O}_9=\gamma_\nu\gamma_5,\;\; \mathcal{O}_{10}=\not{P}\gamma_\nu\gamma_5,\;\; \mathcal{O}_{11}=P_\nu\gamma_5,\;\; \mathcal{O}_{12}=\not{P}P_\nu\gamma_5,\;\; \mathcal{O}_{13}=L_\nu\gamma_5,\;\; \mathcal{O}_{14}=\not{P}L_\nu\gamma_5,\;\; \mathcal{O}_{15}=Q_\nu\gamma_5,\;\; \mathcal{O}_{16}=\not{P}Q_\nu\gamma_5.
\end{aligned}
$$

Thus, there are sixteen independent Lorentz-Dirac structures in the amplitude. The c_j^i can be written

$$
c_j^i=g_{\Lambda_iBM}\sum_k(C_{jk}^{i,F}F_k+C_{jk}^{i,G}G_k). \tag{E12}
$$

where k runs from 1 to 3 for spin $\frac{1}{2}$ states and from 1 to 4 for states with higher spin.

The hadron tensor arising from a single intermediate Λ_i can be written

$$
H_{\mu\nu}^i=\sum_{\text{spins}}M_\mu^{i\dagger}M_\nu^i=\alpha^ig_{\mu\nu}+\beta_{PP}^iP_\mu P_\nu+\beta_{PQ}^iP_\mu Q_\nu+\beta_{QP}^iQ_\mu P_\nu+\beta_{QQ}^iQ_\mu Q_\nu
$$

$$+\beta^i_{QL}Q_\mu L_\nu+\beta^i_{LQ}L_\mu Q_\nu+\beta^i_{LL}L_\mu L_\nu+\beta^i_{PL}P_\mu L_\nu+\beta^i_{PL}P_\mu L_\nu$$
$$+i\gamma^i_a\epsilon^{\mu\nu\rho\delta}P_\rho Q_\delta+i\gamma^i_b\epsilon^{\mu\nu\rho\delta}L_\rho P_\delta+i\gamma^i_c\epsilon^{\mu\nu\rho\delta}L_\rho Q_\delta+i\gamma^i_d\epsilon^{\sigma\mu\rho\delta}L_\sigma P_\rho Q_\delta P_\nu+i\gamma^i_e\epsilon^{\sigma\mu\rho\delta}L_\sigma P_\rho Q_\delta Q_\nu$$
$$+i\gamma^i_f\epsilon^{\sigma\mu\rho\delta}L_\sigma P_\rho Q_\delta L_\nu+i\gamma^i_g\epsilon^{\sigma\nu\rho\delta}L_\sigma P_\rho Q_\delta P_\mu+i\gamma^i_h\epsilon^{\sigma\nu\rho\delta}L_\sigma P_\rho Q_\delta Q_\mu+i\gamma^i_k\epsilon^{\sigma\nu\rho\delta}L_\sigma P_\rho Q_\delta L_\mu. \tag{E13}$$

The terms in γ^i do not contribute to the decay width. Because they are proportional to at least one power of the lepton mass, contributions from β^i_{PL}, β^i_{QL}, β^i_{LL}, β^i_{LP}, β^i_{LQ} are small. The α^i from each intermediate state considered takes the form

$$\alpha^i=\sum_{j,k=1}^{16}a^i_{jk}c^{\dagger i}_j c^i_k. \tag{E14}$$

Similarly, for the $\beta^i_{P_1P_2}$ (P_1 and P_2 denotes P, Q or L),

$$\beta^i_{P_1P_2}=\sum_{j,k=1}^{16}b^i_{jk}c^{\dagger i}_j c^i_k. \tag{E15}$$

When we treat the coherent sum of the contributions from all the states we consider, we write

$$M_\nu=\bar{u}(p_B)\left(\sum_{j=1}^{16}\mathcal{C}_j\mathcal{O}_j\right)u(p_{\Lambda_c})=\bar{u}(p_B)\sum_{i=1}^{6}\left(\sum_{j=1}^{16}c^i_j\mathcal{O}_j\right)u(p_{\Lambda_c}), \tag{E16}$$

which ultimately leads to

$$\mathcal{C}_j=\sum_{i=1}^{6}c^i_j. \tag{E17}$$

In this case, the hadron tensor takes the same form as in eq. E13 with the superscripts i removed. The coefficients contributing to the differential decay widths we consider are then

$$\alpha=\sum_{j,k=1}^{16}a_{jk}\mathcal{C}^\dagger_j\mathcal{C}_k,$$
$$\beta_{P_1P_2}=\sum_{j,k=1}^{16}b_{jk}\mathcal{C}^\dagger_j\mathcal{C}_k. \tag{E18}$$

For each intermediate Λ_i we consider, the c^i_j can be written

$$c^i_j=g_{\Lambda_i BM}\sum_k(C^{i,F}_{jk}F_k+C^{i,G}_{jk}G_k). \tag{E19}$$

where k runs from 1 to 3 for spin $\frac{1}{2}$ states and from 1 to 4 for states with higher spin. Here, $g_{\Lambda\Sigma\pi}$ is the strong coupling constant for the decay $\Lambda^i\to BM$.

For future convenience, we define

$$B=\frac{1}{6m^2_\Lambda m_{\Lambda_c}},\qquad D=\frac{1}{20m^6_\Lambda m^2_{\Lambda_c}},\qquad C_1=(P\cdot Q),\qquad C_2=(P\cdot L),\qquad C_3=(Q\cdot L).$$

The nonzero coefficients a_{jk}s and b_{jk}s for α and βs are listed in the next few subsections.

a. α

$$a_{1,1}=2(2m_{\Lambda_c}m_\Sigma-C_2-C_1-C_3-S_{\Sigma\pi}),\qquad a_{9,9}=2(2m_{\Lambda_c}m_\Sigma+C_2+C_1+C_3+S_{\Sigma\pi}),$$
$$a_{1,2}=2(m_{\Lambda_c}(C_1+S_{\Sigma\pi})-2m_\Sigma C_2-2m_\Sigma S_{\Sigma\pi}),\qquad a_{9,10}=2(m_{\Lambda_c}C_1+m_{\Lambda_c}S_{\Sigma\pi}+2m_\Sigma C_2+2m_\Sigma S_{\Sigma\pi}),$$

$$a_{2,2}=2\big((2m_{\Lambda_c}m_\Sigma+C_2-C_1+C_3-S_{\Sigma\pi})S_{\Sigma\pi}-2C_1C_2\big),$$
$$a_{10,10}=-2\big((2m_{\Lambda_c}m_\Sigma+C_2+C_1-C_3+S_{\Sigma\pi})S_{\Sigma\pi}+2C_1C_2\big).$$

b. β_{PP}

$$b_{1,1}=b_{9,9}=4,\quad b_{1,2}=b_{2,1}=-b_{9,10}=-b_{10,9}=16m_\Sigma,\quad b_{2,2}=b_{10,10}=4(2C_1+S_{\Sigma\pi}),$$

$$\begin{aligned}
b_{3,3}&=2(2m_{\Lambda_c}m_\Sigma+C_2+C_1+C_3+S_{\Sigma\pi}),\\
b_{3,4}=b_{4,3}&=2(4m_\Sigma C_2+2m_{\Lambda_c}C_1+2m_{\Lambda_c}S_{\Sigma\pi}+4m_\Sigma S_{\Sigma\pi}),\\
b_{4,4}&=2\big(2C_2C_1+(2m_{\Lambda_c}m_\Sigma+C_2+C_1-C_3+S_{\Sigma\pi})S_{\Sigma\pi}\big),\\
b_{11,11}&=2(-2m_{\Lambda_c}m_\Sigma+C_2+C_1+C_3+S_{\Sigma\pi}),\\
b_{11,12}=b_{12,11}&=2(-4m_\Sigma C_2+2m_{\Lambda_c}C_1+2m_{\Lambda_c}S_{\Sigma\pi}-4m_\Sigma S_{\Sigma\pi}),\\
b_{12,12}&=2\big(2C_2C_1+(-2m_{\Lambda_c}m_\Sigma+C_2+C_1-C_3+S_{\Sigma\pi})S_{\Sigma\pi}\big),
\end{aligned}$$

$$\begin{aligned}
b_{1,3}&=2(m_{\Lambda_c}+2m_\Sigma), & b_{9,11}&=2(m_{\Lambda_c}-2m_\Sigma),\\
b_{1,4}&=2(2m_{\Lambda_c}m_\Sigma-C_3+S_{\Sigma\pi}), & b_{9,12}&=2(-2m_{\Lambda_c}m_\Sigma-C_3+S_{\Sigma\pi}),\\
b_{2,3}&=2(2m_{\Lambda_c}m_\Sigma+2C_1+C_3+S_{\Sigma\pi}), & b_{10,11}&=2(-2m_{\Lambda_c}m_\Sigma+2C_1+C_3+S_{\Sigma\pi}),\\
b_{2,4}&=2(2m_{\Lambda_c}C_1+m_{\Lambda_c}S_{\Sigma\pi}+2m_\Sigma S_{\Sigma\pi}), & b_{10,12}&=2(2m_{\Lambda_c}C_1+m_{\Lambda_c}S_{\Sigma\pi}-2m_\Sigma S_{\Sigma\pi}),
\end{aligned}$$

c. β_{PQ}, β_{QP}

$$\begin{aligned}
&b_{1,1}=b_{9,9}=2,\quad b_{2,2}=b_{10,10}=-2S_{\Sigma\pi},\quad b_{1,3}=b_{9,11}=2m_{\Lambda_c},\\
&b_{1,4}=-b_{2,1}=b_{9,12}=-b_{10,11}=2(C_2+S_{\Sigma\pi}),\quad b_{2,4}=b_{10,12}=-2m_{\Lambda_c}S_{\Sigma\pi},
\end{aligned}$$

$$\begin{aligned}
b_{1,7}&=2(m_{\Lambda_c}+2m_\Sigma), & b_{9,15}&=2(m_{\Lambda_c}-2m_\Sigma),\\
b_{1,8}&=2(2m_{\Lambda_c}m_\Sigma-C_3+S_{\Sigma\pi}), & b_{9,16}&=2(-2m_{\Lambda_c}m_\Sigma-C_3+S_{\Sigma\pi}),\\
b_{2,7}&=2(2m_{\Lambda_c}m_\Sigma+2C_1+C_3+S_{\Sigma\pi}), & b_{10,15}&=2(-2m_{\Lambda_c}m_\Sigma+2C_1+C_3+S_{\Sigma\pi}),\\
b_{2,8}&=2(2m_{\Lambda_c}C_1+m_{\Lambda_c}S_{\Sigma\pi}+2m_\Sigma S_{\Sigma\pi}), & b_{10,16}&=2(2m_{\Lambda_c}C_1+m_{\Lambda_c}S_{\Sigma\pi}-2m_\Sigma S_{\Sigma\pi}),\\
b_{3,7}&=2(2m_{\Lambda_c}m_\Sigma+C_2+C_1+C_3+S_{\Sigma\pi}), & b_{11,15}&=2(-2m_{\Lambda_c}m_\Sigma+C_2+C_1+C_3+S_{\Sigma\pi}),
\end{aligned}$$

$$\begin{aligned}
b_{3,8}=b_{4,7}&=2(2m_\Sigma C_2+m_{\Lambda_c}C_1+m_{\Lambda_c}S_{\Sigma\pi}+2m_\Sigma S_{\Sigma\pi}),\\
b_{4,8}&=2\big(2C_2C_1+(2m_{\Lambda_c}m_\Sigma+C_2+C_1-C_3+S_{\Sigma\pi})S_{\Sigma\pi}\big),\\
b_{11,16}=b_{12,15}&=2(-2m_\Sigma C_2+m_{\Lambda_c}C_1+m_{\Lambda_c}S_{\Sigma\pi}-2m_\Sigma S_{\Sigma\pi}),\\
b_{12,16}&=2\big(2C_2C_1+(-2m_{\Lambda_c}m_\Sigma+C_2+C_1-C_3+S_{\Sigma\pi})S_{\Sigma\pi}\big).
\end{aligned}$$

d. β_{QQ}

$$b_{1,7}=b_{9,15}=4m_{\Lambda_c},\quad b_{1,8}=-b_{2,7}=b_{9,16}=-b_{10,15}=2(2C_2+2S_{\Sigma\pi}),\quad b_{2,8}=b_{10,16}=-4m_{\Lambda_c}S_{\Sigma\pi},$$

$$\begin{aligned}
b_{7,7}&=2(2m_{\Lambda_c}m_\Sigma+C_2+C_1+C_3+S_{\Sigma\pi}),\\
b_{7,8}=b_{8,7}&=2(4m_\Sigma C_2+2m_{\Lambda_c}C_1+2m_{\Lambda_c}S_{\Sigma\pi}+4m_\Sigma S_{\Sigma\pi}),\\
b_{8,8}&=2\big(2C_2C_1+(2m_{\Lambda_c}m_\Sigma+C_2+C_1-C_3+S_{\Sigma\pi})S_{\Sigma\pi}\big),\\
b_{15,15}&=2(-2m_{\Lambda_c}m_\Sigma+C_2+C_1+C_3+S_{\Sigma\pi}),\\
b_{15,16}=b_{16,15}&=2(-4m_\Sigma C_2+2m_{\Lambda_c}C_1+2m_{\Lambda_c}S_{\Sigma\pi}-4m_\Sigma S_{\Sigma\pi}),\\
b_{16,16}&=2\big(2C_2C_1+(-2m_{\Lambda_c}m_\Sigma+C_2+C_1-C_3+S_{\Sigma\pi})S_{\Sigma\pi}\big),
\end{aligned}$$

e. β_{QL}, β_{LQ}

$$\begin{aligned}
b_{1,1} &= b_{9,9} = 2,\\
b_{2,2} &= b_{10,10} = -2S_{\Sigma\pi},\\
b_{1,5} &= b_{9,13} = 2m_{\Lambda_c},\\
b_{1,6} &= -b_{2,5} = b_{9,14} = -b_{10,13} = 2(C_2 + S_{\Sigma\pi}),\\
b_{2,6} &= b_{10,14} = -2m_{\Lambda_c}S_{\Sigma\pi},\\
b_{1,7} &= -b_{9,15} = 4m_\Sigma,\\
b_{1,8} &= b_{2,7} = b_{9,16} = b_{10,15} = 2(C_1 + S_{\Sigma\pi}),\\
b_{2,8} &= -b_{10,16} = 4m_\Sigma S_{\Sigma\pi},
\end{aligned}$$

$$\begin{aligned}
b_{5,7} &= b_{13,15} = 2(2m_{\Lambda_c}m_\Sigma + C_2 + C_1 + C_3 + S_{\Sigma\pi}),\\
b_{5,8} &= b_{6,7} = 2(2m_\Sigma C_2 + m_{\Lambda_c}C_1 + (m_{\Lambda_c} + 2m_\Sigma)S_{\Sigma\pi}),\\
b_{6,8} &= 2(2C_2C_1 + (2m_{\Lambda_c}m_\Sigma + C_2 + C_1 - C_3)S_{\Sigma\pi} + S_{\Sigma\pi}^2),\\
b_{13,16} &= b_{14,15} = 2(-2m_\Sigma C_2 + m_{\Lambda_c}C_1 + (m_{\Lambda_c} - 2m_\Sigma)S_{\Sigma\pi}),\\
b_{14,16} &= 2(2C_2C_1 + (-2m_{\Lambda_c}m_\Sigma + C_2 + C_1 - C_3)S_{\Sigma\pi} + S_{\Sigma\pi}^2).
\end{aligned}$$

f. β_{LL}

$$b_{1,5} = -b_{9,13} = 8m_\Sigma, \quad b_{1,6} = b_{2,5} = b_{9,14} = b_{10,13} = 2(2C_1 + 2S_{\Sigma\pi}), \quad b_{2,6} = -b_{10,14} = 8m_\Sigma S_{\Sigma\pi}.$$

$$\begin{aligned}
b_{5,5} &= 2(2m_{\Lambda_c}m_\Sigma + C_2 + C_1 + C_3 + S_{\Sigma\pi}),\\
b_{5,6} &= b_{6,5} = 2(4m_\Sigma C_2 + 2m_{\Lambda_c}C_1 + 2m_{\Lambda_c}S_{\Sigma\pi} + 4m_\Sigma S_{\Sigma\pi}),\\
b_{6,6} &= 2\big(2C_2C_1 + (2m_{\Lambda_c}m_\Sigma + C_2 + C_1 - C_3 + S_{\Sigma\pi})S_{\Sigma\pi}\big),\\
b_{13,13} &= 2(-2m_{\Lambda_c}m_\Sigma + C_2 + C_1 + C_3 + S_{\Sigma\pi}),\\
b_{13,14} &= b_{14,13} = 2(-4m_\Sigma C_2 + 2m_{\Lambda_c}C_1 + 2m_{\Lambda_c}S_{\Sigma\pi} - 4m_\Sigma S_{\Sigma\pi}),\\
b_{14,14} &= 2\big(2C_2C_1 - (2m_{\Lambda_c}m_\Sigma - C_2 - C_1 + C_3 - S_{\Sigma\pi})S_{\Sigma\pi}\big).
\end{aligned}$$

g. β_{PL}, β_{LP}

$$\begin{aligned}
&b_{1,1} = b_{9,9} = 2, \quad b_{1,2} = b_{2,1} = -b_{9,10} = -b_{10,9} = 8m_\Sigma, \quad b_{2,2} = b_{10,10} = 2(2C_1 + S_{\Sigma\pi}),\\
&b_{1,3} = -b_{9,11} = 4m_\Sigma, \quad b_{1,4} = b_{2,3} = b_{9,12} = b_{10,11} = 2(C_1 + S_{\Sigma\pi}), \quad b_{2,4} = -b_{10,12} = 4m_\Sigma S_{\Sigma\pi},
\end{aligned}$$

$$\begin{aligned}
b_{3,5} &= 2(2m_{\Lambda_c}m_\Sigma + C_2 + C_1 + C_3 + S_{\Sigma\pi}),\\
b_{3,6} &= b_{4,5} = 2(2m_\Sigma C_2 + m_{\Lambda_c}C_1 + m_{\Lambda_c}S_{\Sigma\pi} + 2m_\Sigma S_{\Sigma\pi}),\\
b_{4,6} &= 2\big(2C_1C_2 + (2m_{\Lambda_c}m_\Sigma + C_2 - C_3 + S_{\Sigma\pi} + C_1)S_{\Sigma\pi}\big),\\
b_{11,13} &= 2(-2m_{\Lambda_c}m_\Sigma + C_2 + C_1 + C_3 + S_{\Sigma\pi}),\\
b_{11,14} &= b_{12,13} = 2(-2m_\Sigma C_2 + m_{\Lambda_c}C_1 + m_{\Lambda_c}S_{\Sigma\pi} - 2m_\Sigma S_{\Sigma\pi}),\\
b_{12,14} &= 2\big(2C_1C_2 - (2m_{\Lambda_c}m_\Sigma - C_2 + C_3 - C_1 - S_{\Sigma\pi})S_{\Sigma\pi}\big),
\end{aligned}$$

$$\begin{aligned}
b_{1,5} &= 2(m_{\Lambda_c} + 2m_\Sigma), & b_{9,13} &= 2(m_{\Lambda_c} - 2m_\Sigma),\\
b_{1,6} &= 2(2m_{\Lambda_c}m_\Sigma - C_3 + S_{\Sigma\pi}), & b_{9,14} &= 2(-2m_{\Lambda_c}m_\Sigma - C_3 + S_{\Sigma\pi}),\\
b_{2,5} &= 2(2m_{\Lambda_c}m_\Sigma + 2C_1 + C_3 + S_{\Sigma\pi}), & b_{10,13} &= 2(-2m_{\Lambda_c}m_\Sigma + 2C_1 + C_3 + S_{\Sigma\pi}),\\
b_{2,6} &= 2(2m_{\Lambda_c}C_1 + m_{\Lambda_c}S_{\Sigma\pi} + 2m_\Sigma S_{\Sigma\pi}), & b_{10,14} &= 2(2m_{\Lambda_c}C_1 + m_{\Lambda_c}S_{\Sigma\pi} - 2m_\Sigma S_{\Sigma\pi}).
\end{aligned}$$

The nonzero terms in the coefficients c_j^i are listed in the following subsections.

h. $\Lambda_1 = \Lambda\frac{1}{2}^+(1115),\ \Lambda_2 = \Lambda\frac{1}{2}^+(1600)$

$$C^G_{1,1} = -C^F_{9,1} = M_\Gamma, \quad C^G_{3,2} = -C^F_{11,2} = C^G_{5,2} = -C^F_{13,2} = -\frac{M_\Gamma}{m_{\Lambda_c}}, \quad C^G_{3,3} = -C^F_{11,3} = -\frac{M_\Gamma}{m_\Lambda},$$
$$C^G_{2,1} = -C^F_{10,1} = -1, \quad C^G_{4,2} = -C^F_{12,2} = C^G_{6,2} = -C^F_{14,2} = \frac{1}{m_{\Lambda_c}}, \quad C^G_{4,3} = -C^F_{12,3} = \frac{1}{m_\Lambda}.$$

i. $\Lambda_3 = \Lambda\frac{1}{2}^-(1405)$

$$C^G_{1,1} = -C^F_{9,1} = -M_\Gamma, \quad C^G_{3,2} = -C^F_{11,2} = C^G_{5,2} = -C^F_{13,2} = -\frac{M_\Gamma}{m_{\Lambda_c}}, \quad C^G_{3,3} = -C^F_{11,3} = -\frac{M_\Gamma}{m_\Lambda},$$
$$C^G_{2,1} = -C^F_{10,1} = -1, \quad C^G_{4,2} = C^F_{12,2} = -C^G_{6,2} = C^F_{14,2} = \frac{1}{m_{\Lambda_c}}, \quad C^G_{4,3} = -C^F_{12,3} = -\frac{1}{m_\Lambda}.$$

j. $\Lambda_4 = \Lambda\frac{3}{2}^-(1520)$

$$\begin{aligned}
C^G_{1,1} &= -C^F_{9,1} = B\Big[\big(2M_\Gamma - m_\Lambda\big)S^2_{\Sigma\pi} + \big[M_\Gamma\big[m_\Lambda(-m_\Lambda + 2m_\Sigma) - 2(C_1 - C_2)\big] - m_\Lambda(2m_\Lambda m_\Sigma + C_1)\big]S_{\Sigma\pi} \\
&\quad + M_\Gamma\big[3m^2_\Lambda(C_1 - C_2 + C_3) + 2C_2(m_\Lambda m_\Sigma - C_1)\big] - 2m_\Lambda C_2 C_1\Big] \\
C^G_{1,4} &= -C^F_{9,4} = m_\Lambda m_{\Lambda_c} B\Big[-M_\Gamma S_{\Sigma\pi} + M_\Gamma(2m_\Lambda m_\Sigma + C_1) - 2m_\Lambda C_1\Big] \\
C^G_{2,1} &= -C^F_{10,1} = B\Big[-2S^2_{\Sigma\pi} + \Big[-M_\Gamma m_\Lambda + m_\Lambda(3m_\Lambda + 2m_\Sigma) + 2(C_1 - C_2)\Big]S_{\Sigma\pi} \\
&\quad + m_\Lambda\big[-M_\Gamma(2m_\Lambda m_\Sigma + 2C_2 + C_1) + m_\Lambda(3C_2 - C_1 - 3C_3) + 2m_\Sigma C_2\big] + 2C_2C_1\Big] \\
C^G_{2,4} &= -C^F_{10,4} = m_\Lambda m_{\Lambda_c} B\Big[S_{\Sigma\pi} - 2M_\Gamma m_\Lambda + 2m_\Lambda m_\Sigma - C_1\Big] \\
C^G_{3,2} &= \frac{m_\Lambda}{m_{\Lambda_c}}C^G_{3,3} = C^G_{5,2} = \frac{B}{m_{\Lambda_c}}\Big[-2M_\Gamma S^2_{\Sigma\pi} + \big[M_\Gamma\big[m_\Lambda(3m_\Lambda - 2m_\Sigma) - m_\Lambda m_{\Lambda_c} + 2(C_1 - C_2)\big] + 2m_\Lambda C_1\big]S_{\Sigma\pi} \\
&\quad + \Big[M_\Gamma\big[3m^2_\Lambda(C_2 - C_1 - C_3) + m_{\Lambda_c} m_\Lambda(2m_\Lambda m_\Sigma + C_1) - 2C_2(m_\Lambda m_\Sigma - C_1)\big] - 2m_\Lambda C_1(m_\Lambda m_{\Lambda_c} - C_2)\Big]\Big] \\
C^G_{3,4} &= -C^F_{11,4} = m_{\Lambda_c} B\Big[-2M_\Gamma S_{\Sigma\pi} + M_\Gamma\big[m_\Lambda(3m_\Lambda - 2m_\Sigma) + 2C_1\big] + 2m_\Lambda C_1\Big] \\
C^G_{4,2} &= \frac{m_\Lambda}{m_{\Lambda_c}}C^G_{4,3} = C^G_{6,2} = \frac{B}{m_{\Lambda_c}}\Big[2S^2_{\Sigma\pi} + \big[m_\Lambda(2M_\Gamma - 3m_\Lambda - 2m_\Sigma + m_{\Lambda_c}) - 2(C_1 - C_2)\big]S_{\Sigma\pi} \\
&\quad + m_\Lambda\big[-2(M_\Gamma + m_\Sigma)(m_{\Lambda_c} m_\Lambda - C_2) - 3m_\Lambda(C_2 - C_1 - C_3) - m_{\Lambda_c} C_1\big] - 2C_2C_1\Big] \\
C^G_{4,4} &= -C^F_{12,4} = m_{\Lambda_c} B\Big[2S_{\Sigma\pi} + m_\Lambda\big[2M_\Gamma - 3m_\Lambda - 2m_\Sigma\big] - 2C_1\Big] \\
C^F_{11,2} &= \frac{m_\Lambda}{m_{\Lambda_c}}C^F_{11,3} = C^F_{13,2} = \frac{B}{m_{\Lambda_c}}\Big[2M_\Gamma S^2_{\Sigma\pi} - \Big[M_\Gamma\big[m_\Lambda(3m_\Lambda - 2m_\Sigma) + m_\Lambda m_{\Lambda_c} + 2(C_1 - C_2)\big] - 2m_\Lambda C_1\Big]S_{\Sigma\pi} \\
&\quad - M_\Gamma\big[3m^2_\Lambda(C_2 - C_1 - C_3) - m_{\Lambda_c} m_\Lambda(2m_\Lambda m_\Sigma + C_1) - 2C_2(m_\Lambda m_\Sigma + C_1)\big] - 2m_\Lambda C_1(m_\Lambda m_{\Lambda_c} + C_2)\Big]
\end{aligned}$$

$$C^F_{12,2} = \frac{m_\Lambda}{m_{\Lambda_c}} C^F_{12,3} = C^F_{14,2} = -\frac{B}{m_{\Lambda_c}} \Big[2S^2_{\Sigma\pi} + \big[m_\Lambda(2M_\Gamma - 3m_\Lambda - 2m_\Sigma - m_{\Lambda_c}) - 2(C_1 - C_2) \big] S_{\Sigma\pi}$$
$$+ m_\Lambda \big[2(M_\Gamma - m_\Sigma)(m_{\Lambda_c} m_\Lambda + C_2) - 3m_\Lambda(C_2 - C_1 - C_3) + m_{\Lambda_c} C_1 \big] - 2C_2C_1 \Big]$$
$$C^G_{7,4} = -C^F_{15,4} = -\frac{M_\Gamma}{2}, \qquad C^G_{8,4} = -C^F_{16,4} = \frac{1}{2}.$$

k. $\Lambda_5 = \Lambda\frac{3}{2}^+(1890)$

$$C^G_{1,1} = -C^F_{9,1} = B \Big[\big[m_\Lambda - 2M_\Gamma \big] S^2_{\Sigma\pi} + \big[M_\Gamma \big[m_\Lambda(m_\Lambda + 2m_\Sigma) + 2(C_1 - C_2) \big] - m_\Lambda(2m_\Lambda m_\Sigma - C_1) \big] S_{\Sigma\pi}$$
$$+ M_\Gamma \big[3m^2_\Lambda(C_2 - C_1 - C_3) + 2C_2(m_\Lambda m_\Sigma + C_1) \big] + 2m_\Lambda C_2 C_1 \Big],$$
$$C^G_{1,4} = -C^F_{9,4} = m_\Lambda m_{\Lambda_c} B \Big[M_\Gamma S_{\Sigma\pi} + M_\Gamma(2m_\Lambda m_\Sigma - C_1) + 2m_\Lambda C_1 \Big],$$
$$C^G_{2,1} = -C^F_{10,1} = B \Big[-2S^2_{\Sigma\pi} + \Big[-M_\Gamma m_\Lambda + m_\Lambda(3m_\Lambda - 2m_\Sigma) + 2(C_1 - C_2) \Big] S_{\Sigma\pi}$$
$$+ m_\Lambda \big[M_\Gamma(2m_\Lambda m_\Sigma - 2C_2 - C_1) + m_\Lambda(3C_2 - C_1 - 3C_3) - 2m_\Sigma C_2 \big] + 2C_2C_1 \Big],$$
$$C^G_{2,4} = -C^F_{10,4} = m_\Lambda m_{\Lambda_c} B \Big[S_{\Sigma\pi} - 2M_\Gamma m_\Lambda - 2m_\Lambda m_\Sigma - C_1 \Big],$$
$$C^G_{3,2} = \frac{m_\Lambda}{m_{\Lambda_c}} C^G_{3,3} = C^G_{5,2} = \frac{B}{m_{\Lambda_c}} \Big[-2M_\Gamma S^2_{\Sigma\pi} + \Big[M_\Gamma \big[m_\Lambda(3m_\Lambda + 2m_\Sigma) + m_\Lambda m_{\Lambda_c} + 2(C_1 - C_2) \big] + 2m_\Lambda C_1 \Big] S_{\Sigma\pi}$$
$$+ \Big[M_\Gamma \big[3m^2_\Lambda(C_2 - C_1 - C_3) + m_{\Lambda_c} m_\Lambda(2m_\Lambda m_\Sigma - C_1) + 2C_2(m_\Lambda m_\Sigma + C_1) \big] + 2m_\Lambda C_1(m_\Lambda m_{\Lambda_c} + C_2) \Big] \Big],$$
$$C^G_{3,4} = -C^F_{11,4} = m_{\Lambda_c} B \Big[-2M_\Gamma S_{\Sigma\pi} + M_\Gamma \big[m_\Lambda(3m_\Lambda + 2m_\Sigma) + 2C_1 \big] + 2m_\Lambda C_1 \Big],$$
$$C^G_{4,2} = \frac{m_\Lambda}{m_{\Lambda_c}} C^G_{4,3} = C^G_{6,2} = \frac{B}{m_{\Lambda_c}} \Big[-2S^2_{\Sigma\pi} + \big[m_\Lambda(-2M_\Gamma + 3m_\Lambda - 2m_\Sigma + m_{\Lambda_c}) + 2(C_1 - C_2) \big] S_{\Sigma\pi}$$
$$+ m_\Lambda \big[-2(M_\Gamma + m_\Sigma)(m_{\Lambda_c} m_\Lambda + C_2) + 3m_\Lambda(C_2 - C_1 - C_3) - m_{\Lambda_c} C_1 \big] + 2C_2C_1 \Big],$$
$$C^G_{4,4} = -C^F_{12,4} = m_{\Lambda_c} B \Big[-2S_{\Sigma\pi} + m_\Lambda \big[-2M_\Gamma + 3m_\Lambda - 2m_\Sigma \big] + 2C_1 \Big],$$
$$C^F_{11,2} = \frac{m_\Lambda}{m_{\Lambda_c}} C^F_{11,3} = C^F_{13,2} = \frac{B}{m_{\Lambda_c}} \Big[2M_\Gamma S^2_{\Sigma\pi} - \Big[M_\Gamma \big[m_\Lambda(3m_\Lambda + 2m_\Sigma) - m_\Lambda m_{\Lambda_c} - 2(C_1 - C_2) \big] - 2m_\Lambda C_1 \Big] S_{\Sigma\pi}$$
$$- M_\Gamma \big[3m^2_\Lambda(C_2 - C_1 - C_3) - m_{\Lambda_c} m_\Lambda(2m_\Lambda m_\Sigma - C_1) + 2C_2(m_\Lambda m_\Sigma + C_1) \big] + 2m_\Lambda C_1(m_\Lambda m_{\Lambda_c} - C_2) \Big],$$
$$C^F_{12,2} = \frac{m_\Lambda}{m_{\Lambda_c}} C^F_{12,3} = C^F_{14,2} = -\frac{B}{m_{\Lambda_c}} \Big[-2S^2_{\Sigma\pi} + \big[m_\Lambda(-2M_\Gamma + 3m_\Lambda - 2m_\Sigma - m_{\Lambda_c}) + 2(C_1 - C_2) \big] S_{\Sigma\pi}$$
$$+ m_\Lambda \big[2(M_\Gamma + m_\Sigma)(m_{\Lambda_c} m_\Lambda - C_2) + 3m_\Lambda(C_2 - C_1 - C_3) + m_{\Lambda_c} C_1 \big] + 2C_2C_1 \Big],$$
$$C^G_{7,4} = -C^F_{15,4} = -\frac{M_\Gamma}{2}, \qquad C^G_{8,4} = -C^F_{16,4} = -\frac{1}{2}.$$

l. $\Lambda_6 = \Lambda\frac{5}{2}^+(1820)$

The coefficients take the form

$$C_{ij}^{F(G)} = D\Big[T_5 S_{\Sigma\pi}^5 + T_4 S_{\Sigma\pi}^4 + T_3 S_{\Sigma\pi}^3 + T_2 S_{\Sigma\pi}^2 + T_1 S_{\Sigma\pi} + T_0\Big],$$

$$C_{1,1}^G = -C_{9,1}^F = D\Big[2M_\Gamma S_{\Sigma\pi}^5 - 4\Big[m_\Lambda^2(M_\Gamma - m_\Sigma) + M_\Gamma(C_1 - C_2)\Big]S_{\Sigma\pi}^4 + T_3 S_{\Sigma\pi}^3 + T_2 S_{\Sigma\pi}^2 + T_1 S_{\Sigma\pi} + T_0\Big],$$

where

$$T_3 = m_\Lambda^4(M_\Gamma - 8m_\Sigma) + m_\Lambda^2(2M_\Gamma(2C_1 - 3C_2 + C_3) + 4m_\Sigma(2C_2 - C_1) + 2M_\Gamma(C_1^2 + C_2^2 - 4C_1C_2),$$

$$T_2 = m_\Lambda^6(M_\Gamma + 4m_\Sigma) + m_\Lambda^4\Big[M_\Gamma\big[4m_\Sigma^2 + 2(C_1 - 2C_2 + 2C_3) + q^2\big] + 4m_\Sigma\big[2C_1 - 3C_2 + C_3\big]\Big]$$
$$+ m_\Lambda^2\Big[2M_\Gamma\big[C_2(C_1 - C_2 + C_3) - C_1C_3\big] + 4m_\Sigma C_2\big[C_2 - 2C_1\big]\Big] + 4M_\Gamma C_2 C_1(C_1 - C_2),$$

$$T_1 = -m_\Lambda^6\Big[2M_\Gamma\big[2m_\Sigma^2 - 3C_2 + C_1 + 3C_3\big] + 4m_\Sigma\big[C_1 - C_2 + C_3\big]\Big]$$
$$+ m_\Lambda^4\Big[M_\Gamma\big[8m_\Sigma^2 C_2 - 6C_2^2 + 8C_2C_1 + 6C_2C_3 - 7C_1^2 - 8C_1C_3 - 2C_1q^2\big] + 4m_\Sigma C_2\big[2C_1 - C_2 + C_3\big]\Big]$$
$$+ 2m_\Lambda^2 C_2 C_1\Big[M_\Gamma\big[2C_1 - C_2 - C_3\big] - 2m_\Sigma C_2\Big] + 2M_\Gamma C_2^2 C_1^2,$$

$$T_0 = M_\Gamma m_\Lambda^2\Big[m_\Lambda^4\Big(- 4m_\Sigma^2\big[2C_2 + q^2\big] + 5\big[C_1^2 + C_2^2 + C_3^2\big] - 2C_1\big[C_2 - 2q^2\big] + 10\big[C_1 - C_2\big]C_3\Big)$$
$$+ m_\Lambda^2\Big(4m_\Sigma^2 C_2^2 + 2C_2C_1\big[3C_2 - 4C_1 - 5C_3\big] + C_1^2 q^2\Big) + 2C_2^2 C_1^2\Big].$$

$$C_{1,4}^G = -C_{9,4}^F = m_\Lambda^2 m_{\Lambda_c} D\Big[- S_{\Sigma\pi}^4 + \big[m_\Lambda^2 + (2C_1 - C_2)\big]S_{\Sigma\pi}^3 + T_2 S_{\Sigma\pi}^2 + T_1 S_{\Sigma\pi} + T_0\Big],$$

where

$$T_2 = \big[m_\Lambda^2(2M_\Gamma m_\Sigma + C_2 - 4C_1 - C_3) + C_1(2C_2 - C_1)\big],$$

$$T_1 = 2m_\Lambda^4(C_1 - M_\Gamma m_\Sigma) + m_\Lambda^2\Big[2M_\Gamma m_\Sigma\big[C_2 - C_1\big] + C_1\big[3C_1 - 3C_2 + C_3\big]\Big] - C_2C_1^2,$$

$$T_0 = m_\Lambda^4\big[M_\Gamma m_\Sigma - C_1\big]\big[C_1 - C_2 + C_3\big] + m_\Lambda^2 C_2 C_1\big[C_1 - M_\Gamma m_\Sigma\big].$$

$$C_{2,1}^G = -C_{10,1}^F = D\Big[- 2S_{\Sigma\pi}^5 - 4(C_2 - C_1)S_{\Sigma\pi}^4 + T_3 S_{\Sigma\pi}^3 + T_2 S_{\Sigma\pi}^2 + T_1 S_{\Sigma\pi} + T_0\Big],$$

where

$$T_3 = \Big[7m_\Lambda^4 + m_\Lambda^2(4M_\Gamma m_\Sigma - 2(C_2 + 2C_1C_3) - 2(C_1^2 + C_2^2 - 4C_1C_2)\Big],$$

$$T_2 = -5m_\Lambda^6 + m_\Lambda^4\Big[- 4m_\Sigma\big[2M_\Gamma + m_\Sigma\big] + 2(C_2 - C_1 - 4C_3) - q^2\Big]$$
$$+ m_\Lambda^2\Big[4M_\Gamma m_\Sigma\big[2C_2 - C_1\big] - 2C_2(C_1 + C_2 + C_3) + 2C_1(2C_1 + C_3)\Big] + 4C_2C_1\big[C_2 - C_1\big],$$

$$T_1 = m_\Lambda^6\Big[4m_\Sigma\big[M_\Gamma + m_\Sigma\big] + 2\big[C_1 - 5C_2 + 5C_3\big]\Big]$$
$$+ m_\Lambda^4\Big[4M_\Gamma m_\Sigma\big[2C_1 - 3C_2 + C_3\big] - 8m_\Sigma^2 C_2 + 2C_2\big[5C_2 - 2C_1 - 5C_3\big] - C_1\big[C_1 - 4C_3 + 2q^2\big]\Big]$$
$$+ m_\Lambda^2\Big[4M_\Gamma m_\Sigma C_2\big[C_2 - 2C_1\big] + 2C_2C_1\big[C_2 + 2C_1 + C_3\big]\Big] - 2C_2^2 C_1^2,$$

$$T_0 = m_\Lambda^6\Big[4M_\Gamma m_\Sigma\big[C_2 - C_1 - C_3\big] + 4m_\Sigma^2\big[2C_2 + q^2\big] - \big[C_1^2 + 5C_2^2 + 5C_3^2\big] - 2C_2\big[C_1 - 5C_3\big]$$
$$- 2C_1\big[3C_3 - 2q^2\big]\Big] + m_\Lambda^4\Big[4M_\Gamma m_\Sigma C_2\big[2C_1 - C_2 + C_3\big] - 4m_\Sigma^2 C_2^2$$
$$+ C_1\big[2C_2(3C_3 - C_2) - C_1q^2\big]\Big] - 2M_\Gamma m_\Lambda^2 m_\Sigma C_2^2 C_1.$$

$$C_{24}^G = -C_{10,4}^F = m_\Lambda^2 m_{\Lambda_c} D\Big[M_\Gamma S_{\Sigma\pi}^3 + \Big[m_\Lambda^2(2m_\Sigma - 3M_\Gamma) + M_\Gamma(C_2 - 2C_1)\Big]S_{\Sigma\pi}^2 + T_1 S_{\Sigma\pi} + T_0\Big],$$

where

$T_1 = 2m_\Lambda^4(M_\Gamma - m_\Sigma) + m_\Lambda^2 M_\Gamma(4C_1 - 3C_2 + C_3) + 2m_\Sigma(C_2 - C_1) + M_\Gamma(C_1 - 2C_2),$

$T_0 = 2m_\Lambda^6(M_\Gamma - m_\Sigma)(C_2 - C_1 - C_3) + m_\Lambda^4\Big[M_\Gamma C_1(3C_2 - C_1 - C_3) - 2m_\Sigma C_2 C_1\Big] + M_\Gamma m_\Lambda^2 C_2 C_1^2.$

$$C^G_{3,2} = \frac{m_\Lambda}{m_{\Lambda_c}} C^G_{3,3} = C^G_{5,2} = \frac{D}{m_{\Lambda_c}}\Big[- 2M_\Gamma S^5_{\Sigma\pi} + T_4 S^4_{\Sigma\pi} + T_3 S^3_{\Sigma\pi} + T_2 S^2_{\Sigma\pi} + T_1 S_{\Sigma\pi} + T_0\Big],$$

where

$T_4 = \Big[2m_\Lambda^2(M_\Gamma - 2m_\Sigma - m_{\Lambda_c}) + 4M_\Gamma(C_1 - C_2)\Big],$

$T_3 = m_\Lambda^4\Big[5M_\Gamma + 4m_\Sigma + 2m_{\Lambda_c}\Big] + 2m_\Lambda^2\Big[M_\Gamma(2C_2 - C_3) + 2m_\Sigma(C_1 - 2C_2) + m_{\Lambda_c}(2C_1 - C_2)\Big]$
$\quad - 2M_\Gamma(C_1^2 + C_2^2 - 4C_1C_2),$

$T_2 = -5M_\Gamma m_\Lambda^6 - m_\Lambda^4\Big[M_\Gamma\big[4m_\Sigma^2 + 2(5C_1 - 5C_2 - 3C_3) + q^2 - 4m_\Sigma m_{\Lambda_c}\big] + 4m_\Sigma(C_1 - 2C_2 + C_3)$
$\quad + 2(4C_1 - C_2 + C_3)\Big] + m_\Lambda^2\Big[2M_\Gamma\big[C_2(C_2 + C_1 - C_3) - C_1(C_1 - C_3)\big]$
$\quad + 4m_\Sigma C_2(2C_1 - C_2) - 2m_{\Lambda_c} C_1(C_1 - 2C_2)\Big] - 4M_\Gamma C_1 C_2(C_1 - C_2),$

$T_1 = 2m_\Lambda^6\Big[M_\Gamma\big[2m_\Sigma^2 - 5C_2 + 3C_1 + 5C_3 - 2m_\Sigma m_{\Lambda_c}\big] + 2C_1 m_{\Lambda_c}\Big]$
$\quad + m_\Lambda^4\Big[M_\Gamma\big[- 8m_\Sigma^2 C_2 + 2C_2(3C_2 - 7C_1 - 3C_3) + C_1(9C_1 + 10C_3) + 2C_1 q^2 + 4m_\Sigma m_{\Lambda_c}(C_2 - C_1)\big]$
$\quad + 4m_\Sigma C_2[C_2 - C_1 - C_3] + 2m_{\Lambda_c} C_1(-3C_2 + 3C_1 + C_3)\Big]$
$\quad + 2m_\Lambda^2 C_1 C_2\Big[M_\Gamma(C_2 - 3C_1 + C_3) + 2m_\Sigma C_2 - m_{\Lambda_c} C_1\Big] - 2M_\Gamma C_2^2 C_1^2,$

$T_0 = m_\Lambda^6\Big[M_\Gamma\big[4m_\Sigma^2(2C_2 + q^2) + C_2(2C_1 - 5C_2 + 10C_3) - C_1(5C_1 + 10C_3 + 4q^2) - 5C_3^2$
$\quad + 4m_{\Lambda_c} m_\Sigma(C_1 - C_2 + C_3)\big] + 4C_1(C_2 - C_1 - C_3)\Big]$
$\quad + m_\Lambda^4\Big[M_\Gamma\big[- 4m_\Sigma^2 C_2^2 + 2C_1C_2(-3C_2 + 4C_1 + 5C_3) - C_1^2 q^2 - 4m_\Sigma m_{\Lambda_c} C_2 C_1\big] + 4m_{\Lambda_c} C_2 C_1^2\Big]$
$\quad - 4M_\Gamma m_\Lambda^2 C_2^2 C_1^2.$

$$C^G_{3,4} = C^F_{11,4} = m_{\Lambda_c} D\Big[- 2M_\Gamma S^4_{\Sigma\pi} + \Big[m_\Lambda^2(2M_\Gamma - m_{\Lambda_c} - 4m_\Sigma + 2M_\Gamma(2C_1 - C_2)\Big]S^3_{\Sigma\pi} + T_2 S^2_{\Sigma\pi} + T_1 S_{\Sigma\pi} + T_0\Big],$$

where

$T_2 = m_\Lambda^4\Big[5M_\Gamma + m_{\Lambda_c} + 4m_\Sigma\Big] + m_\Lambda^2\Big[M_\Gamma\big[2C_2 + C_1 - C_3\big] + 2m_{\Lambda_c} C_1 + 4m_\Sigma(C_1 - C_2)\Big]$
$\quad + 4M_\Gamma C_1(2C_2 - C_1),$

$T_1 = -5M_\Gamma m_\Lambda^6 + m_\Lambda^4\Big[M_\Gamma\big[2m_\Sigma(m_{\Lambda_c} - 2m_\Sigma) + 6C_2 - 7C_1 - 3C_3\big] - 3m_{\Lambda_c} C_1 + 2m_\Sigma(2C_2 - C_1 - C_3)\Big]$
$\quad + m_\Lambda^2\Big[M_\Gamma C_1\big[2C_2 - 3C_1 + C_3\big] - m_{\Lambda_c} C_1^2 + 4m_\Sigma C_2 C_1\Big] - 2M_\Gamma C_2 C_1^2,$

$T_0 = m_\Lambda^6\Big[M_\Gamma\big[- 2m_\Sigma(m_{\Lambda_c} - 2m_\Sigma) - 5C_2 + C_1 + 5C_3\big] + 2m_{\Lambda_c} C_1\Big]$
$\quad + m_\Lambda^4\Big[M_\Gamma\big[- 2m_\Sigma(m_{\Lambda_c} C_1 + 2m_\Sigma C_2) + C_1(4C_1 - 6C_2 + 5C_3)\big] + 2m_{\Lambda_C} C_1^2\Big] - 4M_\Gamma m_\Lambda^2 C_2 C_1^2.$

$$C^G_{4,2} = \frac{m_\Lambda}{m_{\Lambda_c}} C^G_{4,3} = C^G_{6,2} = \frac{D}{m_{\Lambda_c}}\Big[2S^5_{\Sigma\pi} + \Big[2m_\Lambda^2 + 4(C_2 - C_1)\Big]S^4_{\Sigma\pi} + T_3 S^3_{\Sigma\pi} + T_2 S^2_{\Sigma\pi} + T_1 S_{\Sigma\pi} + T_0\Big],$$

where

$T_3 = -9m_\Lambda^4 + 2m_\Lambda^2\Big[M_\Gamma(m_{\Lambda_c} - 2m_\Sigma) + (2C_2 + C_3)\Big] + 2(C_1^2 + C_2^2 - 4C_1C_2),$

$T_2 = 5m_\Lambda^6 + m_\Lambda^4\Big[2M_\Gamma(2m_\Sigma - 3m_{\Lambda_c}) + 4m_\Sigma(m_\Sigma + m_{\Lambda_c}) + 2(5C_1 - 9C_2 + 5C_3) + q^2\Big]$
$\quad + 2m_\Lambda^2\Big[M_\Gamma\big[2m_\Sigma(C_1 - 2C_2) + m_{\Lambda_c}(C_2 - 2C_1)\big] + C_2(C_2 - C_1 + C_3) - C_1(C_1 + C_3)\Big]$

$$+ 4C_1C_2(C_1 - C_2),$$

$$T_1 = m_\Lambda^6\Big[4M_\Gamma m_{\Lambda_c} - 4m_\Sigma(m_\Sigma + m_{\Lambda_c}) + 2(5C_2 - 3C_1 - 5C_3)\Big]$$

$$+ m_\Lambda^4\Big[M_\Gamma\big[4m_\Sigma(2C_2 - C_1 - C_3) + 2m_{\Lambda_c}(4C_1 - 3C_2 + C_3)\big] + 8m_\Sigma^2C_2 + 10C_2(C_1 - C_2 + C_3)$$

$$- C_1(5C_1 - 6C_3 - 2q^2) + 4m_\Sigma m_{\Lambda_c}(C_2 - C_1)\Big]$$

$$+ 2m_\Lambda^2\Big[M_\Gamma\big[2m_\Sigma C_2(2C_1 - C_2) + m_{\Lambda_c}C_1(C_1 - 2C_2)\big] - C_2C_1(C_2 + C_1 + C_3)\Big] + 2C_2^2C_1^2,$$

$$T_0 = m_\Lambda^6\Big[4M_\Gamma m_{\Lambda_c}(C_2 - C_1 - C_3) - 4m_\Sigma^2(2C_2 - q^2) + 5(C_2^2 + C_1^2 + C_3^2) - 2C_1(C_2 - 2q^2)$$

$$+ 10(C_1 - C_2)C_3 + 4m_\Sigma m_{\Lambda_c}(C_1 - C_2 + C_3)\Big]$$

$$+ m_\Lambda^4\Big[2M_\Gamma\big[2m_\Sigma C_2(C_2 - C_1 - C_3) + m_{\Lambda_c}C_1(3C_2 - C_1 - C_3) + 4m_\Sigma C_2(m_\Sigma C_2 - m_{\Lambda_c}C_1)$$

$$+ 2C_1C_2(C_2 - 2C_1 - 3C_3) + C_1^2q^2\Big] + 2m_\Lambda^2 M_\Gamma C_2C_1\Big[2m_\Sigma C_2 + m_{\Lambda_c}C_1\Big].$$

$$C^G_{4,4} = m_{\Lambda_c}D\Big[2S_{\Sigma\pi}^4 + 2\Big[m_\Lambda^2 + (C_2 - 2C_1)\Big]S_{\Sigma\pi}^3 + S_{\Sigma\pi}^2 + T_1S_{\Sigma\pi} + T_0\Big],$$

where

$$T_2 = -9m_\Lambda^4 + m_\Lambda^2\big[M_\Gamma(m_{\Lambda_c} - 4m_\Sigma) + 2C_2 - C_1 + C_3\big] + 2C_1(C_1 - 2C_2),$$

$$T_1 = m_\Lambda^4\big[M_\Gamma(-3m_{\Lambda_c} + 4m_\Sigma) + 2m_\Sigma(m_{\Lambda_c} + 2m_\Sigma) + 5(C_1 - 2C_2 + C_3)\big]$$

$$+ m_\Lambda^2\big[2M_\Gamma\big(2m_\Sigma(C_1 - C_2) - m_{\Lambda_c}C_1\big) - C_1(2C_2 + C_1 + C_3)\big] + 5m_\Lambda^6 + 2C_2C_1^2,$$

$$T_0 = m_\Lambda^6\big[2M_\Gamma m_{\Lambda_c} - 2m_\Sigma(m_{\Lambda_c} + 2m_\Sigma) + 5C_2 - C_1 - 5C_3\big] + m_\Lambda^4\big[M_\Gamma\big(3m_{\Lambda_c}C_1 + 2m_\Sigma(2C_2 - C_1 - C_3)\big)$$

$$- 2m_\Sigma(m_{\Lambda_c}C_1 - 2m_\Sigma C_2) + C_1(2C_2 - 2C_1 - 3C_3)\big] + m_\Lambda^2M_\Gamma C_1(m_{\Lambda_c}C_1 + 4m_\Sigma C_2).$$

$$C^F_{11,2} = C^F_{13,2} = \frac{m_\Lambda}{m_{\Lambda_c}}C^F_{11,3} = \frac{D}{m_{\Lambda_c}}\Big[2M_\Gamma S_{\Sigma\pi}^5 + T_4S_{\Sigma\pi}^4 + T_3S_{\Sigma\pi}^3 + T_2S_{\Sigma\pi}^2 + T_1S_{\Sigma\pi} + T_0\Big],$$

where

$$T_4 = -2\Big[m_\Lambda^2(M_\Gamma - 2m_\Sigma + m_{\Lambda_c}) + 2M_\Gamma(C_1 - C_2)\Big],$$

$$T_3 = -m_\Lambda^4\Big[5M_\Gamma + 4m_\Sigma - 2m_{\Lambda_c}\Big] - 2m_\Lambda^2\Big[M_\Gamma(2C_2 - C_3) + 2m_\Sigma(C_1 - 2C_2) + m_{\Lambda_c}(C_2 - 2C_1)\Big]$$

$$+ 2M_\Gamma(C_1^2 + C_2^2 - 4C_1C_2),$$

$$T_2 = 5M_\Gamma m_\Lambda^6 + m_\Lambda^4\Big[M_\Gamma\big[4m_\Sigma(m_\Sigma + m_{\Lambda_c}) + 10(C_1 - C_2) + 6C_3 + q^2\big]$$

$$+ 4m_\Sigma(C_1 - 2C_2 + C_3) + 2m_{\Lambda_c}(C_2 - 4C_1 - C_3)\Big]$$

$$+ 2m_\Lambda^2\Big[- M_\Gamma\big[C_2(C_2 + C_1 - C_3) - C_1(C_1 - C_3)\big] + 2m_\Sigma C_2(C_2 - 2C_1) + m_{\Lambda_c}C_1(2C_2 - C_1)\Big]$$

$$+ 4M_\Gamma C_2C_1(C_1 - C_2),$$

$$T_1 = 2m_\Lambda^6\Big[M_\Gamma\big[- 2m_\Sigma(m_\Sigma + m_{\Lambda_c}) + 5(C_2 - C_3) - 3C_1\big] + 2m_{\Lambda_c}C_1\Big]$$

$$+ m_\Lambda^4\Big[M_\Gamma\big[8m_\Sigma^2C_2 - 2C_2(3C_2 - 7C_1 - 3C_3) - C_1(9C_1 + 10C_3) - 2C_1q^2 + 4m_\Sigma m_{\Lambda_c}(C_2 - C_1)\big]$$

$$+ 4m_\Sigma C_2(C_1 - C_2 + C_3) + 2m_{\Lambda_c}C_1(3C_1 - 3C_2 + 2C_3)\Big]$$

$$+ 2m_\Lambda^2\Big[M_\Gamma(-C_2 + 3C_1 - C_3) - 2m_\Sigma C_2 - m_{\Lambda_c}C_1\Big] + 2M_\Gamma C_2^2C_1^2,$$

$$T_0 = m_\Lambda^6\Big[M_\Gamma\big[- m_\Sigma^2(2C_2 + q^2) + 5(C_3^2 + C_2^2 + C_3^2) + 10(C_1 - C_2)C_3 - 2C_1(C_2 - 2q^2)$$

$$+ 4m_\Sigma(C_1 - C_2 + C_3)\big] + 4m_{\Lambda_c}(C_2 - C_1 - C_3)\Big]$$

$$+ m_\Lambda^4\Big[M_\Gamma\big[4m_\Sigma C_2(m_\Sigma C_2 - m_{\Lambda_c}C_1) + 2C_2C_1(3C_2 - 4C_1 - 5C_3) + C_1^2q^2\big] + 4m_{\Lambda_c}C_2C_1^2\Big]$$

$$+ 4M_\Gamma m_\Lambda^2C_2^2C_1^2,$$

$$C^F_{12,2} = \frac{m_\Lambda}{m_{\Lambda_c}} C^F_{12,3} = C^F_{14,2} = -\frac{D}{m_{\Lambda_c}}\Big[2S^5_{\Sigma\pi} + 2\Big[m^2_\Lambda + 2(C_2 - C_1)\Big]S^4_{\Sigma\pi} + S^3_{\Sigma\pi} + T_2 S^2_{\Sigma\pi} + T_1 S_{\Sigma\pi} + T_0\Big],$$

where

$$T_3 = -9m^4_\Lambda + 2m^2_\Lambda\big[-M_\Gamma(2m_\Sigma + m_{\Lambda_c}) + (2C_2 + C_3)\big] + 2(C_1^2 + C_2^2 - 4C_1C_2),$$

$$T_2 = 5m^6_\Lambda + m^4_\Lambda\Big[2M_\Gamma(2m_\Sigma + 3m_{\Lambda_c}) + 4m_\Sigma(m_\Sigma - m_{\Lambda_c}) + 2(5C_1 + 5C_3 - 9C_2) + q^2\Big]$$
$$+ m^2_\Lambda\Big[M_\Gamma\big[4m_\Sigma(C_1 - 2C_2) + 2m_{\Lambda_c}(2C_1 - C_2)\big] + 2C_2(C_2 - C_1 + C_3) - 2C_1(C_1 + C_3)\Big]$$
$$+ 4C_2C_1(C_1 - C_2),$$

$$T_1 = m^6_\Lambda\Big[-4M_\Gamma m_{\Lambda_c} + 4m_\Sigma(m_{\Lambda_c} - m_\Sigma) + 2(5C_2 - 3C_1 - 5C_3)\Big]$$
$$+ m^4_\Lambda\Big[M_\Gamma\big[4m_\Sigma(2C_2 - C_1 - C_3) + 2m_{\Lambda_c}(3C_2 - 4C_1 - C_3)\big]$$
$$+ 8m^2_\Sigma C_2 + 10C_2(C_1 - C_2 + C_3) - C_1(5C_1 + 6C_3 - 2q^2) + 4m_\Sigma m_{\Lambda_c}(C_1 - C_2)\Big]$$
$$+ 2m^2_\Lambda\Big[M_\Gamma\big[2m_\Sigma C_2(2C_1 - C_2) + m_{\Lambda_c}C_1(2C_2 - C_1)\big] - C_2C_1(C_1 + C_2 + C_3)\Big] + 2C_2^2C_1^2,$$

$$T_0 = m^6_\Lambda\Big[4M_\Gamma m_{\Lambda_c}(C_1 - C_2 + C_3) - 4m^2_\Sigma(2C_2 + q^2) + 5(C_1^2 + C_2^2 + C_3^2) + 10(C_1 - C_2)C_3$$
$$- 2C_1(C_2 - 2q^2) + 4m_\Sigma(C_2 - C_1 - C_3)\Big]$$
$$+ m^4_\Lambda\Big[2M_\Gamma\big[2m_\Sigma C_2(C_2 - C_1 - C_3) + m_{\Lambda_c}C_1(C_1 - 3C_2 + C_3)\big]$$
$$+ 4m_\Sigma C_2(m_\Sigma C_2 + m_{\Lambda_c}C_1) + 2C_2C_1(C_2 - 2C_1 - 3C_3) + C_1^2q^2\Big] + 2m^2_\Lambda M_\Gamma C_2C_1\Big[2m_\Sigma C_2 - m_{\Lambda_c}C_1\Big],$$

$$C^F_{12,4} = -m_{\Lambda_c}D\Big[2S^4_{\Sigma\pi} + 2\Big[m^2_\Lambda + (C_2 - 2C_1)\Big]S^3_{\Sigma\pi} + T_2S^2_{\Sigma\pi} + T_1S_{\Sigma\pi} + T_0\Big],$$

where

$$T_2 = -9m^4_\Lambda - m^2_\Lambda\big[M_\Gamma(m_{\Lambda_c} + 4m_\Sigma) - 2C_2 + C_1 - C_3\big] + 2C_1(C_1 - 2C_2),$$
$$T_1 = 5m^6_\Lambda + m^4_\Lambda\big[M_\Gamma(3m_{\Lambda_c} + 4m_\Sigma) + 2m_\Sigma(2m_\Sigma - m_{\Lambda_c}) + 5(C_1 - 2C_2 + C_3\big]$$
$$+ m^2_\Lambda\big[2M_\Gamma\big(m_{\Lambda_c}C_1 + m_\Sigma(2C_1 - C_2)\big) - C_1(2C_2 + C_1 + C_3)\big] + 2C_2C_1^2,$$
$$T_0 = m^6_\Lambda\Big[-2M_\Gamma m_{\Lambda_c} + 2m_\Sigma(m_{\Lambda_c} - 2m_\Sigma) + 5C_2 - C_1 - 5C_3\Big]$$
$$+ m^4_\Lambda\Big[M_\Gamma\big[-3m_{\Lambda_c}C_1 + 2m_\Sigma(2C_2 - C_1 - C_3)\big] + 2m_\Sigma(m_{\Lambda_c}C_1 + 2m_\Sigma C_2) + C_1(2C_2 - 2C_1 - 3C_3)\Big]$$
$$+ m^2_\Lambda M_\Gamma C_1\Big[4m_\Sigma C_2 - m_{\Lambda_c}C_1\Big],$$

$$C^G_{5,4} = -C^G_{13,4} = m^4_\Lambda m_{\Lambda_c}D\Big[-M_\Gamma S^2_{\Sigma\pi} + 2M_\Gamma C_1S_{\Sigma\pi} + 4m^2_\Lambda M_\Gamma(m^2_\Sigma - C_1) - M_\Gamma C_1^2\Big],$$

$$C^G_{6,4} = -C^F_{14,4} = m^4_\Lambda m_{\Lambda_c}D\Big[S^2_{\Sigma\pi} - 2C_1S_{\Sigma\pi} + \Big[4m^2_\Lambda(C_1 - m^2_\Sigma) + C_1^2\Big]\Big],$$

$$C^G_{7,4} = m^2_\Lambda m_{\Lambda_c}D\Big[-M_\Gamma S^3_{\Sigma\pi} - \Big[m^2_\Lambda(3M_\Gamma + m_{\Lambda_c} + 2m_\Sigma) - M_\Gamma(C_1 - C_2)\Big]S^2_{\Sigma\pi}$$
$$+ \Big[5M_\Gamma m^4_\Lambda - m^2_\Lambda\big[M_\Gamma(3C_2 - 5C_1) - m_{\Lambda_c}C_1 + 2m_\Sigma C_2\big] + M_\Gamma C_2C_1\Big]S_{\Sigma\pi}$$
$$+ m^4_\Lambda\big(M_\Gamma\big[2m_{\Lambda_c}m_\Sigma + 5(C_2 - C_1 - C_3)\big] - 2m_{\Lambda_c}C_1\big) + 5M_\Gamma m^2_\Lambda C_2C_1\Big],$$

$$C^G_{8,4} = m^2_\Lambda m_{\Lambda_c}D\Big[S^3_{\Sigma\pi} + \Big[5m^2_\Lambda + (C_2 - C_1)\Big]S^2_{\Sigma\pi}$$
$$+ \Big[-5m^4_\Lambda + m^2_\Lambda\big[M_\Gamma(m_{\Lambda_c} - 2m_\Sigma) + 5C_2 - 3C_1\big] - C_2C_1\Big]S_{\Sigma\pi}$$
$$+ m^4_\Lambda\big[-2M_\Gamma m_{\Lambda_c} + 2m_{\Lambda_c}m_\Sigma + 5(C_1 - C_2 + C_3)\big] - m^2_\Lambda\big[M_\Gamma(m_{\Lambda_c}C_1 + 2m_\Sigma C_2) + 3C_2C_1\big]\Big],$$

$$C^F_{15,4} = m^2_\Lambda m_{\Lambda_c} D\Big[M_\Gamma S^3_{\Sigma\pi} + \Big[m^2_\Lambda(3M_\Gamma - m_{\Lambda_c} + 2m_\Sigma) - M_\Gamma(C_1 - C_2)\Big]S^2_{\Sigma\pi}$$
$$- \Big[5M_\Gamma m^4_\Lambda - m^2_\Lambda\big[M_\Gamma(3C_2 - 5C_1) + m_{\Lambda_c}C_1 + 2m_\Sigma C_2\big] + M_\Gamma C_2 C_1\Big]S_{\Sigma\pi}$$
$$+ \Big[m^4_\Lambda\big(M_\Gamma\big[2m_{\Lambda_c}m_\Sigma - 5(C_2 - C_1 - C_3)\big] - 2m_{\Lambda_c}C_1\big) - 5M_\Gamma m^2_\Lambda C_2 C_1\Big]\Big],$$

$$C^F_{16,4} = -m^2_\Lambda m_{\Lambda_c} D\Big[S^3_{\Sigma\pi} + \Big[5m^2_\Lambda + (C_2 - C_1)\Big]S^2_{\Sigma\pi}$$
$$+ \Big[-5m^4_\Lambda + m^2_\Lambda\big[-M_\Gamma(m_{\Lambda_c} + 2m_\Sigma) + 5C_2 - 3C_1\big] - C_2C_1\Big]S_{\Sigma\pi}$$
$$+ m^4_\Lambda\big[2M_\Gamma m_{\Lambda_c} - 2m_{\Lambda_c}m_\Sigma + 5(C_1 - C_2 + C_3)\big] + m^2_\Lambda\big[M_\Gamma(m_{\Lambda_c}C_1 - 2m_\Sigma C_2) - 3C_2C_1\big]\Big].$$

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