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New astrophysical bounds on ultralight axionlike particles (ULALPs)

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Motivated by tension between the predictions of ordinary cold dark matter (CDM) and observations at galactic scales, ultralight axionlike particles (ULALPs) with mass of the order 10^{-22} eV have been proposed as an alternative CDM candidate. We consider cold and collisionless ULALPs produced in the early universe by the vacuum realignment mechanism and constituting most of CDM. The ULALP fluid is commonly described by classical field equations. However, we show that, like QCD axions, the ULALPs thermalize by gravitational self-interactions and form a Bose-Einstein condensate (BEC), a quantum phenomenon. ULALPs, like QCD axions, explain the observational evidence for caustic rings of dark matter because they thermalize and go to the lowest energy state available to them. This is one of rigid rotation on the turnaround sphere. By studying the heating effect of infalling ULALPs on galactic disk stars and the thickness of the nearby caustic ring as observed from a triangular feature in the IRAS map of our galactic disk, we obtain lower mass bounds on the ULALP mass of order 10^{-23} eV and 10^{-20} eV respectively.

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I. INTRODUCTION

Although the existence of dark matter is very well supported by data collected from various sources, there is no general agreement at present on what constitutes the dark matter. Two major candidates are the class of weakly interacting massive particles (or WIMPs), and the QCD axion. The latter is a favored extension of the standard model of particle physics that solves the strong CP problem [1–3] in addition to being a viable dark matter candidate [4–6].

Ultralight axionlike particles (ULALPs) with a wide range of mass between 10^{-33} eV $\leq m \leq 10^{-18}$ eV, the so called “Axiverse”, [7–9] are predicted in string theory based extensions of the standard model. They are dark matter candidates as well, with properties similar to the QCD axion but much lighter. If sufficiently light, they suppress structure formation on small scales [10–17] because they have a Jeans length [18, 19]

$$\ell_J = (16\pi G\rho m^2)^{-\frac{1}{4}} = 1.01 \times 10^{14} \text{ cm} \left(\frac{10^{-5} \text{ eV}}{m} \right)^{\frac{1}{2}} \left(\frac{10^{-29} \text{ g/cm}^3}{\rho} \right)^{\frac{1}{4}} \quad (1)$$

where ρ is the ULALP mass density.

Numerous bounds have been placed on the ULALP mass using observational data. V. Lora et al. [20] found that the mass range $0.3 \times 10^{-22} < m < 10^{-22}$ eV provides a best fit to the properties of dwarf spheroidal galaxies. The cosmic microwave background anisotropy observations require $m > 10^{-24}$ eV [21]. Numerical simulations of structure formation with ULALP dark matter have been carried out and found to give a good description of the core properties of dwarf galaxies when $m \sim 10^{-22}$ eV [22, 23]. ULALP dark matter with mass $m \sim 10^{-21}$ eV was found to alleviate the problems of excess small scale structure that plague ordinary cold dark matter [24]. Recent work finds that data on the Draco II and Triangulum II dwarf galaxies is best fit by a ULALP with mass $m \sim 3.7 - 5.6 \times 10^{-22}$ eV [25], while other authors obtain $m < 0.4 \times 10^{-22}$ eV from Fornax and Sculptor data [26]. Because ULALPs cause reionization to occur at a lower redshift, it has been argued that reconstruction of the UV-luminosity function restricts $m \gtrsim 10^{-22}$ eV [27].

L. Hui et al. [28] have recently written a comprehensive overview of the ULALP literature, including a discussion of the relaxation of ULALP dark matter in gravitationally bound objects. Relaxation (a.k.a. thermalization) is the key ingredient for Bose-Einstein condensation to occur. We show in Section III how the analysis of ref. [28] relates to the earlier results in refs. [29–32].

A relic ULALP population can be produced thermally (generated from the radiation bath) or non-thermally by the vacuum realignment mechanism. The former class of ULALPs behave like dark radiation. In this paper, we are solely interested in ULALPs generated by the vacuum realignment mechanism, which behave like cold dark matter.

In Section II, we show that ULALPs thermalize via gravitational self-interactions and hence form a BEC, in an analogous manner to QCD axions. This is something that the previous works have not taken into account. In Section III, we estimate a lower mass bound on ULALPs by requiring that the infalling ULALPs do not excessively heat the

galactic disk stars. In Section IV, we obtain a stronger bound from the sharpness of the fifth caustic ring of the Milky Way galaxy which appears as a triangular feature in IRAS and PLANCK maps. Section V provides a summary.

II. BOSE-EINSTEIN CONDENSATION OF ULALPS

Bose-Einstein condensation occurs if the following conditions are satisfied: i) the system is composed of a huge number of identical bosons, ii) these particles are highly degenerate, iii) their number is conserved, and iv) they thermalize. Bose-Einstein condensation means that most of the particles go to the lowest energy state available through the thermalizing interactions. We show below that the cold ULALP fluid produced by the vacuum realignment mechanism satisfies all four conditions.

The ULALP is described by two parameters: its mass m , which sets the time when the ULALP field begins to oscillate in the early universe and its decay constant f , with dimensions of energy, which sets the magnitude of its initial misalignment and the strength of its interactions. Unlike the case of QCD axions, m and f are independent parameters. Also, unlike the case of QCD axions, the ULALP mass is taken to be temperature independent. The time when the ULALP field starts to oscillate is of order $t_1 \equiv 1/m$. The ULALPs produced by the vacuum realignment mechanism [4–6] have at that time number density $n(t_1) \sim m\phi_1^2$, where ϕ_1 is the value of the field then. ϕ_1 is of order the decay constant f . After t_1 the number of ULALPs is conserved. So their number density at later times is

$$n(t) \sim m\phi_1^2 \left(\frac{a(t_1)}{a(t)} \right)^3, \quad (2)$$

where $a(t)$ is the cosmic scale factor. By demanding that the ULALPs make up the majority of CDM, we relate their initial number density to their mass

$$n(t_1) \sim \frac{\rho_{c,0}}{m} \left(\frac{t_0}{t_{\text{eq}}} \right)^2 \left(\frac{t_{\text{eq}}}{t_1} \right)^{3/2}, \quad (3)$$

where t_0 and $\rho_{c,0}$ are the age of the Universe and cold dark matter density today, respectively. We assumed that t_1 is in the radiation dominated era which requires $m > 2 \times 10^{-28}$ eV. We have therefore

$$\phi_1 \sim \frac{\sqrt{\rho_{c,0}t_0}}{(mt_{\text{eq}})^{1/4}} \sim 3 \times 10^{17} \text{ GeV} \left(\frac{10^{-22} \text{ eV}}{m} \right)^{1/4}. \quad (4)$$

If inflation does not homogenize the ULALP field, the cold ULALPs produced by vacuum realignment have momentum dispersion of order

$$\delta p(t) \sim \frac{1}{t_1} \left(\frac{a(t_1)}{a(t)} \right), \quad (5)$$

and hence their average state occupation number is

$$\mathcal{N} \sim \frac{(2\pi)^3}{4\pi/3} \frac{n(t)}{(\delta p)^3} \sim 5 \times 10^{98} \left(\frac{10^{-22} \text{ eV}}{m} \right)^{5/2}. \quad (6)$$

If inflation does homogenize the ULALP field, the momentum dispersion is smaller yet and the average quantum state occupation number higher. Therefore, the ULALPs certainly form a highly degenerate Bose gas.

The decay rate of ULALPs into two photons, in analogy with the QCD axion, is of order

$$\Gamma_{a\gamma\gamma} \sim \frac{1}{64\pi} \left(\frac{\alpha}{\pi} \right)^2 \frac{m^3}{f^2} = \frac{1}{2.5 \times 10^{110} \text{ sec}} \left(\frac{m}{10^{-22} \text{ eV}} \right)^3 \left(\frac{10^{17} \text{ GeV}}{f} \right)^2 \quad (7)$$

where α is the fine structure constant. The ULALP is thus stable on the time scale of the age of the universe. More generally, because all ULALP number changing interactions are suppressed by one or more powers of $1/f$, the number of ULALPs is conserved on timescales of order the age of the universe. So the third condition for ULALP Bose-Einstein condensation is satisfied as well.

The fourth condition is that ULALPs thermalize on a time scale shorter than the age of the universe. We call the time scale over which the momentum distribution of the ULALPs changes completely as a result of their self-interactions the relaxation time τ . The relaxation rate is $\Gamma \equiv 1/\tau$. ULALPs certainly have gravitational self-interactions but

perhaps also $\lambda\phi^4$ self-interactions. If thermalization occurs, it occurs in a regime where the energy dispersion $\delta\omega$ is smaller than the relaxation rate Γ . Indeed, since Eq. (5) gives an upper limit on the momentum dispersion, we have

$$\begin{aligned} \delta\omega(t) &\sim \frac{(\delta p(t))^2}{2m} < \frac{1}{2mt_1^2} \left(\frac{a(t_1)}{a(t)} \right)^2 = H && \text{for } t < t_{\text{eq}} \\ &= \frac{3}{4} \left(\frac{t_{\text{eq}}}{t} \right)^{\frac{1}{3}} H && \text{for } t > t_{\text{eq}} \end{aligned} \quad (8)$$

and $H < \Gamma$ is necessary for thermalization to occur. The condition $\delta\omega < \Gamma$ defines the ‘‘condensed regime’’.

In the condensed regime the relaxation rate due to $\lambda\phi^4$ self-interactions is of order [29, 30]

$$\Gamma_\lambda \sim \frac{|\lambda|n}{4m^2} . \quad (9)$$

For QCD axions, $|\lambda| \sim \frac{m^2}{f^2}$. If we assume the same holds true for ULALPs and furthermore that $\phi_1 \sim f$, we have

$$\begin{aligned} \Gamma_\lambda(t)/H(t) &\sim \frac{n(t_1)}{2f^2} \left(\frac{a(t_1)}{a(t)} \right)^3 t \sim \left(\frac{t_1}{t} \right)^{\frac{1}{2}} && \text{for } t < t_{\text{eq}} \\ &\sim \frac{\sqrt{t_1 t_{\text{eq}}}}{t} && \text{for } t > t_{\text{eq}} . \end{aligned} \quad (10)$$

Thus we find that, like QCD axions, the ULALPs may briefly thermalize through their $\lambda\phi^4$ self-interactions when they are first produced by vacuum realignment in the early universe but that they will at any rate stop doing so shortly thereafter.

In the condensed regime, the relaxation rate due to gravitational self-interactions is of order [29, 30]

$$\Gamma_g \sim 4\pi G n m^2 \ell^2 \quad (11)$$

where $\ell \equiv 1/\delta p$ is the correlation length. Since Eq. (5) gives an upper limit on the momentum dispersion, and we assume that ULALPs constitute most of the dark matter, we have

$$\begin{aligned} \Gamma_g(t)/H(t) &\sim \frac{3H}{2} \frac{n(t)m^2}{\rho_{\text{tot}}(t)} \frac{1}{(\delta p(t))^2} \gtrsim \frac{3H}{2m} \frac{\rho(t)}{\rho_{\text{tot}}(t)} \left(\frac{a(t)}{a(t_1)} \right)^2 \\ &\gtrsim \sqrt{\frac{t}{t_{\text{eq}}}} && \text{for } t < t_{\text{eq}} \\ &\gtrsim \left(\frac{t}{t_{\text{eq}}} \right)^{\frac{1}{3}} && \text{for } t > t_{\text{eq}} , \end{aligned} \quad (12)$$

where $\rho = nm$. We used the Friedmann equation to relate the total density ρ_{tot} to the Hubble constant. Eq. (12) shows that, independently of the ULALP mass, the ULALP fluid thermalizes by gravitational self-interactions at the time of equality, or earlier if the ULALP field was homogenized by inflation.

It was shown in ref. [30] that QCD axions that are about to fall into a galactic halo thermalize sufficiently fast by gravitational self-interactions that they almost all go to the lowest energy state available to them consistent with the total angular momentum they acquired from tidal torquing by galactic neighbors. That lowest energy state is one of rigid rotation on the turnaround sphere. It was shown in ref. [33] that this redistribution of angular momentum among infalling dark matter axions explains precisely and in all respects the properties of caustic rings of dark matter for which observational evidence had been found earlier. The observational evidence for caustic ring of dark matter in the Milky Way and other isolated disk galaxies is summarized in ref. [34]. Furthermore it was shown in ref. [35] that this redistribution of angular momentum solves the galactic angular momentum problem which is the tendency of cold dark matter, in numerical simulations of structure formation, to be too concentrated at galactic centers. The fact that Bose-Einstein condensation of the dark matter particles explains the observational evidence for caustic rings and solves the galactic angular momentum problem, and the fact that Bose-Einstein condensation is a property of dark matter axions but not of the other dark matter candidates, constitutes an argument that the dark matter is axions, at least in part [36]. Although these studies [30, 33, 35] were motivated primarily by QCD axions, they do not depend in an essential way on the axion mass and apply equally well to any axionlike dark matter candidate produced by the vacuum realignment mechanism, including ULALPs. To summarize, if ULALPs are the dark matter,

they thermalize by gravitational interactions and form a Bose-Einstein condensate at or before the time of equality between matter and radiation. They thermalize sufficiently fast before falling onto galactic halos to acquire quasi-rigid rotation on the turnaround sphere. They then form caustic rings with properties that are in accord with observations. The observational evidence for caustic rings therefore supports the hypothesis that the dark matter is ULALPs but also, as we will see, constrains that hypothesis.

We conclude this section with a discussion of the results of ref. [28] on ULALP relaxation in gravitationally bound objects. We will show that the relaxation rate obtained there is consistent with the earlier results on axion relaxation in refs. [29, 30]. The relaxation rate in the particle kinetic regime is [30]

$$\Gamma \sim n \sigma \delta v \mathcal{N} \quad (13)$$

where n is particle density, σ is the scattering cross-section, δv is velocity dispersion, and \mathcal{N} is degeneracy (i.e. the average occupation number of those particle states that are occupied). The relevant cross-section for relaxation by gravitational interactions in the particle kinetic regime is [30]

$$\sigma_g \sim \frac{4G^2 m^2}{(\delta v)^4} \quad (14)$$

The average quantum degeneracy is

$$\mathcal{N} \sim \frac{(2\pi)^3 n}{\frac{4\pi}{3} (m\delta v)^3} \quad (15)$$

Combining Eqs. (13 - 15) gives the relaxation rate by gravitational interactions in the particle kinetic regime

$$\Gamma_g \sim 24\pi^2 \frac{(G\rho)^2}{m^3(\delta v)^6} \quad (16)$$

For the special case of a gravitationally bound object, the crossing time is of order $t_{\text{cr}} \sim \sqrt{\frac{3\pi}{4G\rho}}$ and the size of the object is of order $r \sim \delta v t_{\text{cr}}/\pi$. The relaxation time is then of order

$$\tau_g = \frac{1}{\Gamma_g} \sim \frac{2}{27} r^4 m^3 (\delta v)^2 \quad (17)$$

Ignoring the numerical prefactor, which is at any rate poorly known, this is the result given in ref. [28] for gravitationally bound systems. Hui et al. consider several systems that may relax in the particle kinetic regime and find either that they do not relax or that relaxation has no observable consequences. Whether the ULALP fluid relaxes or not is of paramount importance in determining its behavior because it does not obey classical field equations when it relaxes. We found earlier in this section, following the discussion in ref. [30], that the ULALP fluid does relax before it falls into galactic halos. It was our purpose in this paragraph to show that the general discussion of cosmic axion relaxation in ref. [30] is consistent both with the relaxation rate for gravitationally bound objects given in ref. [28] and our claim that the ULALP fluid relaxes before falling into galactic halos.

III. BOUND FROM ULALP INFALL

Provided no thermalization is occurring [30, 37], ULALP dark matter is represented by a wavefunction solving the Schrödinger-Poisson equations. The wavefunction may be written in terms of a real amplitude A and a phase β :

$$\Psi(\vec{r}, t) = A(\vec{r}, t) e^{i\beta(\vec{r}, t)} \quad (18)$$

In the linear regime of the growth of density perturbations, before multi-streaming occurs, Eq. (18) describes a flow of density

$$n(\vec{r}, t) = (A(\vec{r}, t))^2 \quad (19)$$

and velocity

$$\vec{v}(\vec{r}, t) = \frac{1}{m} \vec{\nabla} \beta(\vec{r}, t) \quad (20)$$

In particular, a homogeneous flow of density n and velocity \vec{v} is described by

$$\Psi(\vec{r}, t) = \sqrt{n} e^{i(\vec{p}\cdot\vec{r} - \omega t)} \quad (21)$$

where $\vec{p} = m\vec{v}$ is the momentum and $\omega = p^2/2m$ the energy of each particle. An attractive feature of the wavefunction description is that it readily accomodates multi-streaming [38]. In a region with two homogeneous cold dark matter flows, with densities n_i and velocities \vec{v}_i ($i = 1, 2$), the wavefunction is

$$\Psi(\vec{r}, t) = \sqrt{n_1} e^{i(\vec{p}_1\cdot\vec{r} - \omega_1 t)} + \sqrt{n_2} e^{i(\vec{p}_2\cdot\vec{r} - \omega_2 t - \delta)} \quad (22)$$

The associated density is

$$n(\vec{r}, t) = |\Psi(\vec{r}, t)|^2 = n_1 + n_2 + 2\sqrt{n_1 n_2} \cos(\Delta\vec{p}\cdot\vec{r} - \Delta\omega t - \delta) \quad (23)$$

where $\Delta\vec{p} = \vec{p}_2 - \vec{p}_1$ and $\Delta\omega = \omega_2 - \omega_1$. The interference term is important for ULALP dark matter because the correlation length $\ell = \frac{1}{|\Delta\vec{p}|}$ is large (since m is small and $\Delta\vec{v}$ is generally fixed by observation) and gravity is long range. The gravitational field sourced by the interference term in Eq. (23) is

$$\vec{g} = -8\pi G \sqrt{n_1 n_2} m \ell \sin(\Delta\vec{p}\cdot\vec{r} - \Delta\omega t - \delta) \hat{n} \quad (24)$$

where $\hat{n} = \ell\Delta\vec{p}$. It is the gravitational fields associated with the interference terms in the ULALP fluid density that cause the ULALP dark matter thermalization discussed in the previous section [30].

An isolated galaxy such as our own Milky Way continually accretes surrounding dark matter. There are on Earth two flows of dark matter falling in and out of the Galaxy for the first time ($n = 1$), two flows falling in and out for the second time ($n = 2$), two flows falling in and out for the third time ($n = 3$), and so forth. The total number of flows on Earth is of order the age of the universe divided by the time it takes a particle initially at rest at our galactocentric distance to fall through the Galaxy and reach the opposite side, i.e. 10^{10} years/ 10^8 years = 100. The flows of particles that fell into the halo relatively recently ($n \lesssim 10$) have not been thermalized as a result of gravitational scattering off inhomogeneities in the Galaxy [39]. Those flows and their associated caustics are a robust prediction [40] of galactic halo formation with cold dark matter.

For $n \lesssim 10$, the density of each flow is of order two or three percent [34, 39, 41] of the total average dark matter density at our distance from the galactic center, i.e. each flow has density of order $\rho_{\text{fl}} \sim 10^{-26}$ gr/cm³. Since the typical difference in velocities between pairs of flows is of order $10^{-3}c$, the coherence lengths associated with the interference terms are of order

$$\ell \sim \frac{1}{10^{-3}m} \sim 64 \text{ pc} \left(\frac{10^{-22} \text{ eV}}{m} \right) \quad (25)$$

and their correlation times are of order

$$T \sim \frac{1}{10^{-6}m} \sim 2 \times 10^5 \text{ year} \left(\frac{10^{-22} \text{ eV}}{m} \right) \quad (26)$$

The gravitational fields sourced by the interference terms are of order

$$g \sim 4\pi G \rho_{\text{fl}} \ell \sim 0.5 \frac{\text{km}}{\text{sec} \times \text{Gyear}} \left(\frac{10^{-22} \text{ eV}}{m} \right) \quad (27)$$

A star in the galactic disk acquires from each pair of flows a velocity increment of order gT after a coherence time T . The total velocity acquired by a star is the result of a random walk in its velocity space, and of order

$$\Delta v \sim gT \sqrt{\frac{t_0}{T}} \sqrt{\frac{1}{2}N(N-1)} \sim 0.5 \frac{\text{km}}{\text{sec}} \left(\frac{10^{-22} \text{ eV}}{m} \right)^{\frac{3}{2}} \quad (28)$$

where $t_0 \sim 10^{10}$ year is the age of the universe and $N \sim 20$ the number of flows with density ρ_{fl} . Note that Δv is not sharply sensitive to our assumption on the number of flows since ρ_{fl} and g are inversely proportional to N whereas the number of flow pairs is proportional to N^2 . The thin disk of the Milky Way is made of stars with vertical velocity dispersion $\sigma_z \sim 20$ km/s⁻¹. We obtain a lower bound on the ULALP mass

$$m \gtrsim 10^{-23} \text{ eV} \quad (29)$$

by requiring $\Delta v < \sigma_z$. The result obtained here is broadly consistent with the discussion of the thickening of the Galactic disk by ULALP dark matter in ref. [28].

It was observed in ref. [42] that ULALP dark matter has an oscillating pressure/tension with angular frequency equal to twice the ULALP mass and that this oscillating pressure/tension, being a source of gravity in General Relativity, has measurable effects. The idea presented here is similar but makes use of Newtonian gravity and multi-streaming instead.

IV. BOUND FROM THE SHARPNESS OF THE NEARBY CAUSTIC RING

Cold, collisionless dark matter lies in 6-dimensional phase space on a thin 3-dimensional hypersurface, the thickness of which is the velocity dispersion of the dark matter particles. We refer to this hypersurface as the “phase space sheet”. As dark matter particles fall in and out of a galactic gravitational potential well their phase space sheet wraps up. Locations where the phase space sheet folds back onto itself have large particle density in physical space [41], [43], [44], [40]. Indeed, the phase space sheet is tangent to velocity space at the location of such folds and therefore the physical space density diverges there in the limit of vanishing sheet thickness. For small velocity dispersion, the physical density at the location of a fold is finite but very large. These locations of high density are called caustics. They are generically 2-dimensional surfaces in physical space. The kinds of caustics that appear are classified by Catastrophe Theory.

It was shown [40] that galactic halos built up by the infall of cold collisionless dark matter have two sets of caustics, inner and outer. There is one inner and one outer caustic for each value of the integer n introduced in the previous section. The catastrophe structure of the inner caustics depends on the angular momentum distribution of the infalling particles. If the velocity field of the infalling particles is dominated by net overall rotation ($\vec{\nabla} \times \vec{v} \neq 0$), the inner caustics are closed tubes whose cross-section is a section of the elliptic umbilic (D_{-4}) catastrophe [44], called caustic rings for short. Evidence for caustic rings of dark matter was derived from a variety of observations. The evidence is summarized in ref. [34]. The evidence is explained in all its aspects if the dark matter is axions or axionlike particles such as ULALPs [33].

The caustic ring model of the Milky Way halo was independently tested against observations in ref. [45]. Recently, Y. Huang et al. [46] found evidence for the second and third ($n = 2,3$) caustic rings in our galaxy from their measurement of the Milky Way rotation curve out to very large (100 kpc) radii.

Caustic rings of dark matter produce sharp rises in galactic rotation curves [44]. Part of the observational evidence for caustic rings is that sharp rises appear in the Milky Way rotation curve [47] at radii consistent with the predictions for caustic ring radii by the self-similar infall model [48]. Furthermore, there is a triangular feature in the IRAS map [49] of the Galactic plane whose position on the sky matches the position of the sharp rise in the Milky Way rotation attributed to the caustic ring ($n = 5$) nearest to us. The triangular feature appears in the direction of galactic coordinates $(l, b) = (80, 0)$. It is seen also in the recent Planck observations [50]; see Fig. 1. In Fig. 2 we show the panoramic view of the Milky Way provided by the IRAS $12\mu m$ observations. The triangular feature is clearly visible on the left hand side of the image. It can be understood as the imprint of the nearby caustic ring of dark matter upon the gas and dust in the Galactic disk [48].

The sharpness of the triangular feature implies that its edges are not smoothed on distances larger than approximately 10 pc. Because velocity dispersion smooths caustics, the sharpness of the triangular feature implies an upper limit of order 50 m/s [48, 51] on the velocity dispersion of the flow ($n = 5$) that produces the nearby caustic ring. As we show now, it also implies a lower limit on the mass of ULALP dark matter, because the wave description smooths caustics as well.

Consider the simple fold caustic that occurs at a caustic ring radius, where the particles that fall in along the galactic plane reach their distance of closest approach to the galactic center before falling back out. In the particle description, the particles satisfy the equation of motion

$$m \frac{d^2 r}{dt^2} = - \frac{dV_{\text{eff}}(r)}{dr} \quad (30)$$

where r is distance to the galactic center and $V_{\text{eff}}(r)$ the effective potential for radial motion, including the repulsive angular momentum barrier. In the neighborhood of the caustic, the radial velocity of the particles is

$$v_r(r) = \pm \sqrt{-\frac{2}{m} \frac{dV_{\text{eff}}}{dr}(a)(r-a)} \quad (31)$$

where a is the caustic ring radius. Conservation of the number of particles implies that the density near the caustic is proportional to

$$n(r) \propto \frac{1}{r^2 |v_r(r)|} \quad (32)$$

The density diverges therefore at $r = a$ as $1/\sqrt{r-a}$ which is the characteristic behavior of a simple fold caustic. Spreading of the caustic due to velocity dispersion is discussed in ref. [51].

In the wave description, the behavior of the dark matter fluid is given by a wavefunction $\Psi(\vec{r})$ that satisfies the Schrödinger equation. At the location of the fold discussed in the previous paragraph, the radial part of the

wavefunction satisfies

$$-\frac{\hbar^2}{2m} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d\Psi(r)}{dr} + V_{\text{eff}}(r)\Psi(r) = E\Psi(r) \quad (33)$$

with $E = V_{\text{eff}}(a)$. Near $r = a$, the wavefunction is proportional to an Airy function [52]

$$\Psi(r) = C \frac{1}{r} \text{Ai}(\gamma(a-r)) \quad (34)$$

with

$$\gamma = \left[-\frac{2m}{\hbar^2} \frac{dV_{\text{eff}}}{dr}(a) \right]^{\frac{1}{3}}. \quad (35)$$

The wavefunction smooths the fold caustic over a distance scale γ^{-1} . The sharpness of the triangular feature in the IRAS map associated with the 5th caustic ring implies an upper bound on γ^{-1} of order 10 pc. The dominant contribution to $-\frac{dV_{\text{eff}}}{dr}(a)$ is the centrifugal force $mv_\phi(a)^2/a$ where $v_\phi(a) \simeq 500$ km/s is the velocity component of the flow producing the 5th caustic ring in the direction of galactic rotation. We obtain the bound

$$m \gtrsim 10^{-20} \text{ eV} \quad (36)$$

by requiring $\gamma^{-1} \lesssim 10$ pc. Finally we note that high resolution observation of the nearby caustic provides, in principle, a means to determine the ULALP mass since the gravitational field in the plane of the ring near $r = a$ has characteristic structure on the length scale $\gamma^{-1} \propto m^{-2/3}$.

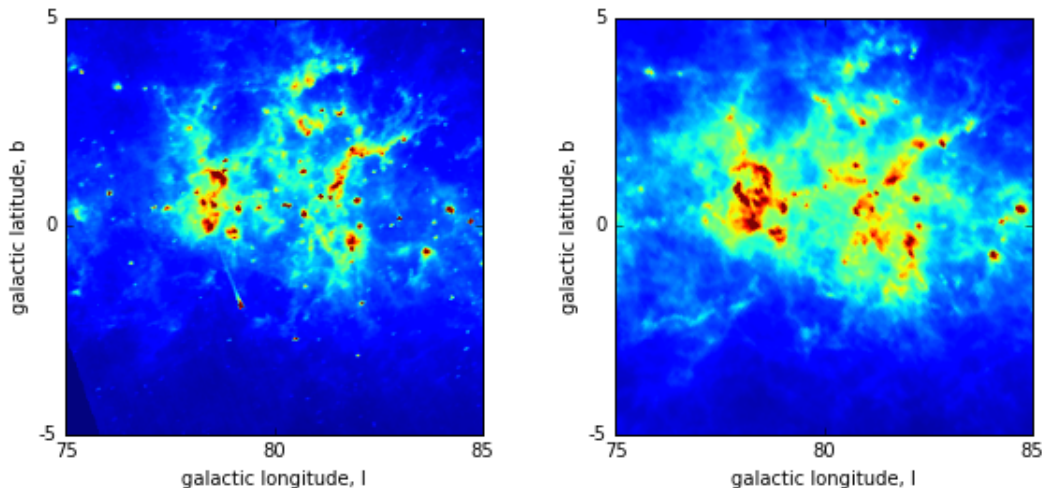


FIG. 1: Images of the region around galactic coordinates (80, 0) from the IRAS 12 μm and Planck 857 GHz observations. The triangular feature, indicative of a tri-cusp caustic ring, is prominent in both data sets.

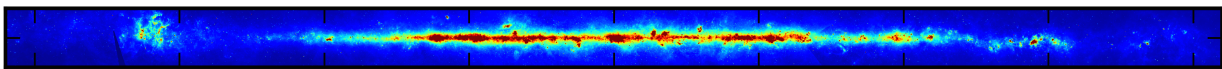


FIG. 2: Panoramic view of the Milky Way galactic plane from the IRAS experiment in the 12 μm band.

V. SUMMARY

In this paper we have explored ULALPs, an alternative cold dark matter candidate discussed by many authors. We showed that ULALPs thermalize through gravitational self-interactions and form a Bose-Einstein condensate in a manner analogous to QCD axions. This was not taken into account in the previous literature.

We placed new constraints on the mass of ULALPs. First, we considered the heating effect of infalling ULALPs on the disk stars of the Milky Way galaxy. We derived therefrom a lower bound of the ULALP mass which is comparable in strength to the previous bounds from structure formation and CMB data.

Our tightest bound comes from taking into account the Bose-Einstein condensate nature of ULALPs. In a way analogous to the QCD axion, Bose-Einstein condensation enables the ULALP dark matter to acquire net overall rotation and hence form caustic rings with tricusp cross section. From the observed sharpness of the triangular feature due to the fifth caustic ring in the Milky Way, we derive a bound on the spread of the ULALP wavefunction and, in turn, on the ULALP mass. The bound we obtain, $m_a \gtrsim 10^{-20}$ eV, disfavors a large fraction of the ULALPs from being the dark matter. It is in tension with recent works that find $m \sim 10^{-22}$ eV fits the data best.

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